CHAPTER 3. Static Electric Fields

Reading assignments: Cheng Ch.3, Ulaby Ch.3, Hayt Chs.2-4, 6, 7
Halliday Chs.21-25

1. Electrostatics in Free Space (or in Vacuum or in ~Air)

Steady-state (time-independent) electric phenomena caused by electric charges at rest

Deductive Approach:
Define $\boldsymbol{E}$ $\Rightarrow$ Fundamental postulates
$\Rightarrow$ Derive other laws, theorems, and relations (Coulomb's law, Gauss's law, Electric potential, ...), which are verified by experiments

A. Fundamental Postulates

to represent the physical laws of electrostatics in free space

1) Electric field intensity $\boldsymbol{E}$

= force per unit charge

$$\boldsymbol{E} \equiv \lim_{q \to 0} \frac{\boldsymbol{F}}{q} \quad \text{(V/m or N/C)} \qquad \text{(3-1)}$$

where $\boldsymbol{F}$ is the electric force on a stationary charge q in the field

$$\boldsymbol{F} = q\boldsymbol{E} \qquad \text{(3-2)}$$

2) Differential form of postulates

Point relations which hold at every point in space:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_v}{\epsilon_o} \qquad \text{non-solenoidal } \boldsymbol{E} \text{ field, charge source } \rho_v \qquad \text{(3-3)}$$

$$\nabla \times \boldsymbol{E} = 0 \qquad \text{irrotational } \boldsymbol{E} \text{ field (no vortex source)} \qquad \text{(3-4)}$$

3) Integral form of postulates

Global relations which hold over the whole space considered:

$$\int_V \text{(3-3)}\, dv \Rightarrow \int_V \nabla \cdot \boldsymbol{E}\, dv = \frac{1}{\epsilon_o} \int_V \rho_v\, dv \qquad \text{(3-5)}$$

divergence theorem $(2-75)$

$$\Rightarrow \oint_S \boldsymbol{E} \cdot d\boldsymbol{s} = \frac{Q}{\epsilon_o} \qquad \text{Gauss's law} \qquad \text{(3-6)}$$

where Q = total charge in V bounded by S

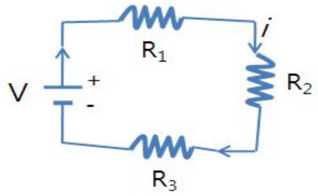
$$\int_S \text{(3-4)} \cdot d\boldsymbol{s} \Rightarrow \int_S (\nabla \times \boldsymbol{E}) \cdot d\boldsymbol{s} = 0$$

Stokes's theorem $(2-103)$

$$\Rightarrow \oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = 0 \qquad \text{(3-7)}$$

conservative field,
path-independent integral
(only end-point dependent)

(cf.) $V = \int \boldsymbol{E} \cdot d\boldsymbol{l}$ in (3-7) for circuit analysis:



$\sum_i V_i = 0$ Kirchhoff's 2nd law (voltage theorem) (3-7)*

: algebraic sum of voltages (emf, ohmic) around any closed circuit is zero

B. *Electric Field Intensities and Coulomb's Law*

1) Electric field intensity due to a point charge

For a point charge q at the origin,

Gauss's law (3-6):

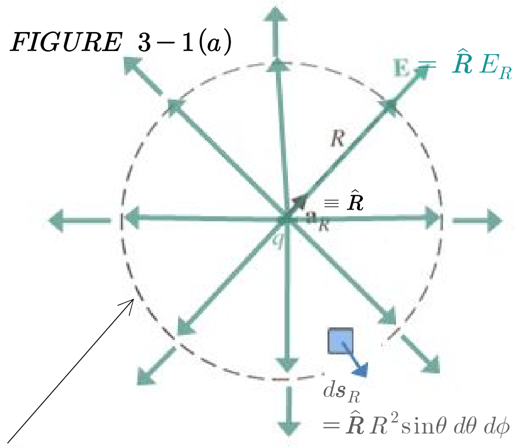


FIGURE $3-1(a)$

Gaussian surface : A hypothetical enclosed surface over which the normal comp. of **E** is a constant

$$\oint_S \boldsymbol{E} \cdot d\boldsymbol{s} = \int_S (\hat{\boldsymbol{R}} E_R) \cdot d\boldsymbol{s}_{\boldsymbol{R}} = \frac{q}{\epsilon_o}$$

$$\Rightarrow \int_S (\hat{\boldsymbol{R}} E_R) \cdot (\hat{\boldsymbol{R}} R^2 \sin\theta\, d\theta\, d\phi) = \frac{q}{\epsilon_o}$$

$$\Rightarrow E_R R^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = \frac{q}{\epsilon_o}$$

$$\Rightarrow E_R R^2 (2)(2\pi) = \frac{q}{\epsilon_o}$$

$$\Rightarrow E_R = \frac{q}{4\pi\epsilon_o R^2}$$

$$\therefore \quad \boldsymbol{E}(\boldsymbol{R}) = \hat{\boldsymbol{R}}\, E_R = \hat{\boldsymbol{R}} \frac{1}{4\pi\epsilon_o} \frac{q}{R^2} \qquad (3\text{-}8)$$

where $\dfrac{1}{4\pi\epsilon_o} = \dfrac{\mu_o c^2}{4\pi} = 10^{-7} c^2 = 9 \times 10^9$ (m/F) (3-12)

Note) $\boldsymbol{E}$ in (3-8) is an irrotational (or conservative) field.

(proof) By using (2-99),

$$\nabla \times \boldsymbol{E} = \frac{1}{R^2 \sin\theta} \begin{vmatrix} \hat{R} & \hat{\theta} R & \hat{\phi} R\sin\theta \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ E_R & R\underbrace{E_\theta}_{=0} & (R\sin\theta)\underbrace{E_\phi}_{=0} \end{vmatrix} = \hat{\theta} \frac{1}{R\sin\theta} \overset{0}{\cancel{\frac{\partial E_R}{\partial \phi}}} - \hat{\phi} \frac{1}{R} \overset{0}{\cancel{\frac{\partial E_R}{\partial \theta}}} = 0$$

For a point charge q at an arbitrary location,

by (3-8)

