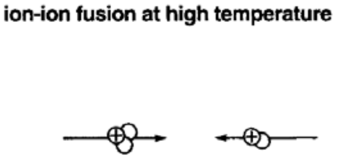
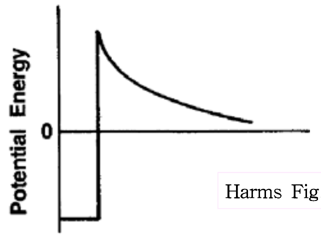
2. Methods of harnessing fusion energy

Reading assignments: Harms Chaps. 7.6, 7.7, 12

A. High temperature fusion

1) Thermonuclear fusion

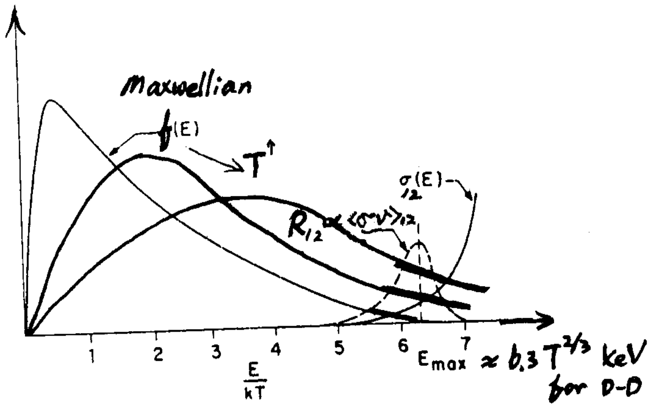




Harms Fig. 12.1 b)

A high reactant temperature is required for fusion reactions to allow a sufficient number of ions to overcome the Coulomb barrier or to penetrate it by tunneling effect.

Effect of Maxwellian distribution on fusion reaction rate R_{12}



Only fast particles in the tail of ion distribution function contribute to fusion

⇒ Thermal conditions with high temperature are needed for high R_{12}

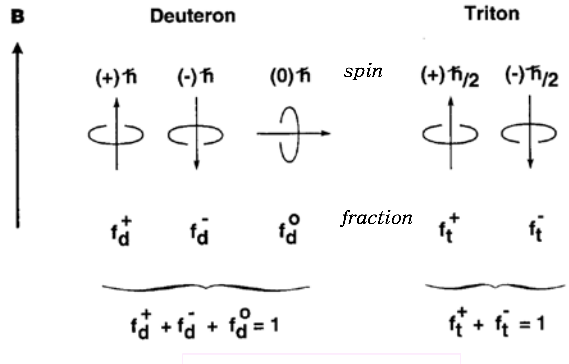
⇒ **Thermonuclear Fusion**

: Main approach to development of fusion power reactors

2) Spin polarized fusion

Spin polarized in a magnetic field:

D^+ : $\hbar$
T^+ : $\hbar/2$
n : $\hbar/2$
$\alpha(^4_2He)$: 0



Harms Fig. 7.6

In a reaction process with the maximum spin of 3/2 $\hbar$ for $({}^5_2He)$,

$$D + T \;\rightarrow\; \underset{\textit{compound nucleus}}{({}^5_2He)^*} \;\rightarrow\; \alpha + n \;:\quad \textit{cross section } \sigma_o \textit{ (maximum)}$$

For D-T reactions,

$$D + T \;\rightarrow\; \alpha + n \;:\quad \textit{cross section } \sigma_{DT}$$

where σ_{DT} can be represented by

$$\sigma_{DT} = [(f_d^+ f_t^+ + f_d^- f_t^-) + \frac{2}{3} f_d^o + \frac{1}{3}(f_d^+ f_t^- + f_d^- f_t^+)]\sigma_o \qquad (38)$$

For random polarization ($f_d^+ = f_d^- = f_d^o \equiv 1/3$, $f_t^+ = f_t^- \equiv 1/2$),

$$(38) \Rightarrow \quad (\sigma_{DT})_{r,r} = \frac{2}{3}\sigma_o \qquad (39)$$

For spin polarized D & T ($f_d^+ = f_t^+ = 1$, $others = 0$ or $f_d^- = f_t^- = 1$, $others = 0$),

$$(38) \Rightarrow \quad (\sigma_{DT})_{+,+} = (\sigma_{DT})_{-,-} = \sigma_o \qquad (40)$$

Comparison between (39) and (40) indicates that a **50 % increase of fusion cross section**, consequently, fusion power density could be achieved **by spin polarized ions** (D^+, T^+ or $D^+, {}^3He^+$) supplied by accelerators for time periods up to 10 s.

