2007 Fall: Electronic Circuits 2

CHAPTER 12
Filters and Tuned Amplifiers

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Introduction

- Passive LC Filters

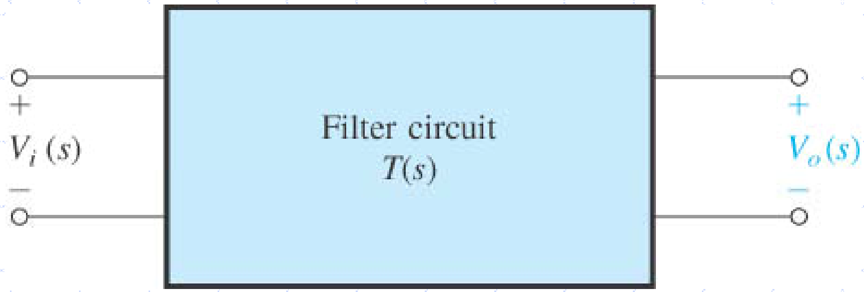
- Electronic Filter – Active Filter
  - Active RC Filters
  - Switched capacitor circuits

→ Advantages : **No inductors**

Inductors are large and physically bulky for low frequency applications

12.1.1 Filter Transmission

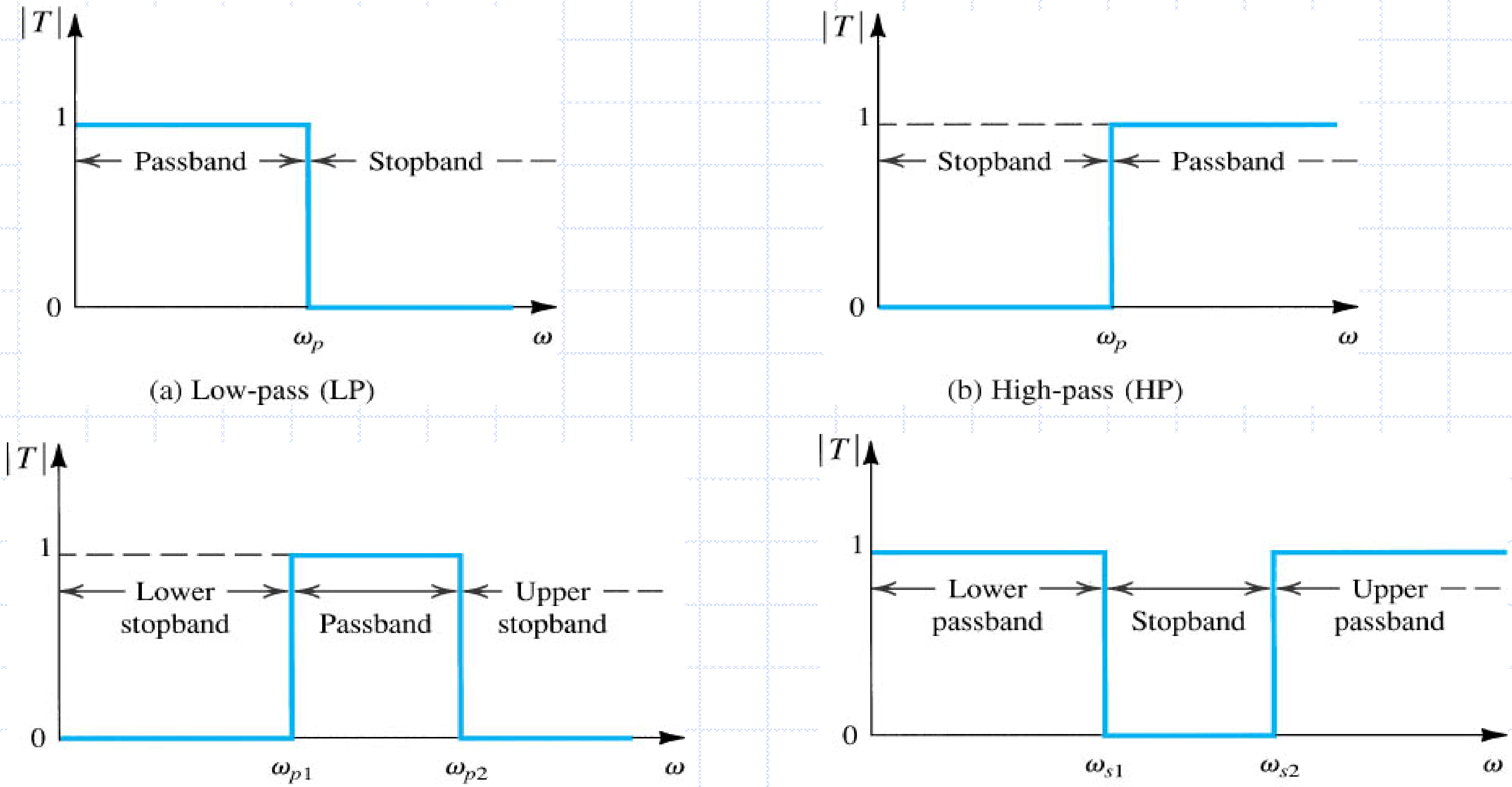
- Filter – a two port device



- Transfer function: $T(s) \equiv \dfrac{V_o(s)}{V_i(s)}$
- The filter transmission: $T(j\omega) = \underbrace{|T(j\omega)|}_{\text{magnitude}} e^{\overbrace{j\phi(\omega)}^{\text{phase}}}$ $\quad (s = j\omega)$
- Gain function: $G(\omega) \equiv 20\log|T(j\omega)|$ dB
- Attenuation function: $A(\omega) \equiv -20\log|T(j\omega)|$ dB
- Input Output relation: $|V_o(j\omega)| = |T(j\omega)||V_i(j\omega)|$

12.1.2 Filter Types

- Frequency-selection Function
  - Passing : Passband, |T|=1
  - Stopping : Stopband, |T|=0
- Brick-Wall response

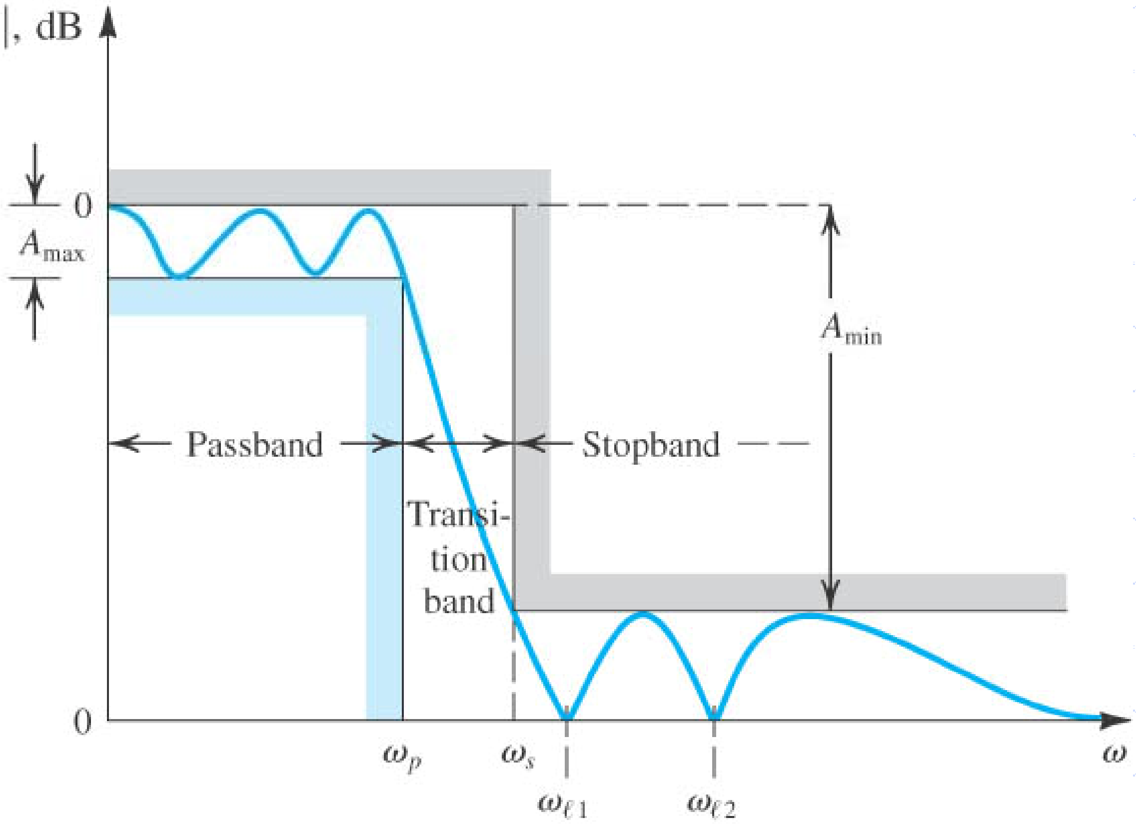


(a) Low-pass (LP) (b) High-pass (HP)

(c) Bandpass (BP) (d) Bandstop (BS)

12.1.3 Filter Specification

Passband Ripple →
range: 0.05 dB ~ 3 dB

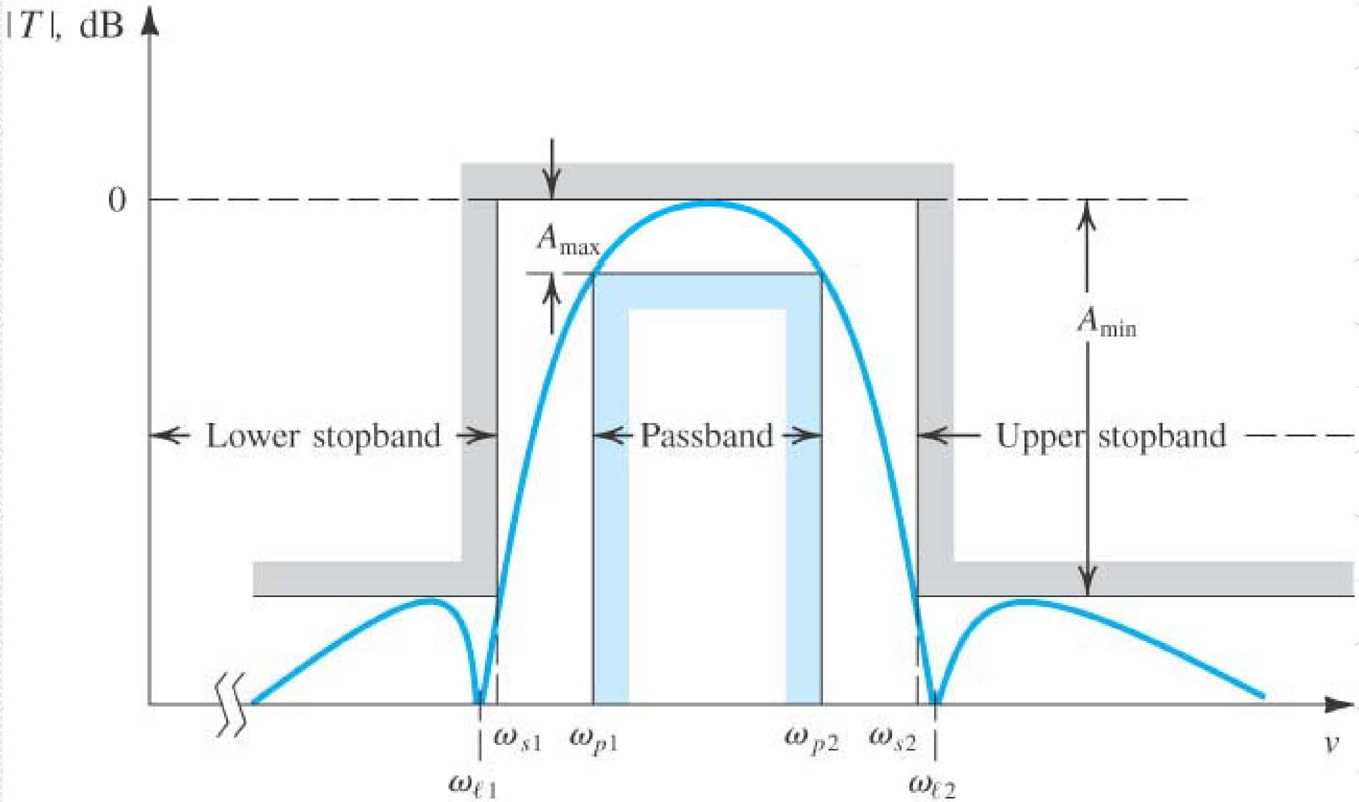


- w_p : Passband edge
- A_{max} : Maximum allowed variation in passband transmission
- w_s : Stopband edge
- A_{min} : Minimum required stopband attenuation
- w_s/w_p : Selectivity factor

12.1.3 Filter Specification

Filter Approximation

- The process of obtaining a transfer function that meets given specifications
- Performed using computer programs(Snelgrove, 1982;Ouslis and Sedra, 1995), filter design table(Zverev, 1967) or closed-form expressions(Section 12.3)



12.2 The Filter Transfer Function

Filter Transfer Function *T(s)*

- $$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0}$$

  - N: filter order
  - if N≥M, stable
  - $a_0,\cdots,a_M$ & $b_0,\cdots,b_{N-1}$: real numbers

- $$T(s) = \frac{a_M (s - z_1)(s - z_2)\cdots(s - z_M)}{(s - p_1)(s - p_2)\cdots(s - p_N)}$$

  - $z_1,\cdots,z_M$: transfer function zeros = transmission zeros
  - $p_1,\cdots,p_N$: transfer function poles = natural modes
  - real or complex number(conjugate pair)

12.2 The Filter Transfer Function

Filter Transfer Function *T(s)*

- Since in the stopband the transmission is zero or small
  – the zeros are usually, placed on the jω axis at stopband frequencies
- Infinite attenuation at ω_{l1} and ω_{l2}
  1. zeros at $s = +j\omega_{l1}$ & $+j\omega_{l2}$
     also at $s = -j\omega_{l1}$ & $-j\omega_{l2}$

  Numerator polynomial

  $(s + j\omega_{l1})(s - j\omega_{l1})(s + j\omega_{l2})(s - j\omega_{l2})$
  $= (s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)$

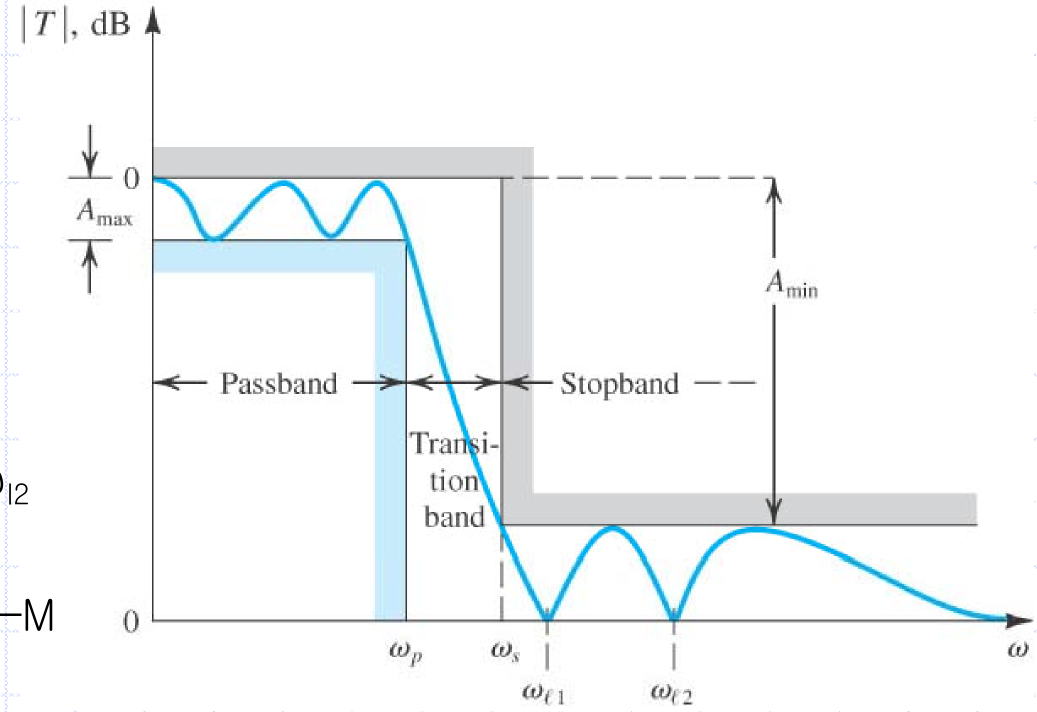
  → for $s = j\omega$,

  $(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)$
  $= (-\omega^2 + \omega_{l1}^2)(-\omega^2 + \omega_{l2}^2)$

  which is zero at $\omega=\omega_{l1}$ and $\omega=\omega_{l2}$

  2. zeros at $s=\infty$
     the numbers of zeros at $s=\infty=N-M$

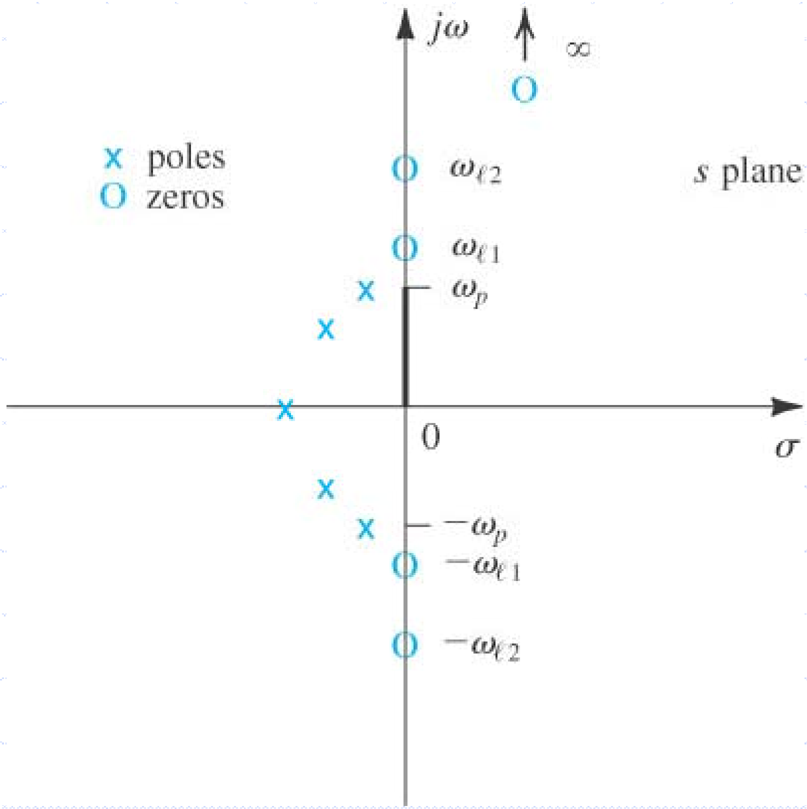
$$\because \text{as } s \to \infty, \quad T(s) \to \frac{a_M}{s^{N-M}}$$



12.2 The Filter Transfer Function

◆ Pole-zero pattern for a 5th-order LPF(N=5)

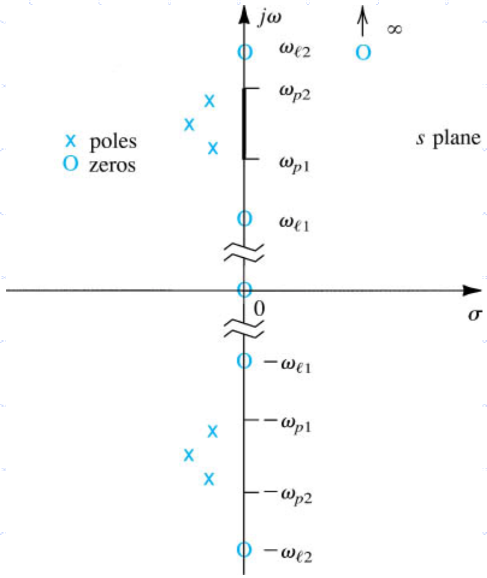
- Two pairs of complex-conjugate poles and real-axis pole
  → all the poles lie in the vicinity of passband
  → high transmission at pass band frequencies

