Chapter 6 Mixing of Decaying Substances

Contents

Objectives

6.1 Non-Conservative Pollutants

6.1.1 Category of Non-Conservative Pollutants

1) Toxic Substance

  - Metals: mercury, cadmium, lead
  - Industrial chemicals: toluene, benzenes, phenols, PCB
  - Hydrocarbons: PAH (polycyclic aromatic hydrocarbons)
  - Agricultural chemicals: pesticides, herbicides, DDT
  - Radioactive substances

2) BOD-DO coupled system

3) Temperature

4) Suspended Solids

5) Bacteria and pathogens

6) Nutrients

7) Oil

[Cf] Conservative pollutants

- one which does not undergo any chemical or biochemical changes in transport

- no loss due to chemical reactions or biochemical degradation

- salt, chloride, total dissolved solids, some metals

6.1.2 Transport of Non-Conservative Pollutants

(1) Toxic Substance

Physio-chemical phases of the transport of toxic substances:

- loss of the chemical due to biodegradation, volatilization, photolysis, and other chemical and bio-chemical reactions
- sorption and desorption between dissolved and particulate forms in the water column and bed sediment
- settling and resuspension mechanisms of particulates between water column and bed sediment

Assume only loss of the chemical

$$\frac{\partial(hC)}{\partial t}+\frac{\partial}{\partial x}(uCh)+\frac{\partial}{\partial y}(vCh)=\nabla\cdot(hD\nabla C)+hS$$

where S = sink/source term

Assume first-order decay

- decay rate is proportional to the amount of material present

$$\frac{dC}{dt}=-kC=S$$

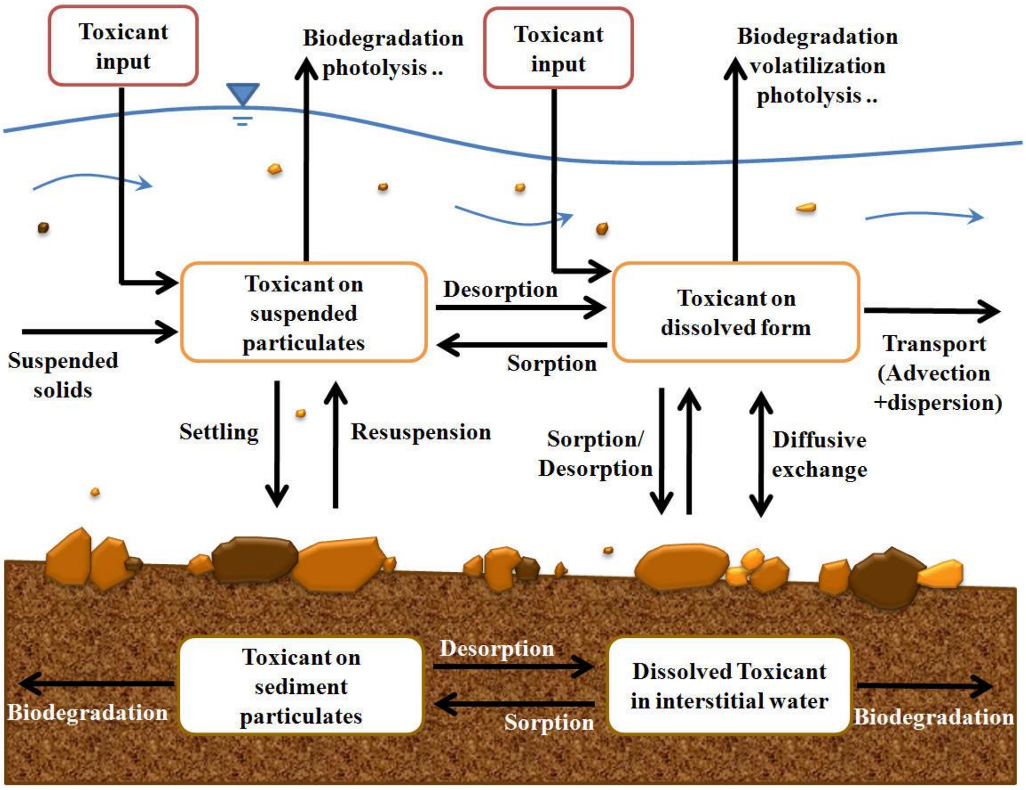
where C = mass/volume; S = mass/(volume · time); k = 1/time = decay rate

- Rate of disappearance of BOD due to biodegradation

- Radioactive substance also decay in strength in this way

- Coliform bacteria and pathogens die away with a rate of first-order decay

$$\frac{\partial(hC)}{\partial t}+\frac{\partial}{\partial x}(uCh)+\frac{\partial}{\partial y}(vCh)=\nabla\cdot(hD\nabla C)-khC$$



