CHAPTER 6. Electromagnetic Wave Equations

Reading assignments: Cheng Ch.6-4~6-5, Ulaby Ch.6-1~6-2,
Halliday Chs.31, 33

1. Review of Maxwell's Equations and Wave Equations

A. Maxwell's Equations

1) Differential and Integral forms of Maxwell's Equations

Differential form (point expression)	Integral Form (global expression)	Physical Law (significance)	
$\nabla\times E = -\dfrac{\partial B}{\partial t}$	$\oint_C E\cdot dl = -\int_S \dfrac{\partial B}{\partial t}\cdot ds$	Faraday's law	$(6-45,46a)$
$\nabla\times H = J + \dfrac{\partial D}{\partial t}$	$\oint_C H\cdot dl = \int_S \left(J + \dfrac{\partial D}{\partial t}\right)\cdot ds$	Ampere's law	$(6-45,46b)$
$\nabla\cdot D = \rho_v$	$\oint_S D\cdot ds = \int_V \rho_v\, dv$	Gauss's law	$(6-45,46c)$
$\nabla\cdot B = 0$	$\oint_S B\cdot ds = 0$	No isolated maget	$(6-45,46d)$

2) Constitutive Relations

For linear isotropic medium

$$D = \varepsilon E = \varepsilon_o\varepsilon_r E = \varepsilon_o(1+\chi_e)E = \varepsilon_o E + P \qquad (3\text{-}67,\ 62)$$

$$B = \mu H = \mu_o\mu_r H = \mu_o(1+\chi_m)H = \mu_o H + \mu_o M \qquad (5\text{-}65,\ 60)$$

($J = \sigma E$: Ohm's law) (4-10)

In general,

$$D = \overleftrightarrow{\varepsilon}\cdot E, \qquad B = \overleftrightarrow{\mu}\cdot H, \quad (J = \overleftrightarrow{\sigma}\cdot E)$$

3) Relationship between Sources and Fields

a) Stationary charges ⇒ Electric fields

$\nabla\cdot D = \rho_v$ ($\rho_v = \rho_f$ free charge) (3-63)

$\nabla\cdot P = -\rho_{pv}$ ($\rho_{pv} = \rho_b$ bound polarization charge) (3-59)

$\nabla\cdot E = (\rho_v + \rho_{pv})/\epsilon_o = \rho_t/\epsilon_o$ (total charge) (3-60)

b) Moving charges (Currents) ⇒ Magnetic fields

$\nabla\times H = J + \dfrac{\partial D}{\partial t} = J + J_d$ ($J = J_f$ free + J_d displacement currents) (6-44)

$\nabla\times M = J_{mv}$ ($J_{mv} = J_b$ bound magnetization current) (5-51)

$\nabla\times B = \mu_o(J + J_{mv} + J_d)$ (total current) (5-56)

4) Conservation of electric charge (Equation of current continuity)

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \qquad (4\text{-}20),\quad (6\text{-}41)$$

B. Boundary Conditions

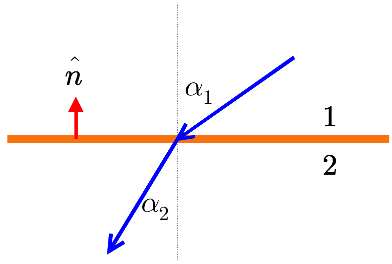
$\hat{n} \times (E_1 - E_2) = 0$: tangential E (3-79)(6-47a)

$\hat{n} \times (H_1 - H_2) = J_s$: tangential H (5-71)(6-47b)

$\hat{n} \cdot (D_1 - D_2) = \rho_s$: normal D (3-80)(6-47c)

$\hat{n} \cdot (B_1 - B_2) = 0$: normal B (5-68)(6-47d)

$\hat{n} \cdot (J_1 - J_2) = 0$: normal J (4-34)



Note) $\dfrac{\tan\alpha_1}{\tan\alpha_2} = \dfrac{\epsilon_1}{\epsilon_2} = \dfrac{\mu_1}{\mu_2} = \dfrac{\sigma_1}{\sigma_2}$ (3-83),

C. Properties of Electromagnetic Fields

1) Electromagnetic Forces

$F = F_e + F_m = q(E + u \times B)$: Lorentz's force on an individual charge (5-5)

$f = f_e + f_m = \rho_v E + J \times B$: e.m. body force in plasmas

2) Electromagnetic Torques

$T = p \times E$: electric torque (5-124)*

$T = m \times B$: magnetic torque (5-124)

3) Electromagnetic Energy

$w = \dfrac{1}{2}(D \cdot E + H \cdot B) = \dfrac{\epsilon E^2}{2} + \dfrac{B^2}{2\mu}$: e.m. energy density (3-108), (5-108)