⇒ Reduced temperature(T_{ign}) and confinement condition($n\tau$) are required

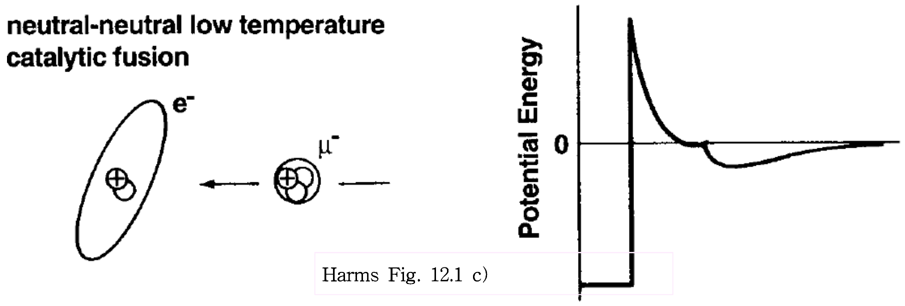
B. Low temperature fusion

1) Muon catalyzed fusion

Properties of muon μ are similar to those of electrons except mass;

$m_\mu = 207\, m_e$, lifetime $\tau_\mu = 2.2 \times 10^{-6}\ s$

Therefore, a muonic hydrogen atom has a radius 207 times smaller than that of a conventional hydrogen atom($r_\mu = \frac{1}{207} r_e$), and appears like an oversized and overweighted neutron

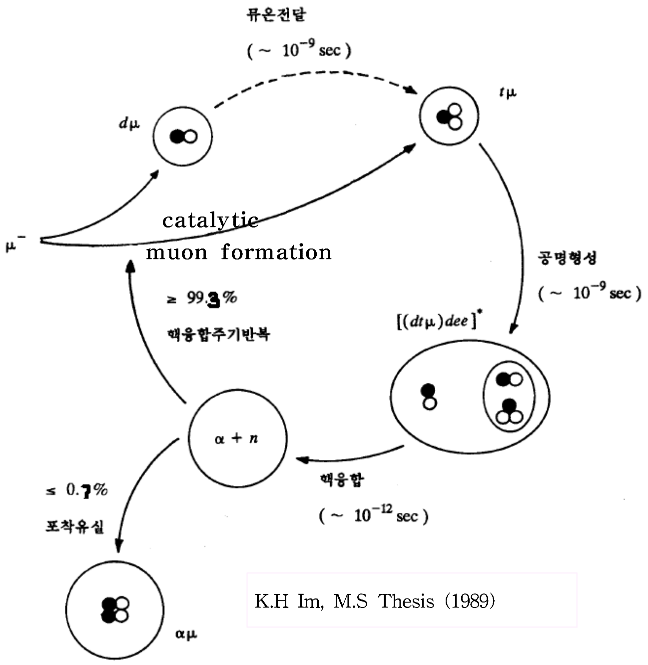


Harms Fig. 12.1 c)

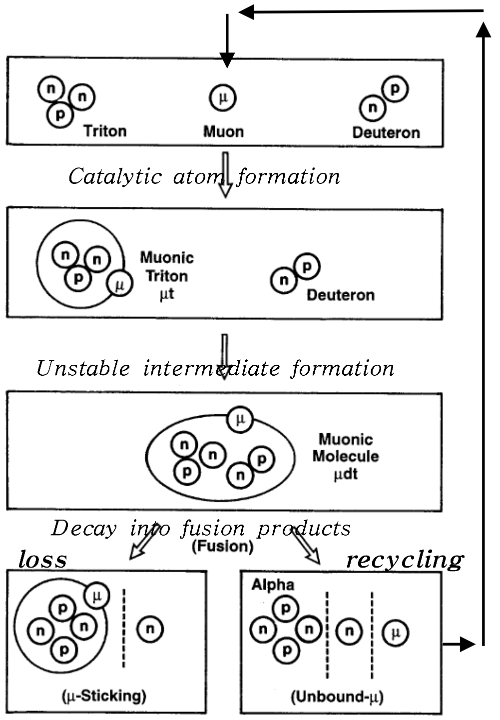
⇒ Reduction of the repulsive Coulomb forces

⇒ **Muon catalyzed fusion can be sustained in a low temperature** $\lesssim 10^3 K$

Experimentally confirmed in a liquid hydrogen
at temperatures 300 $\lesssim$ T $\lesssim$ 900 K



K.H Im, M.S Thesis (1989)



Harms Fig. 7.8

Advantages :