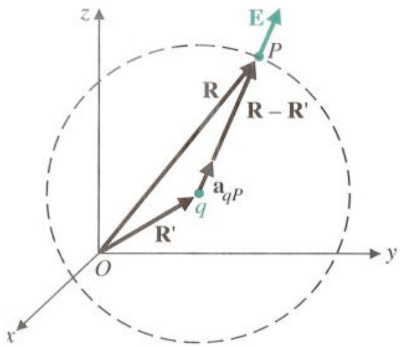


FIGURE $3-1(b)$

$$\boldsymbol{E}_P(\boldsymbol{R}) = \frac{\boldsymbol{R} - \boldsymbol{R}'}{|\boldsymbol{R} - \boldsymbol{R}'|} \frac{1}{4\pi\epsilon_o} \frac{q}{|\boldsymbol{R} - \boldsymbol{R}'|^2} \qquad (3\text{-}10,\ 9)$$

$$= \frac{q}{4\pi\epsilon_o} \frac{\boldsymbol{R} - \boldsymbol{R}'}{|\boldsymbol{R} - \boldsymbol{R}'|^3} \qquad (3\text{-}11)$$

2) Coulomb's law

: Electrostatic force between two point charge

(measured by Coulomb in 1785)

A force F_{12} exerted on q_2

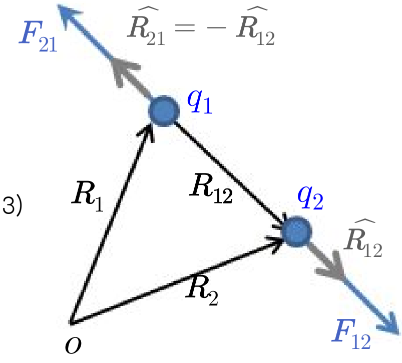
due to E_{12} generated by q_1:

(3-2) (3-8)

$$\boldsymbol{F}_{12}(\boldsymbol{R}_2) = q_2\boldsymbol{E}_{12} = \widehat{\boldsymbol{R}_{12}}\frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{R_{12}^2} \qquad \text{(3-13)}$$

$$= -\widehat{\boldsymbol{R}_{21}}\frac{1}{4\pi\epsilon_o}\frac{q_2q_1}{R_{21}^2}$$

$$= -\boldsymbol{F}_{21}(\boldsymbol{R}_1)$$



Notes) i) Mutual force: $\boldsymbol{F}_{12}(\boldsymbol{R}_2) = -\boldsymbol{F}_{21}(\boldsymbol{R}_1)$

ii) $q_1q_2 > 0 \quad \Rightarrow$ repulsive force between two same charges

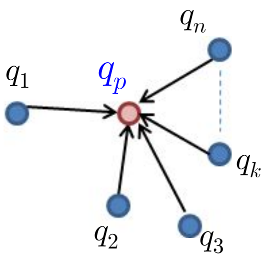
$q_1q_2 < 0 \quad \Rightarrow$ attractive force between two opposite charges

(cf) Law of universal gravitation (Newton, 1687)

$$\boldsymbol{F}_g = -\widehat{\boldsymbol{R}_{12}}G\frac{M_1M_2}{R_{12}^2} < 0 \text{ : attractive force}$$

iii) F (or E) is a linear function about charge q.

$$\begin{cases} F(a\,q_1) = a\,F(q_1) & \text{: associative} \\ F(a\,q_1 + b\,q_2) = a\,F(q_1) + b\,F(q_2) & \text{: distributive} \end{cases}$$



$\Longrightarrow$ Linear (or Fourier) superposition principle:

$$\boldsymbol{E}(\boldsymbol{R}_p) = \sum_{k=1}^{n}\boldsymbol{E}_k(\boldsymbol{R}_p) = \sum_{\substack{k=1\\k\neq p}}^{n}\widehat{\boldsymbol{R}_{kp}}\frac{1}{4\pi\epsilon_o}\frac{q_k}{R_{kp}^2} \qquad \text{(3-11)*}$$

$$\boldsymbol{F}(\boldsymbol{R}_p) = \sum_{k=1}^{n}\boldsymbol{F}_k(\boldsymbol{R}_p) = \sum_{\substack{k=1\\k\neq p}}^{n}\widehat{\boldsymbol{R}_{kp}}\frac{1}{4\pi\epsilon_o}\frac{q_kq_p}{R_{kp}^2} \qquad \text{(3-13)*}$$

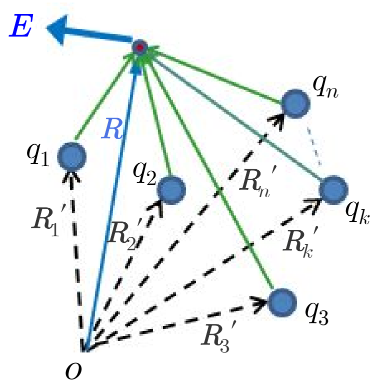
3) Electric field intensity due to charge distributions

For a system of discrete point charges,

E at R by the superposition principle

using (3-11):

$$\boldsymbol{E}(\boldsymbol{R}) = \frac{1}{4\pi\epsilon_o}\sum_{k=1}^{n}\frac{q_k(\boldsymbol{R}-\boldsymbol{R}_k{}')}{|\boldsymbol{R}-\boldsymbol{R}_k{}'|^3} \qquad \text{(3-14)}$$



For a system of volume charge distribution,

$$dE(R)= \frac{1}{4\pi\epsilon_o}\frac{(R-R')}{|R-R'|}\frac{\rho_v(R')}{|R-R'|^2}dv'$$

$$E(R) = \int_{V'} dE$$

$$= \frac{1}{4\pi\epsilon_o}\int_{V'}\frac{(R-R')}{|R-R'|}\frac{\rho_v(R')}{|R-R'|^2}dv' \qquad \text{(3-16)}$$

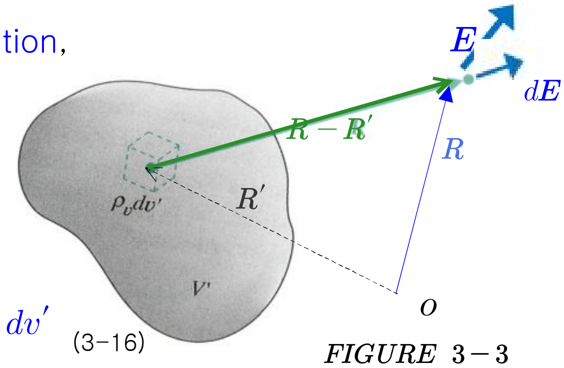


FIGURE 3−3

For a system of surface charge distribution,

$$E(R) = \frac{1}{4\pi\epsilon_o}\int_{S'}\frac{(R-R')}{|R-R'|}\frac{\rho_s(R')}{|R-R'|^2}ds' \qquad \text{(3-17)}$$