Note) Conservation of electromagnetic energy = Poynting's theorem:

$$\nabla \cdot (E \times H) + \frac{\partial}{\partial t}\left(\frac{E \cdot D}{2} + \frac{H \cdot B}{2}\right) + E \cdot J = 0 \qquad (7\text{-}64)$$

D. Electromagnetic Potential Functions and Wave Equations

1) Electromagnetic Potential Functions

$B = \nabla \times A$ $\Rightarrow$ vector magnetic potential A (6-50)

$E = -\nabla V - \dfrac{\partial A}{\partial t}$ $\Rightarrow$ scalar electric potential V (6-53)

2) Nonhomogeneous Wave Equations for A and V

(6-53) in (6-45c) : $\nabla^2 V = -\dfrac{\rho_v}{\epsilon} - \dfrac{\partial}{\partial t}(\nabla \cdot A)$ (6-45c)*

(6-50, 53) in (6-45b) : $\nabla^2 A - \mu\epsilon \dfrac{\partial^2 A}{\partial t^2} = -\mu J + \nabla(\nabla \cdot A + \mu\epsilon \dfrac{\partial V}{\partial t})$ (6-55)

With Lorentz condition, $\nabla \cdot \boldsymbol{A} + \mu\epsilon \dfrac{\partial V}{\partial t} = 0$ (6-56)

$\Rightarrow$ Nonhomogeneous wave equations :

$$\nabla^2 \boldsymbol{A} - \mu\epsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J} \qquad (6\text{-}57)$$

$$\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\epsilon} \qquad (6\text{-}58)$$

3) Solutions of Nonhomogeneous Wave Equations

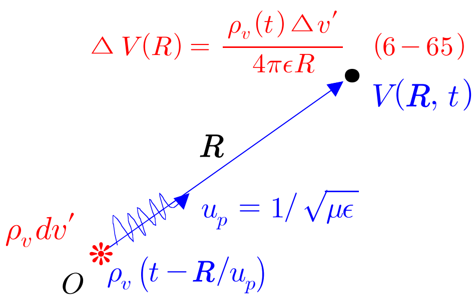
a) Time-independent solutions :

$$V(\boldsymbol{R}) = \frac{1}{4\pi\epsilon}\int_{V'} \frac{\rho_v}{R} dv', \quad \boldsymbol{E}(\boldsymbol{R}) = \frac{1}{4\pi\epsilon}\int_{V'} \hat{\boldsymbol{R}} \frac{\rho_v}{R^2} dv' \qquad (3\text{-}38,\ 16)$$

$$\boldsymbol{A}(\boldsymbol{R}) = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}}{R} dv', \quad \boldsymbol{B}(\boldsymbol{R}) = \frac{\mu I}{4\pi}\oint_{C'} \frac{d\boldsymbol{l}' \times \hat{\boldsymbol{R}}}{R^2}$$: Biot-Savart law (5-22, 31)

b) Time-dependent solutions $\Rightarrow$ Retarded potentials

Consider an element point charge $\rho_v dv'$ at the origin at time t.



At a distance R far away from O in spherical symmetry,

(6-58) $\Rightarrow$

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial V}{\partial R}\right) - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = 0 \qquad (6\text{-}59)$$

Change of variable: $V(R,t) = U(R,t)/R$ (6-60)

(6-59) $\Rightarrow$ $\dfrac{\partial^2 U}{\partial R^2} - \mu\epsilon \dfrac{\partial^2 U}{\partial t^2} = 0$: 1-D homogeneous wave equation (6-61)

Solution: $U(R,t) = f(t - R\sqrt{\mu\epsilon}) = f(t - R/u_p)$: ftn. of retarded time (6-62)

where $u_p = 1/\sqrt{\mu\epsilon}$: wave phase velocity (6-63)

$\therefore$ Solution of (6-59) : $V(R,t) = f(t - R\sqrt{\mu\epsilon})/R = f(t - R/u_p)/R$ (6-64)

(6-64) in (6-65) : $\Delta f(t - R/u_p) = \dfrac{\rho_v(t - R/u_p)\Delta v'}{4\pi\epsilon}$ (6-66)

Retarded scalar potential at R, t due to a charge distribution over V':

$$V(R,t) = \frac{1}{4\pi\epsilon}\int_{V'} \frac{\rho_v(t - R/u_p)}{R} dv' \quad \text{(V)} \qquad (6\text{-}67)$$

Similarly, retarded vector magnetic potential :

$$\boldsymbol{A}(R,t) = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}(t - R/u_p)}{R} dv' \quad \text{(Wb/m)} \qquad (6\text{-}68)$$