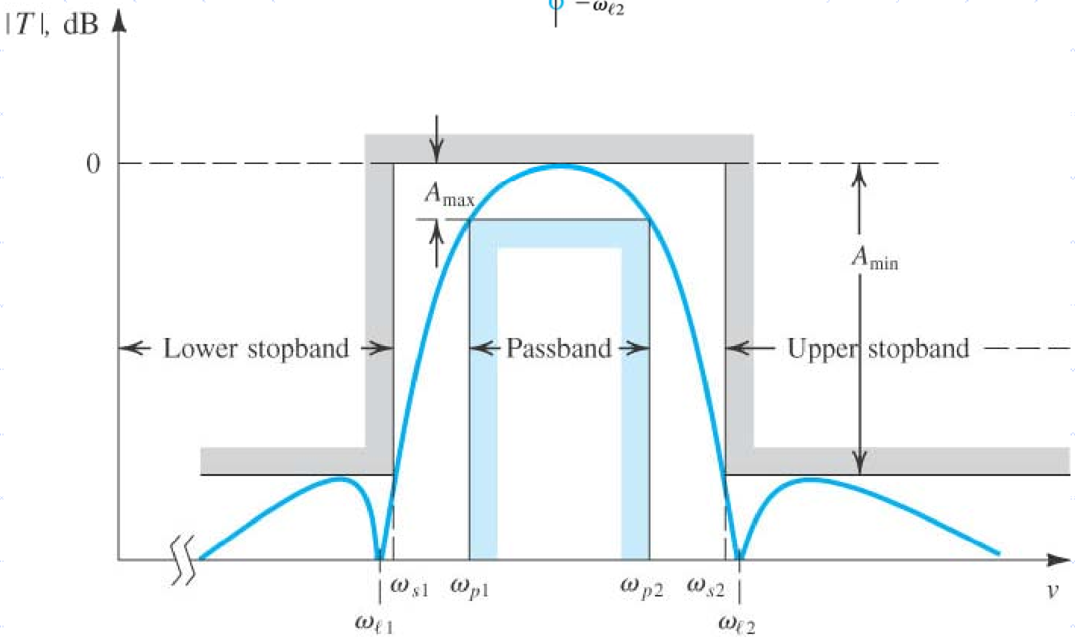


- $s = \pm j\omega_{l1}$ & $\pm j\omega_{l2}$ & ∞
- $T(s) = \dfrac{a_4(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)}{s^5+b_4s^4+b_3s^3+b_2s^2+b_1s+b_0}$

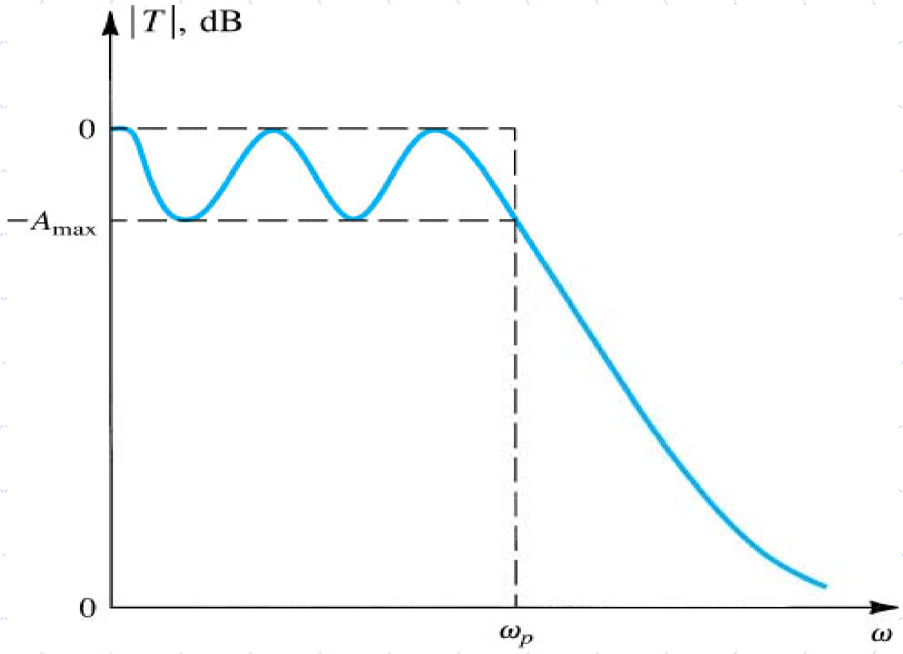
12.2 The Filter Transfer Function

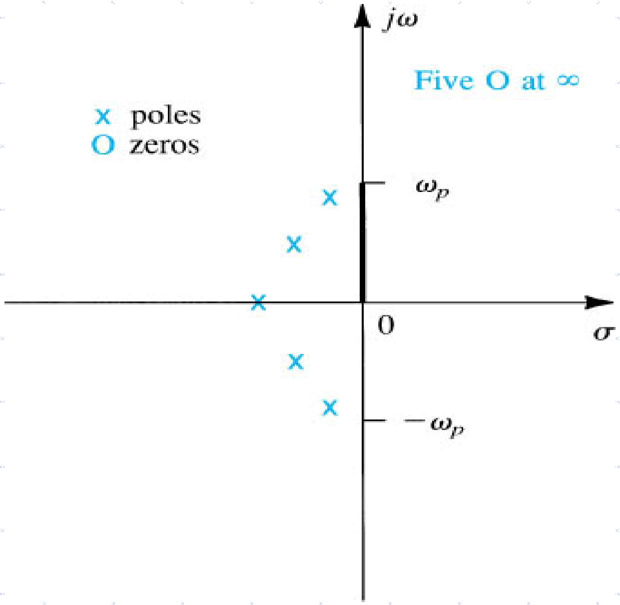
- It has one or more zeros at s = 0 and one or more zeros at s = ∞
- Assuming that only one zero exists at s = 0 & s = ∞
- N=6
- $s = \pm j\omega_{l1}, s = \pm j\omega_{l2}, s = 0, s = \infty$
- $T(s) = \dfrac{a_5 s(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^6 + b_5 s^5 + \ldots + b_0}$





12.2 The Filter Transfer Function





- It is possible that all zeros are at s=∞
- The more selective the required filter response is, the higher its order must be, and the closer its natural modes are to the jw axis

12.2 The Filter Transfer Functions

- Problem 12.9

A third-order low-pass filter has transmission zeros at ω=2rad/s and ω=∞. Its natural modes are at s=-1 and s=-0.5±j0.8. The dc gain is unity. Find T(s)

- Poles at -1 and -0.5±j0.8 : denominator $D(s)=(s+1)(s^2+s+0.89)$
- Zeros at ∞ and ±j2 : numerator $N(s)=k(s+j2)(s-j2)=k(s^2+4)$
- There is one zero at ∞ because $Degree(D(s))- Degree(N(s))=1$. Thus,

$$T(s)=\frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

- DC gain = 1 : $T(j0)=4k/0.89=1 \rightarrow k=0.2225$
- T(s) is,

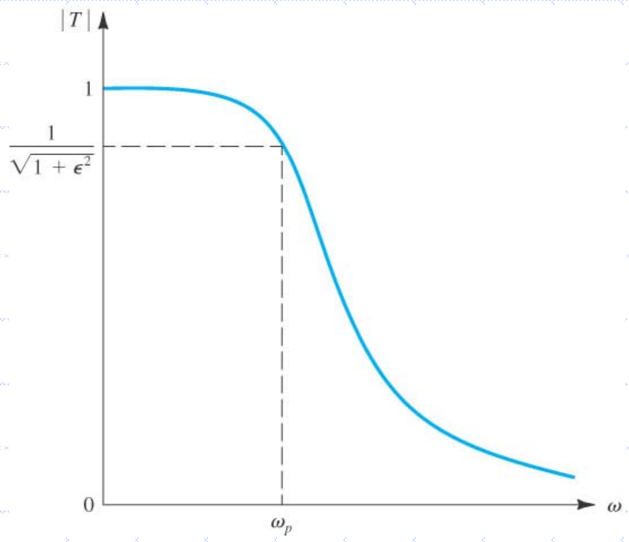
$$T(s)=\frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

12.3 Butterworth and Chebyshev Filters

- In this section, we present two functions that are frequently used in **approximating the transmission characteristics** of low-pass filters.
  : Closed-form expressions

12.3.1 The Butterworth Filter

Filter Transfer Function *T(s)*



- Monotonically decreasing transmission
- All the transmission zero at ω = ∞
- The magnitude function for an N[th]-order
- Butterworth filter with a passband edge ω_P is

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_P}\right)^{2N}}}$$

at $\omega = \omega_P$,

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}}$$

- Thus, the parameter ε determines the maximum variation in passband transmission,

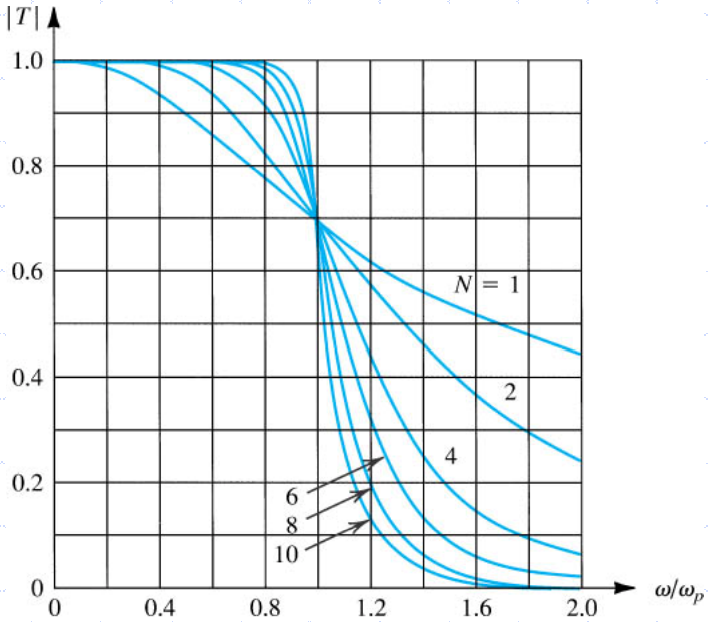
$$A_{\max} = 20\log\sqrt{1+\varepsilon^2}$$

- Conversely, given A_{max},

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

12.3.1 The Butterworth Filter

Filter Transfer Function *T(s) (cont.)*



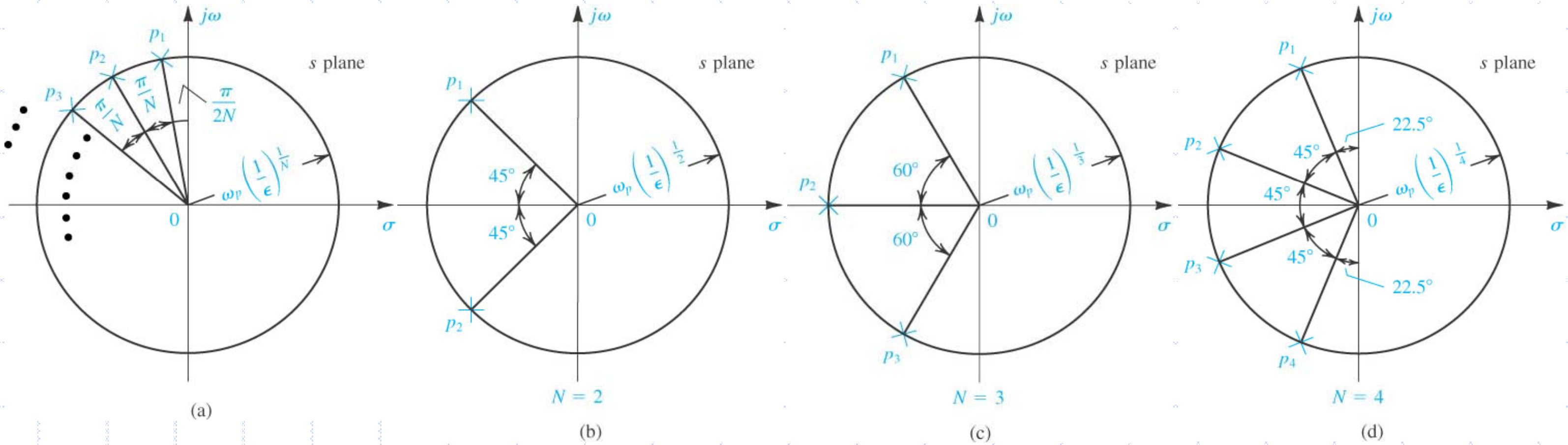
- In the Butterworth response the maximum deviation in passband transmission occurs at the passband edge, ω_P, only
- The first 2N—1 derivatives of |T| relative to ω are zero at ω = 0
  → very flat near ω = 0 (maximally flat response)
- The degree of passband flatness increases as the order N is increased
  → as the order N is increased the filter response approaches the ideal brick-wall type of response
- The edge of the stopband, $\omega = \omega_S$, attenuation is

$$A(\omega_S) = -20\log\left[1\Big/\sqrt{1+\varepsilon^2(\omega_S/\omega_P)^{2N}}\right] = 10\log\left[1+\varepsilon^2(\omega_S/\omega_P)^{2N}\right]$$

- This equation can be used to determine the filter order required, which is the lowest integer value of N that yields $A(\omega_S) \geq A_{min}$

12.3.1 The Butterworth Filter

Filter Transfer Function *T(s) (cont.)*



- The natural modes of an N^{th}-order Butterworth filter can be determined from the graphical construction above.
- Natural modes lies on a circle of radius $\boldsymbol{\omega_P(1/\varepsilon)^{1/N}}$

  → **same frequency of** $\boldsymbol{\omega_0 = \omega_P(1/\varepsilon)^{1/N}}$
- Space by equal angles of π/N, with the first mode at an angle π/2N from the +jw axis.
- Transfer function is

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$ → *K is a constant dc gain of the filter*

12.3.1 The Butterworth Filter

How to find a Butterworth transfer function

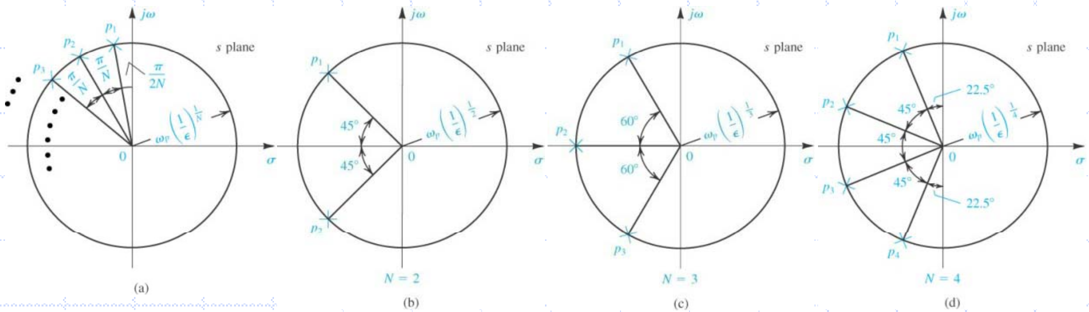
- Determine ε.

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

- Determine the required filter order as the lowest integer value of N that results in $A(\omega_S) \geq A_{min}$.

$$A(\omega_S) = -20\log\left[1 \middle/ \sqrt{1+\varepsilon^2(\omega_S/\omega_P)^{2N}}\right] = 10\log\left[1+\varepsilon^2(\omega_S/\omega_P)^{2N}\right]$$

- Determine the N natural modes



- Determine $T(s)$

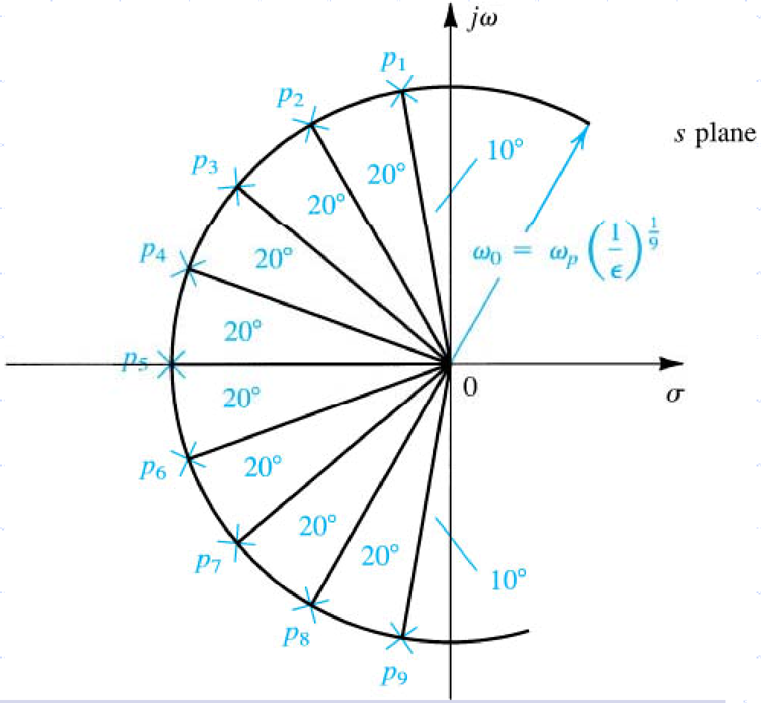
$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

12.3.1 The Butterworth Filter

- Example 12.1

Find the Butterworth transfer function that meets the following low-pass filter specifications: f_p=10kHz, A_{max}=1dB, f_s=15kHz, A_{min}=25dB, dc gain=1

- $\varepsilon = \sqrt{10^{A_{\max}/10} - 1} = 0.5088$
- $A(\omega_S) = 10\log\left[1 + \varepsilon^2 (\omega_S/\omega_P)^{2N}\right]$
  $= 22.3dB$ (when N=8)
  $= 25.3dB$ (when N=9)



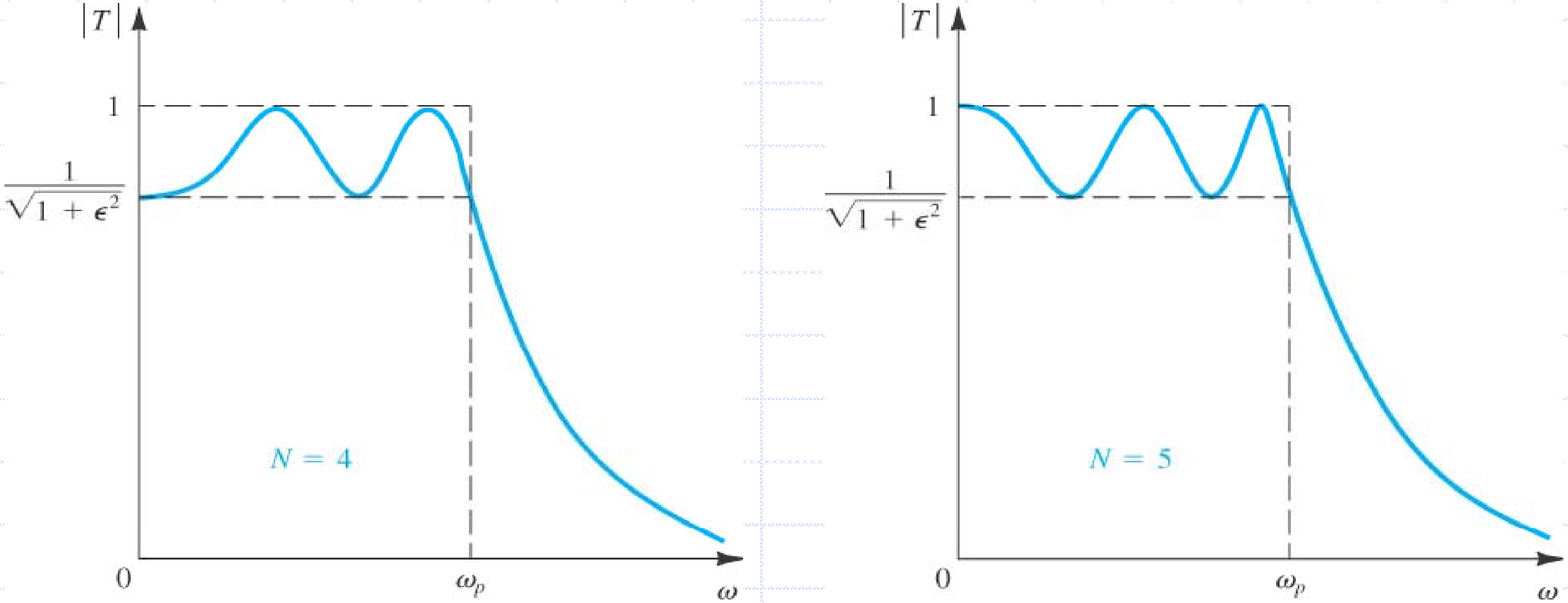
$\omega_0 = \omega_p(1/\varepsilon)^{1/N} = 6.773 \times 10^4 rad/s$
$p_1 = \omega_0(-\cos 80° + j\sin 80°) = \omega_0(-0.1736 + j0.9848)$