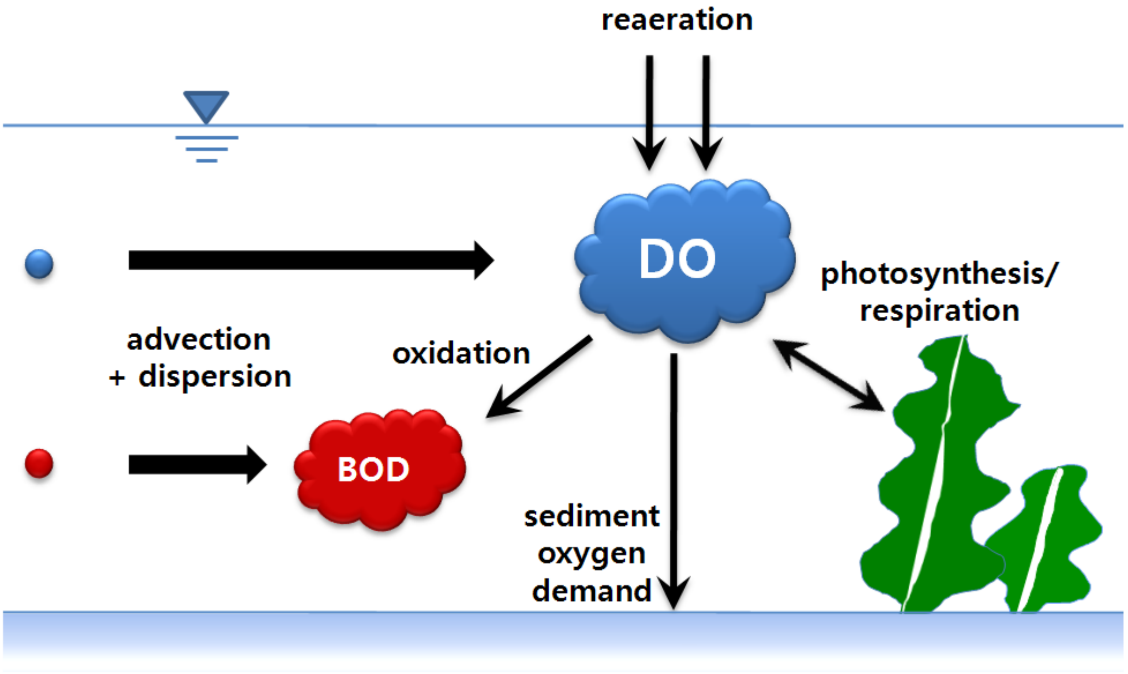
(2) BOD-DO

- Linked materials

- Behavior of one material depends upon the amount of another- Conc. of dissolved oxygen depends not only on transport of DO but also on the amount of BOD present

- Biodegradable substances undergo biochemical reactions

- Oxygen is used up in aerobic decomposition



(3) Heat transport

$$\frac{\partial hT}{\partial t}+\frac{\partial}{\partial x}(uTh)+\frac{\partial}{\partial y}(vTh)=\nabla\cdot(hD\nabla T)+hS$$

$$S=\frac{dT}{dt}=\frac{q_{net}}{\rho c_p h}=\frac{K(T-T_e)}{\rho c_p h}$$

K = atmospheric heat exchange coefficient ($W/m^2\,°C$)

c_p = specific heat of water

T_e = equilibrium temperature

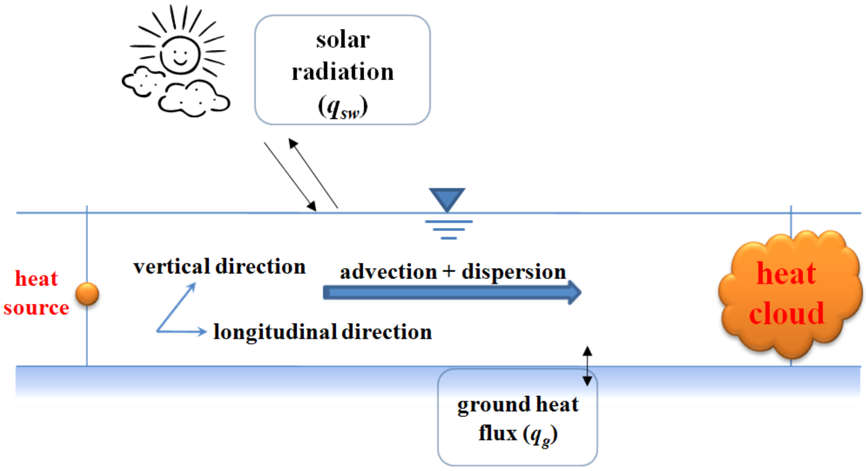
= temperature that a body of water would reach if all meteorological conditions were constant in time

Assume that u, v, h satisfy the continuity eq.