E. Homogeneous Electromagnetic Wave Equations

Maxwell's equations in source-free nonconducting media :

$$\nabla\times E = -\mu\frac{\partial H}{\partial t} \qquad \text{(6-94a)}$$

$$\nabla\times H = \varepsilon\frac{\partial E}{\partial t} \qquad \text{(6-94b)}$$

$$\nabla\cdot E = 0 \qquad \text{(6-94c)}$$

$$\nabla\cdot B = 0 \qquad \text{(6-94d)}$$

(6-94b) in $\nabla\times(6-94a)$ using $\nabla\times\nabla\times\boldsymbol{E} = \nabla(\cancel{\nabla\cdot\boldsymbol{E}}) - \nabla^2\boldsymbol{E} = -\nabla^2\boldsymbol{E}$:

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{(6-96)}$$: electric wave equation

In a similar way, (6-94a) in $\nabla\times(6-94b)$:

$$\nabla^2 H - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = 0 \qquad \text{(6-97)}$$: magnetic wave equation

2. Review of Field-Circuit Relations and Phasors

A. Relations between Field and Circuit Theory

1) Basic relations

Field quantities: $\boldsymbol{E}, \boldsymbol{D}, \boldsymbol{B}, \boldsymbol{H}, \boldsymbol{J}$ $\quad\epsilon, \mu, \sigma$ $\rightarrow$ function of $\boldsymbol{r}$ and t;

More general in distributed medium

Circuit quantities: V, I $\quad C, L, R$ $\rightarrow$ function of two-terminal ($L_s \ll \lambda$) and t;

Simple in lumped medium

$$V = -\int \boldsymbol{E}\cdot d\boldsymbol{l} \qquad \text{(3-28)}$$

$$I = \oint \boldsymbol{H}\cdot d\boldsymbol{l} = \int_S \boldsymbol{J}\cdot d\boldsymbol{s} \qquad \text{(5-10) (4-5)}$$

$$C = \frac{Q}{V} = \frac{\oint_S \boldsymbol{D}\cdot d\boldsymbol{s}}{-\int \boldsymbol{E}\cdot d\boldsymbol{l}} = \epsilon\frac{S}{d} \qquad \text{(3-87)(4-36)}$$

$$L = \frac{\Lambda}{I} = \frac{\int \boldsymbol{B}\cdot d\boldsymbol{s}}{\oint \boldsymbol{H}\cdot d\boldsymbol{l}} = \mu\frac{S}{d} \qquad \text{(5-77)}$$

$$R = \frac{V}{I} = \frac{-\int \boldsymbol{E}\cdot d\boldsymbol{l}}{\oint \boldsymbol{H}\cdot d\boldsymbol{l}} = \frac{-\int \boldsymbol{E}\cdot d\boldsymbol{l}}{\oint_S \sigma\boldsymbol{E}\cdot d\boldsymbol{s}} = \frac{1}{\sigma}\frac{d}{S} \qquad \text{(4-37)}$$

2) Energy relations

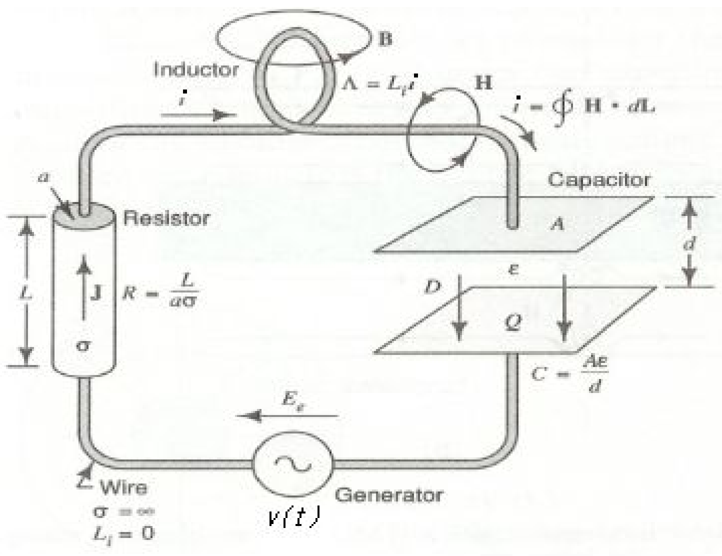
$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\int_V \boldsymbol{D}\cdot\boldsymbol{E}\,dv = \frac{1}{2}\int_V \epsilon E^2\,dv \qquad \text{(3-110, 105, 106)}$$