Combining ω_0, $\omega_9 \rightarrow s^2 + s0.3472\omega_0 + \omega_0^2$

$$T(s) = \frac{K\omega_0^9}{(s+\omega_0)(s^2 + s1.8792\omega_0 + \omega_0^2)(s^2 + s1.5321\omega_0 + \omega_0^2)(s^2 + s\omega_0 + \omega_0^2)(s^2 + s0.3472\omega_0 + \omega_0^2)}$$

12.3.2 The Chebyshev Filter

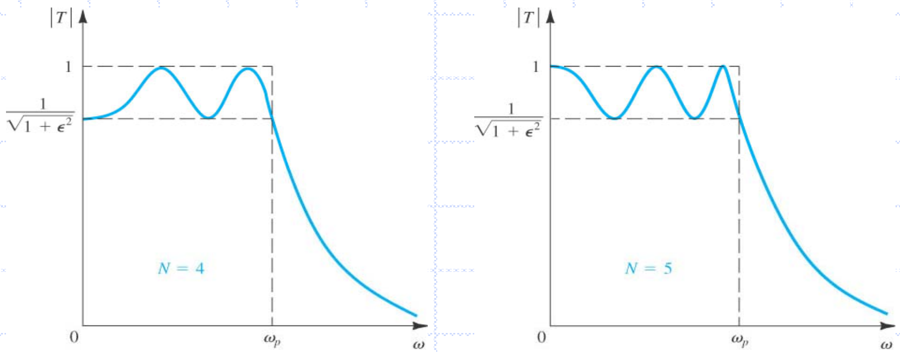
◆ The Chebyshev Filter



- **Equiripple response** (A_{max} = the peak ripple) in the passband and a monotonically decreasing transmission in the stopband.
- The odd-order filter, |T(0)|=1
  The even-order filter exhibits its maximum magnitude deviation at w = 0.
- Total number of passband maxima and minima equals the order of the filter, N.
- All the zeros are at w = ∞.

12.3.2 The Chebyshev Filter

The Chebyshev Filter (cont.)



- The magnitude of the transfer function with a passband edge ω_P is

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cos^2[N\cos^{-1}(\omega/\omega_P)]}} \qquad \text{for} \quad \omega \le \omega_P$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cosh^2[N\cosh^{-1}(\omega/\omega_P)]}} \qquad \text{for} \quad \omega \ge \omega_P$$

$$|T(j\omega_P)| = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad \text{for} \quad \omega = \omega_P$$

- Thus, the parameter ε determines the passband ripple according to

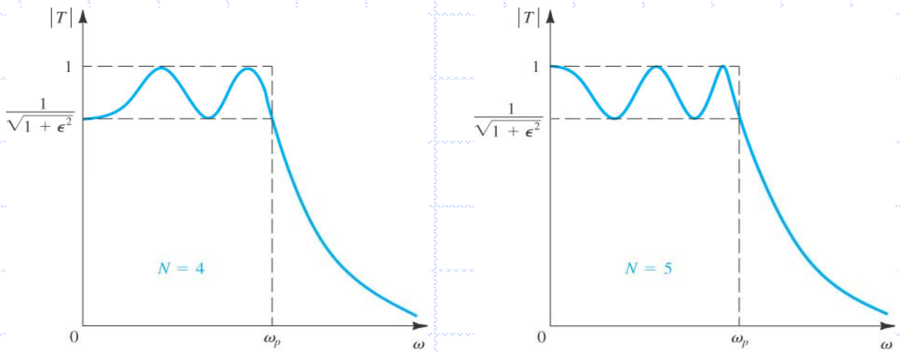
$$A_{\max} = 10\log(1+\varepsilon^2)$$

conversely,

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

12.3.2 The Chebyshev Filter

The Chebyshev Filter (cont.)



- The attenuation at the stopband edge($\omega = \omega_S$) is

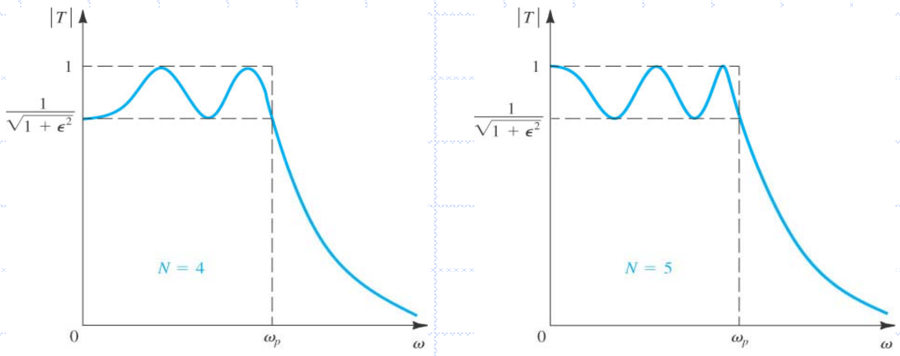
  $$A(\omega_S) = 10\log[1+\varepsilon^2\cosh^2(N\cosh^{-1}(\omega_S/\omega_P))]$$

  → this equation can be used to determine the order N required to obtain a specified A_{min} by finding the lowest integer value of N that yields $A(\omega_S) \geq A_{min}$.

- Increasing the order N of the Chebyshev filter causes its magnitude function to approach the ideal brick-wall low-pass response.

12.3.2 The Chebyshev Filter

◆ The Chebyshev Filter (cont.)



- The poles are

$$p_k = -\omega_P \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_P \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) \quad k = 1,2,\cdots,N$$

- The transfer function is

$$T(s) = \frac{K\omega_P^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)}$$

12.3.2 The Chebyshev Filter

How to find the transfer function

1. Determine ε

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

2. Determine the order required, $A(\omega_S)$

$$A(\omega_S) = 10\log[1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(\omega_S/\omega_P))]$$

3. Determine the poles, p_k

$$p_k = -\omega_P \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_P \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) \quad k = 1,2,\cdots,N$$

4. Determine the transfer function, T(s)

$$T(s) = \frac{K\omega_P^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)}$$

12.3.2 The Chebyshev Filter

- Example 12.2

Find the Chebyshev transfer function that meets the following low-pass filter specifications: f_p=10kHz, A_{max}=1dB, f_s=15kHz, A_{min}=25dB, dc gain=1

- $\varepsilon = \sqrt{10^{A_{\max}/10} - 1} = 0.5088$
- $A(\omega_S) = 10\log[1+\varepsilon^2 \cosh^2(N\cosh^{-1}(\omega_S/\omega_P))]$

  $= 21.6dB$ (when N=4)

  $= 29.9dB$ (when N=5)
- $p_k = -\omega_P \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_P \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) \quad k = 1,2,\cdots,N$

  $p_1, p_5 = \omega_p(-0.0895 \pm j0.9901)$, $p_2, p_4 = \omega_p(-0.2342 \pm j0.6119)$, $p_5 = \omega_p(-j0.2895)$

$$T(s) = \frac{\omega_P^5}{8.1408(s+0.2895\omega_P)(s^2+s0.4684\omega_P+0.4293\omega_P^2)(s^2+s0.1789\omega_P+0.9883\omega_P^2)}$$

12.4 First-Order and Second-Order Filter Functions

- Study the simplest filter transfer functions
  - first and second order
- Cascade design
  - realize a high-order filter.

    → design of active filters (utilizing op amps and RC circuits)
- Filter poles occur in complex-conjugate pairs
  - a high-order transfer function T(s) is factored into the product of second-order functions.
- If T(s) is odd there will also be a first-order function in the factorization.
- Overall transfer function of the cascade
  - simply the product of the transfer functions of the individual blocks.

12.4 First-Order and Second-Order Filter Functions

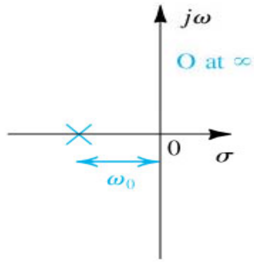
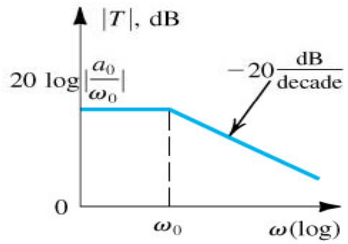
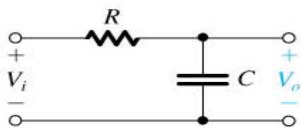
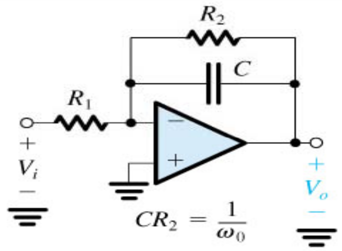
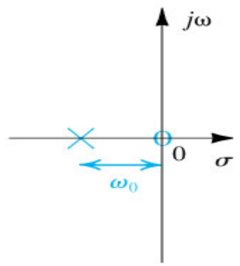
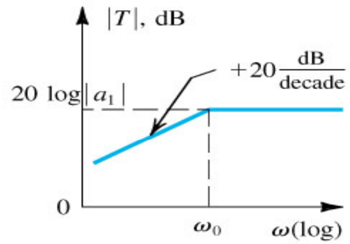
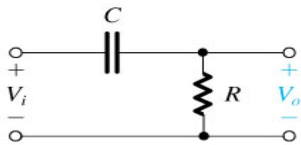
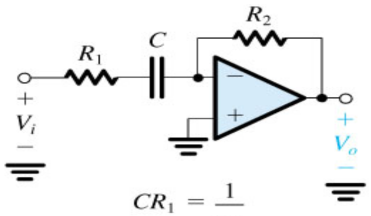
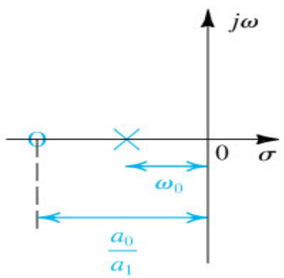
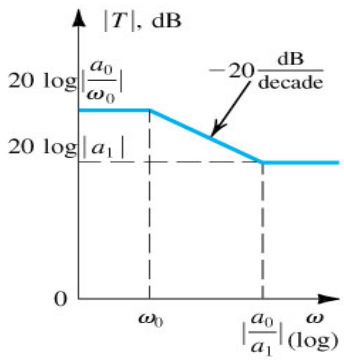
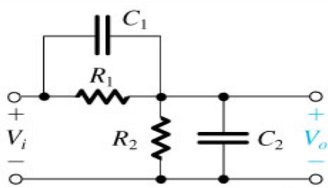
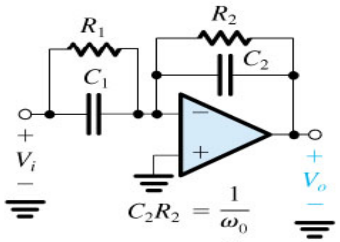
◈ General First-Order Transfer Function

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \rightarrow \text{bilinear transfer function}$$

- A natural mode at $s=-\omega_0$
- A transsmission zero at $s=-a_0/a_1$
- High frequency gain → a_1
- The numerator coefficients, a0 and a1, determine the type of filter(e.g., low pass, high pass, etc.)
- Active circuit
  Low output impedance
  Limits the high-frequency operation(→ op amp)

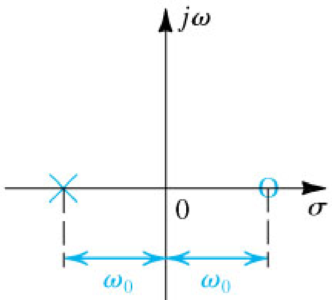
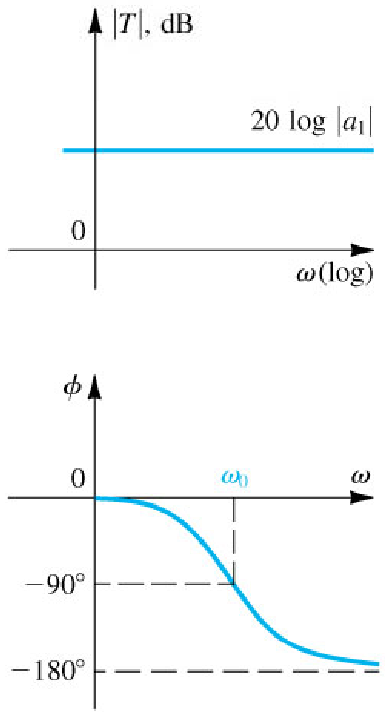
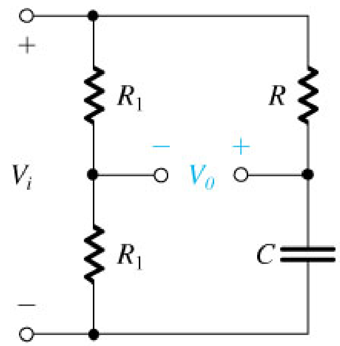
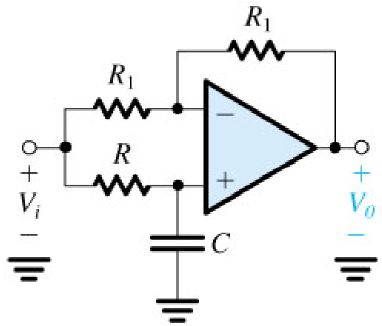
12.4 First-Order and Second-Order Filter Functions

◆First-Order Filters

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\|T\|$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \frac{a_0}{s+\omega_0}$			 $CR = \frac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \frac{a_1 s}{s+\omega_0}$			 $CR = \frac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s+\omega_0}$			 $(C_1 + C_2)(R_1 /\!/ R_2) = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1+R_2}$ HF gain $= \frac{C_1}{C_1+C_2}$	 $C_2R_2 = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

12.4 First-Order and Second-Order Filter Functions

◆First-Order Filters (cont.)

$T(s)$	Singularities	$\|T\|$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1 \frac{s-\omega_0}{s+\omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

- Although the transmission is constant, its phase shows frequency selectivity
- All-pass filters are used as phase shifters and in systems that require phase shaping

12.4 First-Order and Second-Order Filter Functions

Second-Order Filter Functions

- The general second order (or biquadratic) filter transfer function

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega^2{}_0}$$

- ω_0 and Q determine the natural modes(poles) according to

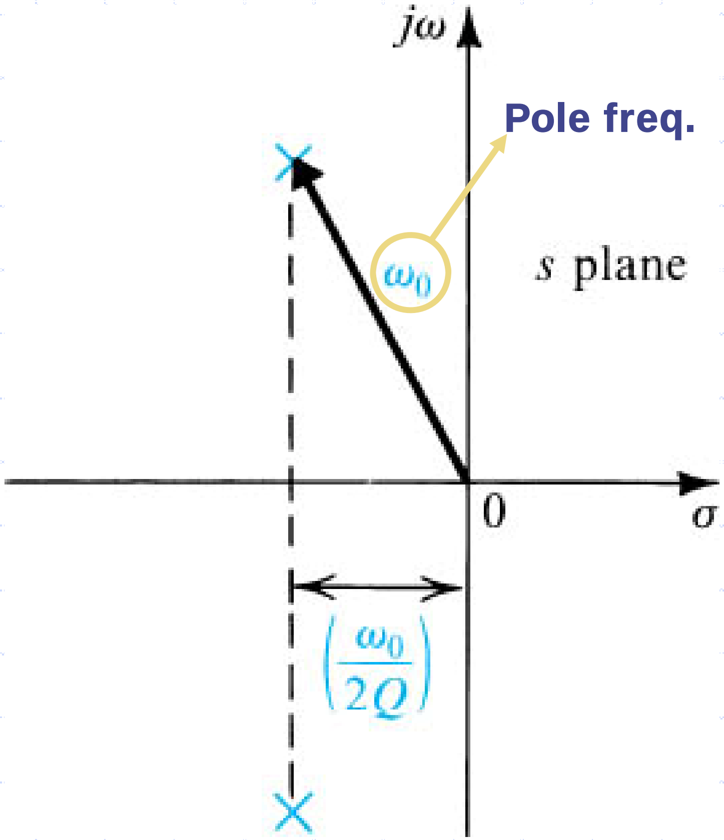
$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - (1/4Q^2)}$$

- Q>0.5 : complex-conjugate natural modes.

12.4 First-Order and Second-Order Filter Functions

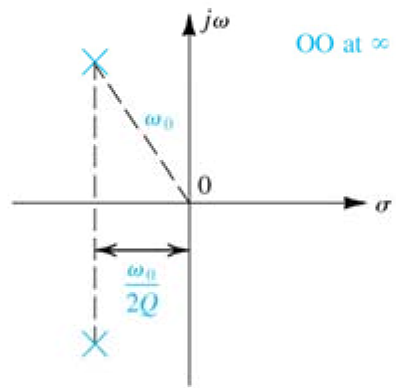
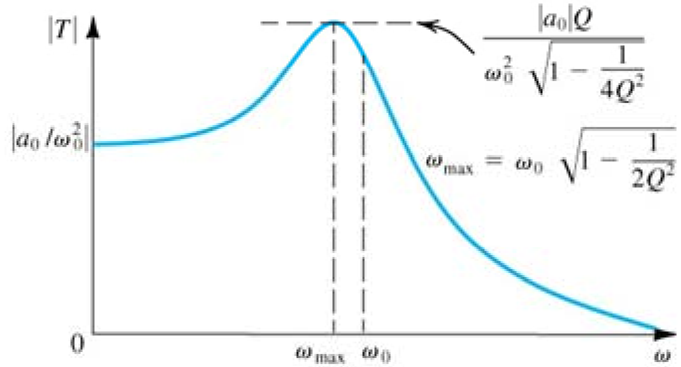
◆ Second-Order Filter Functions (cont.)