Low temperature (room temperature ~1000 K) reaction,

Liquid fuel is usable

Energy viability :

Muon production energy $E_{\mu c}$ by the accelerator ≈ 3 GeV

Energy multiplication ratio : *muon slowing down energy*

$$M_E \equiv \frac{E^*_{out}}{E^*_{in}} = \left(\frac{Q_{DT}}{E_{\mu c}}\right)\chi_\mu + \frac{E_{\mu s}}{E_{\mu c}} \approx \left(\frac{17.6}{3000}\right)(34) + 0.1 \approx 0.3$$

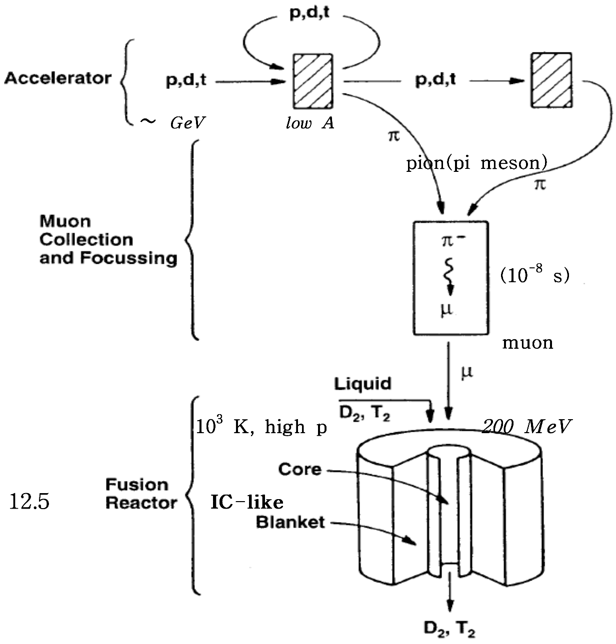
muon recycle efficiency = # of fusion events by one muon

Major research subjects :

Improvement of resonance formation rates of $dt\mu$ molecule ions

Reduction method of μ-sticking losses

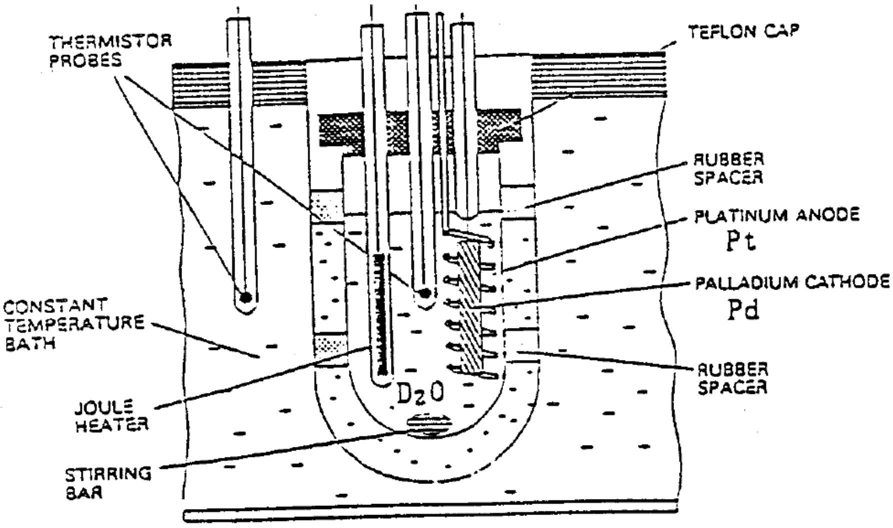
$\Rightarrow \chi_\mu^{\uparrow}$



Muon catalyzed reactor system

Harms Fig. 12.5

2) Cold fusion



Pons & Fleischmann (March 1989) reported that, in the experiments for electrolytic cell with Pt anode and Pd cathode in D_2O, **excessive heat** generation and **n & T** are observed.

⇒ *Result of D-T fusion reaction in the room temperature* (Cold fusion)

Problems :

Lack of consistency in experimental results
No definite evidence of detection of fusion products n, T, ^{3}He, p
Might be an other form of chemical reactions
with excessive reaction energy ?

3) Sonoluminescence (Sono-fusion or Bubble fusion)

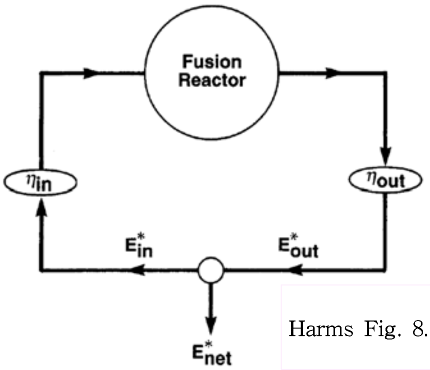
R. Taleyarkhan of ORNL (January 2001) reported that his sono-luminescence (10^{-9} s) experiments with ultrasonic waves passing through bubbles in deuterium-aceton mixture observed fusion neutrons.