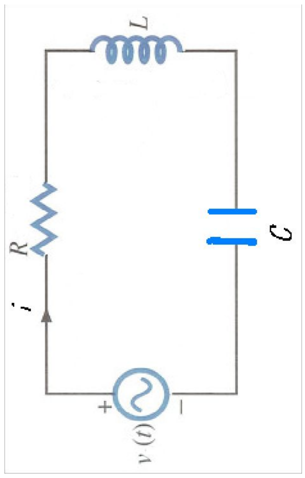
$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\int_V \boldsymbol{H}\cdot\boldsymbol{B}\,dv = \frac{1}{2}\int_V \mu H^2\,dv \qquad \text{(5-104, 105, 106)}$$

$$P_{ohmic} = I^2 R = \int_V \boldsymbol{E}\cdot\boldsymbol{J}\,dv = \int_V \sigma E^2\,dv \qquad \text{(4-31, 30)}$$

3) Comparison of an RLC circuit

Consider a series circuit of resistor, inductor, and capacitor connected to a generator





Equivalent RLC circuit

electric fields in R = emf + induced fields by currents and charges

$$\Rightarrow \quad \boldsymbol{E} = \boldsymbol{E}_e + \boldsymbol{E}_A + \boldsymbol{E}_V$$

$$\Rightarrow \quad \boldsymbol{E}_e = \frac{\boldsymbol{J}}{\sigma} - \frac{\partial \boldsymbol{A}}{\partial t} - \nabla V \quad : \text{ Field equation} \qquad (1)$$

$$\Rightarrow \quad \oint \boldsymbol{E}_e\cdot d\boldsymbol{l} = \oint\left(\frac{\boldsymbol{J}}{\sigma} - \frac{\partial \boldsymbol{A}}{\partial t} - \nabla V\right)\cdot d\boldsymbol{l}$$

Assume $i(t)$ = same at all points of circuit at any t $(L_s \ll \lambda)$.

then, $$v(t) = \frac{JL}{\sigma} + \frac{d}{dt}\oint \boldsymbol{A}\cdot d\boldsymbol{l} + Ed$$

$$\frac{JL}{\sigma} = \frac{iL}{a\sigma} = iR$$

$$\frac{d}{dt}\oint \boldsymbol{A}\cdot d\boldsymbol{l} = \frac{d}{dt}\int_S (\nabla\times\boldsymbol{A})\cdot d\boldsymbol{s} = \frac{d}{dt}\int_S \boldsymbol{B}\cdot d\boldsymbol{s} = \frac{d\Phi}{dt} = -L\frac{di}{dt}$$

$$Ed = \frac{D}{\epsilon}d = \frac{Q}{\epsilon A/d} = \frac{Q}{C} = \frac{1}{C}\int i\,dt$$

$$\therefore \quad \underline{v(t) = Ri} + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad : \text{ Circuit equation} \qquad (6\text{-}70)$$

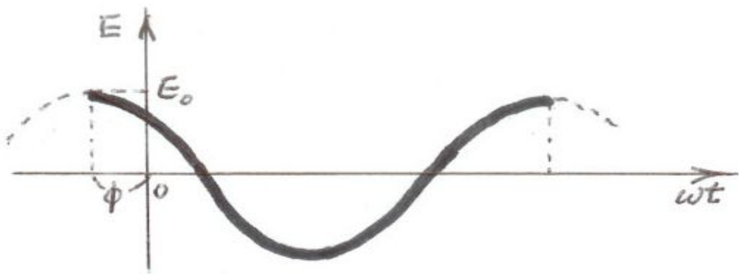
→ Extension of (dc) Ohm's law to time-varying cases

B. Review of Phasors

1) Phasor representation of time-harmonic fields

Electric field varying sinusoidally with time (Instantaneous time-varying field):

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_o(\boldsymbol{r})\cos(\omega t+\phi) = Re\left[\boldsymbol{E}_o(\boldsymbol{r})\, e^{j(\omega t+\phi)}\right] \qquad \text{(6-69)*}$$



where $|\boldsymbol{E}_o(\boldsymbol{r})|$ = magnitude at $\boldsymbol{r}$

$\omega = 2\pi f = 2\pi/T$: angular frequency (rad/s)

$f = 1/T$: frequency (Hz) $\quad T$ = (time) period (s)

ϕ = phase angle (rad or deg)