- Q determines the distance of the poles from the jω axis
  : the higher the value of Q, the closer the poles are to the jω axis
  → more selective
- Q < 0 → poles are in the RHP
  → oscillation
- Q = pole quality factor = pole Q



12.4 First-Order and Second-Order Filter Functions

Second-Order Filter Functions – LP case

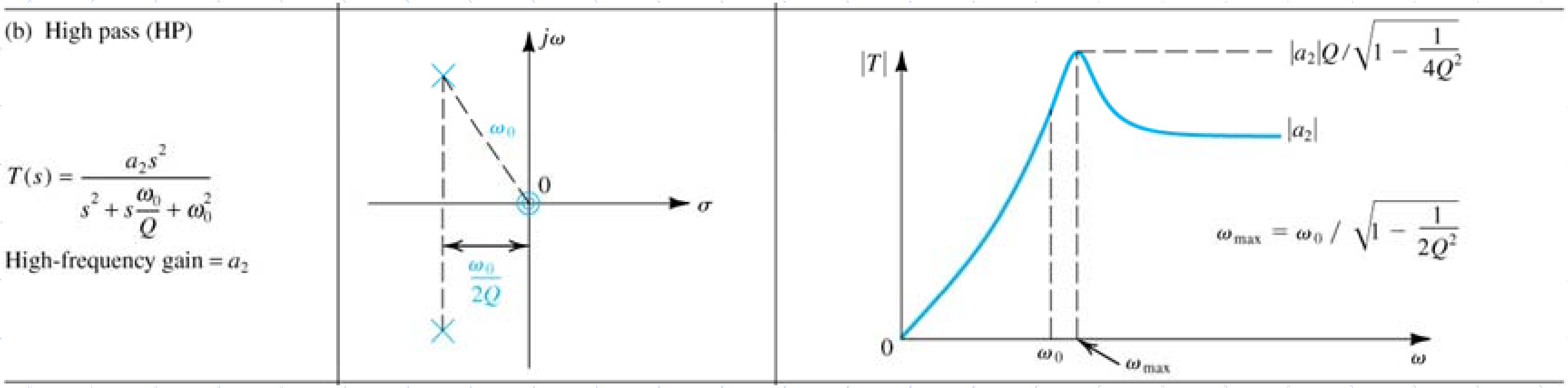
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		

- Lowpass Filter : The peak occurs only for $Q > \dfrac{1}{\sqrt{2}}$

$$Q = \frac{1}{\sqrt{2}} \rightarrow \text{Butterworth, or maximally flat}$$

12.4 First-Order and Second-Order Filter Functions

Second-Order Filter Functions – HP case

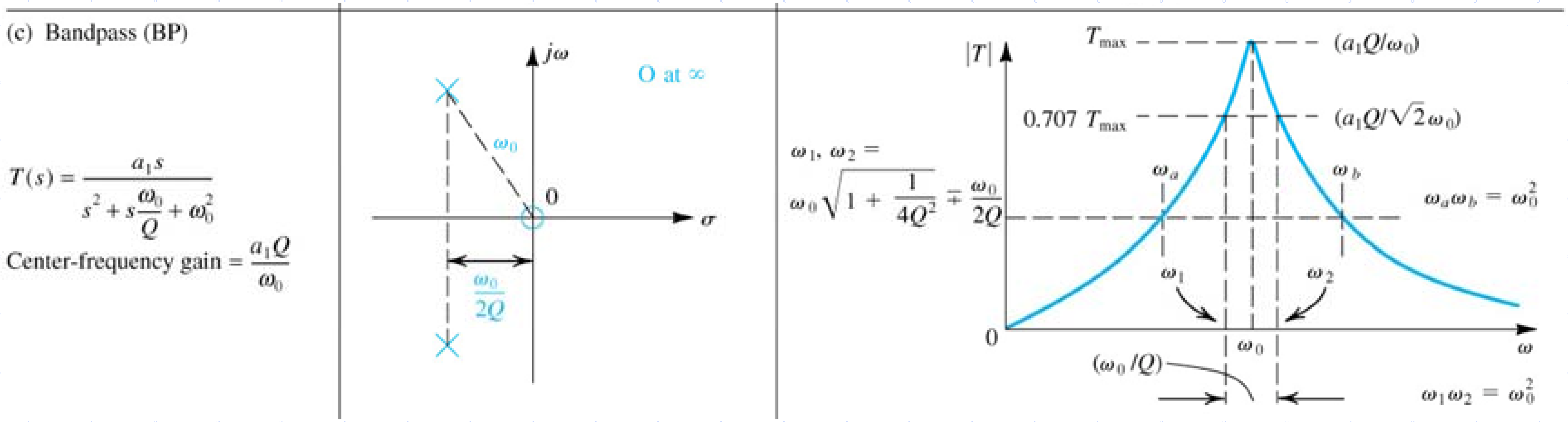


- Highpass Filter : Transmission zeros at s=0

Peak for $Q > \dfrac{1}{\sqrt{2}}$

12.4 First-Order and Second-Order Filter Functions

◆ Second-Order Filter Functions – BP case



- Bandpass Filter : Transmission zeros at s=0 and s=∞

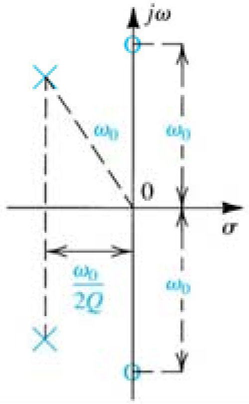
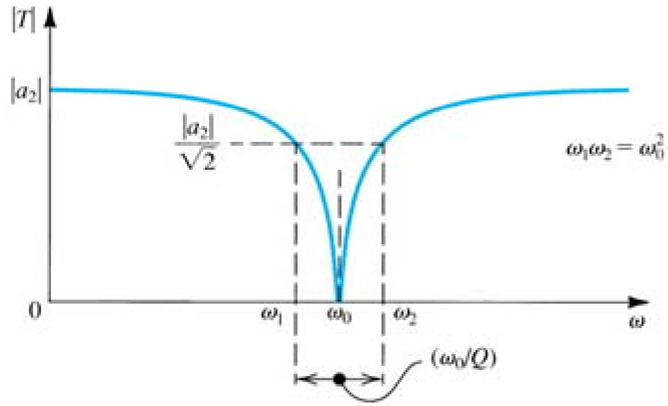
  Magnitude response peaks at $\omega = \omega_o$=center frequency

  3dB: $\omega_1, \omega_2 = \omega_o\sqrt{1+\frac{1}{4Q^2}} \pm \frac{\omega_o}{2Q}$

  BW=$\omega_2 - \omega_1 = \frac{\omega_o}{Q}$:as Q↑, BW↓(more selective)

12.4 First-Order and Second-Order Filter Functions

◆ Second-Order Filter Functions – Notch case

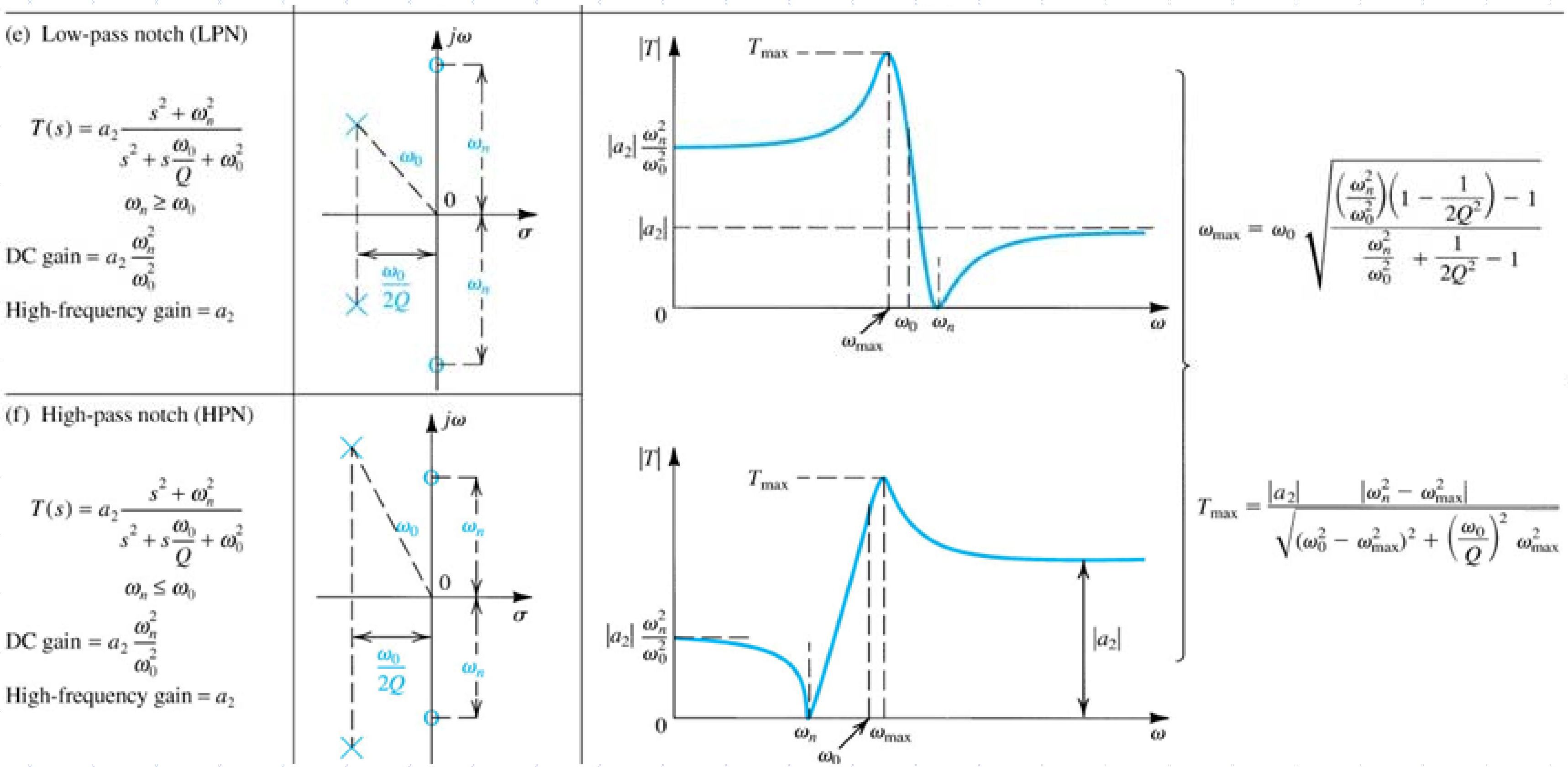
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T \rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain = a_2		

- Notch Filter : Transmission zeros are located on the jω axis, at the complex-conjugate locations $\pm j\omega_n$, then the magnitude response exhibits zero tranmission at $\omega = \omega_n$.

  (notch in the magnitude response occurs at $\omega = \omega_n$, notch frequency)

12.4 First-Order and Second-Order Filter Functions

- Second-Order Filter Functions – LPN, HPN



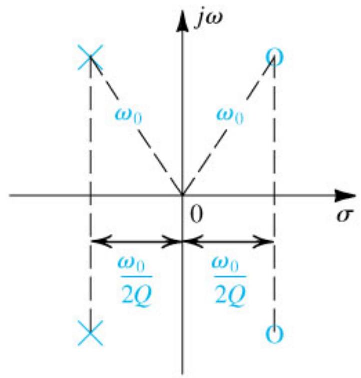
12.4 First-Order and Second-Order Filter Functions

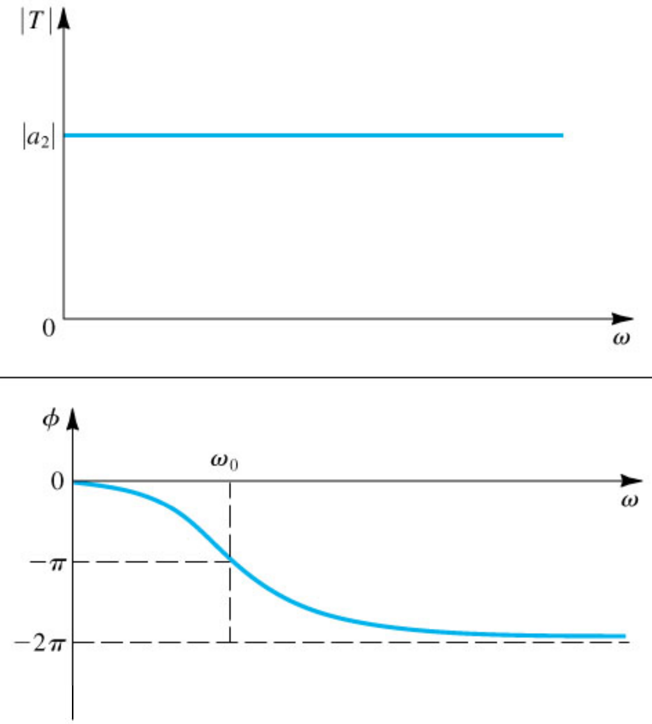
Second-Order Filter Functions – All-pass case

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\dfrac{\omega_0}{Q} + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

Flat gain = a_2



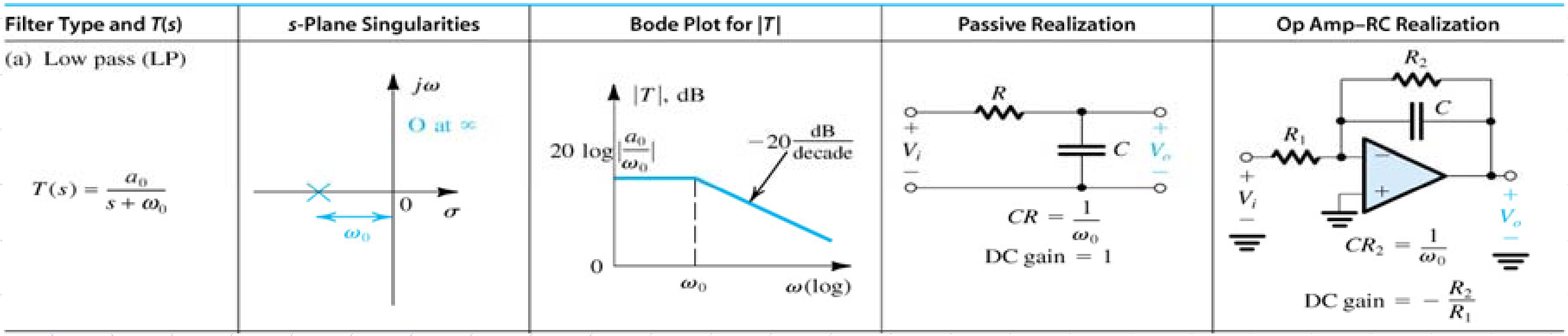


- Allpass Filter : Two transmission zeros are in the right half of the s plane, at the mirror image locations of the poles

12.4 First-Order and Second-Order Filter Functions

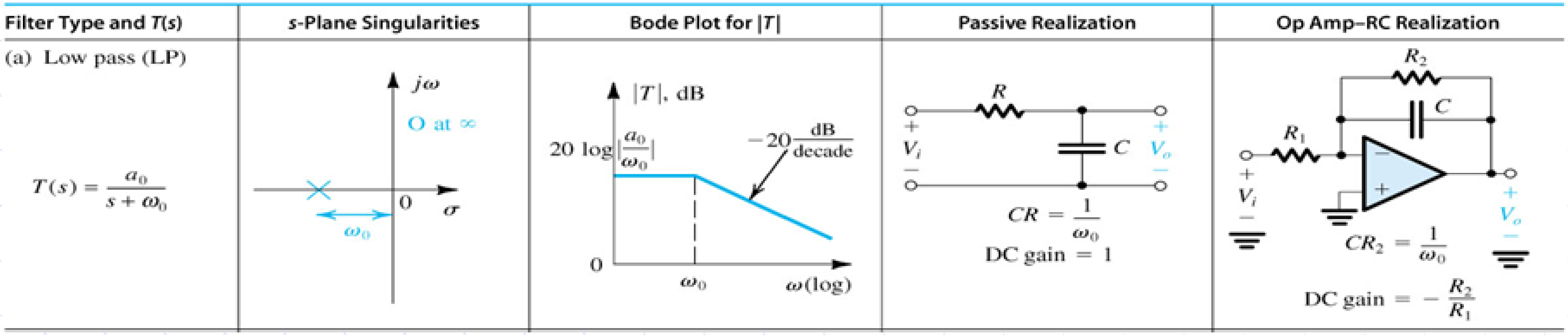
- Problem 12.19

Use the information displayed in below figure to design a first-order op amp-RC low-pass filter having a 3-dB frequency of 10 kHz, a dc gain magnitude of 10, and an input resistance of 10kΩ



12.4 First-Order and Second-Order Filter Functions

- Problem 12.19 (cont.)



- $R_{in} = R_1 = 10\text{k}\Omega$
- DC gain $= -R_2/R_1 = -10$

  $\rightarrow R_2 = 10R_1 = 100\text{k}\Omega$
- $R_2C = 1/w_0$

  $\rightarrow C = 1/w_0R_2 = 0.159\text{nF}$

12.4 First-Order and Second-Order Filter Functions

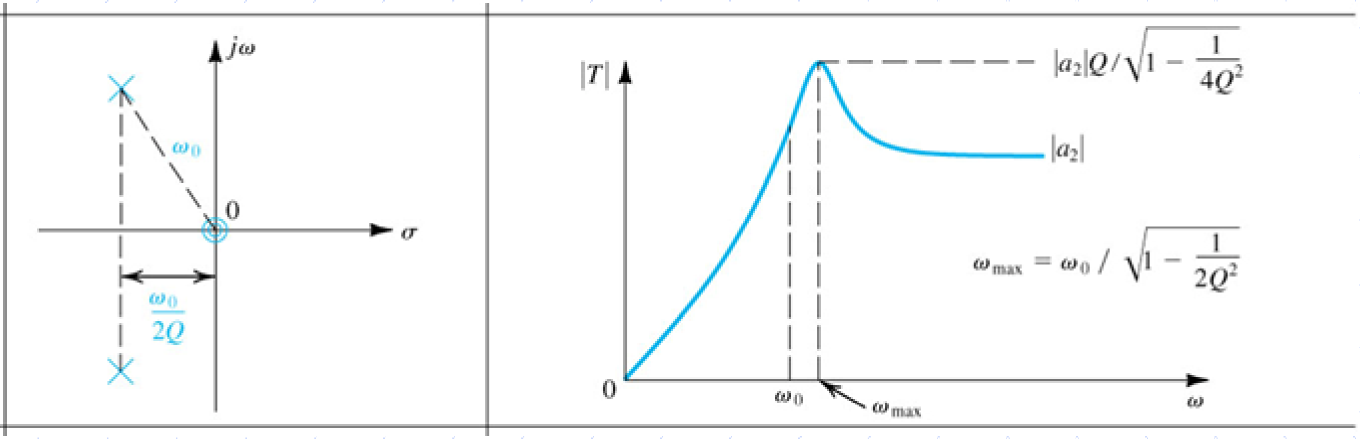
- Problem 12.28

Use the information given in below figure to find the transfer function of a second-order high-pass filter with natural modes at -0.5±j√3/2 and a high frequency gain of unity.

(b) High pass (HP)

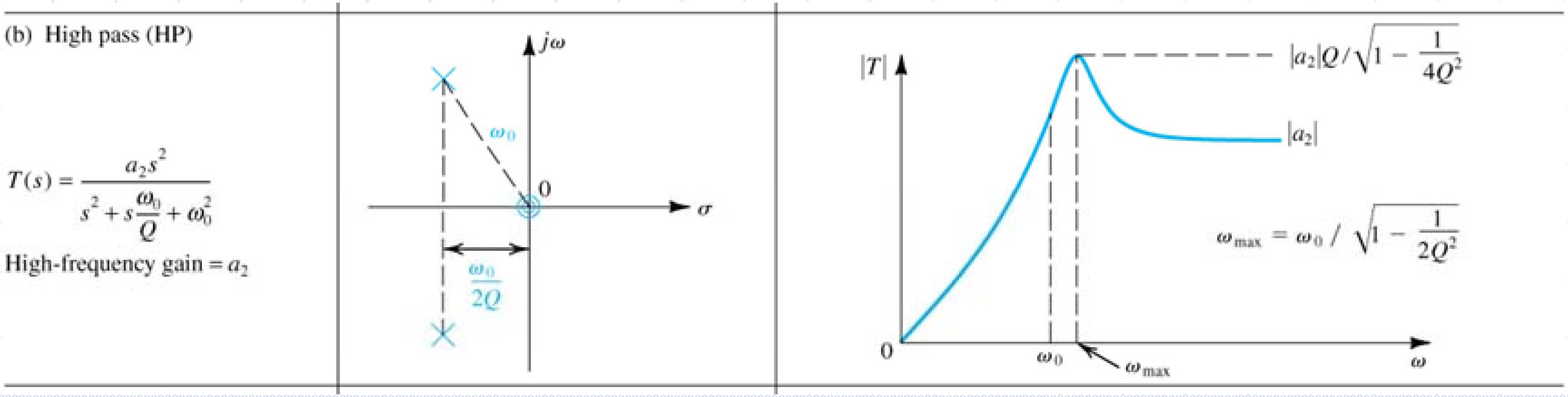
$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

High-frequency gain = a_2



12.4 First-Order and Second-Order Filter Functions

- Problem 12.28 (cont.)