3. Physical conditions for thermonuclear fusion reactors

Reading assignments: Harms Chaps. 4, 7.1, 7.2, 8

A. Fusion reactor energetics

1) System energy balance



Harms Fig. 8.1

Fundamental requirement of any power reactor for the total energy :

$$E^*_{net} = E^*_{out} - E^*_{in} > 0$$

$$\int_0^{\tau}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau}\left(\frac{dE^*}{dt}\right)_{in} dt > 0 \qquad (41)$$

Total thermal energy content in the fusion reactor during a fusion burn time,

$$\int_0^{\tau_b} \frac{dE^*_{th}}{dt}\, dt \;=\; E^*_{heat} + E^*_f - E^*_n - E^*_{rad} - \int_0^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}}\, dt \qquad (42)$$

where $\eta_{in} \equiv \dfrac{E^*_{heat}}{E^*_{in}}$: plasma heating efficiency,

$Q_p \equiv \dfrac{E^*_f}{\eta_{in} E^*_{in}}$: plasma Q-value (energy amplification factor), (43)

$f_c \equiv \dfrac{E^*_f - E^*_n}{E^*_f}$: charged particle energy fraction,

$E^*_{rad} = E^*_{br} + E^*_{cyc}$: radiation energy loss

$\dfrac{E^*_{th}(t)}{\tau_{E^*}(t)} \equiv \int_V d^3r \dfrac{E_{th}(\boldsymbol{r}, t)}{\tau_E(\boldsymbol{r}, t)}$: global energy loss rate by plasma leakage,

For the same energetic state after burn (LHS of(42) = 0),

$$E^*_f = \frac{E^*_{rad} + \int_0^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}}\, dt}{f_c + Q_p^{-1}} \qquad (44)$$

For the ignition ($Q_p \to \infty$, i.e., $E^*_{in} \to 0$, energy self-sufficiency),

$$\text{(42), (43)} \;\Rightarrow\; f_{c,DT} \int_V d^3r \int_0^{\tau_b} R_{DT}(\boldsymbol{r}, t)\, Q_{DT}\, dt =$$

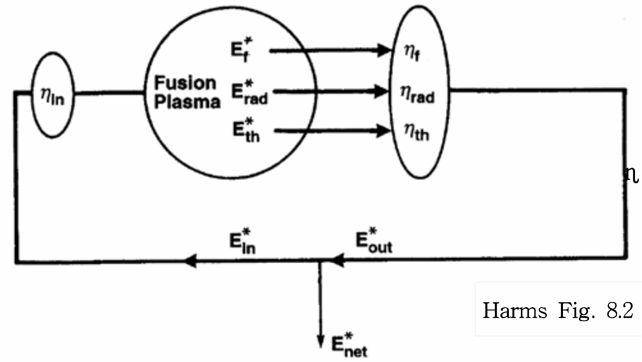
$$\int_V d^3r \left[\int_0^{\tau_b} (P_{br} + P_{cyc})\, dt + \int_0^{\tau_b} \frac{E_{th}(\boldsymbol{r}, t)}{\tau_E(\boldsymbol{r}, t)}\, dt \right] \qquad (45)$$

Ignition condition ($Q_p \to \infty$) for homogeneous plasma in steady-state operation,

$$\text{(45)} \;\Rightarrow\; f_{c,DT} P_{DT}(n_i, T_i) = P_{br}(n_i, n_e, T_e) + P_{cyc}(n_e, T_e) + \frac{3}{2}\,\frac{n_i k T_i + n_e k T_e}{\tau_E} \qquad (46)$$

2) Lawson Criterion

= Practical energy break-even condition for confinement parameter $n\tau_E$ in a fusion reactor of electric power plant



$$\eta_{out} = \frac{\sum_{l=f,\,rad,\,th} \eta_l E_l}{\sum_{l=f,\,rad,\,th} E_l}$$

Harms Fig. 8.2

Operating condition for break-even (energy viability) :