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{h}\nabla\cdot(hD\nabla T)+\frac{A}{\rho c_p}(T-T_e)$$

$$T_e=T_d+\frac{q_{sw}}{K}$$

T_e = dew point temperature



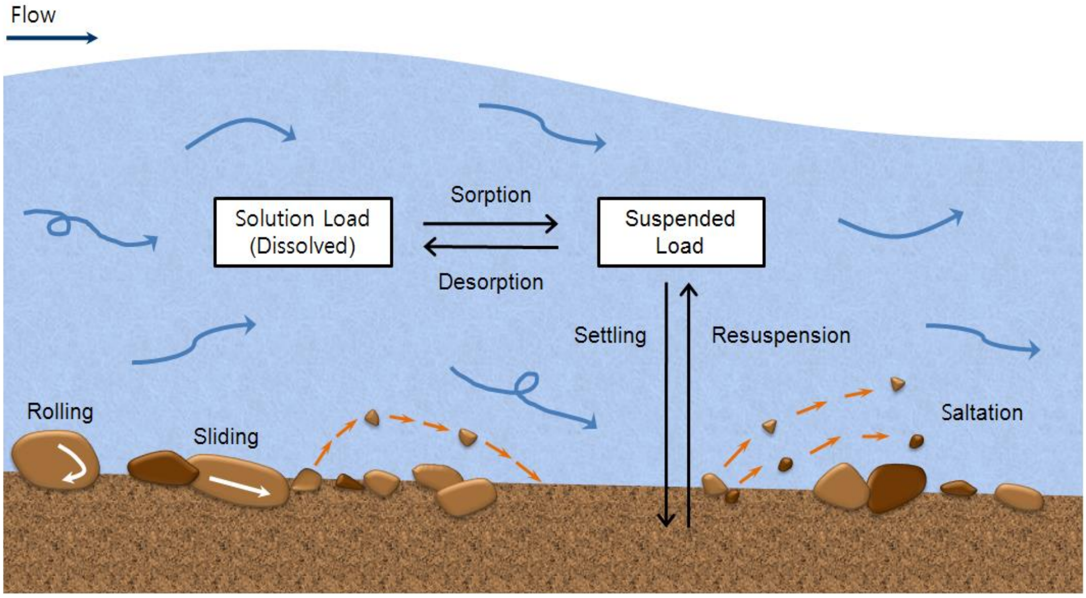
(4) Suspended Solids

Suspended solids= suspended sediments + particulate forms in the water column

$$\frac{\partial (hC)}{\partial t}+\frac{\partial}{\partial x}(uCh)+\frac{\partial}{\partial y}(vCh)=\nabla\cdot(hD\nabla C)+hS$$

$$S=\frac{W_s(C_{eq}-C)}{h}$$

$$C_{eq}=\frac{G_{gr}D\rho_s}{\rho h}$$

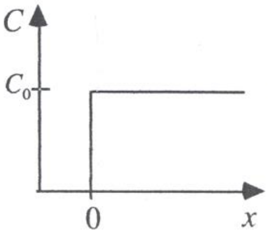
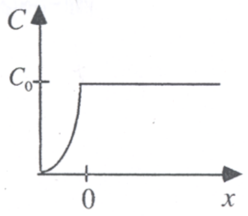


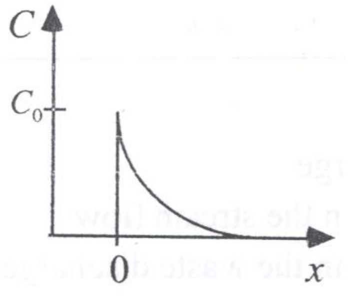
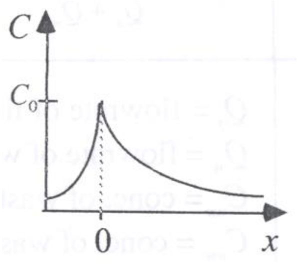
6.2 Analytical Solutions for Non-Conservative Pollutants