- $\omega_0 = \sqrt{(1/2)^2 - (\sqrt{3}/2)^2} = 1, \quad \omega_0 / 2Q = 1/2$

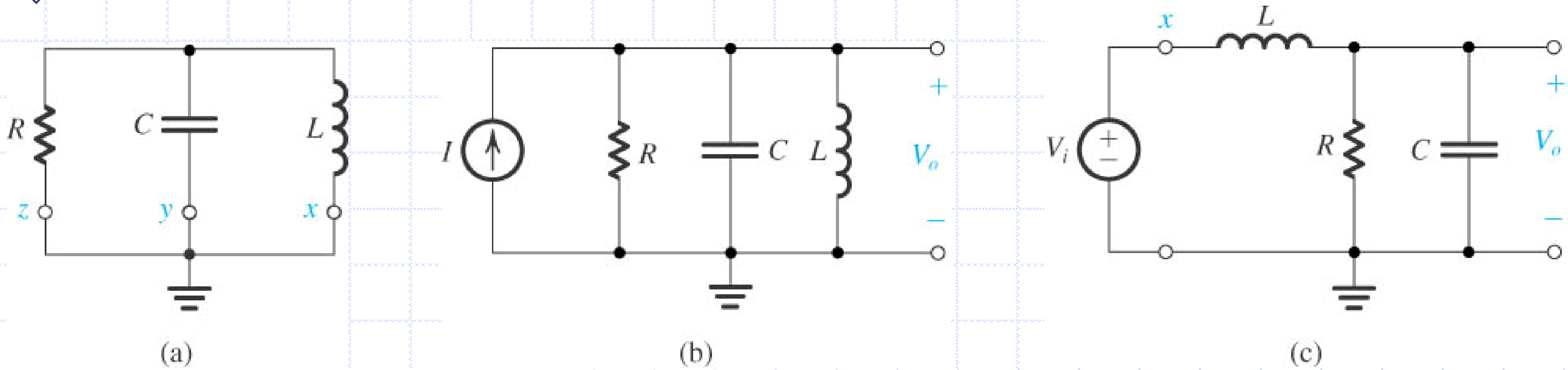
$$\rightarrow T(s) = \frac{a_2 s^2}{(s^2 + s\omega_0 / Q + \omega_0^2)} = \frac{a_2 s^2}{(s^2 + s + 1)}$$

- $|T(j\infty)| = a_2 = 1$. *Thus,*

$$T(s) = \frac{s^2}{(s^2 + s + 1)}$$

12.5 The Second-Order LCR Resonator

The Resonator Natural Modes



- The natural modes can be determined by applying an excitation that does not change the natural structure of the circuit
- In fig(b) the resonator is excited with a current source I.
  : An independent ideal current source is equivalent to an open circuit → does not alter the natural structure of the resonator

12.5 The Second-Order LCR Resonator

◆ The Resonator Natural Modes

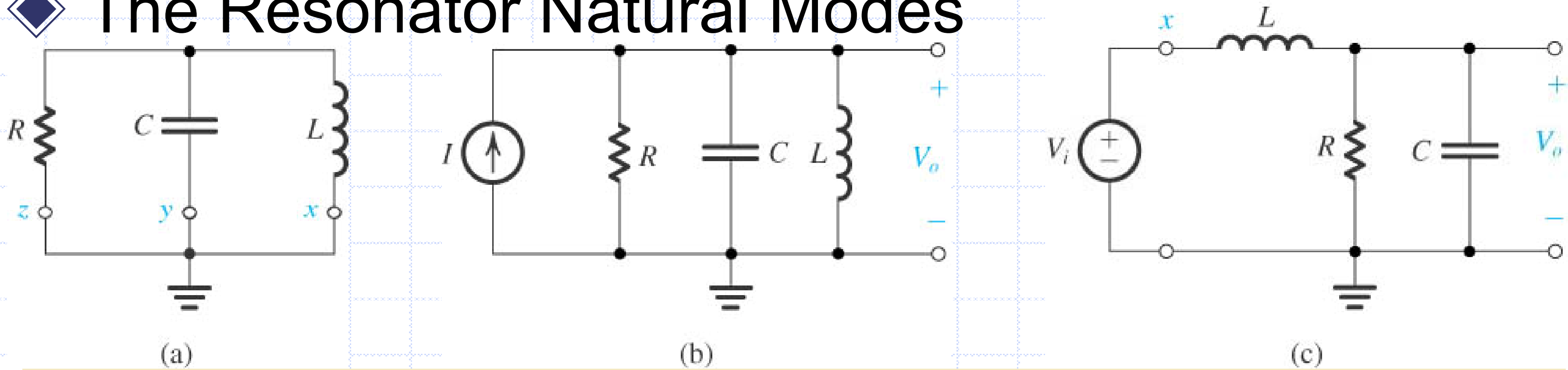
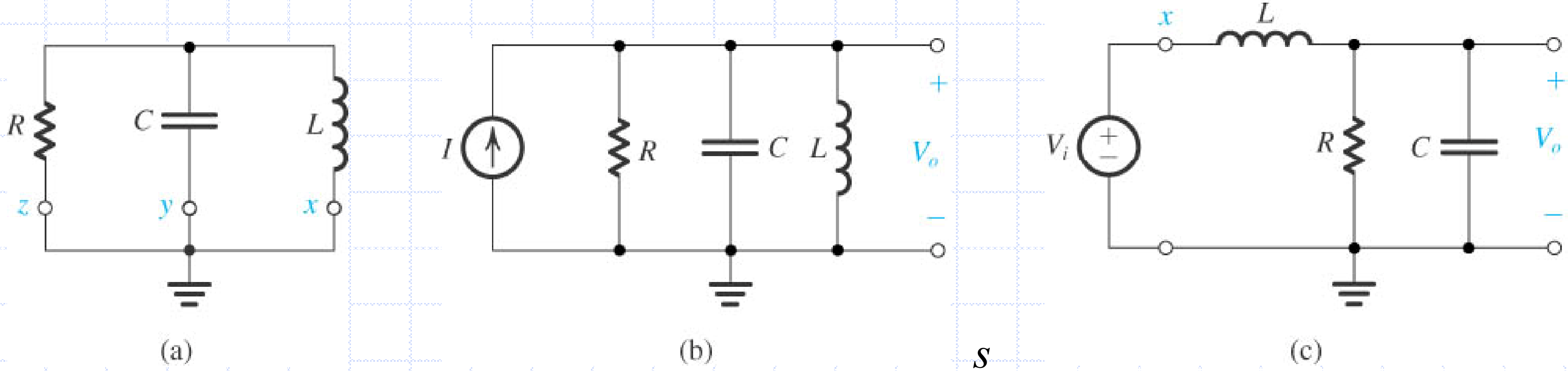


Figure 12.17 (a) The second-order parallel LCR resonator. (b, c) Two ways of exciting the resonator of (a) without changing its *natural structure:* resonator poles are those poles of V_o/I and V_o/V_i.

- The natural modes can be determined by applying an excitation that does not change the natural structure of the circuit
- In fig(b) the resonator is excited with a current source I.
  : An independent ideal current source is equivalent to an open circuit → does not alter the natural structure of the resonator
- An alternative way of exciting the parallel LCR resonator is shown in Fig. (c)

12.5 The Second-Order LCR Resonator

◆ The Resonator Natural Modes (cont.)



$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{\frac{1}{sL} + sC + \frac{1}{R}} = \frac{\frac{s}{C}}{s^2 + s(\frac{1}{CR}) + \frac{1}{LC}}$$

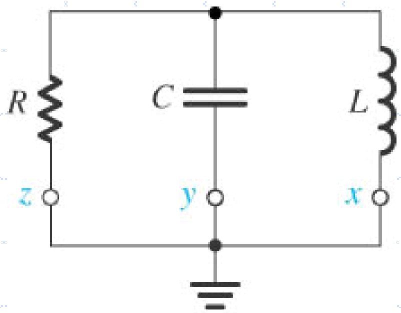
- Equating the denominator to the standard form $[s^2+s(\omega_0/Q)+\omega_0^2]$

$$\omega_o^{\,2} = \frac{1}{LC} \quad ; \quad \frac{\omega_o}{Q} = \frac{1}{CR} \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad ; \quad Q = \omega_o CR$$

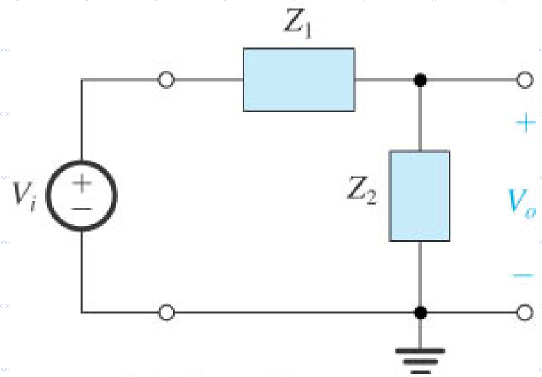
12.5 The Second-Order LCR Resonator

Realization of Transmission Zeros

- Find out where to inject the input voltage signal V_i so that the transfer function V_o/V_i is the desired one



(a)



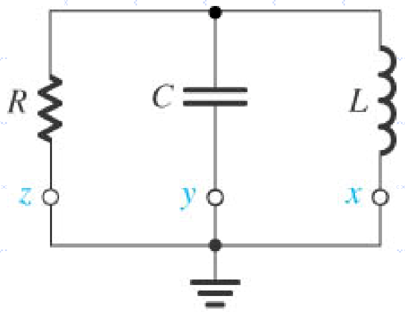
(a) General structure

- Any of the nodes labeled x, y, or z can be disconnected from ground and connected to Vi forming of a voltage divider.

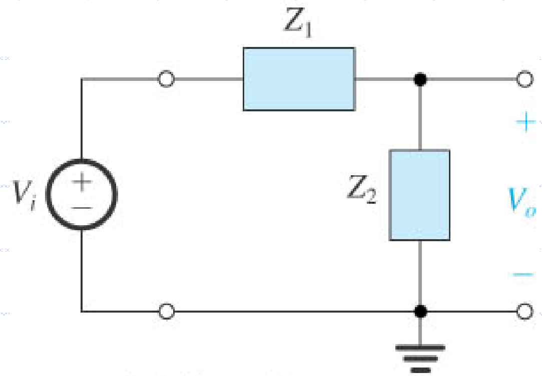
$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

12.5 The Second-Order LCR Resonator

◆ Realization of Transmission Zeros (cont.)



(a)

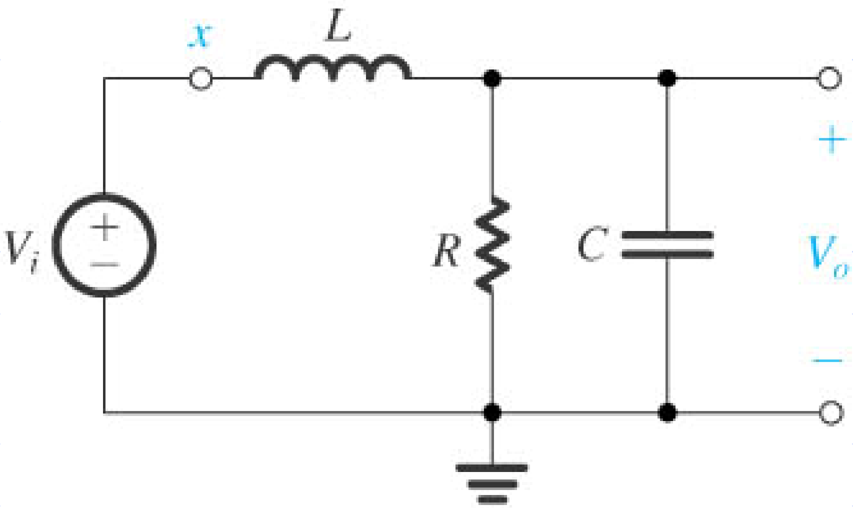


(a) General structure

- The transmission zeros : $Z_2(s)$ = zero & $Z_1(s) \neq$ zero
  or $Z_1(s) \rightarrow$ infinite & $Z_2(s) \rightarrow$ not infinite
- The output will be zero either when $Z_2(s)$ behaves as a short circuit of $Z_1(s)$ behaves as an open circuit.
- If there is a value of s at which both Z_1 and Z_2 are zero, then V_o/V_i will be finite and no transmission zero is obtained

12.5 The Second-Order LCR Resonator

◆ Realization of the Low-Pass Function



(b) LP

zeros $\quad sL = \infty$

$$\frac{1}{(sC + \frac{1}{R})} = 0$$

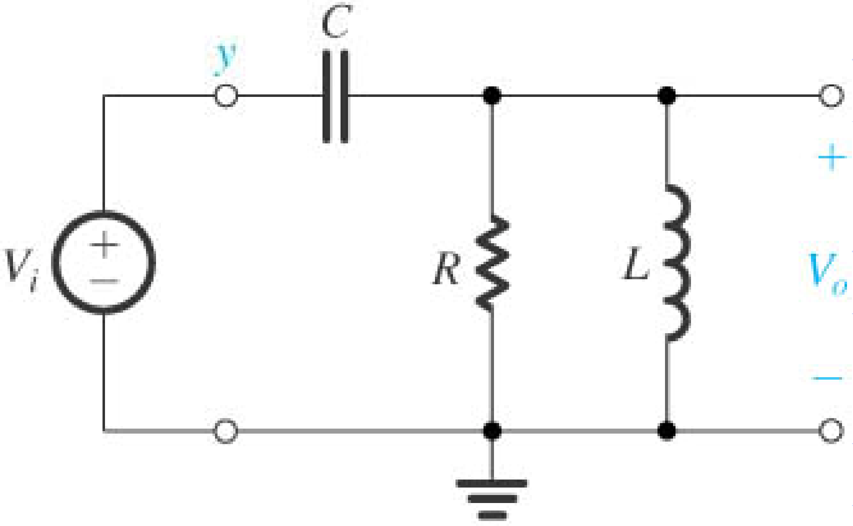
$\Rightarrow$ two zeros at $s = \infty$

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{\frac{1}{sL}}{\frac{1}{sL} + sC + \frac{1}{R}}$$

$$= \frac{1/LC}{s^2 + s(1/CR) + 1/LC}$$

12.5 The Second-Order LCR Resonator

◆ Realization of the High-Pass Function



(c) HP

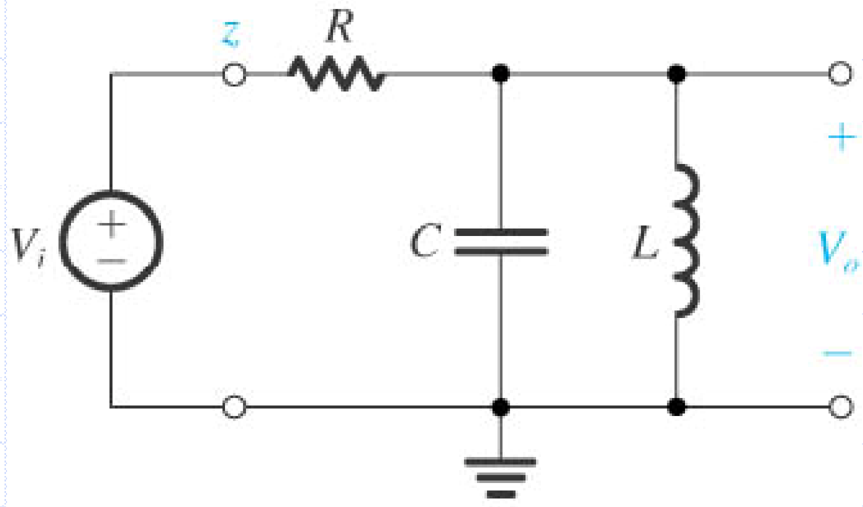
$$\text{zeros} \begin{cases} s = 0 : \text{Capacitor} \\ s = 0 : \text{Inductor} \end{cases}$$

$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s(\frac{\omega_o}{Q}) + \omega_o^2}$$

$$a_2 = 1$$

12.5 The Second-Order LCR Resonator

Realization of the Band-Pass Function



(d) BP

$$\text{zeros}\begin{cases} s = 0 & : \text{Inductor} \\ s = \infty & : \text{Capacitor} \end{cases}$$

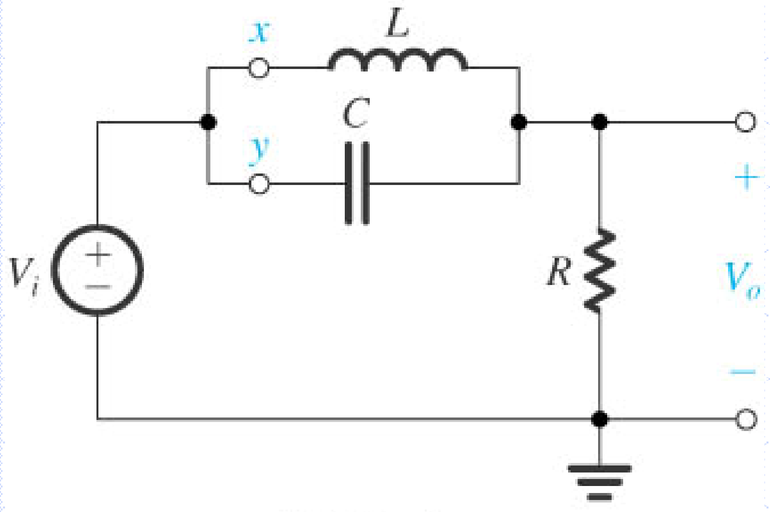
- at w_0, LC-tuned circuit exhibits an infinite impedance
  → no current flows
- the center freq. gain is unity

$$T(s) = \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{sL} + sC}$$

$$= \frac{s(\frac{1}{CR})}{s^2 + s(\frac{1}{CR}) + (\frac{1}{LC})}$$

12.5 The Second-Order LCR Resonator

Realization of the Notch Function



(e) Notch at ω_0

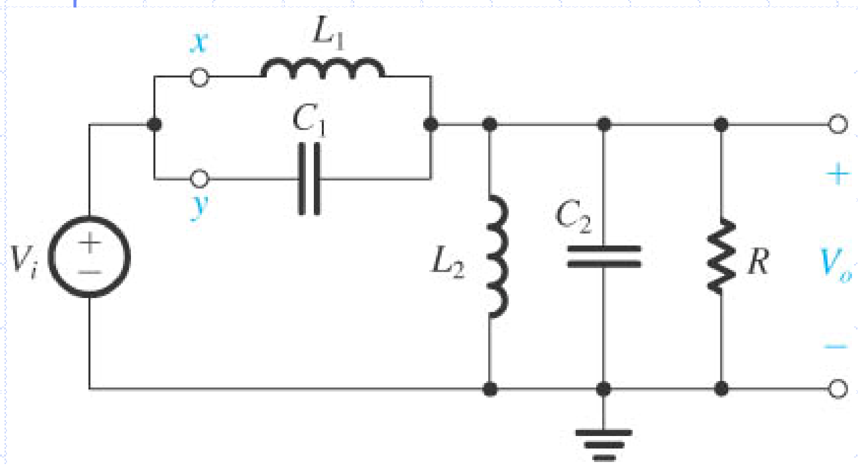
- The impedance of the LC circuit becomes infinite at $\omega_0 = 1/\sqrt{LC}$
  → zero transmission
- The resistor does not introduce zeros.

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

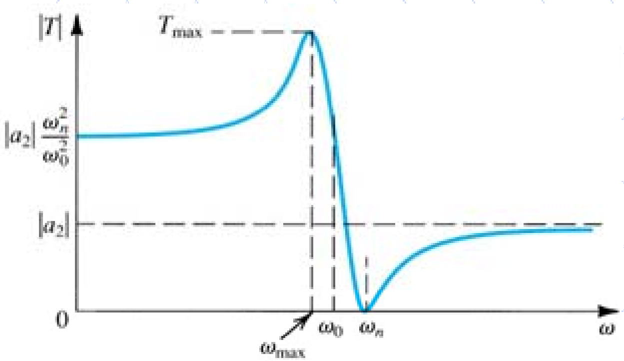
- The high-frequency gain a_2 can be found from the circuit to be unity

12.5 The Second-Order LCR Resonator

◆ Realization of the Notch Function (cont.)



