1. COMMON PROBABILITY DISTRIBUTIONS IN ENGINEERING

1.1 Objectives

- To understand probability distributions relevant to engineering applications
- To investigate statistical fundamentals of probability distributions
- To make use of Matlab statistical toolbox

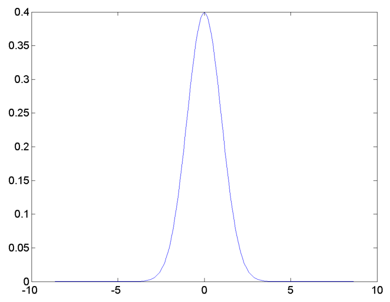
1.2 Types of Probability Distributions

Let X be a random variable in an engineering application. The probability density function (pdf) and cumulative distribution function (cdf) of X are denoted by f_X and F_X, respectively.

1.2.1 Normal Distribution

$$y(x;\mu,\sigma)=f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

```
>> x=[-10:0.1:10];
>> y=normpdf(x,0,1);
>> plot(x,y)
```

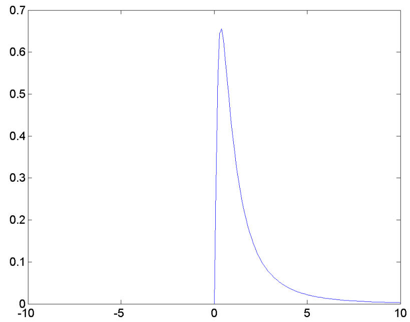


- Symmetric distribution, skewness=0, kurtosis=3
- Central limit theorem states that any distribution with finite mean and standard deviation tends to follow normal distribution
- Special case of chi-squared distribution and gamma distribution
- Dimension of fabricated part

1.2.2 Lognormal Distribution

$$y(x;\mu,\sigma)=f_X(x)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$

```
>> x=[-10:0.1:10];
>> y=lognpdf(x,0,1);
>> plot(x,y)
```

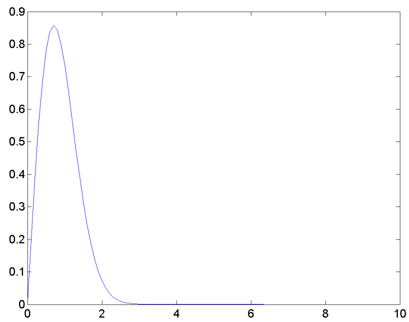


- Limited to a finite value at the lower limit
- Positively skewed
- Strengths of materials, fracture toughness

1.2.3 Weibull Distribution

$$y(x;v,k,a) = f_X(x) = \frac{k-a}{v-a}\left(\frac{x-a}{v-a}\right)^{k-1} e^{-\left(\frac{x-a}{v-a}\right)^k}, \quad \text{2 parameter Weibull if } a=0$$

```
>> x=[0:0.1:10];
>> y=weibpdf(x,1,2);
>> plot(x,y)
```

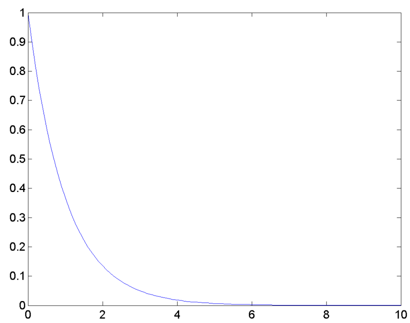


- Originally proposed for fatigue life
- Used in analysis of systems with weakest link
- Wear, fatigue, and fracture

1.2.4 Exponential Distribution

$$y(x;\mu) = f_X(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

```
>> x=[0:0.1:10];
>> y=exppdf(x,1);
>> plot(x,y)
```

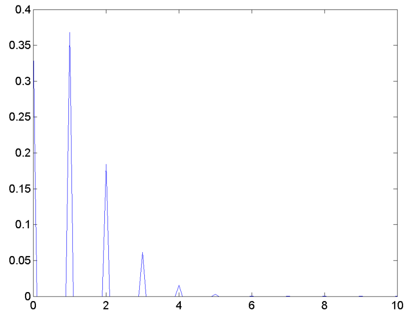


- Amount of time between occurrences
- Called as "memoryless random distribution"
- Continuous version of Poisson distribution to describe the number of occurrences per unit time
- Mean time between failures

1.2.5 Poisson Distribution (Discrete)

$$y(x;\lambda) = f_X(x) = \frac{\lambda^x}{x!}e^{-\lambda}$$

```
>> x=[0:0.1:10];
>> y=poisspdf(x,1);
>> plot(x,y)
```