For a system of line charge distribution,

$$E(R) = \frac{1}{4\pi\epsilon_o}\int_{L'}\frac{(R-R')}{|R-R'|}\frac{\rho_l(R')}{|R-R'|^2}dl' \qquad \text{(3-18)}$$

(e.g. 3-3) Infinitely long uniform line charge:

(3-18) for an cylindrically symmetric field ($\partial/\partial\phi = 0$):

$$E(R) = \frac{1}{4\pi\epsilon_o}\int_{-\infty}^{+\infty}(\hat{r}\,r-\hat{z}\,z')\frac{\rho_l dz'}{(r^2+z'^2)^{3/2}}$$

$$= \frac{\hat{r}}{4\pi\epsilon_o}\int_{-\infty}^{+\infty}\underbrace{\frac{\rho_l\, r dz'}{(r^2+z'^2)^{3/2}}}_{dE_r}$$

$$-\underbrace{\cancel{\frac{\hat{z}}{4\pi\epsilon_o}\int_{-\infty}^{+\infty}\underbrace{\frac{\rho_l\, z' dz'}{(r^2+z'^2)^{3/2}}}_{dE_z}}}_{0}$$

0 (∵ integrand = odd ftn. about z')

$$= \hat{r}\frac{\rho_l r}{4\pi\epsilon_o}\frac{z'}{r^2(r^2+z'^2)^{1/2}}\Bigg|_{z'=-a\to-\infty}^{z'=a\to+\infty}$$

$$= \hat{r}\frac{\rho_l}{4\pi\epsilon_o r}\frac{2}{\sqrt{\underbrace{(r/a)^2}_{0 \text{ for } r \ll a}+1}}\Bigg|_{a\to\infty}$$

$$= \hat{r}\frac{\rho_l}{2\pi\epsilon_o r} \qquad \text{(V/m)} \qquad \text{(3-23)}$$

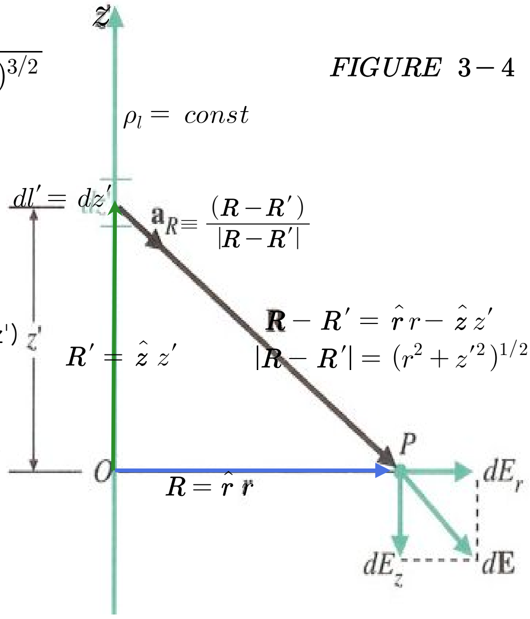


FIGURE 3−4

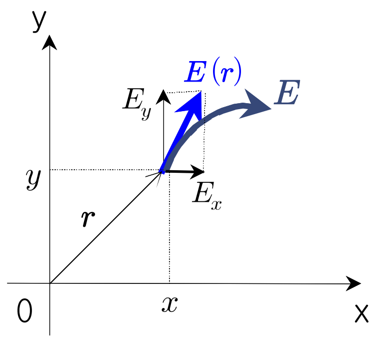
4) Electric field lines

= Electric field flux lines (or streamlines or direction lines)

Functional form of E_x and E_y

$\longrightarrow$ Equation of field lines (streamlines):

$$\frac{E_y}{E_x} = \frac{dy}{dx} \quad \xRightarrow{solution} \quad \text{Eq. of field lines} \qquad (1)$$

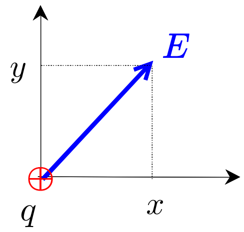


(e.g.1) Point charge field lines

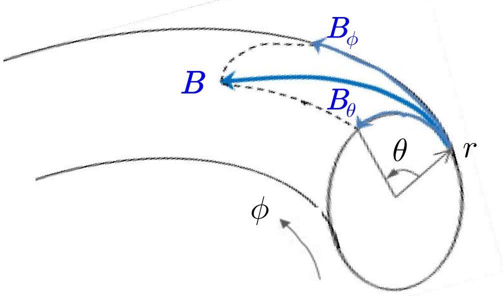
$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \quad \ln x = \ln y - \ln c_1 \quad \Rightarrow \quad y = c_1 x \qquad (2)$$



(e.g.2) Toroidal magnetic field lines



$$\frac{B_\theta}{B_\phi} = \frac{dl_\theta}{dl_\phi} = \frac{r\,d\theta}{R\,d\phi} \qquad (3)$$

Safety factor:

$$q(r) \equiv \frac{d\phi}{d\theta} = \frac{r}{R}\frac{B_\phi}{B_\theta} \qquad (4)$$

C. Gauss's Law and Electric Flux Density

1) Gauss's law in free space

Differential form in free space: (3-3) $\Rightarrow$ $\nabla \cdot (\epsilon_o \boldsymbol{E}) = \rho_v$ (3-3)*

Integral form in free space: (3-6) $\Rightarrow$ $\oint_S (\epsilon_o \boldsymbol{E}) \cdot d\boldsymbol{s} = Q$ (3-6)*

Total flux thru closed surface S

= Total charge in V enclosed by S

<u>Faraday's experiment</u>

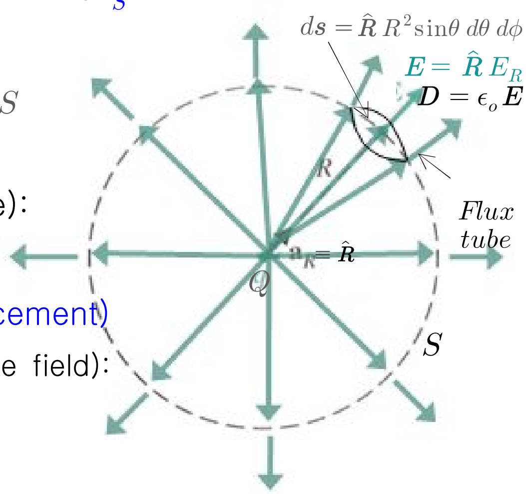
Electric flux (Field lines thru surface):