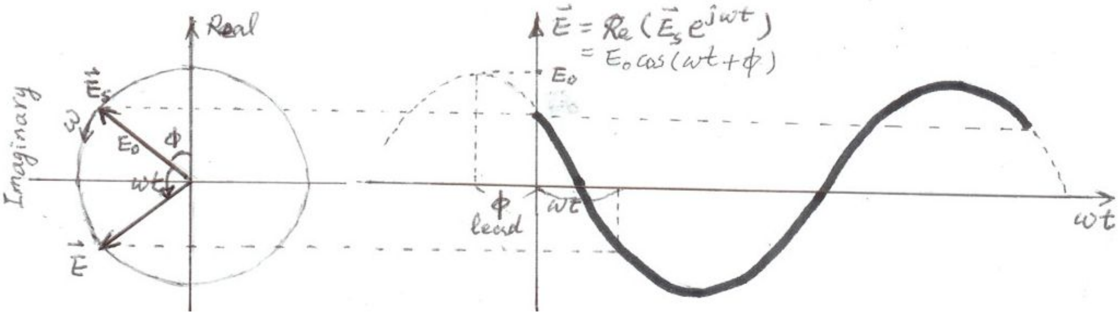
Note) Euler's identity : $e^{j\phi} = \cos\phi + j\sin\phi$ (2)

Phasor notation :

$$\boldsymbol{E}(\boldsymbol{r},t) = Re\left[\boldsymbol{E}_o(\boldsymbol{r})\, e^{j(\omega t+\phi)}\right] = Re\left[\boldsymbol{E}_s\, e^{j\omega t}\right] \qquad \text{(6-79)}$$

where $\boldsymbol{E}_s = \boldsymbol{E}_o(\boldsymbol{r})\, e^{j\phi}$: (vector) phasor (6-79)*

= polar form of complex field containing amplitude and phase; independent of t



Notes) i) $E_s = E_o e^{j\phi} = E_o(\cos\phi + j\sin\phi)$

$$= E_r + jE_i \quad (E_r = E_o\cos\phi,\ E_i = E_o\sin\phi)$$

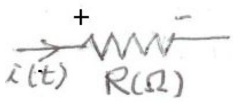
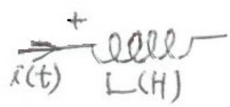
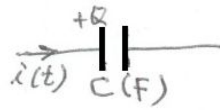
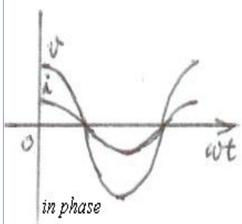
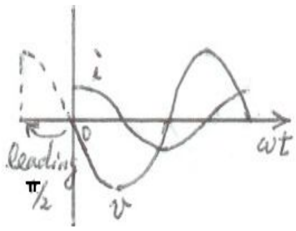
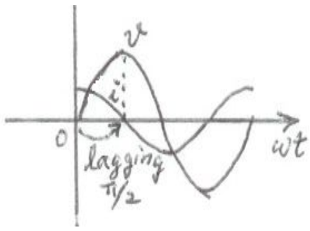
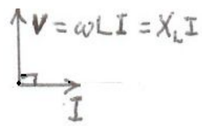
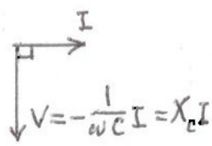
$$= E_o\angle\phi \quad \left(E_o = \sqrt{E_r^2+E_i^2},\ \phi = \tan^{-1}\frac{E_i}{E_r}\right) \qquad (3)$$

ii) $$\frac{\partial \boldsymbol{E}}{\partial t} = Re\left[\frac{\partial(\boldsymbol{E}_s e^{j\omega t})}{\partial t}\right] = Re(j\omega\boldsymbol{E}_s e^{j\omega t}) = Re(j\omega\boldsymbol{E})$$

$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = Re[-\omega^2\boldsymbol{E}] \qquad (4)$$

$$\int \boldsymbol{E}\,dt = Re\left[\frac{1}{j\omega}\boldsymbol{E}_s e^{j\omega t}\right] = Re\left(\frac{1}{j\omega}\boldsymbol{E}\right)$$

2) Time-harmonic responses of circuits

	Resistor	Inductor	Capacitor
			
Current	$i(t) = v(t)/R$	$i(t) = \frac{1}{L}\int v(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$
Voltage	$v(t) = R\,i(t)$	$v(t) = L\frac{di}{dt} = L\frac{d^2Q}{dt^2}$	$v(t) = \frac{1}{C}\int i\,dt = \frac{Q}{C}$
DC response	$V = RI$	$V = 0$	$I = 0$
Time-harm. resp. $i(t) = I\cos\omega t$ $= Re[Ie^{j\omega t}]$	$v(t) = RI\cos\omega t$ $= Re[R\,Ie^{j\omega t}]$ $= Re[R\,Ie^{j\omega t}]$	$v(t) = -\omega LI\sin\omega t$ $= Re[j\omega LIe^{j\omega t}]$ $= Re[jX_L Ie^{j\omega t}]$	$v(t) = \frac{1}{\omega C}I\sin\omega t$ $= Re[-\frac{j}{\omega C}Ie^{j\omega t}]$ $= Re[+jX_C Ie^{j\omega t}]$
Time diagram			
Phasor diagram			
Resistance or Reactance	R	$X_L = \omega L$ (inductive)	$X_C = -\frac{1}{\omega C}$ (capacitive)
Impedance $(Z = V/I)$	$Z = R + j0$ $= R\angle 0$ $= Re^{j0}$	$Z = 0 + jX_L$ $= \omega L\angle\frac{\pi}{2}$ $= \omega Le^{j\frac{\pi}{2}}$	$Z = 0 + jX_C$ $= \frac{1}{\omega C}\angle -\frac{\pi}{2}$ $= +\frac{1}{\omega C}e^{-j\frac{\pi}{2}}$
Power$(p = iv)$	$iv = i^2R$	$\frac{1}{2}L\frac{di^2}{dt}$	$\frac{1}{2}C\frac{dv^2}{dt}$
Energy $(E = \int_0^T p\,dt)$	$\frac{1}{2}RI^2$	$\frac{1}{2}LI^2$	$\frac{1}{2}CV^2$