6.2.1 Steady state equations and solutions

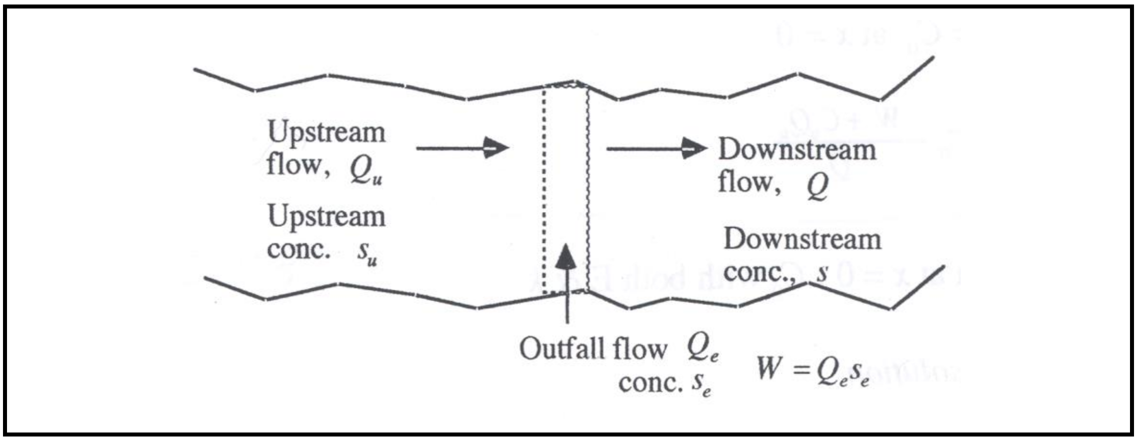
- Pollutant discharges are nearly constant over time (continuous discharge)
- Concentration in the stream will reach a steady state condition soon after the discharge starts.

$$\cancel{\frac{\partial C}{\partial t}} = -u\frac{\partial C}{\partial x} + E\frac{\partial^2 C}{\partial x^2} + S$$

I. Conservative Pollutant		
Item	W/O Dispersion (Case 1)	W/ Dispersion (Case 2)
G.E.	$0 = u\frac{\partial C}{\partial x}$	$0 = -u\frac{\partial C}{\partial x} + E\frac{\partial^2 C}{\partial x^2}$
B.C.'s	$C(0) = C_0$ - continuous discharge	$C(0) = C_0$ - continuous discharge
Solutions	$C(x) = C_0, \quad x \ge 0$ $C_0 = \frac{Q_s C_{ws} + Q_w C_{ww}}{Q_s + Q_w}$	$C = C_0 \exp(\frac{u}{E}x), \quad x < 0$ $C = C_0, \quad (x \ge 0)$
	Q_s = flowrate of the stream; Q_w = flowrate of waste discharge C_{ws} = conc. of waste material in the stream flow C_{ww} = conc. of waste material in the waste discharge	
Graphics		

II. Non-conservative Pollutant		
Item	W/O Dispersion (Case 3)	W/ Dispersion (Case 4)
G.E.	$0=-u\dfrac{\partial C}{\partial x}-kC$	$0=-u\dfrac{\partial C}{\partial x}+E\dfrac{\partial^2 C}{\partial x^2}-kC$
B.C.'s	$C(0)=C_0$ $C_0=\dfrac{W}{Q}=\dfrac{Q_s C_{ws}+Q_w C_{ww}}{Q_s+Q_w}$	$C(-\infty)=0$ $C(\infty)=0$ $C(0)=C_0$ $C_0=\dfrac{W}{Q}\dfrac{1}{\alpha}$
Solutions	$C(x)=C_0\exp(-\dfrac{k}{u}x),\, x\ge 0$	$C(x)=C_0\exp\left[\dfrac{u}{2E}(1+\alpha)x\right],\, x\le 0$ $C(x)=C_0\exp\left[\dfrac{u}{2E}(1-\alpha)x\right],\, x\ge 0$ $\alpha=\sqrt{1+\dfrac{4kE}{u^2}}$
Graphics		

- Boundary condition at the outfall



1) Conservative substance

Consider mass balance at discharge point (outfall)

Mass rate of substance upstream + mass rate added by outfall

= mass rate of substance immediately downstream from outfall assuming complete mixing (using midstream discharge of diffuser)

$$Q_u s_u + Q_e s_e = Qs \qquad \text{(a)}$$

$$W = Q_e s_e = \text{input waste load [M/T]} \qquad \text{(b)}$$

[Re] mass rate = flow rate × concentration

Flow continuity

$$Q_u + Q_e = Q \qquad \text{(c)}$$

Combine (a) ~ (c)

$$s = \frac{Q_u s_u + Q_e s_e}{Q} = \frac{Q_u s_u + W}{Q}$$

If $s_u = 0$

Then $s = \frac{W}{Q}$

Thus, boundary condition at the outfall is give as

$$C = C_0 = \frac{W + C_u Q_u}{Q} \; at \; x = 0$$

2) Non-conservative substance

Derive general solution to the governing equation below

$$0 = -u \frac{\partial C}{\partial x} + E \frac{\partial^2 C}{\partial x^2} - kC \tag{6.32}$$

This G.E. can be regarded as second order ordinary differential equation since C is a function of x.

It is a kind of characteristic equation.