$\psi = Q$ (C) (5)

Electric flux density (Electric displacement)

(Field lines per unit area normal to the field):

$D \equiv \dfrac{\psi}{S}$ (C/m^2) (6)



$$\therefore \quad \psi = \oint_S d\psi = \oint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q \tag{7}$$

(e.g.) For a positive charge Q in free space,

$$\left.\begin{aligned} &\text{(3-8):} \quad \boldsymbol{E}(\boldsymbol{R}) = \hat{\boldsymbol{R}}\, E_R = \hat{\boldsymbol{R}} \frac{Q}{4\pi\epsilon_o R^2} \\ &\text{(7):} \quad \boldsymbol{D}(\boldsymbol{R}) = \hat{\boldsymbol{R}} \frac{\psi}{S} = \hat{\boldsymbol{R}} \frac{Q}{4\pi R^2} \end{aligned}\right\} \Longrightarrow \quad \boldsymbol{D} = \epsilon_o \boldsymbol{E} \tag{8}$$

2) Generalization of Gauss's law in any medium

Integral form: (7) $\Rightarrow$

$$\underline{\oint_S \boldsymbol{D} \cdot d\boldsymbol{s}} = Q = \underline{\int_V \rho_v \, dv} \tag{9}$$

Differential form:

Applying divergence (or Gauss's) theorem to (9),

$$\int_V (\nabla \cdot \boldsymbol{D})\, dv = \int_V \rho_v dv \quad \Rightarrow \quad \int_V (\nabla \cdot \boldsymbol{D} - \rho_v)\, dv = 0$$

$$\Rightarrow \quad \nabla \cdot \boldsymbol{D} = \rho_v \tag{10}$$

3) Electric field intensity due to symmetric charge distributions

(e.g. 3-4)

Infinitely long uniform line charge ρ_l

For a cylindrically symmetric field $(\partial/\partial\phi = 0)$,

Gauss's law (3-6) for $\boldsymbol{E} = \hat{\boldsymbol{r}} E_r$

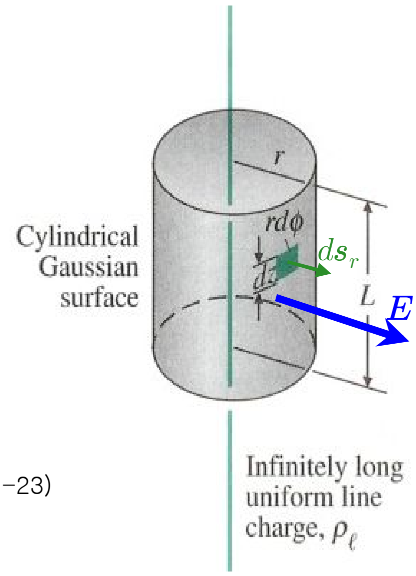
$$\Rightarrow \oint_S \boldsymbol{E} \cdot d\boldsymbol{s} = \frac{Q}{\epsilon_o}$$

$$\Rightarrow \int_{top} \overset{E_z = 0}{\cancel{E_z}} ds_z - \int_{bottom} \cancel{E_z} ds_z + \int_{cylinder} E_r ds_r = \frac{Q}{\epsilon_o}$$

$$\Rightarrow \int_0^L \int_0^{2\pi} E_r r d\phi dz = \frac{\rho_l L}{\epsilon_o}$$

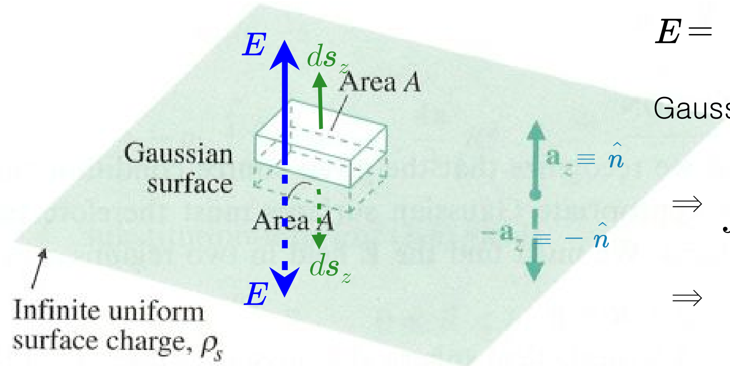
$$\Rightarrow \quad 2\pi r \cancel{L} E_r = \frac{\rho_l \cancel{L}}{\epsilon_o} \quad \Rightarrow \quad E_r = \frac{\rho_l}{2\pi\epsilon_o r}$$

$$\therefore \quad \boldsymbol{E}(r) = \hat{\boldsymbol{r}} \frac{\rho_l}{2\pi\epsilon_o r} \tag{3-23}$$



(e.g. 3-5)

Infinite uniform sheet charge ρ_s



$$\boldsymbol{E} = \pm\, \hat{z}\, E_z$$

Gauss's law: $\oint_S \boldsymbol{E} \cdot d\boldsymbol{s} = \frac{Q}{\epsilon_o}$

$$\Rightarrow \int_{top\,A} E_z\, ds_z + \int_{bottom\,A} E_z\, ds_z = \frac{Q}{\epsilon_o}$$

$$\Rightarrow \quad 2E_z \int_A ds_z = \frac{Q}{\epsilon_o}$$

$$\Rightarrow \quad 2E_z \cancel{A} = \frac{\rho_s \cancel{A}}{\epsilon_o} \quad \Rightarrow \quad E_z = \frac{\rho_s}{2\epsilon_o}$$

$\therefore \quad \boldsymbol{E} = \pm\, \hat{z}\, \frac{\rho_s}{2\epsilon_o}$ (+ for above the sheet, − for below the sheet) (3-25a,b)

Notes) $E \propto 1/r^2$ for a point charge source. (3-8)

$E \propto 1/r$ for a line charge source. (3-23)

E is independent of r for a surface charge source. (3-25)

(e.g. 3-6)