(f) General notch



- To place the notch frequency w_n arbitrarily relative to w_0,

$$L_1C_1=1/w_n^2$$

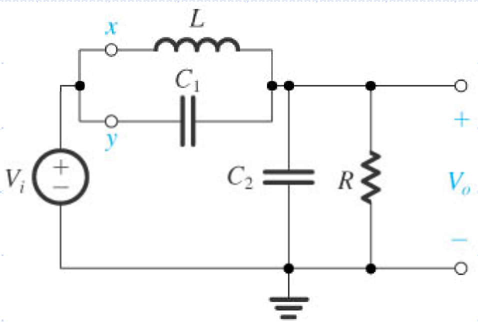
- Thus the L_1C_1 tank circuit introduces a pair of zeros at $\pm jw_n$

- Not to alter the natural modes,

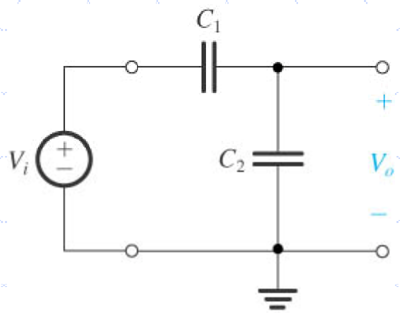
$$C_1+C_2=C \quad \& \quad L_1||L_2=L$$

12.5 The Second-Order LCR Resonator

Realization of the Notch Function - LPN



(g) LPN ($\omega_n > \omega_0$)



(h) LPN as $s \to \infty$

- For the LPN, $w_n > w_o$

  $\rightarrow L_1C_1 < (L_1 || L_2)(C_1 + C_2)$

  This condition can be satisfied with L_2 eliminated (i.e., $L_2 = \infty$ and $L_1 = L$)

- Transfer function

$$T(s) = \frac{s^2 + (1/LC_1)}{s^2 + s(1/CR) + 1/L(C_1 + C_2)}$$

- As $s \rightarrow \infty$

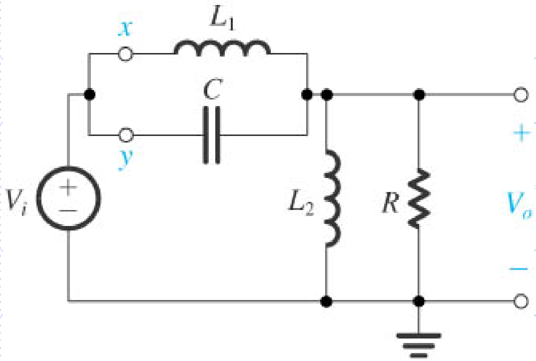
$$V_o/Vi = C_1/(C_1 + C_2)$$

  Thus,

$$a_2 = C_1/(C_1 + C_2)$$

12.5 The Second-Order LCR Resonator

Realization of the Notch Function - HPN



(i) HPN ($\omega_n < \omega_0$)

- For the HPN, $w_n < w_o$

  $\rightarrow L_1C_1 > (L_1||L_2)(C_1+C_2)$

  Which can be satisfied by selecting $C_2=0$

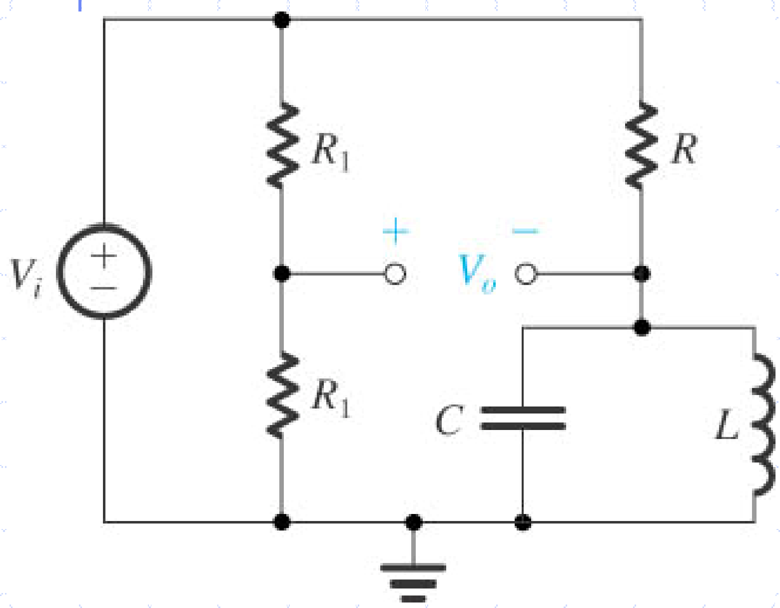
- Transfer function

$$T(s) = \frac{s^2 + (1/L_1C)}{s^2 + s(1/CR) + [1/(L_1 \| L_2)C]}$$

- As s $\rightarrow \infty$ V_o approaches V_i, thus the high frequency gain, a_2, is unity.

12.5 The Second-Order LCR Resonator

◆ Realization of the All-pass Function



- The all-pass transfer function

$$T(s) = \frac{s^2 - s(w_0/Q) + w_0^2}{s^2 + s(w_0/Q) + w_0^2} = 1 - \frac{s2(w_0/Q)}{s^2 + s(w_0/Q) + w_0^2}$$

- The second term is a bandpass function with a center-frequency gain of 2

- All pass realization with a flat gain of 0.5

$$T(s) = 0.5 - \frac{s(w_0/Q)}{s^2 + s(w_0/Q) + w_0^2}$$

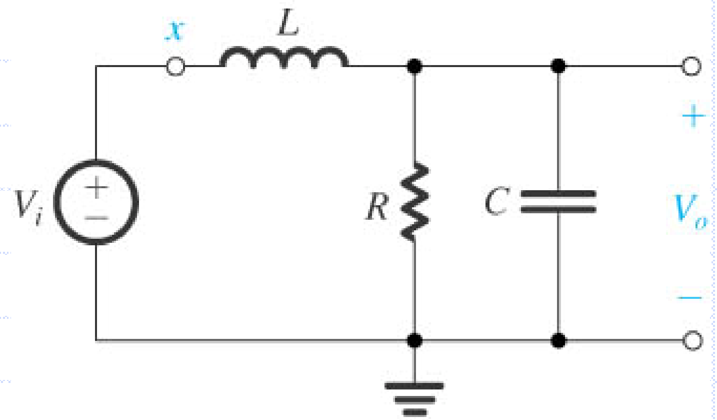
12.5 The Second-Order LCR Resonator

Problem 12.36

Use the circuit of below figure, design a lowpass filter with $\omega_0=10^5$ rad/s and $Q=1/\sqrt{2}$. Utilize a 0.1uF capacitor

$$T(s)=\frac{1/LC}{s^2+s(1/CR)+1/LC}=\frac{a_0}{(s^2+s\omega_0/Q+\omega_0^2)}$$

- $\omega_0 = 1/\sqrt{LC} \;\rightarrow\; L = 1mH$
- $Q = \omega_0 CR$
  $\rightarrow R = Q/\omega_0 C = 70.7\Omega$



(b) LP

12.6 Second-Order Active Filters Based on Inductor Replacement

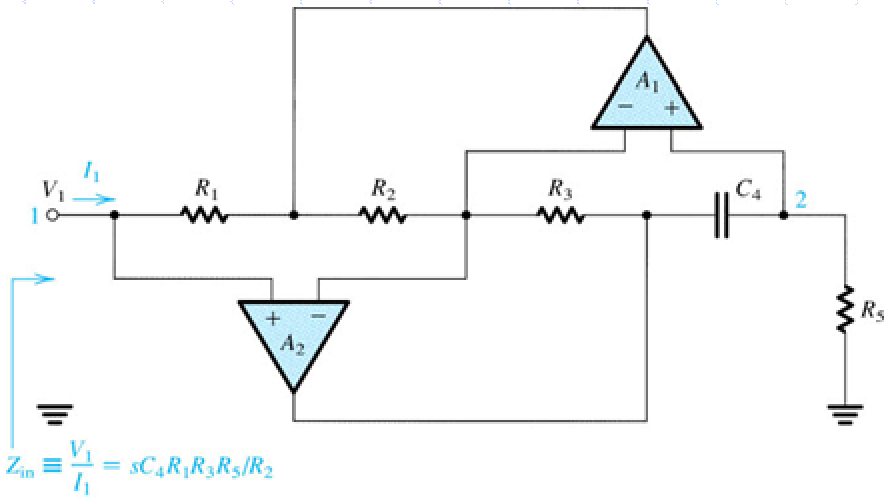
- Study a family of op amp-RC circuits (various second-order filters)

- Based on an op amp-RC resonator

- Obtained by replacing the inductor L, in the LCR resonator with an op amp-RC circuit that has an inductive input impedance

12.6 Second-Order Active Filters Based on Inductor Replacement

The Antoniou Inductance-Simulation Circuit



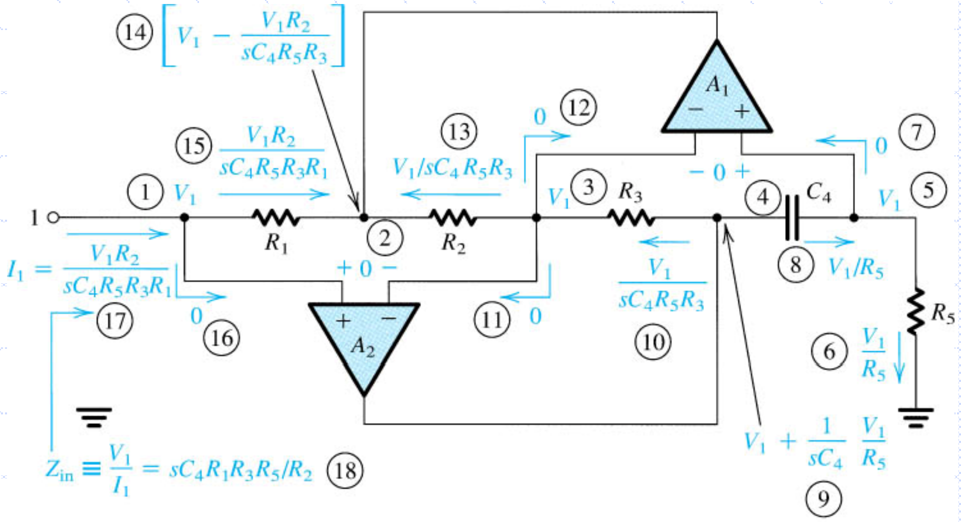
(a)

- Invented by A. Antoniou
- If the circuit is fed at its input (node 1) with a voltage source V_1 and the input current is denoted I_1, (for ideal op amps)

$$Z_{in}=V_1/I_1=sC_4R_1R_3R_5/R_2 \qquad L=C_4R_1R_3R_5/R_2$$

12.6 Second-Order Active Filters Based on Inductor Replacement

◆ The Antoniou Inductance-Simulation Circuit (cont.)



(b)

- Assuming ideal op amps.
- The design of this circuit is usually based on selecting

$$R_1=R_2=R_3=R_5=R \qquad\qquad L=CR^2$$

- Convenient values are selected for C and R to yield the desired inductance value L

12.4 First-Order and Second-Order Filter Functions

◆ Problem 12.40

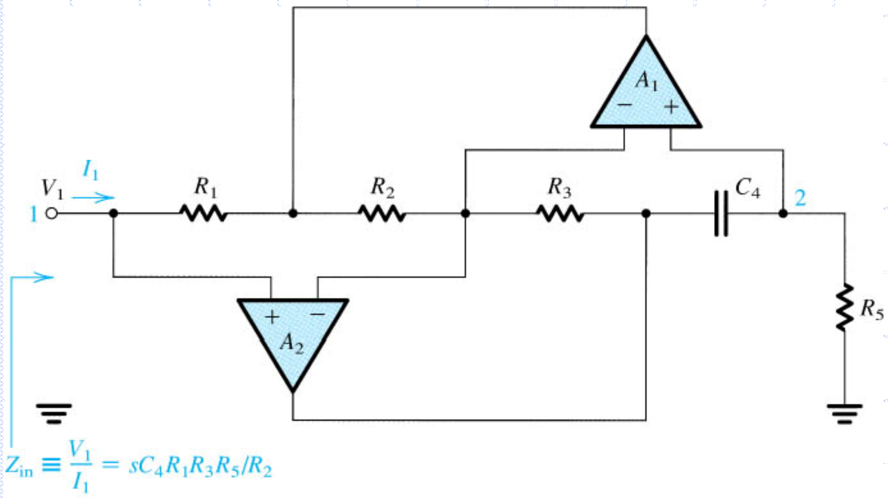
Design the Antoniou inductance-simulation circuit to realize an inductance of 0.1H

- $L = C_4R_1R_3R_5/R_2$
  choose $R_1=R_2=R_3=R_5=10K\ \Omega$

  $\rightarrow L= 10^8C_4H$

- $L=0.1H$

  $\rightarrow C_4=1nF$



(a)

12.6 Second-Order Active Filters Based on Inductor Replacement

The Op Amp-RC Resonator

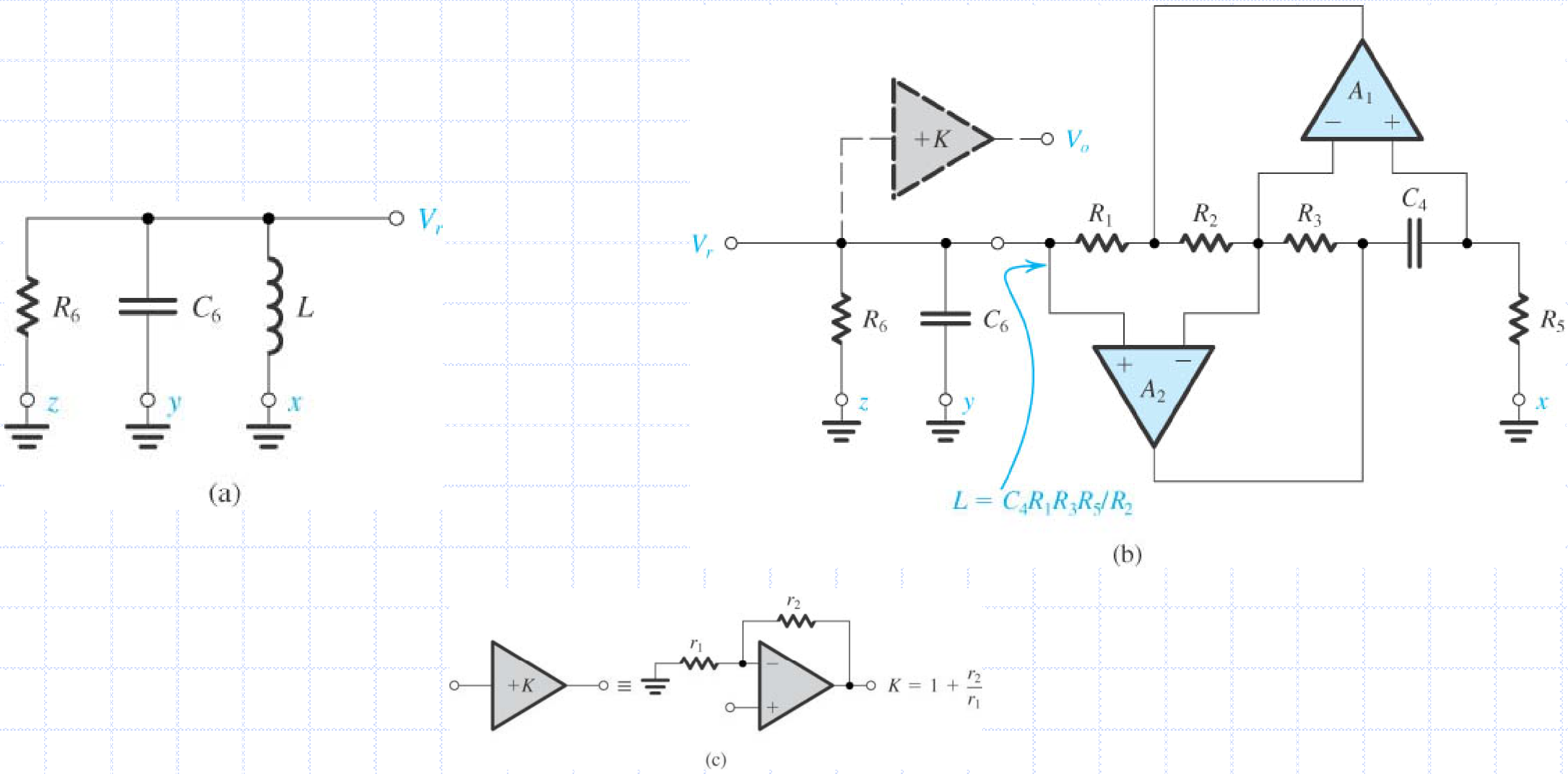
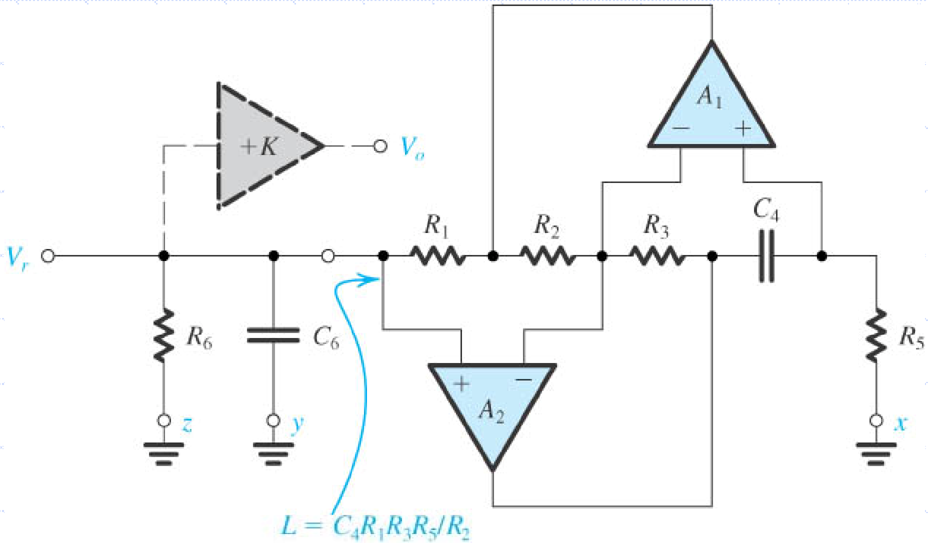


Figure 12.21 (a) An LCR resonator. (b) An op amp–RC resonator obtained by replacing the inductor *L* in the LCR resonator of (a) with a simulated inductance realized by the Antoniou circuit of Fig. 12.20(a). (c) Implementation of the buffer amplifier *K*.

12.6 Second-Order Active Filters Based on Inductor Replacement

◆ The Op Amp-RC Resonator (cont.)



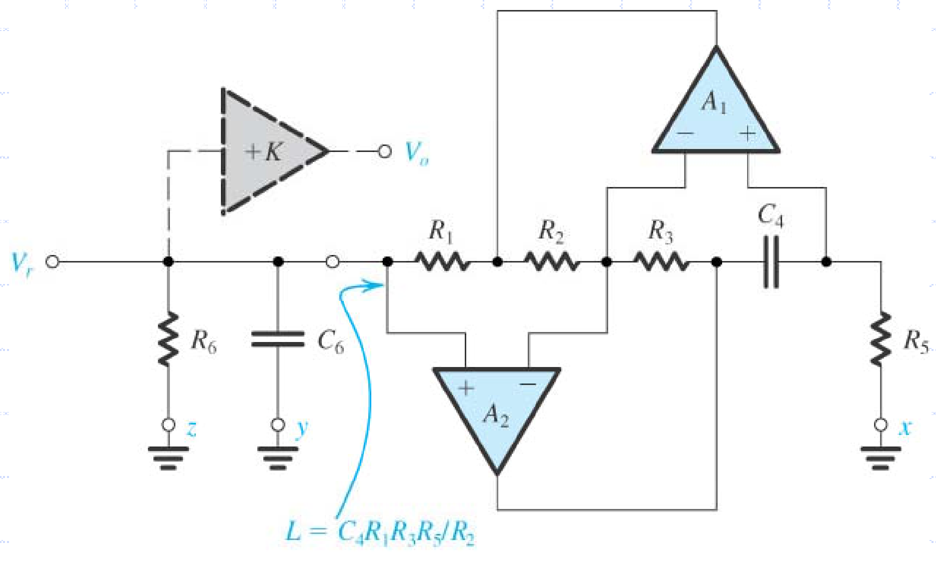
(b)

- Replacing the inductor L with a simulated inductance realized by the Antoniou circuit → second-order resonator.
- Pole frequency

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{C_4C_6R_1R_3R_5/R_6}$$

12.6 Second-Order Active Filters Based on Inductor Replacement

◆ The Op Amp-RC Resonator (cont.)