Thus, we try a solution of the form $C = e^{mx}$, then $\frac{\partial C}{\partial x} = me^{mx}$ and $\frac{\partial^2 C}{\partial x^2} = m^2 e^{mx}$.

Applying these equations into G.E. gives characteristic equation as follow.

$$0 = -u(me^{mx}) + E(m^2 e^{mx}) - k(e^{mx}) = e^{mx}(Em^2 - um - k)$$

$$\therefore m_1,\, m_2 = \frac{u \pm \sqrt{u^2 + 4kE}}{2E} = \frac{u}{2E}\left\{1 \pm \sqrt{1 + \frac{4kE}{u^2}}\right\}$$

$$\therefore \therefore C_1 = e^{m_1 x} \text{ and } C_2 = e^{m_2 x}$$

Let $\alpha = \sqrt{1 + \frac{4kE}{u^2}} > 1$, then $m_1,\, m_2 = \frac{u}{2E}(1 \pm \alpha)$

These functions are linearly independent and hence form a fundamental set of solution. Therefore, general solution of the form of

$$C = K_1 \exp(m_1 x) + K_2 \exp\left(m_2 x\right)$$

K_1 and K_2 can be evaluated from B.C.'s.

Boundary conditions are

$$C = 0 \quad at \quad x = \pm\infty$$

$$C = C_0 \quad at \quad x = 0$$

For $x \le 0$

$$C = 0 = K_1 e^{m_1(-\infty)} + K_2 e^{m_2(-\infty)} = K_1 e^{-\infty} + K_2 e^{+\infty} \rightarrow K_2 = 0$$

$$C = C_0 = K_1 + K_2 = K_1 \qquad \therefore C_0 = K_1$$

For $x \geq 0$

$$C = 0 = K_1 e^{m_1 \infty} + K_2 e^{m_2 \infty} = K_1 e^{+\infty} + K_2 e^{-\infty} \rightarrow K_1 = 0$$

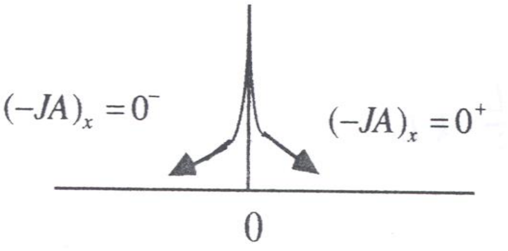
$$C = C_0 = K_1 + K_2 = K_2 \qquad \therefore C_0 = K_2$$

Solutions are

$$C = C_0 e^{m_1 x}, \quad x \leq 0$$

$$C = C_0 e^{m_2 x}, \quad x \geq 0$$

To get C_0, Consider flux



$$W = \left(-JA\right)_{x=0^-} + \left(-JA\right)_{x=0^+} = -\left(uC_0 - E\frac{\partial C}{\partial x}\bigg|_{x=0^-} \right) A + \left(uC_0 - E\frac{\partial C}{\partial x}\bigg|_{x=0^+} \right) A$$

$$\therefore \frac{W}{EA} = \left(\frac{\partial C}{\partial x} \right)_{x=0^-} - \left(\frac{\partial C}{\partial x} \right)_{x=0^+}$$

$x \leq 0: \quad C = C_0 e^{m_1 x}$

$$\frac{\partial C}{\partial x} = C_0 m_1 e^{m_1 x}; \quad at \quad x = 0^-, \quad \frac{\partial C}{\partial x} = C_0 m_1$$

$x \geq 0: \quad C = C_0 e^{m_2 x}$

$$\frac{\partial C}{\partial x} = C_0 m_2 e^{m_2 x}; \quad at \quad x = 0^+, \quad \frac{\partial C}{\partial x} = C_0 m_2$$

$$\therefore \frac{W}{EA} = C_0 m_1 - C_0 m_2 = C_0 \left(m_1 - m_2 \right)$$

$$= C_0 \frac{u}{2E} \left[(1+\alpha) - (1-\alpha) \right] = C_0 \frac{u}{2E} \left[2\alpha \right] = C_0 \frac{u}{E} \alpha$$

$$\boxed{\underline{C_0 = \frac{W}{uA} \frac{1}{\alpha} = \frac{W}{Q} \frac{1}{\alpha}}}$$

Note that

If $k = 0 \;\; or\, E = 0 \rightarrow \;\; C_0 = \dfrac{W}{Q}; \quad \alpha = \sqrt{1 + \dfrac{4kE}{u^2}} = 1$