A spherical electron cloud of volume charge ρ_v

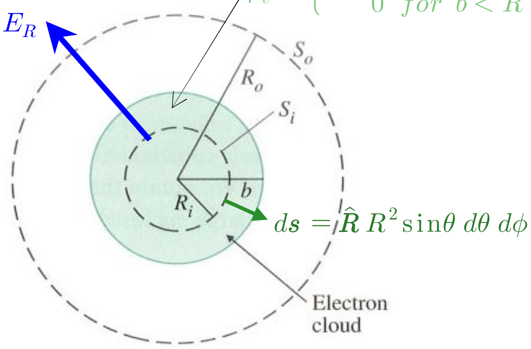
volume charge density

$$\rho_v = \begin{cases} -\rho_o & for\ 0 \le R \le b \\ 0 & for\ b < R < \infty \end{cases}$$

Gauss's law:

$$\oint_S \boldsymbol{E} \cdot d\boldsymbol{s} = \frac{Q}{\epsilon_o} = \frac{1}{\epsilon_o} \int_V \rho_v\, dv$$

$\boldsymbol{E} = \hat{\boldsymbol{R}}\, E_R$

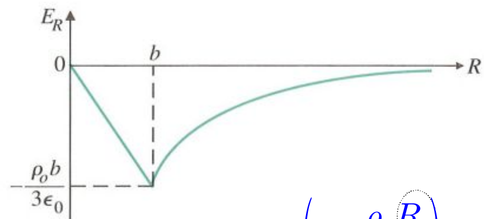


For $0 \le R \le b$,

$$\oint_{S_i} (\hat{\boldsymbol{R}} E_R) \cdot d\boldsymbol{s} = -\frac{1}{\epsilon_o} \int_V \rho_o\, dv$$

$$E_R R^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= -\frac{\rho_o}{\epsilon_o} \int_0^R R^2 dR \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$



$$E_R 4\pi R^2 = -\rho_o 4\pi R^3/3\epsilon_o \quad \Rightarrow \quad E_R = -\rho_o R/3\epsilon_o \quad \Rightarrow \quad \boldsymbol{E} = \hat{\boldsymbol{R}}\left(-\frac{\rho_o R}{3\epsilon_o}\right)$$

Similarly, for $b < R < \infty$,

$$\oint_{S_o} (\hat{\boldsymbol{R}} E_R) \cdot d\boldsymbol{s} = -\frac{\rho_o}{\epsilon_o} \int_0^b R^2 dR \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$E_R 4\pi R^2 = -\rho_o 4\pi b^3/3\epsilon_o \Rightarrow E_R = -\rho_o b^3/3\epsilon_o R^2 \Rightarrow \boldsymbol{E} = \hat{\boldsymbol{R}}\left(-\frac{\rho_o b^3}{3\epsilon_o R^2}\right)$$

D. Electric (Scalar) Potential V

1) Definition of V

The fundamental postulate (3-4) in electrostatics: $\nabla \times \boldsymbol{E} = \mathbf{0}$ (3-4)

Null vector identity (2-105): $\nabla \times (\nabla f) = \mathbf{0}$

Therefore, $\boldsymbol{E}$ can be found by defining an scalar electric potential V such that

$$\boldsymbol{E} = -\nabla V \qquad \text{(3-26)}$$

2) Physical meaning of V

Work done by external force in moving q along $\boldsymbol{dl}$:

$$dW = -\boldsymbol{F} \cdot \boldsymbol{dl} = -q\boldsymbol{E} \cdot \boldsymbol{dl} \qquad (11)$$

Differential electric potential:

$$dV \equiv \frac{dW}{q} = -\boldsymbol{E} \cdot \boldsymbol{dl} \qquad (12)$$

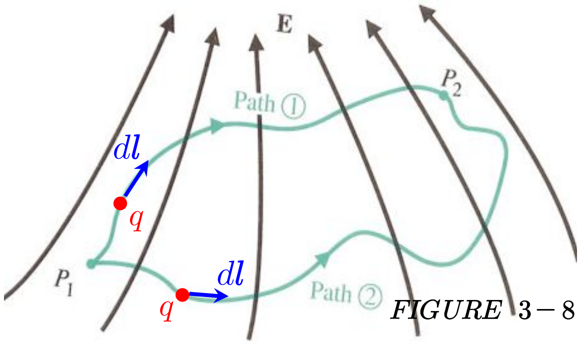


FIGURE 3−8

Electric potential difference (Voltage) between points P_1 and P_2 :

$$V_{21} \equiv V_2 - V_1 = -\int_{P_1}^{P_2} \boldsymbol{E} \cdot \boldsymbol{dl} \qquad \text{(V or J/C)} \qquad \text{(3-28)}$$

= Work done in moving a unit charge from P_1 to P_2 (path independent)

: single-value function, one value independent of path, solely depend on potentials at two points

For a closed path (path ① − path ②),

$$\oint_C \boldsymbol{E} \cdot \boldsymbol{dl} = \int_{P_1}^{P_2} \boldsymbol{E} \cdot \boldsymbol{dl} + \int_{P_2}^{P_1} \boldsymbol{E} \cdot \boldsymbol{dl} = -(V_2 - V_1) - (V_1 - V_2) = 0,$$

which is identical with (3-7) resulted by Stokes's theorem for $\nabla \times \boldsymbol{E} = \mathbf{0}$.

Electric scalar potential at any point

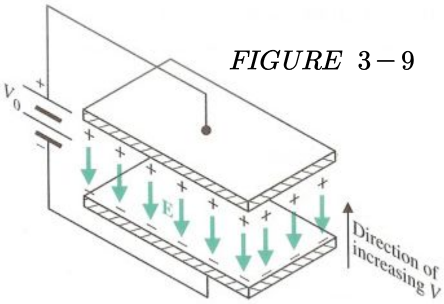
= Work per Coulomb required to move a positive test charge from a reference-zero position to a position $\boldsymbol{R}$ in question

$$V(\boldsymbol{R}) = -\int_{\infty}^{R} \boldsymbol{E} \cdot \boldsymbol{dl} \qquad \text{(V or J/C)} \qquad (13)$$

when a reference zero-potential point is chosen at $R = \infty$, i.e., $V|_{R \to \infty} = 0$.