(b)

- Pole Q factor

$$Q = \omega_0 C_6 R_6 = R_6 \sqrt{C_6 R_2 / C_4 R_1 R_3 R_5}$$

- Usually selects $C_4 = C_6 = C$ and $R_1 = R_2 = R_3 = R_5 = R$, which results in

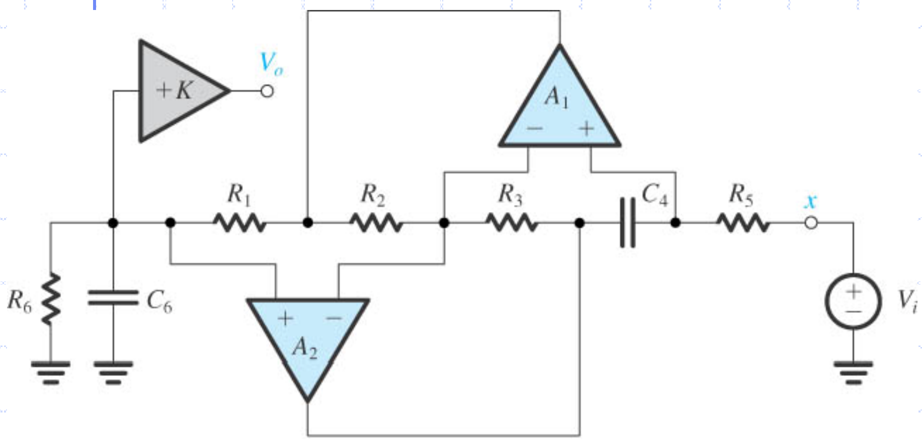
$$\omega_0 = 1/CR$$

$$Q = R_6 / R$$

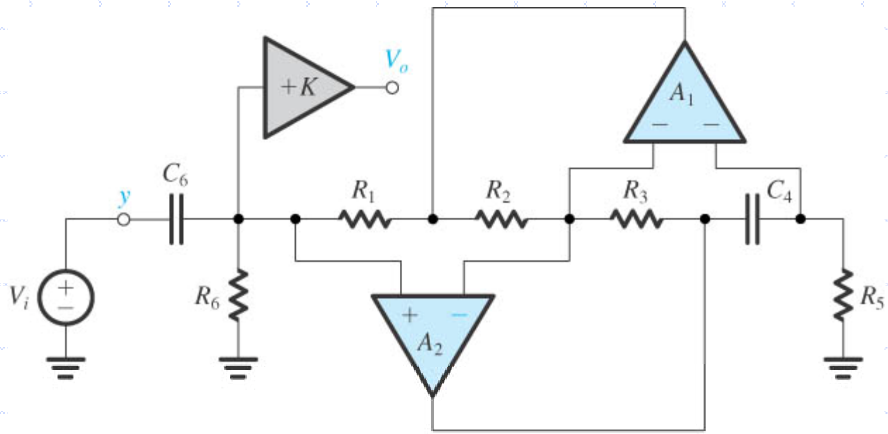
- Select a practically convenient value for C → determine the value of R to realize a given ω_0 → determine the value of R_6 to realize a given Q

12.6 Second-Order Active Filters Based on Inductor Replacement

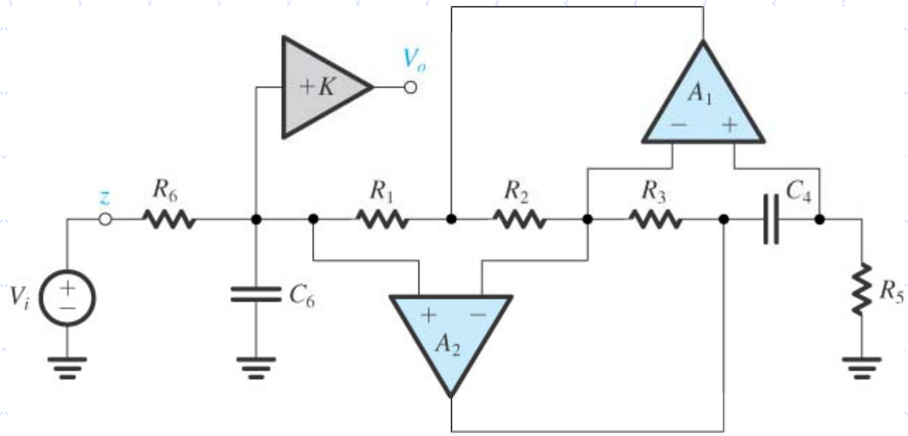
◆ Realization of the Various Filter types



(a) LP



(b) HP

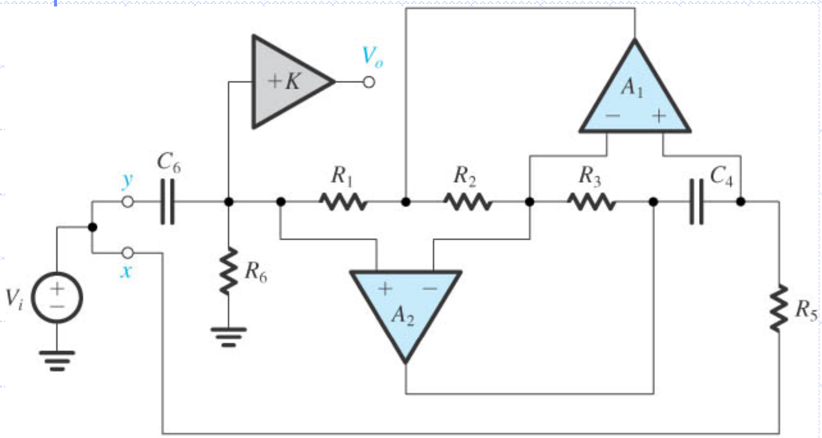


(c) BP

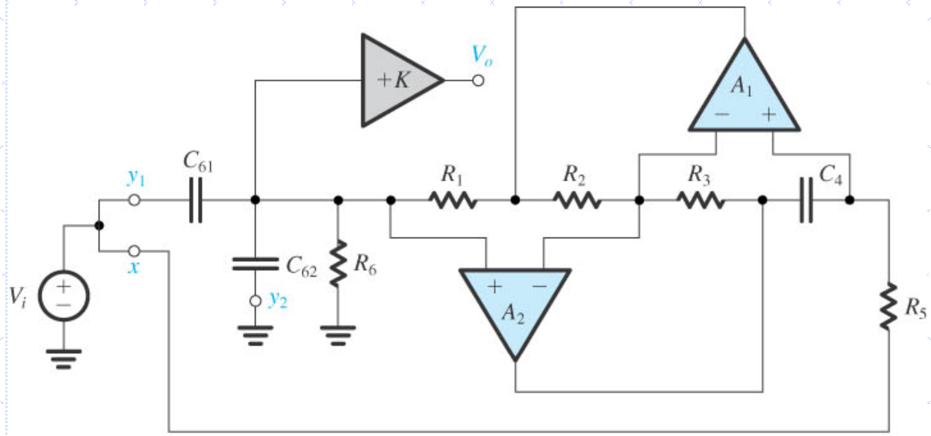
- Low-pass function : inject V_i to node y
- High-pass function : inject V_i to node y
- disconnect node z from ground and connect it to the signal source V_i

12.6 Second-Order Active Filters Based on Inductor Replacement

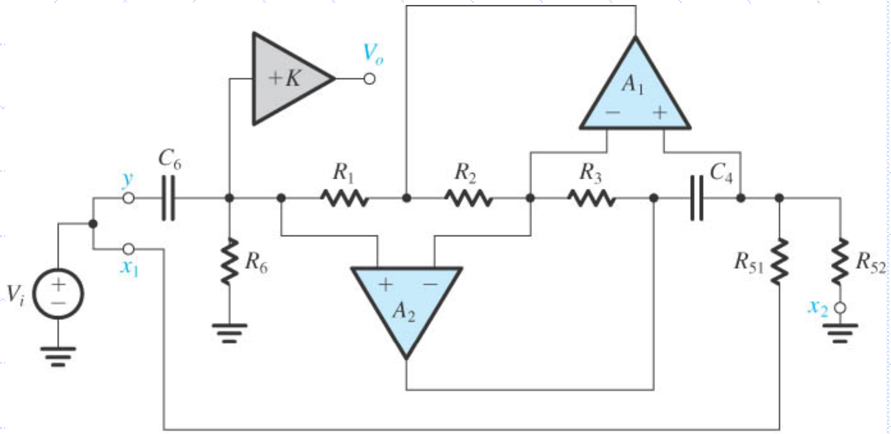
- Realization of the Various Filter types (cont.)



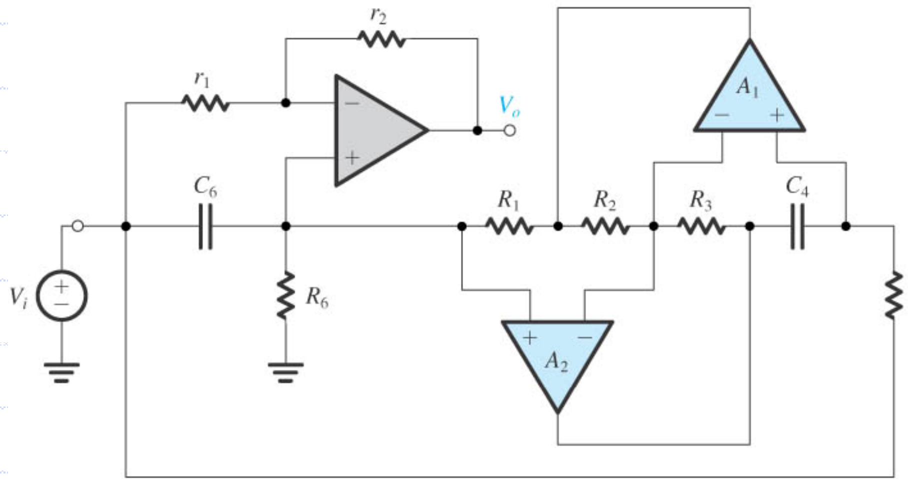
(d) Notch at ω_0



(e) LPN, $\omega_n \geq \omega_0$



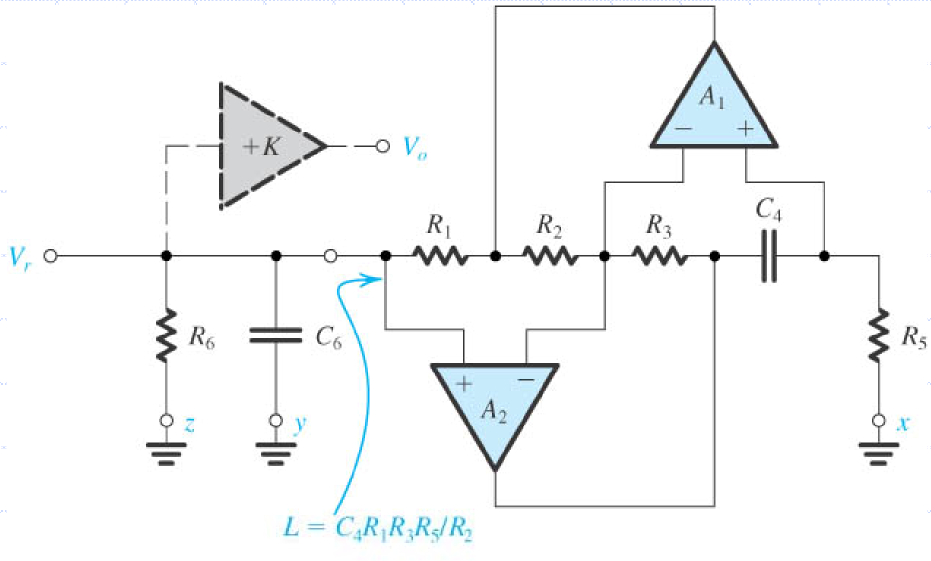
(f) HPN, $\omega_n \leq \omega_0$



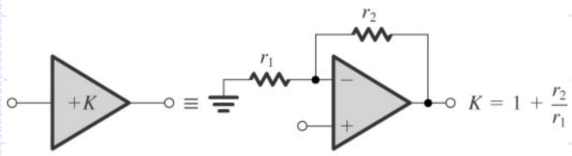
(g) All-pass

12.6 Second-Order Active Filters Based on Inductor Replacement

◆ Realization of the Various Filter types (cont.)



(b)



(c)

- In all cases the output can be taken as the voltage across the resonance circuit, V_r.
- Connecting a load there would change the filter characteristics. → The problem can be solved by utilizing a buffer amplifier.

12.6 Second-Order Active Filters Based on Inductor Replacement

The All- Pass circuit

- An all-pass function with a flat gain of unity

  AP = 1 – (BP with a center frequency gain of 2) → complementary

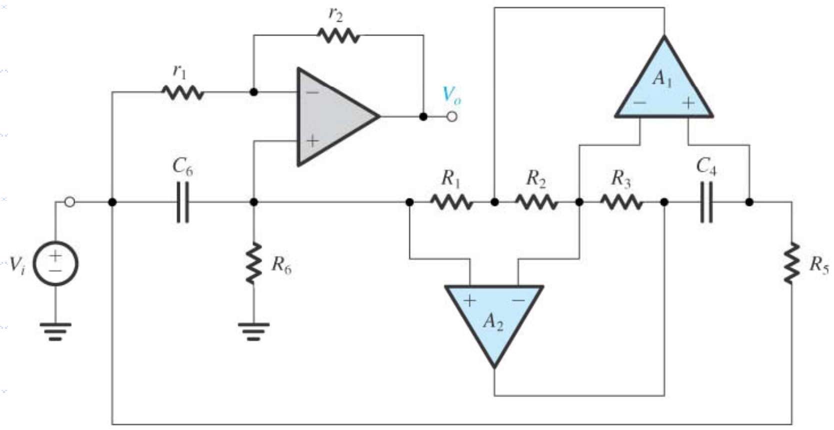
- All-pass circuit with unity flat gain is the complement of the bandpass circuit a center-frequency of 2.

- A simple procedure for obtaining the complement of a given linear circuit : Interchanging input and ground in a linear circuit generates a circuit whose transfer function is the complement of that of the original circuit.

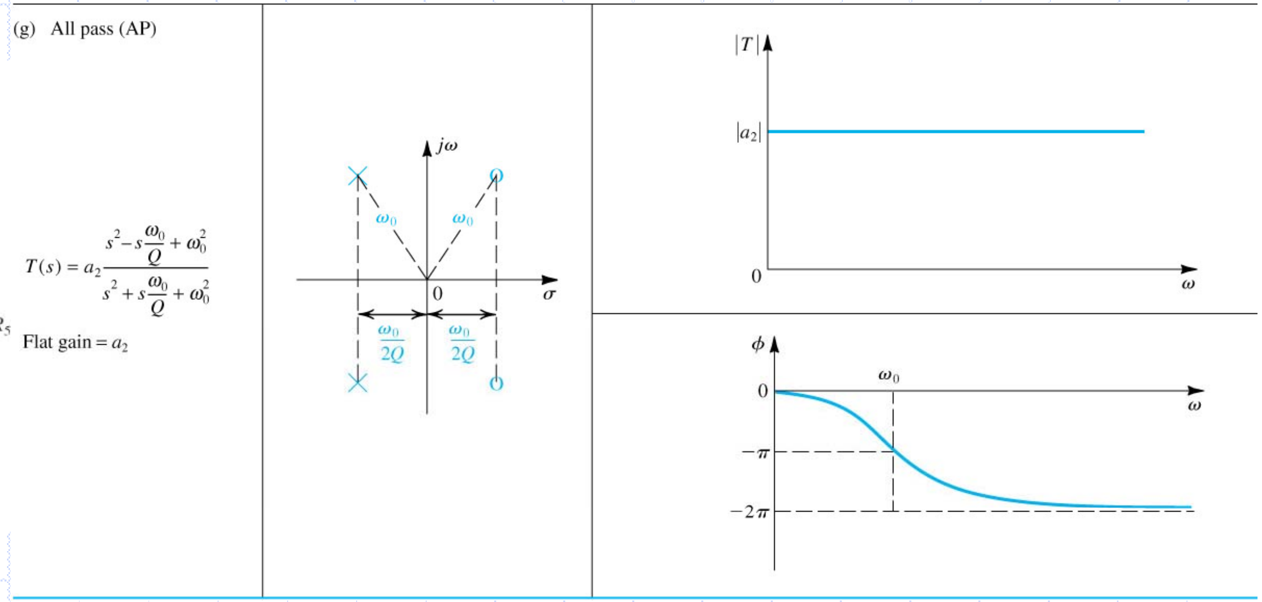
12.6 Second-Order Active Filters Based on Inductor Replacement

- Problem 12.44

Design the all-pass circuit of below figure to provide a phase shift of 180(degree) at f=1 KHz and to have Q=1. Use 1-nF capacitors.



(g) All-pass



12.6 Second-Order Active Filters Based on Inductor Replacement

Problem 12.44 (cont.)