3) Application to RLC circuit

(6-70):

$$v(t) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \qquad \text{in a time domain} \qquad (6\text{-}70)$$

where $v(t) = V_o \cos\omega t = Re[(V_o e^{j0})e^{j\omega t}] = Re(V_s e^{j\omega t})$ (6-72)

$i(t) = I_o \cos(\omega t + \phi) = Re[(I_o e^{j\phi})e^{j\omega t}] = Re(I_s e^{j\omega t})$ (6-69, 73)

(6-72, 73) in (6-70):

$$V_s = RI_s + j\omega L I_s + \frac{I_s}{j\omega C}$$

$$\Rightarrow V_s = \left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] I_s = [R + j(X_L + X_C)] I_s = Z I_s \quad (6\text{-}78)$$

in a cosine-reference phasor domain

$$\Rightarrow I_s = \frac{V_s}{Z} = \frac{V_s}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{V_o}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$= \underbrace{\frac{V_o}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}_{\equiv I_o} \angle \underbrace{\tan^{-1}\left[\frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}\right]}_{\equiv \phi} \equiv I_o e^{j\phi} \quad (6\text{-}78)^*$$

$$\Rightarrow i(t) = Re(I_s e^{j\omega t}) = Re[I_o e^{j(\omega t + \phi)}] = I_o \cos(\omega t + \phi) \quad (6\text{-}78)^{**}$$

3. Time-Harmonic Electromagnetics and Helmholtz's Equations

A. Time-Harmonic Maxwell's Equations

Time-harmonic fields in terms of phasors:

$$\boldsymbol{E}(x,y,z;t) = Re[\boldsymbol{E}_s(x,y,z)\, e^{j\omega t}] \qquad \text{(6-79)}$$

$$\boldsymbol{H}(x,y,z;t) = Re[\boldsymbol{H}_s(x,y,z)\, e^{j\omega t}]$$

$$\boldsymbol{J}(x,y,z;t) = Re[\boldsymbol{J}_s(x,y,z)\, e^{j\omega t}]$$

$$\rho_v(x,y,z;t) = Re[\rho_{vs}(x,y,z)\, e^{j\omega t}]$$

(6-79) in Maxwell's equations (6-45) :

$$\nabla\times E_s = -j\omega\mu\, H_s \qquad \text{(6-80a)}$$

$$\nabla\times H_s = J_s + j\omega\varepsilon E_s \qquad \text{(6-80b)}$$

$$\nabla\cdot E_s = \rho_{vs}/\varepsilon \qquad \text{(6-80c)}$$

$$\nabla\cdot B_s = 0 \qquad \text{(6-80d)}$$

B. Phasor Form of Time-Harmonic Wave Equations

1) Nonhomogeneous Helmholtz's Equations

$V(x,y,z;t) = Re[V_s(x,y,z)\, e^{j\omega t}]$, $\boldsymbol{A}(x,y,z;t) = Re[\boldsymbol{A}_s(x,y,z)\, e^{j\omega t}]$ in (6-57, 58) :

$$\nabla^2 V_s + k^2 V_s = -\frac{\rho_{vs}}{\epsilon} \qquad \text{(6-81)}$$

$$\nabla^2 \boldsymbol{A}_s + k^2 \boldsymbol{A}_s = -\mu \boldsymbol{J}_s \qquad \text{(6-83)}$$

where $k = \omega\sqrt{\mu\epsilon} = \frac{\omega}{u_p} = \frac{2\pi f}{u_p} = \frac{2\pi}{\lambda}$: wavenumber (6-82)

Solutions of (6-81, 83) :