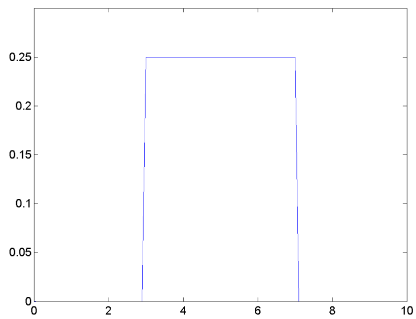


- An event occurrence in a given interval
- The occurrences are independent
- Average number of occurrence is fixed
- Product defections in a given batch

1.2.6 Uniform Distribution

$$y(x;a,b) = f_X(x) = \frac{1}{b-a}$$

```
>> x=[0:0.1:10];
>> y=unifpdf(x,3,7);
>> plot(x,y)
```

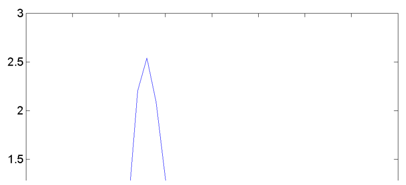


- Symmetric, skewness=0
- Equal occurrence
- Random number generator

1.2.7 Beta Distribution

$$y(x;a,b) = f_X(x) = \frac{1}{B(a,b)}x^{a-1}(1-x)^{b-1}, \quad B(a,b): \text{ Beta function}$$

```
>> x=[-10:0.1:10];
>> y=betapdf(x,3,6);
>> plot(x,y)
```



- `Bounded distributions`
- `Related to Gamma distribution`
- `Manufacturing tolerance`

1.2.8 Other Distributions in Engineering

Rayleigh distribution, Gamma distribution, Extreme Type I, II distributions, etc. Refer to http://mathworld.wolfram.com/topics/ProbabilityandStatistics.html.

Perform a parametric study for normal, lognormal, and Weibull distributions! **(Homework)**

1.3 Multi-variate Random Vector

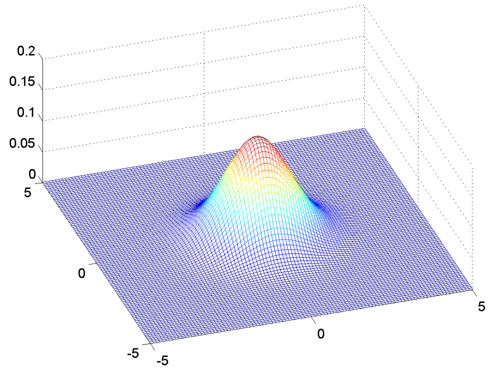
Suppose X_1 and X_2 are jointly distributed, and joint event is defined as $X_1 \le x_1$ and $X_2 \le x_2$. The corresponding bi-variate distribution of random vector is defined as

$$\text{Joint CDF:} \quad F_{X_1X_2}(x_1,x_2) = P(X_1 \le x_1, X_2 \le x_2)$$
$$\text{Joint PDF:} \quad f_{X_1X_2}(x_1,x_2) = \tfrac{\partial^2}{\partial x_1 \partial x_2} F_{X_1X_2}(x_1,x_2) \tag{1}$$

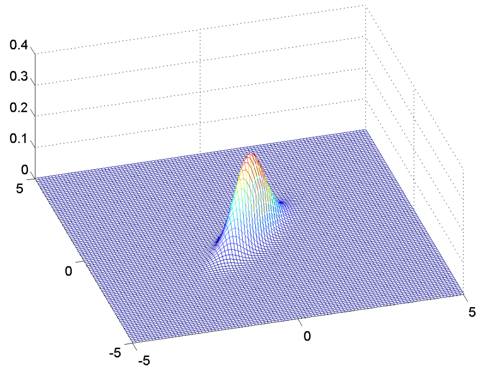
Assume that two random variables are normally distributed. To define the joint PDF of a multivariate distribution, five parameters are required, namely, the mean values of X and Y, μ_{X_1} and μ_{X_2}, their standard deviations σ_{X_1} and σ_{X_2}, and the correlation coefficient $\rho_{X_1X_2}$. The PDF of the bivariate normal distribution can be expressed as

$$f_{X_1X_2}(x_1,x_2) = \frac{1}{2\pi\sigma_{X_1}\sigma_{X_2}\sqrt{1-\rho_{X_1X_2}^2}} \exp\left\{ -\frac{1}{2(1-\rho_{X_1X_2}^2)} \left[\left(\frac{x_1-\mu_{X_1}}{\sigma_{X_1}} \right)^2 -2\rho_{X_1X_2} \frac{(x_1-\mu_{X_1})(x_2-\mu_{X_2})}{\sigma_{X_1}\sigma_{X_2}} + \left(\frac{x_2-\mu_{X_2}}{\sigma_{X_2}} \right)^2 \right] \right\} \tag{2}$$