Required input energy = Output electric energy

$$E^*_{in} = \eta_f E^*_f + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th}$$

$$\Rightarrow \quad \frac{E^*_{rad} + E^*_{th}}{\eta_{in}} = \eta_{out}(E^*_f + E^*_{rad} + E^*_{th}) \qquad (47)$$

$$\Rightarrow \quad P_{rad} + P_{th} = \eta_{in}\eta_{out}(P_f + P_{rad} + P_{th}) \qquad (48)$$

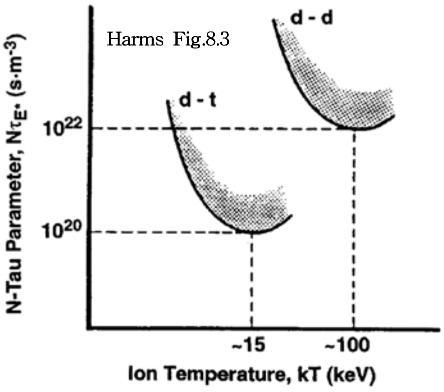
For $n_i = n_e = n,\ \ T_i = T_e = T,\ \ P_{rad} \approx P_{br}$,

$$A_{br} n^2 \sqrt{kT} + \frac{3nkT}{\tau_E} = \eta_{in}\eta_{out}\left(\gamma\, n_1 n_2 \langle \sigma v \rangle_{12} Q_{12} + A_{br} n^2 \sqrt{kT} + \frac{3nkT}{\tau_E}\right) \qquad (49)$$

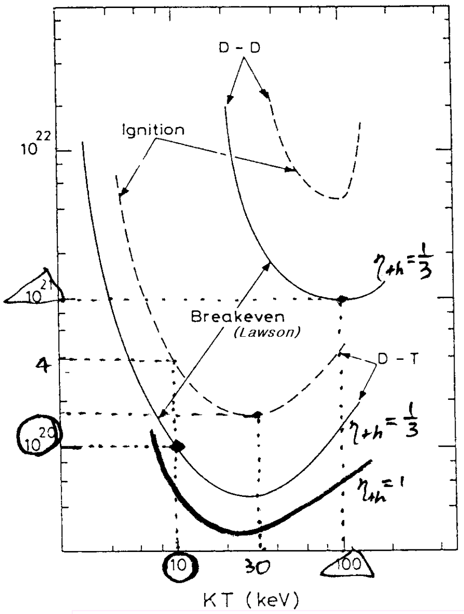
$$\therefore \quad n\tau_E > \frac{3(1-\eta_{in}\eta_{out})kT}{\eta_{in}\eta_{out}\gamma \langle \sigma v \rangle_{12} Q_{12} - (1-\eta_{in}\eta_{out}) A_{br}\sqrt{kT}} = f(\eta_{in}, \eta_{out}, T) \qquad (50)$$

where $\gamma = 1/4$ for 50%-50% D-T, $\gamma = 1/2$ for D-D

(e.g.) For $\eta_{in}\eta_{out} \approx 1/3$, For $\eta_{in} = 1,\ \eta_{out} = \eta_{th}$



Harms Fig.8.3



R.D. Gill, Fig.2 in "P.P. and Nucl. Fus. Res." Academic (1981)

Simplified Lawson conditions

- Goal of scientific feasibility:

$$(n\tau)_{DT} \approx 10^{20}\ m^{-3} \text{ at } 10\ keV$$

$$(n\tau)_{DD} \approx 10^{21}\ m^{-3} \text{ at } 100\ keV$$

(cf) Scientific break-even condition : $Q_p \equiv \dfrac{E_f^*}{\eta_{in} E_{in}^*} = 1$

3) Ignition conditions ($Q_p \rightarrow \infty$)

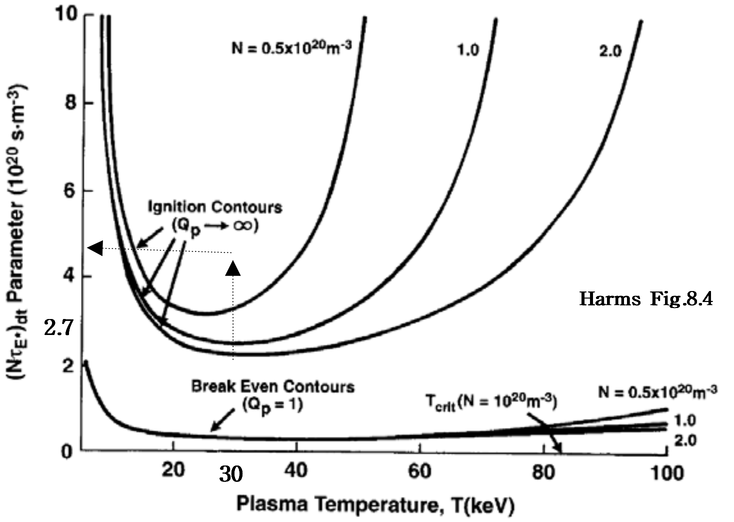
Actual self-sustaining engineering reactor condition with no heating power :