▪Comparison of <u>initial mixing distance</u> X_m with <u>decay distance</u> X_d

Initial mixing distance X_m for steady flow (state)

$$X_m = 0.4\, \bar{u} W^2 / \varepsilon_t \qquad \text{(a)}$$

Distance required for decay of substance to a <u>factor of $\underline{e^{-1}}$ (0.3678)</u>

$$X_d = \bar{u} / k \qquad \text{(b)}$$

Therefore, ratio is given as

$$\frac{X_d}{X_m} = \frac{\bar{u}/k}{0.4\,\bar{u}W^2/\varepsilon_t} = 2.5\,\frac{\varepsilon_t}{kW^2} \qquad \text{(c)}$$

Set $\alpha_1 = \dfrac{4Ek}{\bar{u}^2}$ (d)

Eq. (5.19): $E = 0.011\,\bar{u}^2 W^2 / du^*$ (e)

Eq.(5.6): $\varepsilon_t = 0.6\,du^*$ (f)

Thus, combining these equations gives

$$\alpha_1 = 0.0264\,W^2 k/\varepsilon_t \qquad \text{(g)}$$

Equating (c) and (g) yields

$$\boxed{\frac{X_d}{X_m} = \frac{0.06}{\alpha_1}}$$

Consider analytical solution for non-conservative substance with dispersion and decay terms

$$\begin{aligned} C(x) &= C_0 \exp\left\{-\frac{kx}{\bar{u}}\left[\frac{2}{\alpha_1}\left(\sqrt{\alpha_1+1}-1\right)\right]\right\} \\ &= C_0 \exp\left\{-\frac{\bar{u}x}{2E}\left(\sqrt{\alpha_1+1}-1\right)\right\}, x \ge 0 \end{aligned} \qquad (6.33)$$

If $\frac{X_d}{X_m} > 1 \ \left(X_d > X_m\right)$

Then,

$$\frac{0.06}{\alpha_1} > 1 \ \rightarrow \ \alpha_1 < 0.06 \ \rightarrow \ \left[\left(\frac{2}{\alpha_1}\right)(\sqrt{\alpha_1 + 1} - 1\right] \approx 0.985 \approx 1$$

From Eq. (6.33), this means that

$$C \cong C_0 \exp\left\{-\frac{kx}{\bar{u}}\right\} \quad \rightarrow \text{solution for case w/o dispersion}$$

Therefore, there are two possibilities

(i) $X_d > X_m$

→ The longitudinal dispersion term has a negligible effect.

→ We may drop the dispersion term.

→ We may use an analytical solution. $C = C_0 \exp\left\{-\frac{kx}{\bar{u}}\right\}$

(ii) $X_d < X_m$

→ The material decays before it mixes across the cross section

→ Eq. (6.32) is not a suitable model.

→ The concentration distribution must be computed numerically.

6.2.2 Time Variable Analysis

Time variable behavior of water quality in river downstream of an outfall to describe

1) the downstream transport of peak in a waste-water discharge load;

2) an accidential spill of a chemical;

3) the day-to-day variation in water quality due to day-to-day changes in waste load inputs.

◆ Three types

W/O Dispersion	W/ Dispersion	
	Instantaneous Input	Continuous Input
▪ No mixing in the longitudinal direction ▪ Plug flow ▪ Impulse input remains its shape as it progress downstream	▪ Stream dispersion spreads the response temporally and spatially. ▪ used to estimate the transient effects due to inputs such as BOD from storm water overflow (by pass from combined sewer system) and batch discharges of municipal & industrial wastes	▪ Rectangular input ▪ $\eta = \frac{kE}{u^2}$ →dimensionless ▪ Relative effects of longitudinal mixing for a nonconservative substance ① upland stream $\eta \approx< 0.01$ → not significant ② Main drainage rivers $\eta \approx 0.01 \sim 0.5$ → significant ③ Large rivers $\eta \approx 0.5 \sim 1.0$

Time variable equations & solutions

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + E\frac{\partial^2 C}{\partial x^2} + S$$

$S = -kC$ → first-order decay

Item	W/O Dispersion	W/ Dispersion	
G.E.	$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} - kC$	$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + E\frac{\partial^2 C}{\partial x^2} - kC$	
Case	Case 5	Case 6	Case 7
B.C.'s	$C = (0) = C_0$ $C_0 = \frac{W(t)}{Q}$	Instantaneous Input	Continuous Input over an interval of time
		$M = M\delta(x)$	±Rectangular Input $C(x=0,\ t<\tau) = C_0$ $C(x=0,\ t>\tau) = 0$
Sol.	$C(x,t) = \frac{W(t-t^*)\exp\left(-\frac{kx}{u}\right)}{Q}$	$C(x,t) = \frac{M}{A\sqrt{4\pi Et}}$ $\exp\left[\frac{-(x-ut)^2}{4Et} - kt\right]$ mean = ut variance $\sigma^2 = 4Et$	$C(x,t) = \frac{C_0}{2}\cdot\exp\left(-\frac{kx}{u}\right)$ $\left[erf\left(\frac{x-u(t-\tau)(1+\eta)}{\sqrt{4E(t-\tau)}}\right.\right.$ $\left.\left.-erf\,\frac{x-nt(1+\eta)}{\sqrt{4Et}}\right)\right]$ $\tau =$time interval of input $\eta = \frac{kE}{u^2}$