If X_1 and X_2 are correlated, namely, $\rho_{X_1X_2} \ne 0$, $f_{X_1X_2}(x_1,x_2)$ is not symmetry.




```
>> [x1,x2]=meshgrid(-5:0.1:5);
>> f=1/(2*pi)*exp(-(x1.^2+(x2.).^2)./2);
>> mesh(x1,x2,f)
```




```
>> [x1,x2]=meshgrid(-5:0.1:5);
>> f=1/(2*pi*sqrt(1-0.8^2))*exp(-(x1.^2-
1.6*x1.*x2+x2.^2)./(2*(1-0.8^2)^2));
>> mesh(x1,x2,f)
```

Bivariate distribution of random vector can be generalized for n-dimensional random vector, $\mathbf{X}:\Omega \to R^n$. Joint CDF and PDF for n-dimensional random vector are written as

$$\text{Joint CDF:} \quad F_{\mathbf{X}}(\mathbf{x}) = P\left(\bigcap_{i=1}^{n}\{X_i \le x_i\}\right)$$
$$\text{Joint PDF:} \quad f_{\mathbf{X}}(\mathbf{x}) = \tfrac{\partial^n}{\partial x_1 \cdots \partial x_n} F_{\mathbf{X}}(\mathbf{x}) \tag{3}$$

A multi-variate normal random vector is distributed as

$$f_{\mathbf{X}}(\mathbf{x}) = (2\pi)^{-\frac{n}{2}} \left|\mathbf{\Sigma}_{\mathbf{X}}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_{\mathbf{X}})^T \mathbf{\Sigma}_{\mathbf{X}}^{-1}(\mathbf{x}-\boldsymbol{\mu}_{\mathbf{X}})\right] \tag{4}$$

where $\boldsymbol{\mu}_{\mathbf{X}}$ and $\mathbf{\Sigma}_{\mathbf{X}}$ are mean and covariance matrix of $\mathbf{X}$.

2. Moments of a Random Vector

2.1 Objectives

- To extend statistical moments of a random variable to a random vector
- To apply statistical moments to an uncertain response
- To prepare an uncertainty propagation through system in Section 3

2.2 Definition of Moments of a Random Vector

Let $\mathbf{X}=\{X_1,\cdots,X_n\}^T$ be an n-dimensional random vector and $g(\mathbf{X})$ be a function of **X**. In general, the N^{th} statistical moment of $g(\mathbf{X})$ is defined as

$$E\left[g(\mathbf{X})\right]^N \equiv \int_\Omega g^N(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{5}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of **X** and Ω is a random space.

2.3 Statistical Moments of a Random Vector

First, one special case is considered to find out statistical moments of input random variable, that is, $g(\mathbf{X})=X_i, i=1,\cdots,n$.

2.3.1 Mean of a Random Vector

Let $g(\mathbf{X})=X_1$ and set N=1. The first moment of random variable X_1 is defined as

$$\begin{aligned} E\left[X_1\right]^1 &\equiv \int_{-\infty}^{\infty} x_1 f_{X_1}(x_1) dx_1 \\ &= \mu_{X_1} \end{aligned} \tag{6}$$

Similarly,

$$E\left[X_2\right]^1 \equiv \int_{-\infty}^{\infty} x_2 f_{X_2}(x_2) dx_2 = \mu_{X_2}$$

$$\vdots$$

$$E\left[X_n\right]^1 \equiv \int_{-\infty}^{\infty} x_n f_{X_n}(x_n) dx_n = \mu_{X_n}$$

$$\boldsymbol{\mu}_{\mathbf{X}} = \left\{\mu_{X_1} \quad \cdots \quad \mu_{X_n}\right\}^T$$

2.3.2 Covariance of a Random Vector

Let $g(\mathbf{X})=(X_i-\mu_i)(X_j-\mu_j)$. The statistical moment is defined as

$$\begin{aligned} E\left[(X_i-\mu_i)(X_j-\mu_j)\right] &\equiv \int_{-\infty}^{\infty} (x_i-\mu_i)(x_j-\mu_j) f_{X_iX_j}(x_i,x_j) dx_i dx_j \\ &= \left[\Sigma_{ij}\right] = \boldsymbol{\Sigma} \end{aligned} \tag{7}$$

where $f_{X_iX_j}(x_i, x_j)$ and Σ_{ij} are the joint PDF and the covariance matrix of X_i and X_j, respectively.