Charged-particle self-heating power

$\geq$ (Radiation + Plasma) loss powers

$$f_{c,12} P_{12} \quad \geq \quad (P_{br} + P_{cyc}) + \frac{3nkT}{\tau_E}$$

$$n\tau_E \geq \frac{3kT}{\gamma \langle \sigma v \rangle_{12} f_{c,12} Q_{12} - A_{br}\sqrt{kT} - A_{cyc} B^2 nkT} = f(T) \qquad (51)$$



Harms Fig.8.4

Ignition condition ($Q_p \rightarrow \infty$) for D-T

- Goal of engineering feasibility :

$(n\tau T)_{DT} \approx 5\times10^{21}\ m^{-3} s\ keV$ at 30 keV

B. *Fusion burn*

1) System particle balance

General rate equation:

Accumulation = (Injection - Leakage) + (Generation - Burnup) reaction

$$\frac{dn_j}{dt} = (F_{+j} - F_{-j}) + (R_{+j} - R_{-j}) \qquad (52)$$

Fuel balance equations for D-T burn ($D + T \rightarrow \alpha + n$) :

$$\frac{dn_{D,T}}{dt} = F_{D,T} - \frac{n_{D,T}}{\tau_{D,T}} - n_D n_T \langle \sigma v \rangle_{DT} \quad \text{for j = D, T} \qquad (53)$$

2) D-T burnup fractions

Fuel ion rate equation for 50%-50% D-T fusion burn : $(53)_D + (53)_T$

$$\frac{dn_i}{dt} = F_i - \frac{n_i}{\tau_i} - \frac{n_i^2}{2} \langle \sigma v \rangle_{DT} \qquad (54)$$

where $n_i = n_D + n_T$, $F_i = F_D + F_T$, $\tau_i^{-1} = \tau_D^{-1} + \tau_T^{-1}$

a) Elementary D-T burn ($F_{+j} = F_{-j} = R_{+j} = 0$ case)

$$\frac{dn_i}{dt} \approx -\frac{n_i^2}{2}\langle \sigma v\rangle_{DT} \tag{54}$$

$$\int_{n_{io}}^{n_i(\tau_b)} \frac{dn_i}{n_i^2} = -\frac{1}{2}\int_0^{\tau_b}\langle \sigma v\rangle_{DT}\,dt$$

$$-\frac{1}{n_i(\tau_b)} + \frac{1}{n_{io}} = -\frac{1}{2}\int_0^{\tau_b}\langle \sigma v\rangle_{DT}\,dt$$

For isothermal fuel burn,

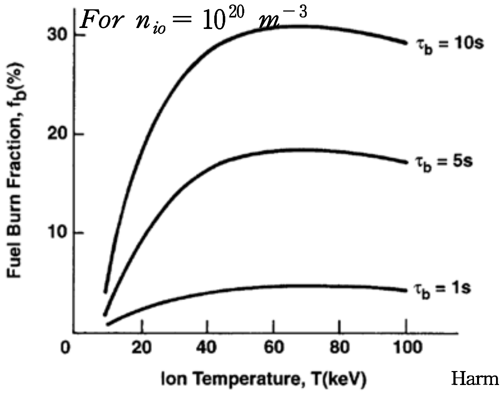
$$n_i(\tau_b) \approx \frac{1}{n_{io}^{-1} + \langle \sigma v\rangle_{DT}\tau_b/2} \tag{55}$$

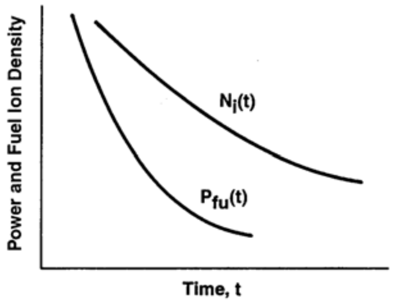
Burnup fraction per pass :

$$f_b \equiv \frac{n_{io} - n_i(\tau_b)}{n_{io}} = \frac{1}{1 + (n_{io}\langle \sigma v\rangle_{DT}\tau_b/2)^{-1}} \tag{56}$$