Notes) ① $\boldsymbol{E} = -\nabla V$ $\Leftrightarrow$ ② $\nabla \times \boldsymbol{E} = 0$ $\Leftrightarrow$ ③ $\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = 0$

FIGURE 3−9



① $\Rightarrow$ Negative sign (−) :

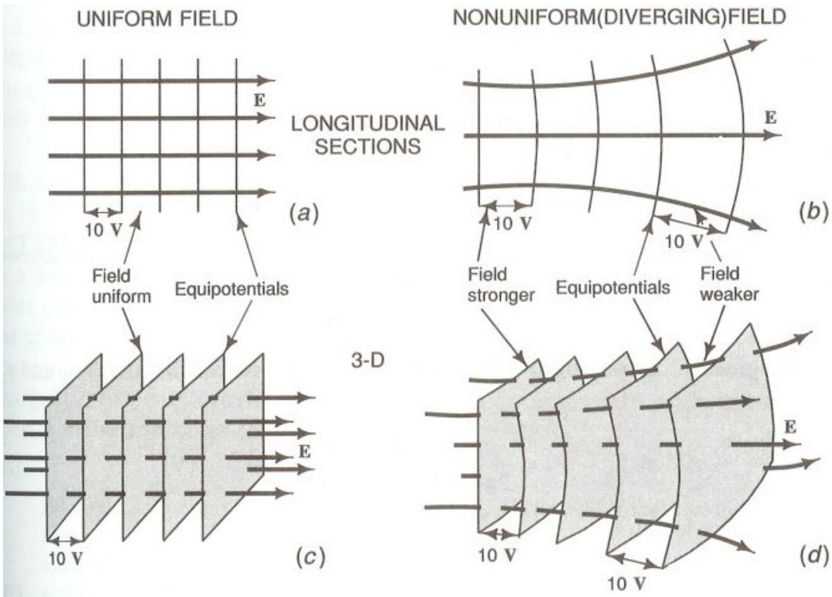
Direction of $\boldsymbol{E}$

= Direction of decreasing V

i.e., $\boldsymbol{E} \downarrow\uparrow \nabla V$

① $\Rightarrow$ $\boldsymbol{E} \propto \nabla V$:

Electric field lines $\perp$ Equipotential lines : Orthogonality



$\boldsymbol{E} \perp$ Conducting surface [$\because$ $V_{cond.\ surf.}$= const.(equipot.)]

② Irrotational (curl-free) field

③ Conservative field (path-independent, only end-point dependent)

3) Electric potential V due to charge distributions

At a distance $\boldsymbol{R}$ from a point charge q,

(3−8) $\boldsymbol{E}(\boldsymbol{R}) = \hat{\boldsymbol{R}} \dfrac{1}{4\pi\epsilon_o} \dfrac{q}{R^2}$ in (13):

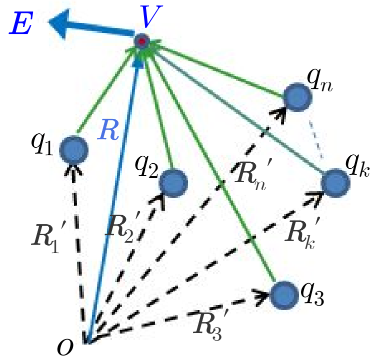
$$V(\boldsymbol{R}) = -\int_{\infty}^{R} \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_{\infty}^{R} \left(\hat{\boldsymbol{R}} \frac{1}{4\pi\epsilon_o} \frac{q}{R^2} \right) \cdot (\hat{\boldsymbol{R}}\, dR) = \frac{1}{4\pi\epsilon_o} \frac{q}{R} \text{ (V)} \qquad (3\text{−}29)$$

Between any two points $P_1(R_1)$ and $P_2(R_2)$,

$$V_{21} \equiv V_2 - V_1 = -\int_{P_1}^{P_2} \boldsymbol{E} \cdot d\boldsymbol{l} = \frac{q}{4\pi\epsilon_o} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$

For a system of discrete point charges,
V at R by the superposition principle
using (3-14):

$$V(\mathbf{R}) = \frac{1}{4\pi\epsilon_o}\sum_{k=1}^{n}\frac{q_k}{|\mathbf{R}-\mathbf{R}_k{}'|} \quad \text{(V)} \qquad \text{(3-31)}$$

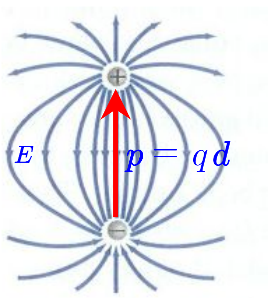


Total electric potential in various charge distributions of points, line, surface and volume by the superposition principle:

$$V(\mathbf{R}) = \frac{1}{4\pi\epsilon_o}\left(\sum_{k=1}^{n}\frac{q_k}{|\mathbf{R}-\mathbf{R}_k{}'|} + \int_{L'}\frac{\rho_l(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|}dl' \right. \qquad \text{(3-31, 3-40)}$$

$$\left. + \int_{S'}\frac{\rho_s(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|}ds' + \int_{V'}\frac{\rho_v(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|}dv'\right) \qquad \text{(3-39, 3-38)}$$

(e.g. 3-7) Electric dipole



Electric-field pattern

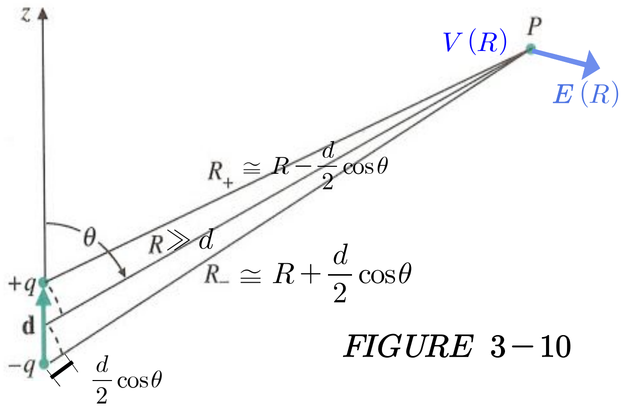


FIGURE 3−10

$$V(R) = \frac{q}{4\pi\epsilon_o}\left(\frac{1}{R_+} - \frac{1}{R_-}\right) = \frac{q}{4\pi\epsilon_o}\left(\frac{1}{R - \frac{d}{2}\cos\theta} - \frac{1}{R + \frac{d}{2}\cos\theta}\right)$$

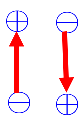
$$= \frac{q}{4\pi\epsilon_o}\left(\frac{d\cos\theta}{R^2 - \cancel{\frac{d^2}{4}\cos^2\theta}}\right) \cong \frac{qd\cos\theta}{4\pi\epsilon_o R^2} \qquad \text{(3-35)}$$

$$\Longrightarrow \quad V(R) = \frac{\mathbf{p}\cdot\hat{\mathbf{R}}}{4\pi\epsilon_o R^2} \qquad \text{(3-36)}$$