When $i = j$, the diagonal terms in the covariance matrix are obtained as

$$\begin{aligned} E\left[X_1-\mu_1\right]^2 &\equiv \int_{-\infty}^{\infty}(x_1-\mu_1)^2 f_{X_1}(x_1)dx_1 \\ &= \sigma_{X_1}^2 = \Sigma_{11} \\ &\vdots \\ E\left[X_n-\mu_n\right]^2 &\equiv \int_{-\infty}^{\infty}(x_n-\mu_n)^2 f_{X_n}(x_n)dx_n \\ &= \sigma_{X_n}^2 = \Sigma_{nn} \end{aligned} \tag{8}$$

If $i \neq j$, the off-diagonal terms in the covariance matrix are obtained as

$$\begin{aligned} E\left[(X_1-\mu_1)(X_2-\mu_2)\right] &\equiv \int_{-\infty}^{\infty}(x_1-\mu_1)(x_2-\mu_2) f_{X_1X_2}(x_1,x_2)dx_1dx_2 \\ &= \Sigma_{12} \\ &\vdots \\ E\left[(X_n-\mu_n)(X_{n-1}-\mu_{n-1})\right] &\equiv \int_{-\infty}^{\infty}(x_n-\mu_n)(x_{n-1}-\mu_{n-1}) f_{X_nX_{n-1}}(x_n,x_{n-1})dx_ndx_{n-1} \\ &= \Sigma_{nn-1} \end{aligned} \tag{9}$$

The covariance matrix is written as

$$\mathbf{\Sigma_X} = \begin{bmatrix} \Sigma_{11} & \cdots & \Sigma_{1n} \\ \vdots & \ddots & \vdots \\ \Sigma_{n1} & \cdots & \Sigma_{nn} \end{bmatrix}$$

2.3.3 Properties of Covariance Matrix, $\mathbf{\Sigma_X}$

- $\mathbf{\Sigma_X}$ is symmetric, i.e., $\mathbf{\Sigma_X} = \mathbf{\Sigma_X}^T$: *Prove this property!* (**Homework**)
- Variance of X_i is the i^{th} diagonal element of $\mathbf{\Sigma_X}$, i.e., $\sigma_{X_i}^2 = \Sigma_{ii}$
- $\mathbf{\Sigma_X}$ is a positive semi-definite matrix, i.e., $\mathbf{A}^T\mathbf{\Sigma_X}\mathbf{A} \geq 0, \quad \forall \mathbf{A} \in R^n$

2.3.4 Correlation Coefficient, ρ_{ij}

The correlation coefficient ρ_{ij} is defined as

$$\rho_{ij} \equiv \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii}}\sqrt{\Sigma_{jj}}} = \frac{\Sigma_{ij}}{\sigma_i \sigma_j} \tag{10}$$

The correlation coefficient ρ_{ij} is a degree of correlation between two random variables. Note that $\Sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ represents the off-diagonal elements of covariance matrix, $\mathbf{\Sigma_X}$.

- If X_i and X_j are independent (*i.e.*, $f_{X_1X_2}(x_1,x_2) = f_{X_1}(x_1)f_{X_2}(x_2)$), then X_i and X_j are uncorreleated (*i.e.* $\rho_{ij} = 0$), but vice versa is not true.
- $-1 \le \rho_{ij} \le +1$
- If $X_i = aX_j + b$, $\rho_{ij} = \pm 1 = \mathrm{sgn}(a)$: *Prove this property!* (**Homework**)

3. UNCERTAINTY PROPAGATION IN ENGINEERING APPLICATIONS

3.1 Objectives

- To understand a mechanism of uncertainty propagation through an engineering system
- To survey methods to identify and manage the mechanism of uncertainty
- To apply the probabilistic approach to design, instead of a safety factor approach

3.2 What is "Uncertainty Propagation"?

- Types of Uncertainty:
  1. "Physical Uncertainty" is the actual variability of physical quantities, such as loads, material properties, and dimensions
  2. "Statistical Uncertainty" arises solely as a result of lack of statistical information
  3. "Model Uncertainty" occurs as a result of simplifying assumptions, approximate mathematical modeling, unknown boundary conditions, etc.

In all engineering applications, uncertainties in inputs are propagated through a system to uncertainties in outputs, as shown Fig. 1.