6.3 Modeling BOD-DO Coupled System

6.3.1 Solutions of BOD-DO Coupled System

◆ Coupled system (BOD, DO)

- determination of dissolved oxygen concentrations downstream of a discharge of BOD

◆ Oxygen Demand

= indirect measure of organic materials (= organic pollutants) in terms of the amount of oxygen required to (completely) oxidize it.

COD - Chemical Oxygen Demand

BOD: CBOD - Carbonaceous BOD

NBOD - Nitrogeneous BOD

Organic matter + $O_2 \rightarrow CO_2 + H_2O$

◆ Importance of dissolved oxygen (DO)

- Anaerobic conditions in a stream are indicative of extreme pollution
- Low dissolved oxygen concentrations have severe effects on the kind of biota which inhabit the stream

◆ Sources and sinks of DO

(a) Sources

- Reaeration from the atmosphere
- Photosynthetic oxygen production
- DO in incoming tributaries

(b) Sinks

- Oxidation of BOD
- Oxygen demand of sediments of water body
- Use of oxygen for respiration by aquatic plants

$\therefore \dfrac{dC}{dt}$ = reaeration + (Photosynthesis-respiration) − oxydation of BOD

− sediment oxygen demand ± oxygen transport (into and out of segment)

Let C = concentration of DO

L = concentration of BOD

(1) rate of utilization of DO by BOD

$$\frac{dL}{dt} = -k_1 L \rightarrow \text{exertion of BOD}$$

= utilization of DO

= depletion of DO

k_1 = deoxygenation coefficient

(2) reaeration from the atmosphere

= diffuse of oxygen into the stream rate of reaeration

$\propto$ degree to which the water is unsaturated with oxygen

Let C_s = DO saturation concentration

then oxygen deficit, DOD = $C_s - C$

$\therefore$ rate of reaeration

$$\frac{dC}{dt} = +k_2 (C_s - C)$$

k_2 = reaeration coefficient (1/T)

$\therefore$ Conservation equation for C

$$\frac{\partial C}{\partial t} = -u \frac{\partial C}{\partial x} + E \frac{\partial^2 C}{\partial x^2} - k_1 L + k_2 (C_s - C)$$

Let $D = C_s - C$

Then $dD = -dC$

$$\therefore -\frac{\partial D}{\partial t} = +u\frac{\partial D}{\partial x} - E\frac{\partial^2 D}{\partial x^2} - k_1 L + k_2 D$$

$$\frac{\partial D}{\partial t} = -u\frac{\partial D}{\partial x} + E\frac{\partial^2 D}{\partial x^2} + k_1 L - k_2 D \Rightarrow \text{G.E. for DO Deficit}$$

For BOD

$$\frac{\partial L}{\partial t} = -u\frac{\partial L}{\partial x} + E\frac{\partial^2 L}{\partial x^2} - k_1 L \Rightarrow \text{G.E. for BOD concentration}$$

Let reaction terms

$$S_L = -k_r L$$

$$S_D = k_d L - k_a D$$

in which

L = concentration of remaining BOD

D = dissolved oxygen deficit (DOD) = $C_s - C$

C_s = DO saturation concentration

C = actual DO concentraion

k_r = BOD removal coefficient = $k_d + k_s$

k_s = settling coefficient

k_d = deoxygenation coefficient = biochemical degradation

k_a = reaeration coefficient

G.E.: unsteady state

BOD $\dfrac{\partial L}{\partial t} = -u\dfrac{\partial L}{\partial x} + E\dfrac{\partial^2 L}{\partial x^2} - k_r L$

DOD $\dfrac{\partial D}{\partial t} = -u\dfrac{\partial D}{\partial x} + E\dfrac{\partial^2 D}{\partial x^2} + k_d L - k_a D$

◆ Steady state, W/ Dispersion (Estuary)

(i) BOD: $0 = -u\dfrac{\partial L}{\partial x} + E\dfrac{\partial^2 L}{\partial x^2} - k_r L \quad \rightarrow$ same as case 4

$$L = L_0 \exp\left[\frac{u}{2E}\left(1+\alpha_r\right)x\right], \quad x \le 0$$

$$= L_0 \exp\left[\frac{u}{2E}\left(1-\alpha_r\right)x\right], \quad x \le 0$$

in which

$$L_0 = \frac{W}{Q\alpha_r}$$

$$\alpha_r = \sqrt{1 + \frac{4k_r E}{u^2}}$$

(ii) DOD: $0 = -u\frac{\partial D}{\partial x} + E\frac{\partial^2 D}{\partial x^2} + k_d L - k_a D$