(6-82) in (6-67, 68) $\Rightarrow$

$$\rho_v(x,y,z;t-R/u_p) = Re[\rho_v(x,y,z)\, e^{j\omega(t-R/u_p)}] = Re[\rho_v(x,y,z)\, e^{-jkR} e^{j\omega t)}]$$

$$\boldsymbol{J}(x,y,z;t-R/u_p) = Re[\boldsymbol{J}(x,y,z)\, e^{j\omega(t-R/u_p)}] = Re[\boldsymbol{J}(x,y,z)\, e^{-jkR} e^{j\omega t)}]$$

$$V_s(\boldsymbol{R}) = \frac{1}{4\pi\epsilon}\int_{V'} \frac{\rho_v e^{-jkR}}{R} dv'$$: retarded electric potential (6-84)

$$\boldsymbol{A}_s(\boldsymbol{R}) = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J} e^{-jkR}}{R} dv'$$: retarded vector magnetic potential (6-85)

Notes)

i) For $R \ll \lambda$, $kR \ll 1$, $i.e.$, $e^{-jkR} \approx 1$

$\Rightarrow$ (6-84, 85) for time-varying $\approx$ (3-38)(5-52) for static

ii) $\boldsymbol{E}_s(R) = -\nabla V_s - j\omega \boldsymbol{A}_s$, $\boldsymbol{B}_s(R) = \nabla \times \boldsymbol{A}_s$ in the phasor domain

$\Rightarrow$ $\boldsymbol{E}(\boldsymbol{R},t) = Re[\boldsymbol{E}_s(\boldsymbol{R})e^{j\omega t}]$, $\boldsymbol{B}(\boldsymbol{R},t) = Re[\boldsymbol{B}_s(\boldsymbol{R})e^{j\omega t}]$ for cosine reference in the time domain

iii) Formal procedure for determining time-harmonic fields

(1) Take cosine (or sine) reference for instantaneous time-varying fields and sources [(6-69), (6-69)*]

(2) Express the fields and sources as phasors [(6-79), (6-79)*]

(3) Recast Maxwell's or wave or circuit equations in phasor forms[(6-80, 6-81, 83, 78)], i.e., time-domain → phasor-domain equations

(4) Find phasors of fields by solving the phasor-domain equations [(6-84, 85), (6-78)*]

(5) Find the instantaneous fields for cosine (or sine) reference in the time domain [(6-79), (6-78)**]

(e.g. 6-9)

In a nonconducting dielectric medium with μ_o and $\epsilon = 9\epsilon_o$,

given $\boldsymbol{E}(z,t) = \hat{\boldsymbol{y}}\, 5\cos(10^9 t - \beta z)$ (V/m) (6-88)

find $\boldsymbol{H}$ and β.

(1) $\boldsymbol{E}(z,t) = \hat{\boldsymbol{y}}\, 5\cos(10^9 t - \beta z) = Re[(\hat{\boldsymbol{y}}\, 5 e^{-j\beta z})\, e^{j10^9 t}]$

(2) $\boldsymbol{E}_s(z) = \hat{\boldsymbol{y}}\, 5\, e^{-j\beta z}$ (6-89)

(3) (6-80a) $\Rightarrow$ $\boldsymbol{H}_s(z) = -\dfrac{1}{j\omega\mu_o}\nabla \times \boldsymbol{E}_s$ (6-80a)*

(6-80b) for nonconducting $\sigma = J = 0$ $\Rightarrow$ $\boldsymbol{E}_s(z) = \dfrac{1}{j\omega\epsilon}\nabla \times \boldsymbol{H}_s$ (6-80b)*

(4) (6-89) in (6-80a)* $\Rightarrow$ $\boldsymbol{H}_s(z) = -\hat{\boldsymbol{x}}\dfrac{\beta}{\omega\mu_o} 5\, e^{-j\beta z}$ (6-90)

(6-90) in (6-80b)* $\Rightarrow$ $\boldsymbol{E}_s(z) = \hat{\boldsymbol{y}}\dfrac{\beta^2}{\omega^2\mu_o\epsilon} 5\, e^{-j\beta z}$ (6-91)

(6-89) = (6-91) $\Rightarrow$ $\beta = \omega\sqrt{\mu_o\epsilon} = \dfrac{3\omega}{c} = \dfrac{3\times 10^9}{3\times 10^8} = 10$ in (6-90)

$\Rightarrow$ $\boldsymbol{H}_s(z) = -\hat{\boldsymbol{x}}\, 0.0398\, e^{-j10z}$ (6-92)

(5) $\boldsymbol{H}(z,t) = Re[\boldsymbol{H}_s(z)\, e^{j10^9 t}] = -\hat{\boldsymbol{x}}\, 0.0398\cos(10^9 t - 10z)$ (6-93)