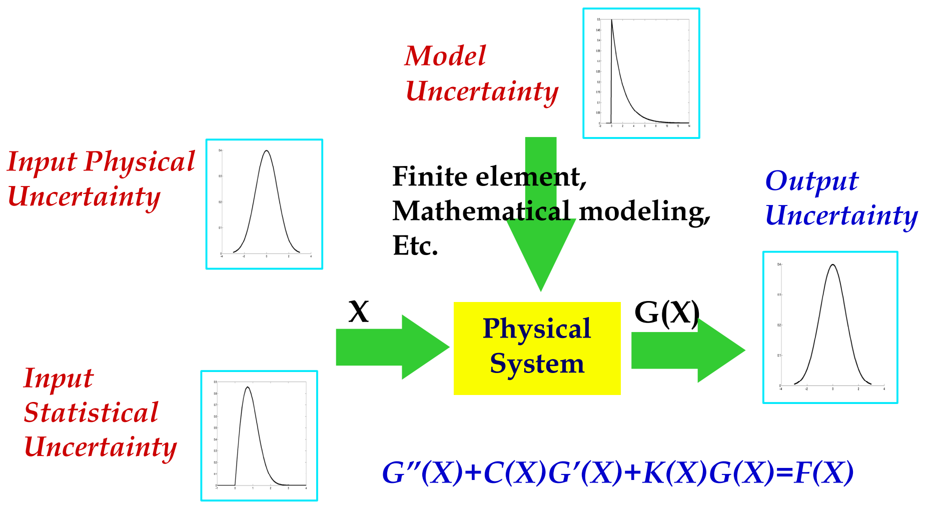


Figure 1. Uncertainty Propagation through Physical System

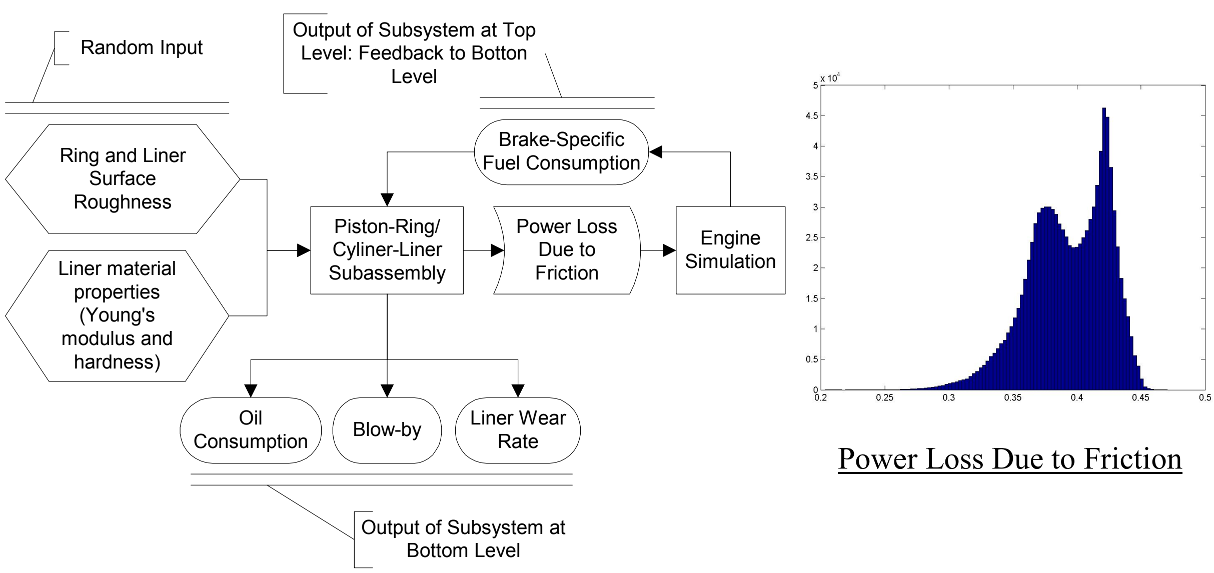


Figure 2. Uncertainty Propagation of V6 Gasoline Engine System

One example of uncertainty propagation is shown in Fig 2. To identify and manage uncertainty propagation in an engineering system, the mechanism of uncertainty propagation must be thoroughly understood. Uncertainty propagation in engineering applications will be discussed in the following sections.

3.3 Uncertainty Propagation for Linear System

Let the response variable Y be linearly related to input random vector $\mathbf{X} = \{X_1, \cdots, X_n\}^T$, i.e.,

$$Y = a_1 X_1 + \cdots + a_n X_n + b \quad \text{or} \quad Y = \mathbf{a}^T \mathbf{X} + b \tag{11}$$

where $\mathbf{a} = \{a_1, \cdots, a_n\}^T$ is an n-dimensional coefficient vector.