Fusion power density released with time :

$$P_{DT} = \frac{n_i^2(t)}{4}\langle \sigma v\rangle_{DT}Q_{DT} = \frac{1}{4}\left[\frac{1}{n_{io}^{-1} + \langle \sigma v\rangle_{DT}t/2}\right]^2\langle \sigma v\rangle_{DT}Q_{DT} \tag{57}$$





Harms Figs. 7.2 &7.3

b) Comprehensive D-T burn

$$\frac{dn_i(t)}{dt} = F_i - \frac{n_i(t)}{\tau_i} - \frac{n_i^2(t)}{2}\langle \sigma v\rangle_{DT}\ , \qquad 0 \le t \le \tau_b \tag{58}$$

Solution of Eq.(58) :

$$\frac{dn}{dt} = a_o + a_1 n + a_2 n^2 \quad : \ \textit{nonlinear 1st-oder ODE}$$

where $a_o = F_i$, $a_1 = -1/\tau_i$,

Slope of n vs. t

$$\approx \begin{array}{ll} a_o > 0 & \textit{for small } n \\ a_2 n^2 < 0 & \textit{for large } n \end{array}$$

Steady-state $\left(\frac{dn}{dt} = 0,\ t \to \infty\right)$ solution :

$$a_o + a_1 n + a_2 n^2 = 0$$

$$\rightarrow n_i^o = \frac{-1/\tau_i + \sqrt{1/\tau_i^2 + 2F_i < \sigma v >_{DT}}}{< \sigma v >_{DT}}$$

For steady-state operation $(t \rightarrow \infty)$,

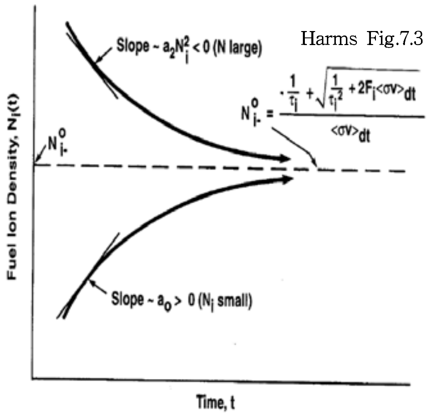
$$F_i - \frac{n_i(t)}{\tau_i} - \frac{n_i^2(t)}{2}\langle \sigma v \rangle_{DT} = 0$$

Burnup fraction per pass

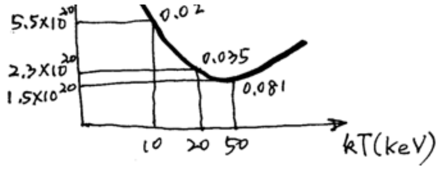
$$f_b \equiv \frac{R_{DT}}{F_i}$$

$$= \frac{(n_i^2/2)\langle \sigma v \rangle_{DT}}{(n_i^2/2)\langle \sigma v \rangle_{DT} + n_i/\tau_i}$$

$$= \frac{1}{1 + (n_i \langle \sigma v \rangle_{DT} \tau_i/2)^{-1}} \qquad (59)$$



Harms Fig.7.3



Typical design value : $f_b \approx$ **0.05**

$\rightarrow$ 20 trip through

C. Methods for realization of Lawson and ignition conditions

1) Plasma confinement ($n\tau$ condition)

a) **Major approach** to development of fusion reactors

Magnetic confinement :

- High-pressure, vacuum magnetic chamber in which plasma motions are suppressed across magnetic field lines
- $n \approx 10^{20} \sim 10^{21}\ m^{-3}$, $\tau_E \approx \sim$ sec
- {Open systems : Magnetic mirrors, Linear pinches, etc.
  Closed systems : Tokamak, Stellarator, Reverse-field pinch, Compact torus, Bumpy torus, etc.

Inertial confinement :

- Fusion reaction of micro H-bomb type in super-high density target plasma implosion by laser or ion beams (direct), or by x rays from laser or ion beams (indirect)
- $n \approx 10^{28} \sim 10^{29}\ m^{-3}$, $\tau_E \approx 10^{-8} \sim 10^{-9}$ sec
- {Laser beam-driven fusion : CO_2, Nd-glass, KrF lasers
  Particle beam-driven fusion : Light or heavy particle accelerators

b) **Alternative approach** to development of fusion reactors

Electrostatic confinement (spherically nested virtual anodes & cathodes)

(cf) **Material confinement** - not realizable due to melting of materials

c) **Naturally occurring fusion** in sun and stars

Gravitational confinement :

- Fusion reaction of hydrogen plasma confined by gravitational forces due to enormous mass
- Energy producing process in the sun and stars, which is impossible to realize on the Earth due to its limited mass