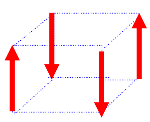
where $\mathbf{p} = q\mathbf{d}$ is the electric dipole moment.

$$\mathbf{E}(R) = -\nabla V = -\hat{\mathbf{R}}\frac{\partial V}{\partial R} - \hat{\boldsymbol{\theta}}\frac{\partial V}{\partial \theta} = \frac{p}{4\pi\epsilon_o R^3}(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta) \qquad \text{(3-37)}$$

Notes) Quadrupole Octopole



$V \propto 1/R^3$



$V \propto 1/R^4$

(e.g. 3-8) Uniformly charged circular disk

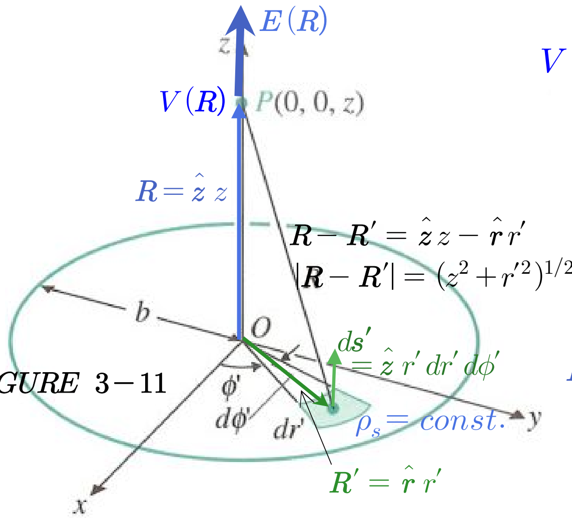


FIGURE 3−11

$$V(R) = \frac{1}{4\pi\epsilon_o}\int_{S'}\frac{\rho_s(R')}{|R - R'|}\, ds'$$

$$= \frac{\rho_s}{4\pi\epsilon_o}\int_0^{2\pi} d\phi' \int_0^b \frac{r'}{(z^2 + r'^2)^{1/2}}\, dr'$$

$$= \frac{\rho_s}{2\,\epsilon_o}\left[(z^2 + b^2)^{1/2} - |z|\right] \qquad (3\text{-}41)$$

$$E(R) = -\nabla V = -\hat{z}\,\frac{\partial V}{\partial z}$$

$$= \pm\,\hat{z}\,\frac{\rho_s}{2\,\epsilon_o}\left[1 - |z|\,(z^2 + b^2)^{-1/2}\right]$$

[+ for $z > 0$ (above the disk), (3-42a)

− for $z < 0$ (below the disk)] (3-42b)

(cf) If $z \ll b \to \infty$, then the disk becomes an infinite sheet and $|z|\,(z^2 + b^2)^{-1/2} \to 0$ in (3.42).

$$\therefore\ E = \pm\,\hat{z}\,\frac{\rho_s}{2\epsilon_o} \quad \equiv \quad (3\text{-}25)$$

E. Electrostatic Systems and Applications

1) CRO (Cathode-Ray Oscilloscope)

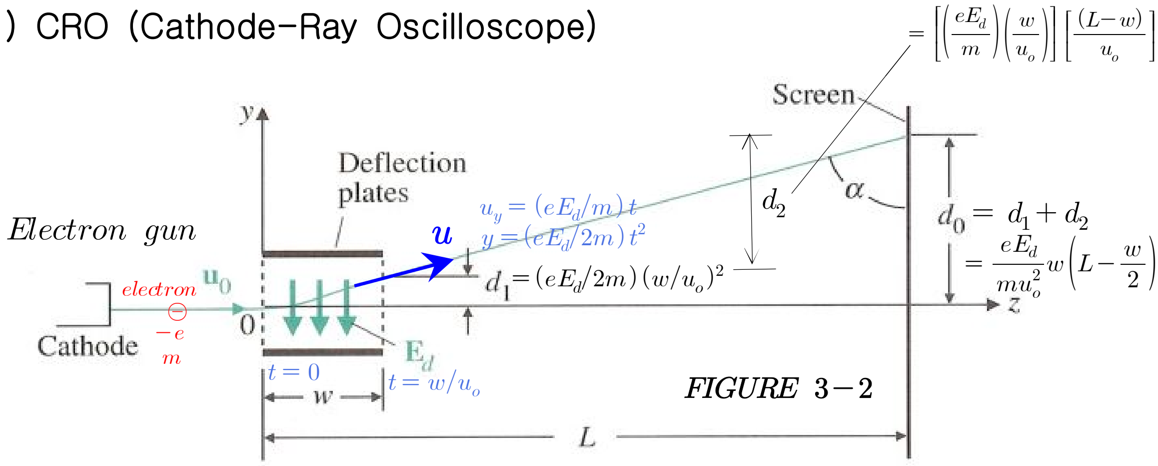
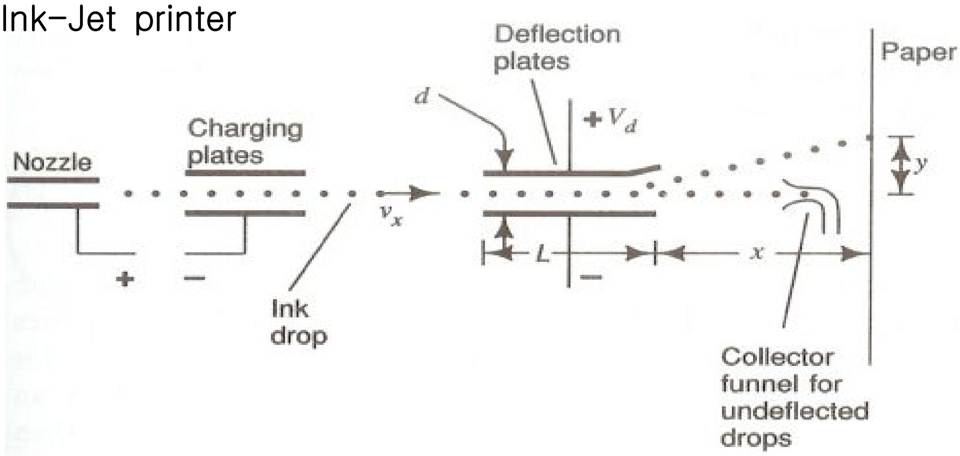
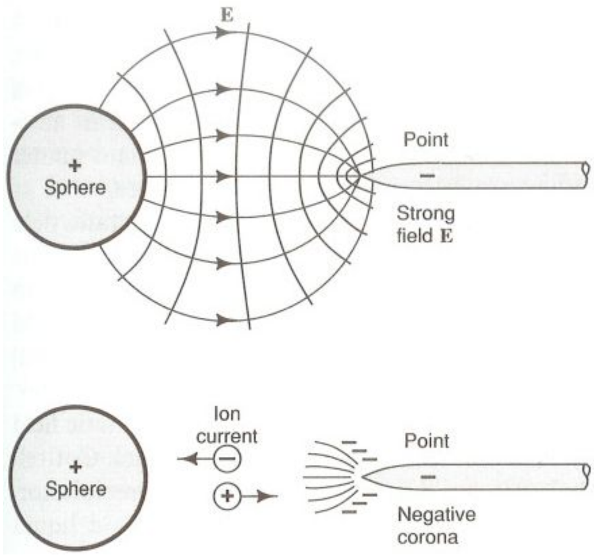