Mean of Y

$$\begin{aligned} E[Y] = \mu_Y &= E[\mathbf{a}^T \mathbf{X} + b] \\ &= E[\mathbf{a}^T \mathbf{X}] + E[b] \\ &= \mathbf{a}^T E[\mathbf{X}] + b \\ &= \mathbf{a}^T \mu_{\mathbf{X}} + b \end{aligned}$$

Variance of Y

$$\begin{aligned} Var[Y] = \sigma_Y^2 &= E[(Y - \mu_Y)^2] \\ &= E[(Y - \mu_Y)(Y - \mu_Y)^T] \\ &= E[(\mathbf{a}^T \mathbf{X} + b - \mathbf{a}^T \boldsymbol{\mu}_{\mathbf{X}} - b)(\mathbf{a}^T \mathbf{X} + b - \mathbf{a}^T \boldsymbol{\mu}_{\mathbf{X}} - b)^T] \\ &= \mathbf{a}^T E[(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})^T]\mathbf{a} \qquad \leftarrow \text{From Eq. (7)} \\ &= \mathbf{a}^T \Sigma_{\mathbf{X}} \mathbf{a} \end{aligned}$$

Generalization

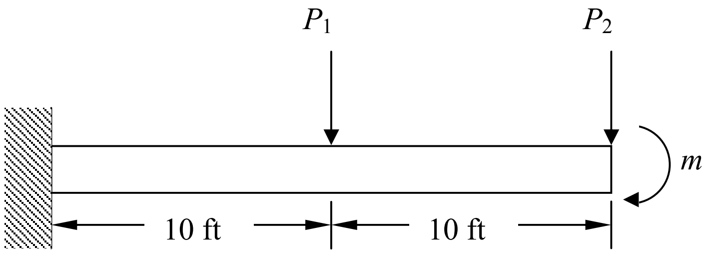
Let $\mathbf{Y} \in R^m$ be a random response vector of interest, which is related to input $\mathbf{X} \in R^n$. The linear system is given in the following equation.

$$\mathbf{Y} = \mathbf{A}^T \mathbf{X} + \mathbf{B}$$

where $\mathbf{A} \in R^n \times R^m$ and $\mathbf{B} \in R^m$ are coefficient matrix and vector, respectively. Let $\boldsymbol{\mu}_{\mathbf{Y}} \in R^m$ and $\boldsymbol{\Sigma}_{\mathbf{Y}} \in R^m \times R^m$ be the mean vector and covariance matrix of output $\mathbf{Y}$. Then,

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{Y}} &= \mathbf{A}^T \boldsymbol{\mu}_{\mathbf{X}} + \mathbf{B} \\ \boldsymbol{\Sigma}_{\mathbf{Y}} &= \mathbf{A}^T \boldsymbol{\Sigma}_{\mathbf{X}} \mathbf{A} \end{aligned} \tag{12}$$

Example 1: Cantilever Beam



Given that:

$$P_1 \sim \left[1000 \text{ lb}, 100^2 \text{ lb}^2\right]$$
$$P_1 \sim \left[500 \text{ lb}, 50^2 \text{ lb}^2\right]$$
$$m \sim 10000 \text{ lb-ft, deterministic}$$

Assume P_1 and P_2 are uncorrelated. Calculate mean, standard deviation, and coefficient of variation (COV) of maximum moment at the fixed end.

Solution: At fixed end, the maximum moment is expressed as

$$M_{\max} = 10P_1 + 20P_2 + 10000 = \mathbf{a}^T\mathbf{X} + b$$

where

$$\mathbf{a}^T = \{10 \quad 20\}, \quad b = 10000 \text{ lb-ft},$$
$$\mathbf{X} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \text{lb}, \quad \mu_X = \begin{Bmatrix} 1000 \\ 500 \end{Bmatrix} \text{lb}, \quad \mathbf{\Sigma_X} = \begin{bmatrix} 100^2 & 0 \\ 0 & 50^2 \end{bmatrix} \text{lb}^2$$

Mean of $M_{\max}$

$$\begin{aligned} \mu_{M_{\max}} &= \mathbf{a}^T \mu_{\mathbf{X}} + b \\ &= \{10 \quad 20\} \begin{Bmatrix} 1000 \\ 500 \end{Bmatrix} + 10000 \\ &= 30,000 \text{ lb-ft} \end{aligned}$$

Variance of $M_{\max}$

$$\begin{aligned}\sigma_{M_{\max}}^2 &= \mathbf{a}^T \Sigma_{\mathbf{X}} \mathbf{a} \\ &= \{10 \quad 20\} \begin{bmatrix} 100^2 & 0 \\ 0 & 50^2 \end{bmatrix} \begin{Bmatrix} 10 \\ 20 \end{Bmatrix} \\ &= 2\times 10^6 \ [\text{lb-ft}]^2 \\ \sigma_{M_{\max}} &= \sqrt{2\times 10^6} = 1414.2 \text{ lb-ft}\end{aligned}$$

Coefficient of Variation (COV) of $M_{\max}$

$$\text{COV} \equiv \frac{\text{standard deviation}}{\text{mean}} = \frac{1414.2}{30000} = 0.047 \text{ or } 4.7\%$$