2) Plasma heating (T condition)

a) Primary heating in current flowing plasma

Ohmic (resistive) heating ($T \lesssim 1 \sim 2\ keV$)

b) Auxiliary heating

Neutral beam injection (NBI)

Radio-frequency(RF) heating : ICRH, LHH, ECRH

Adiabatic compression heating

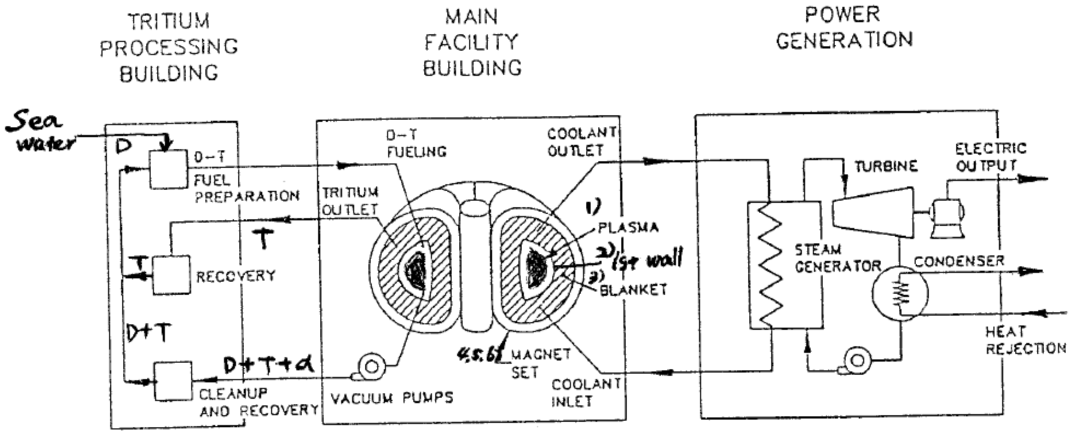
c) Self-heating

Fusion product (α) self-heating

4. Fusion reactor development

A. Fusion science and engineering

Schematic diagram of fusion power plant



Fuel handling →|← Fusion reactor →|←Thermal storage→|← Power conversion

1) Core plasma (50%-50% D-T, $T = 10^8$ K, $f_b \approx 5\%$)

- Plasma generation and current drive (mag. induction, NBI, RF)
- Plasma heating (ohmic, NBI, RF, self-α)
- Plasma equilibrium, stability, transport
- Plasma diagnostics
- Impurity control (Divertor, Limiter) - Edge plasma (SOL)
- Fueling and ash removal

2) First wall (heat resistant, non-magnetic, high strength, radiation resistant)

- SS, Mo, Nb; V, Ta, W, Ni, Al alloys
- Deposition region of high energy particles, radiations, thermal loads
- Plasma-surface interactions
- Impurity control by surface treatment (C, Be, B)
- Radiation damage, neutron loading, welding

3) Blanket (T = 800-1300 K)

- Tritium breeding (Li, Li-Pb, LiO_2, Li_2BeF_4, $LiAlO_2$)
- Coolant for neutron thermalization and heat production (water, He, liquid metal)
- Corrosion, Pumping under magnetic field influence, Radiation, Environmental effects

4) **Shielding & insulation for magnet protection** (Pb, LiH)

5) **Magnets** (TF, PF, VF)
   - Conductor coils with water cooling
   - Superconducting magnets (NbTi, Nb_3Sn)
   - Cryogenic system ($T < 20$ K)

6) **Biological shielding & insulation**

7) **Remote control and maintenance**

8) **Evaluations of environmental impact**
   - Tritium handling and neutron activated structures
   - Pollution problems and stray magnetic field effects
   - Siting and decommissioning

9) **Economics**
   - Electricity costs (capital, fuel, maintenance & repair)
   - Availability and reliability
   - Maintenance problem
   - Fusion-fission hybrid reactor

B. *Research and development stages*

1) **Scientific feasibility *(Break-even):*** Problems in A. 1), 2), 5)
2) **Engineering feasibility *(Ignition):*** Problems in A. 2)-7)
3) **Commercial feasibility *(Steady-state):*** Problems in A. 8)-9)

Scientific feasibility		Engineering feasibility		Commercial feasibility
Break-even	⇒	*Self-ignition*	⇒	*Steady-state*
Confinement Heating Impurity control		High β Material tests Quasi-steady state Reactor engineering		Safety Environment Economics Reliability

Homework :

Harms 4.6, 8.3, 8.4 for $D-{}^3He$ with no P_{cyc}, **8.5(a), 8.6**