- Phase shift = 180^{O} at $f = f_0$

  $\rightarrow f_0 = 1\text{kHz} = R_2/(2\pi C_4 C_6 R_1 R_3 R_5)$

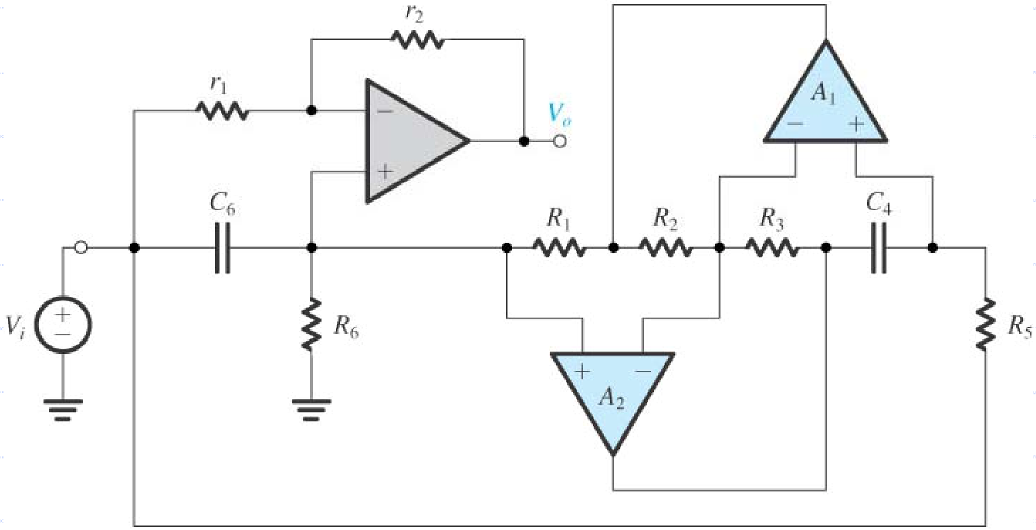
  Let $R_1=R_2=R_3=R_5=R$, $C_4=C_6=C=1\text{nF}$, then

  $f_0 = 2\pi/(CR)^2$

- $R = 1/2\pi f_0 C = 159.16\text{k}\ \Omega = R_1 = R_2 = R_3 = R_5$

- $w_0/Q = 1/R_6 C_6$

  $\rightarrow R_6 = Q/(C_6 w_0) = 159.16\text{k}\Omega$



(g) All-pass

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain = a_2

12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

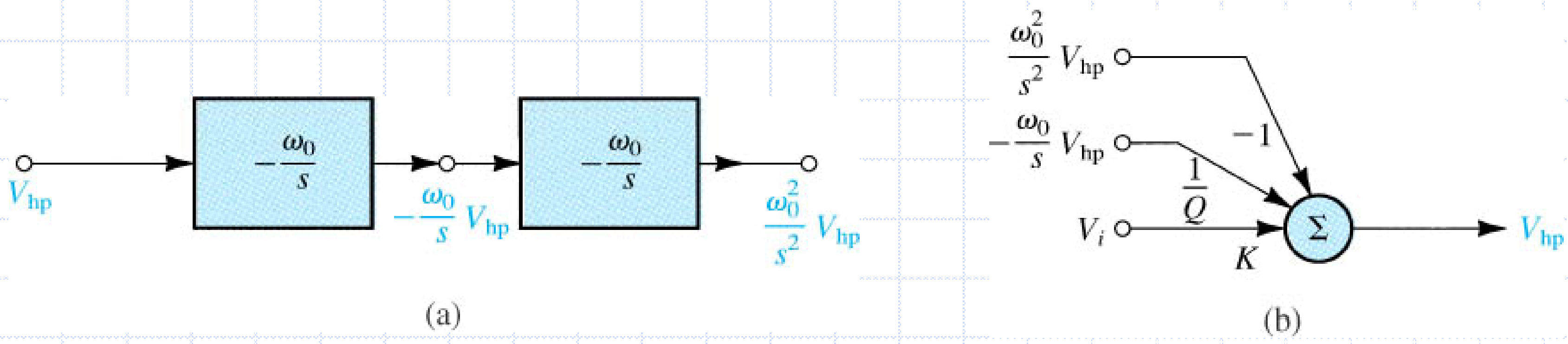
Derivation of the Two-Integrator-Loop Biquad

- To derive the two-integrator-loop biquadratic circuit, start from the second-order high-pass transfer function

$$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(w_0 / Q) + w_0^{\;2}}$$

- Cross-multiplying the equation and dividing both sides by s^2,

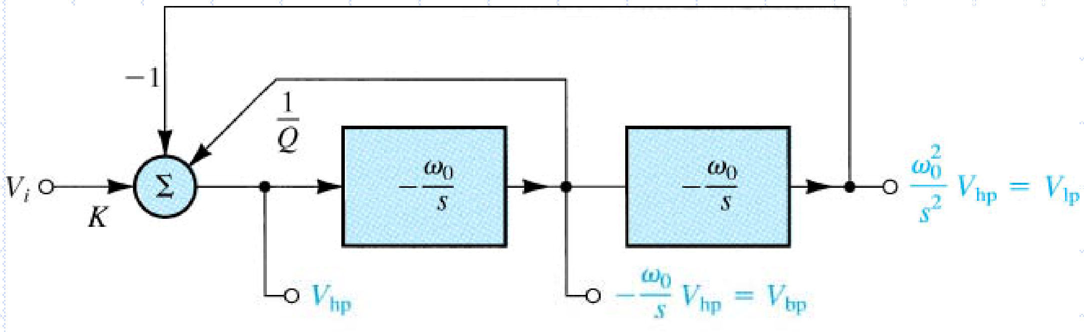
$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_o}{s} V_{hp}\right) + \left(\frac{\omega_o^{\;2}}{s^2} V_{hp}\right) = KV_i$$



12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

◆ Derivation of the Two-Integrator-Loop Biquad(cont)

- A complete block diagram realization



(c)

- From the output of the summer, obtained high-pass transfer function

$$T_{hp} = \frac{V_{hp}}{V_i}$$

- From the output of the first integrator, obtained bandpass function

$$T_{bp}(s) = \frac{(-\omega_0 / s)V_{hp}}{V_i} = \frac{K\omega_0 s}{s^2 + s(\omega_0 / Q) + \omega_0^2}$$

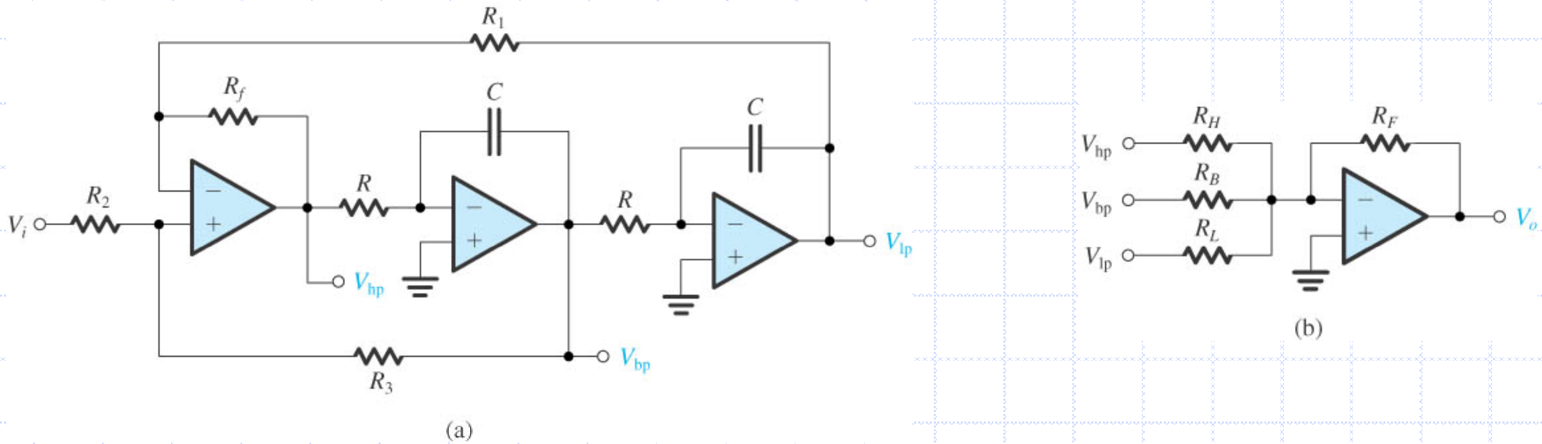
- From the output of the second integrator, obtained lowpass function

$$T_{lp}(s) = \frac{(-\omega_0^2 / s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0 / Q) + \omega_0^2}$$

12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

Circuit Implementation – KHN biquad

- The Kerwin-Huelsman-Newcomb circuit(**KHN biquad**)



- Integrator : Miller integrator circuit having $CR=1/w_0$

- Summer : Op-amp summing circuit

12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

Circuit Implementation – KHN biquad

- Design procedure

  1) Select suitable C and R for $CR=1/\omega_0$

  2) Determine the values of the resistors associated with the summer from

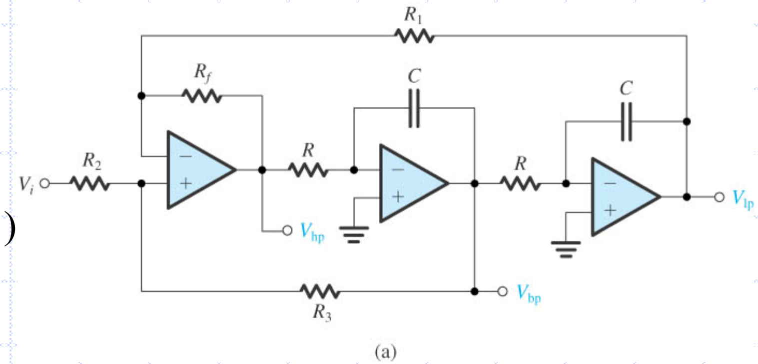
$$V_{hp} = \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_i + \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)\left(-\frac{\omega_o}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_o^2}{s^2}V_{hp}\right)$$

$$V_{hp} = KV_i - \frac{1}{Q}\left(\frac{\omega_o}{s}V_{hp}\right) - \left(\frac{\omega_o^2}{s^2}V_{hp}\right)$$

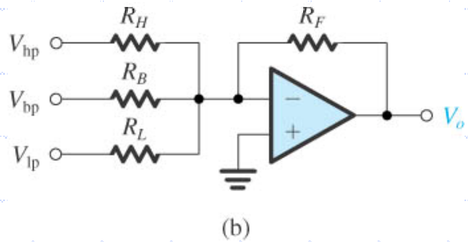
$$\frac{R_f}{R_1} = 1$$

$$\frac{R_3}{R_2} = 2Q-1 \quad (\because R_f = R_1)$$

$$K = 2-(1/Q) \quad \left(\because R_f = R_1, \frac{R_3}{R_2} = 2Q-1\right)$$



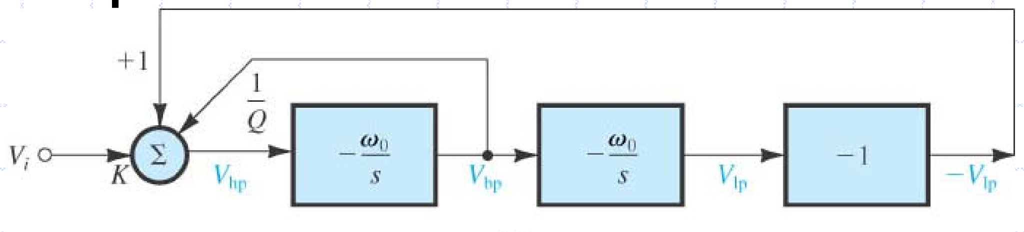
(a)



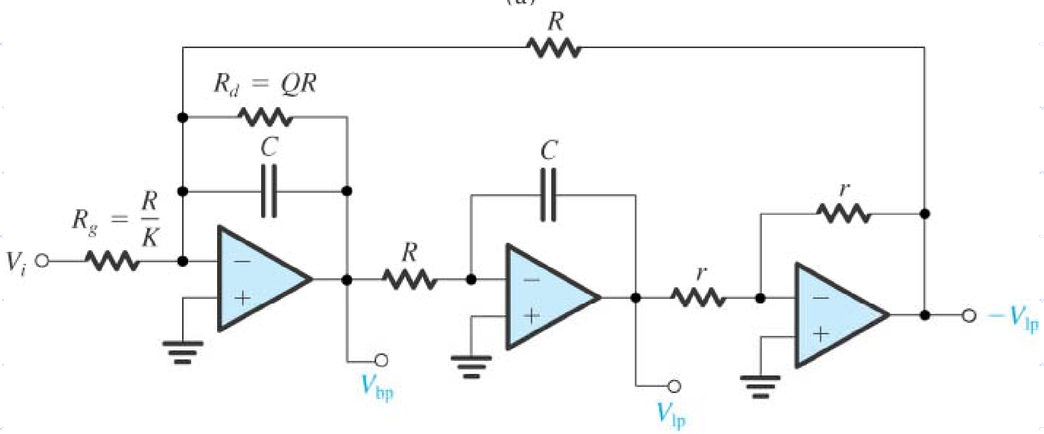
(b)

12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

◆ Circuit Implementation – Tow-Thomas Biquad



(a)

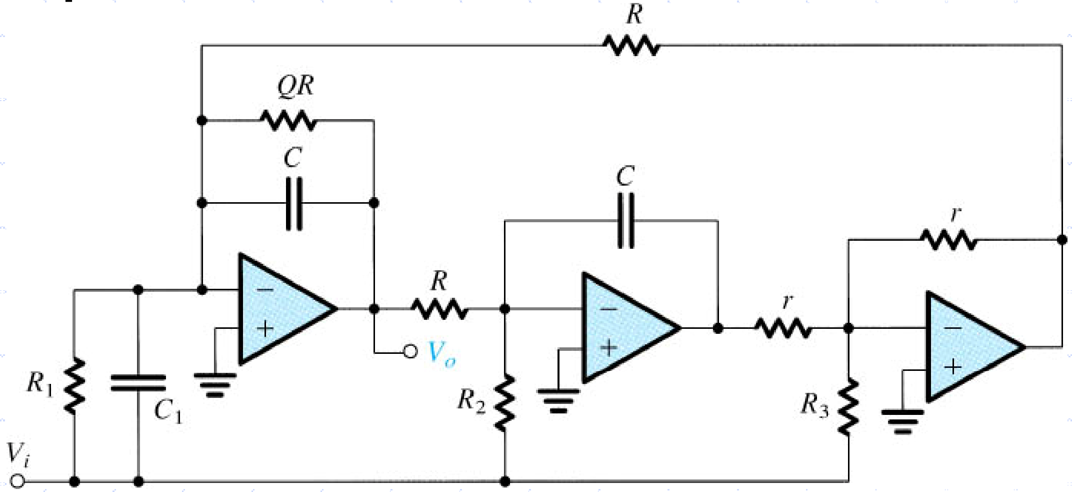


(b)

- All three op amp are used in a single-ended mode.
- All the coefficients of the summer have the same sign.
- High-pass function is no longer available.

12.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

◆ Circuit Implementation – Tow-Thomas Biquad



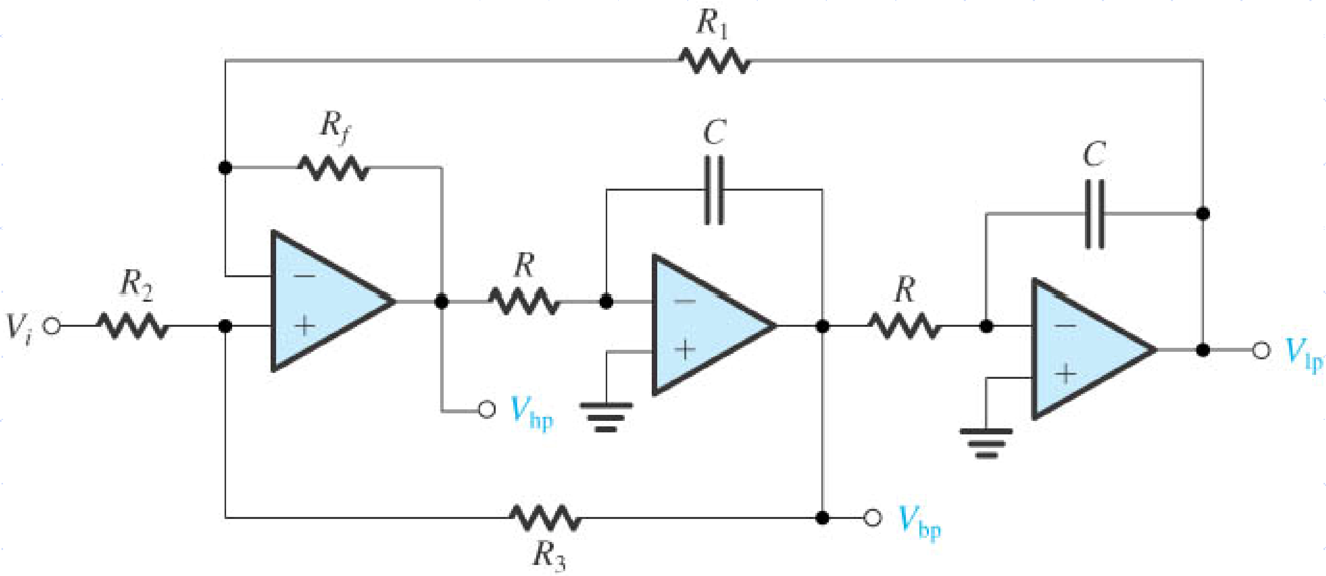
- Feedforward scheme is employed to realize the finite transmission zeros required for the notch and all-pass functions
- The virtual ground at the input of each of the three op amps permits the input signal to be fed to all three op amps.
- Transfer functions is

$$\frac{V_o}{V_i} = \frac{s^2(C_1/C) + s(1/C)(1/R_1 - r/RR_3) + 1/(C^2RR_2)}{s^2 + s(1/QCR) + 1/(CR)^2}$$

12.6 Second-Order Active Filters Based on Inductor Replacement

- Problem 12.49

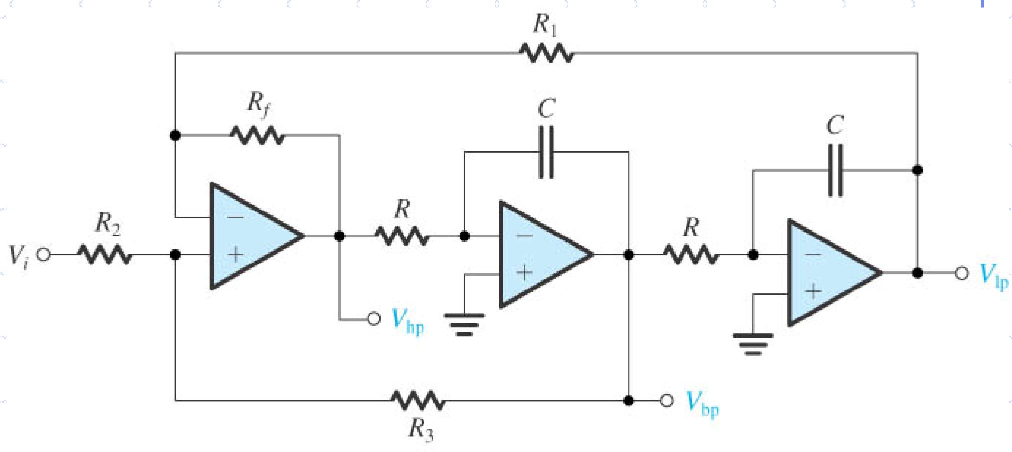
Design the KHN circuit to realize a bandpass filter with a center frequency of 1kHz and a 3-dB bandwidth of 50Hz. Use 10-nF capacitors.



(a)

12.6 Second-Order Active Filters Based on Inductor Replacement

- Problem 12.49 (cont.)
  - $B=w_0/Q$
    $\rightarrow Q=(2\pi 10^3)/(2\pi 50)=50$
  - Choose C=10nF
    $\rightarrow R=1/w_0C=15.92\text{k}\Omega$
  - Use $R_1=R_f=10\text{k}\Omega$,
    $R_3/R_2=2Q-1=39$
  - Choose $R_2=10\text{k}\Omega \rightarrow R_3=390\text{k}\Omega$



(a)

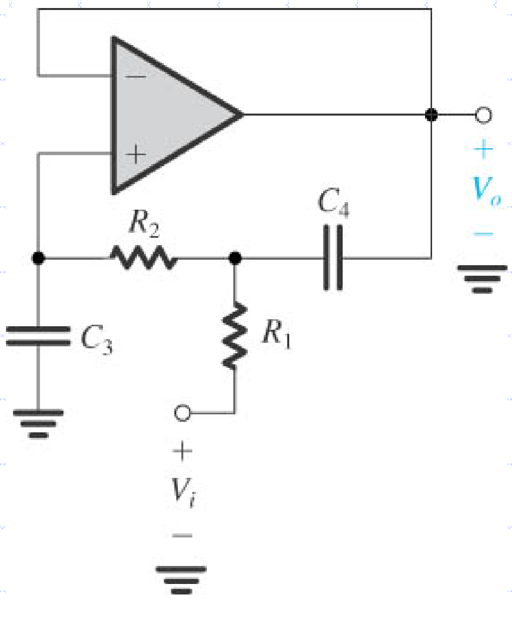
$$\frac{R_f}{R_1}=1$$

$$\frac{R_3}{R_2}=2Q-1 \ (\because R_f=R_1)$$

$$K=2-(1/Q) \ (\because R_f=R_1, \frac{R_3}{R_2}=2Q-1)$$

12.8 Single-Amplifier Biquadratic Active Filters

◆ the Sallen-and-Key circuits (p. 1132)



(c)

$$\frac{V_o}{V_i} = \frac{1/(C_3C_4R_1R_2)}{s^2 + s\frac{1}{C_4R_2}\left(1+\frac{R_2}{R_1}\right) + \frac{1}{C_3C_4R_1R_2}}$$

$$\omega_0 = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2}}\frac{C_4}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$DC\ gain = 1$$