3.4 Uncertainty Propagation for Nonlinear System

Let the response variable Y be nonlinearly related to input random vector $\mathbf{X} = \{X_1, \cdots, X_n\}^T$, i.e.,

$$Y = g(\mathbf{X}), \text{ where } Y \in R \text{ and } \mathbf{X} \in R^n \tag{13}$$

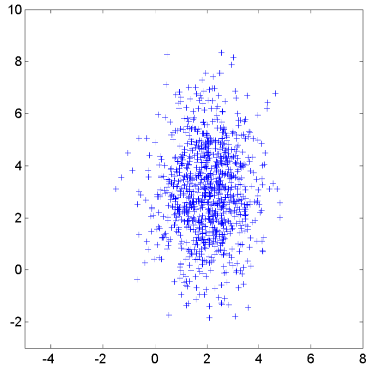
where $g(\mathbf{X})$ is a nonlinear response. For the nonlinear response $g(\mathbf{X})$, it is very difficult or almost impossible to determine the mean and variance of Y exactly. There are several approximate methods available to identify uncertainty propagation for nonlinear system.

3.4.1 Sampling Method: Monte Carlo simulation

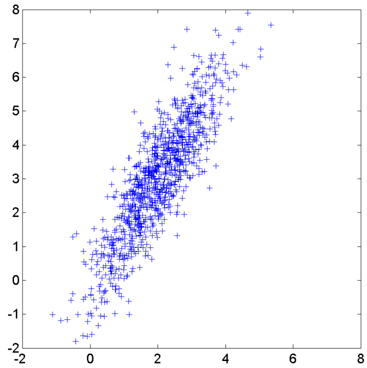
- Simple but numerically expensive
- Seldom used due to its computational intensiveness, but used for a benchmark study
- To estimate a failure rate,

$$p_f = \frac{N_f}{N}, \quad \begin{aligned} N_f &: \text{No of Failed Simulation} \\ N &: \text{ No of Total Simulation} \end{aligned}$$

Random Number Generator



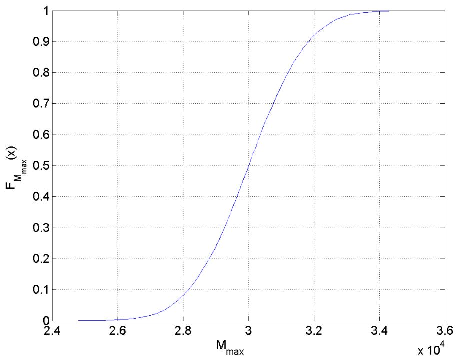
```
>> m=[2 3];
>> s=[1 0;0 3];
>> r=mvnrnd(m,s,1000);
>> plot(r(:,1),r(:,2),'+')
```



```
>> m=[2 3];
>> s=[1 1.5;1.5 3];
>> r=mvnrnd(m,s,1000);
>> plot(r(:,1),r(:,2),'+')
```

<u>Monte Carlo Simulation: Example 1</u>

```
>> p1=normrnd(1000,100,10000,1);
>> p2=normrnd(500,50,10000,1);
>> Mmax=10*p1+20*p2+10000;
>> cdfplot(Mmax)
>> mean(Mmax)
ans =
   3.0006e+004
>> std(Mmax)
ans =
   1.4271e+003
```



3.4.3 First-Order Second Moment (FOSM) Method or Moment Matching Method

Using the first-order Taylor series expansion, a nonlinear response $Y = g(\mathbf{X})$ can be linearized as

$$Y = g(\mathbf{X}) = g(\boldsymbol{\mu}_{\mathbf{X}}) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mathbf{X}=\boldsymbol{\mu}_{\mathbf{X}}} (X_i - \mu_{X_i}) + H.O.T. \tag{14}$$

Assume that the first-order expansion is adequate for approximating Y, i.e.,

$$
\begin{aligned}
Y &\cong \hat{Y} \\
&= g(\boldsymbol{\mu}_{\mathbf{X}}) + \sum_{i=1}^{n} \overbrace{\left.\frac{\partial g}{\partial X_i}\right|_{\mathbf{X}=\boldsymbol{\mu}_{\mathbf{X}}}}^{\alpha_i} (X_i - \mu_{X_i}) \\
&= \sum_{i=1}^{n} \alpha_i X_i + \underbrace{g(\boldsymbol{\mu}_{\mathbf{X}}) - \sum_{i=1}^{n} \alpha_i \mu_{X_i}}_{b} \\
&= \boldsymbol{\alpha}^T \mathbf{X} + b
\end{aligned}
\tag{15}
$$

where $\boldsymbol{\alpha} = \{\alpha_1, \cdots, \alpha_n\}^T$ is a vector of gradients evaluated at mean of **X**. From a linear transformation theory in Section 3.3,

$$
\begin{aligned}
\mu_Y &\cong E[\hat{Y}] = \boldsymbol{\alpha}^T \boldsymbol{\mu}_{\mathbf{X}} + b \\
\sigma_Y^2 &\cong Var[\hat{Y}] = \boldsymbol{\alpha}^T \boldsymbol{\Sigma}_{\mathbf{X}} \boldsymbol{\alpha}
\end{aligned}
\tag{16}
$$