FIGURE 3−2

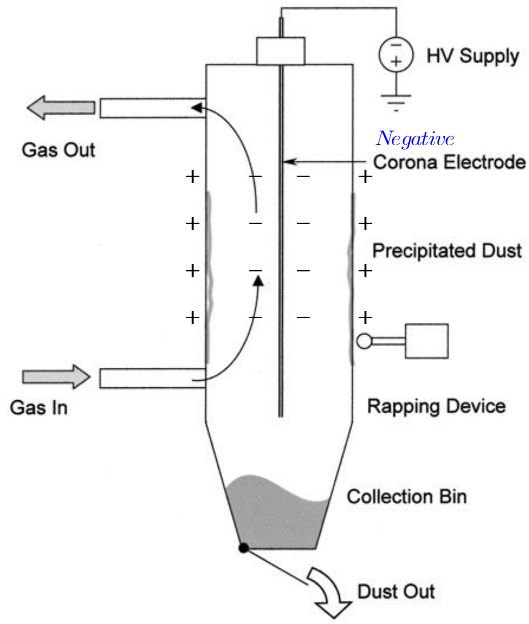
2) Ink-Jet printer



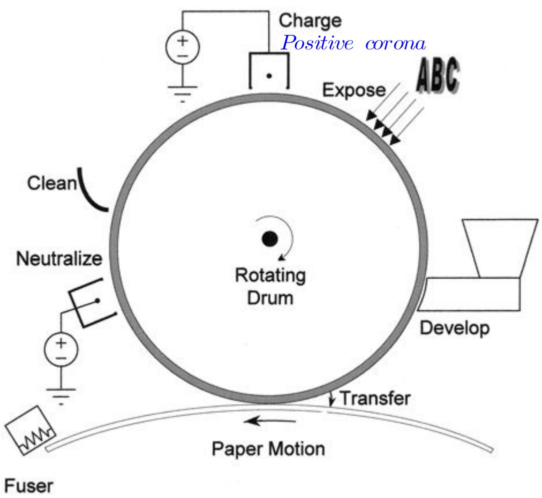
3) Corona discharge



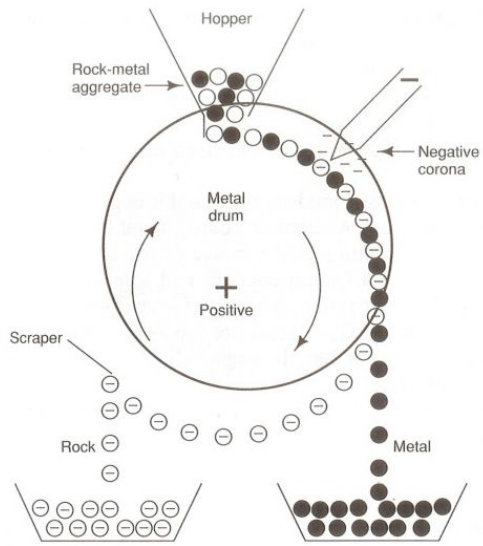
4) Electrostatic precipitator



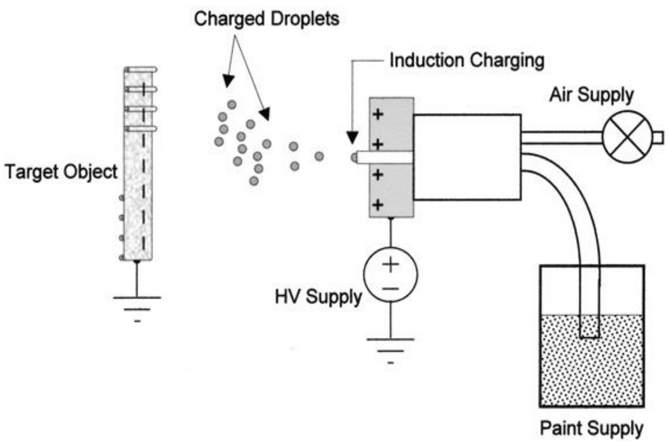
5) Xerographic copying machine



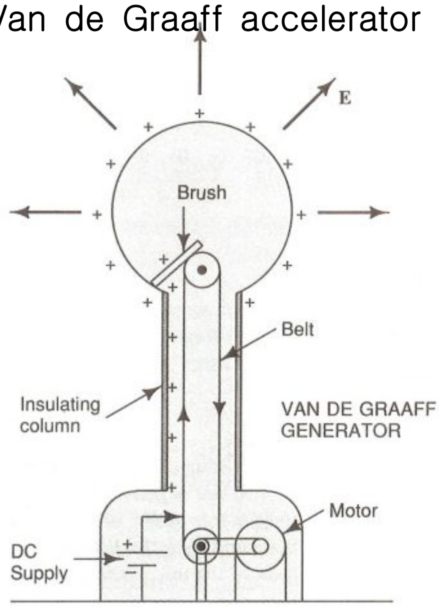
6) Electrostatic separator



7) Electrostatic painting



8) Van de Graaff accelerator



Homework Set 3

1) P.3-1
2) P.3-4
3) P.3-7
4) P.3-8
5) P.3-11
6) P.3-12