2) Homogeneous Helmholtz's Equations

In source-free nonconducting media,

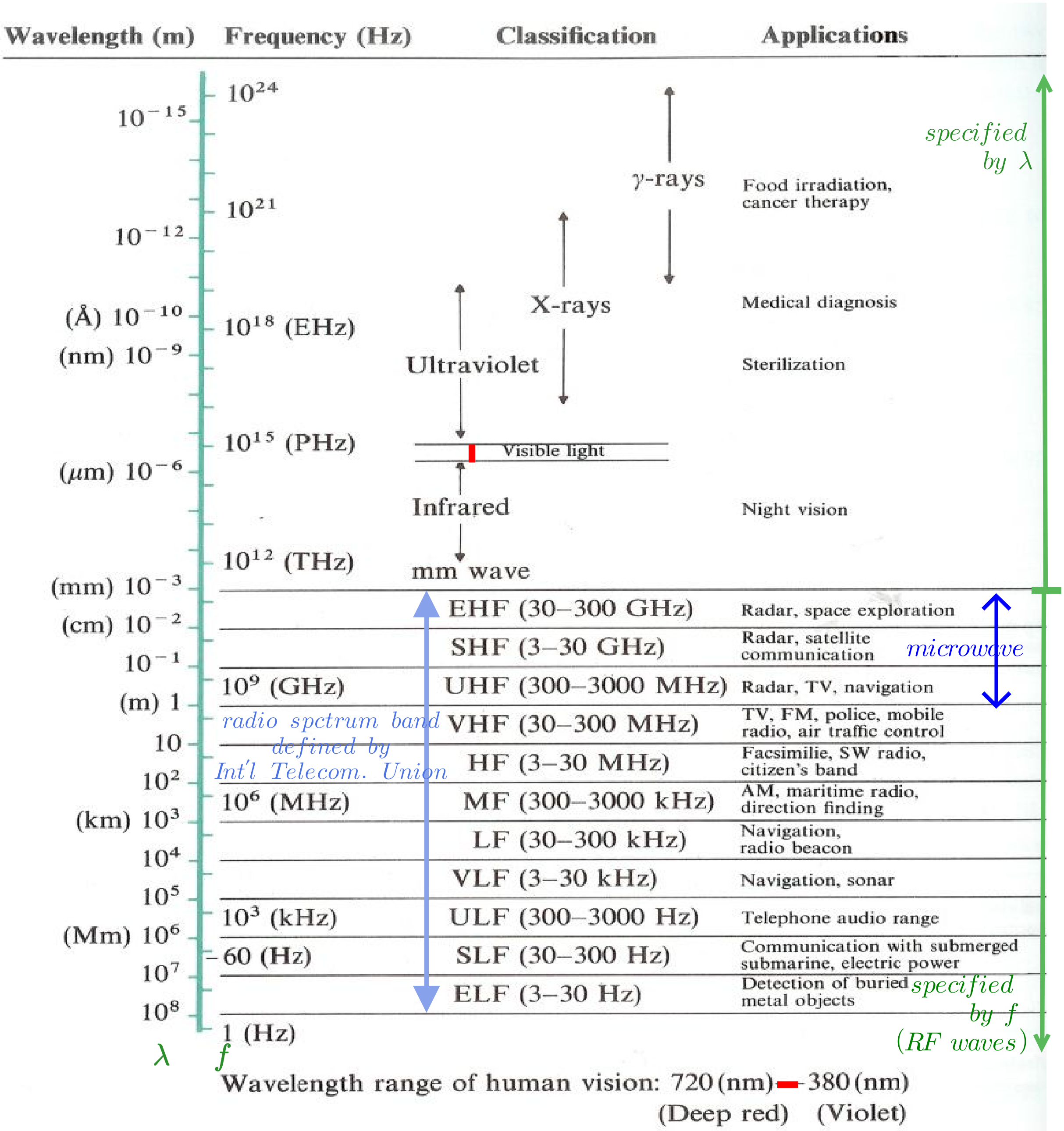
phasor representation of e.m. wave equations (6-96,97) with $\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$:

$$\nabla^2 E_s + \mu\varepsilon\omega^2 E_s = 0 \quad \Rightarrow \quad \nabla^2 E_s + k^2 E_s = 0 \qquad (6\text{-}98)$$

$$\nabla^2 H_s + \mu\varepsilon\omega^2 H_s = 0 \quad \Rightarrow \quad \nabla^2 H_s + k^2 H_s = 0 \qquad (6\text{-}99)$$

where $k^2 = \mu\epsilon\omega^2 = \frac{\omega^2}{u_p^2} = \left(\frac{2\pi}{\lambda}\right)^2$ (6-82)

C. Electromagnetic Spectrum and Applications



Homework Set 1 1) P.6-17 2) P.6-18 3) P.6-19 4) P.6-20