Even if this method is easy to use, it has a drawback of numerical accuracy. Therefore, an advanced first-order second moment (AFOSM) method has recently been developed to identify the mechanism of uncertainty propagation.

3.4.4 Advanced First-Order Second Moment (AFOSM) Method

Hasofer and Lind (1978) introduced a rotationally invariant reliability measure, which allows identifying uncertainty propagation more accurately. This is called a first-order reliability method (FORM) or an asymptotic second-order reliability method (asymptotic SORM). FORM or asymptotic SORM requires a transformation of any random space to a standard normal space. Uncertainty propagation of the nonlinear response (pubic force) in side impact crash is accurately identified using FORM, as depicted in Fig. 3. More detail descriptions will be presented in Lecture 6.

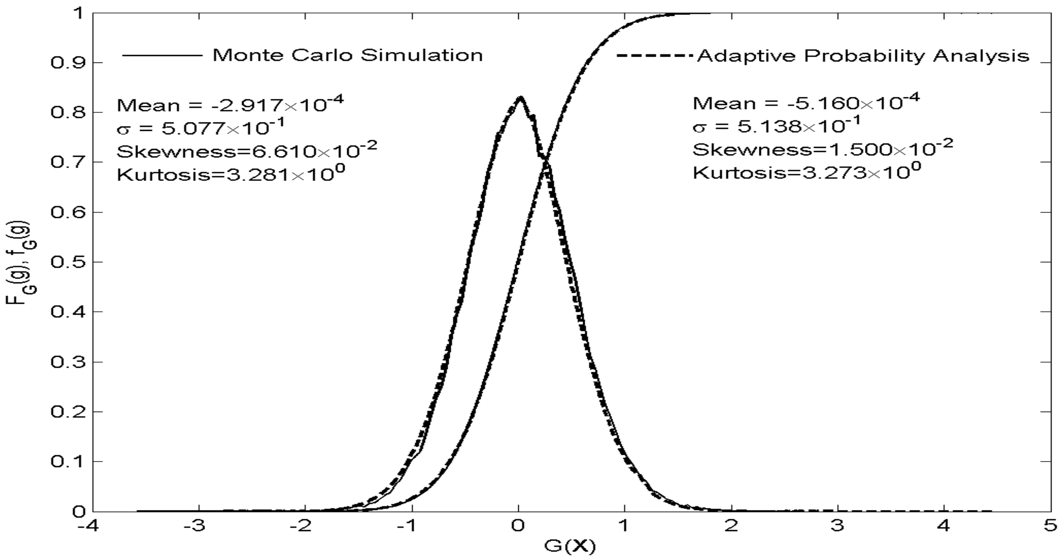


Figure 3. PDF and CDF of pubic force in side impact crash

Homework

Consider the following simply supported beam subject to a uniform load, as illustrated in Fig. 4.

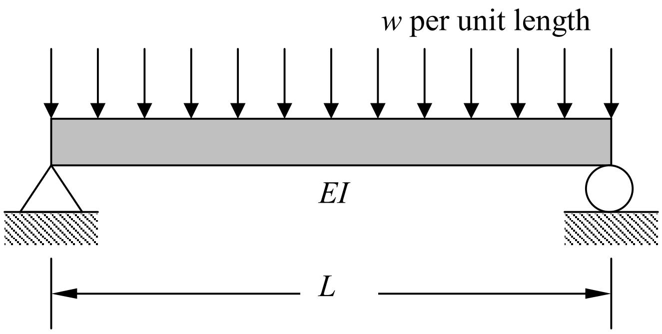


Figure 4. Simply Supported Beam

Random Vector:

$$EI = X_1 \sim N(\mu_{X_1} = 3\times10^7, \sigma_{X_1} = 10^5)$$
$$w = X_2 \sim N(\mu_{X_2} = 10, \sigma_{X_2} = 1)$$

The maximum deflection of the beam is shown as

$$Y = g(X_1, X_2) = -\frac{5X_2L^4}{384X_1}$$

Using Monte Carlo simulation and FOSM, identify uncertainty propagation to the maximum deflection by plotting PDF or CDF, and estimating mean and standard deviation of the maximum deflection.