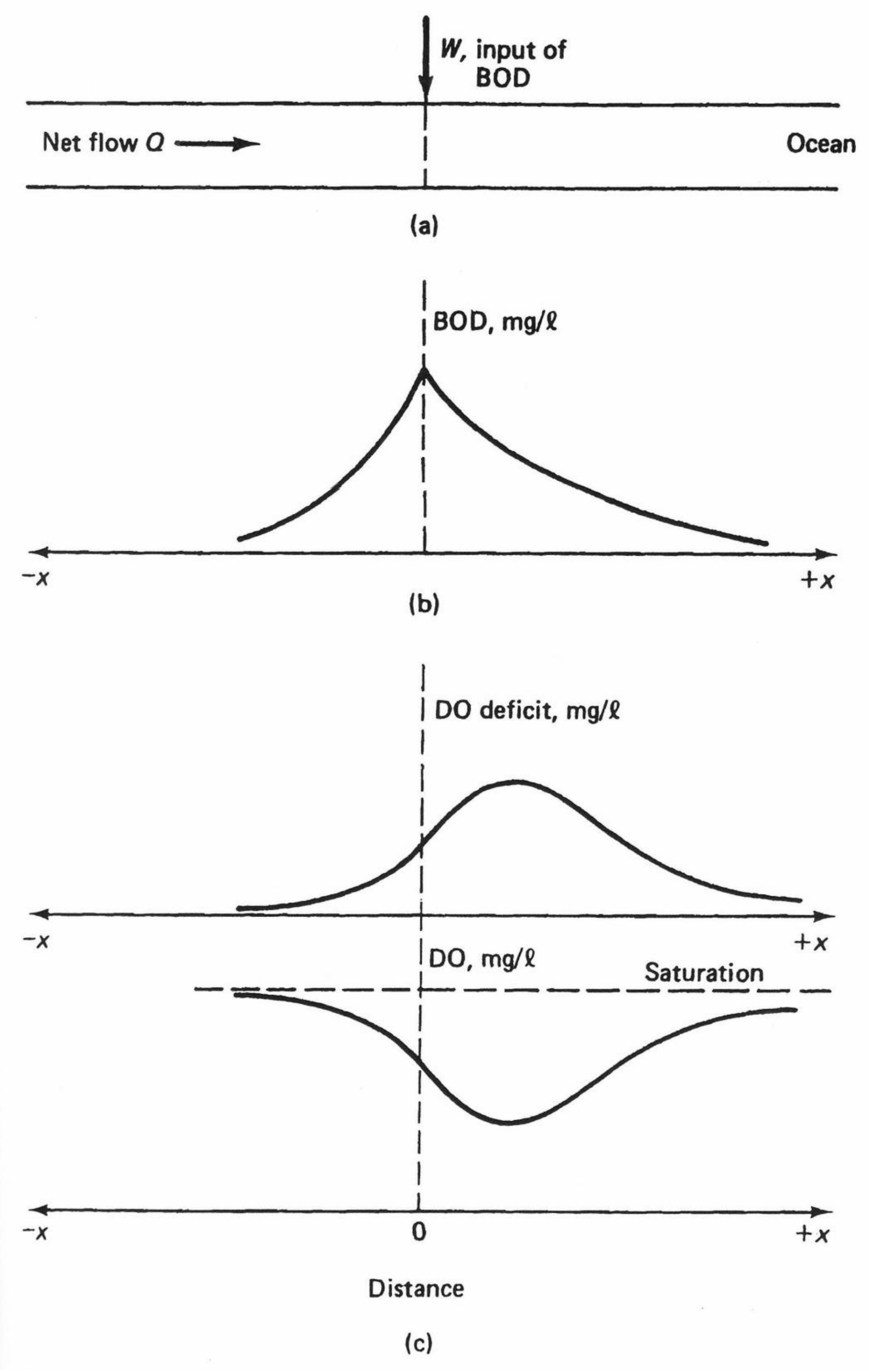
$$D = \frac{W}{Q}\frac{k_d}{k_a - k_r}\left\{\frac{\exp\left[\frac{u}{2E}(1+\alpha_r)x\right]}{\alpha_r} - \frac{\exp\left[\frac{u}{2E}\left(1-\alpha_a\right)x\right]}{\alpha_a}\right\}, \quad x \le 0$$

$$D = \frac{W}{Q}\frac{k_d}{k_a - k_r}\left\{\frac{\exp\left[\frac{u}{2E}(1-\alpha_r)x\right]}{\alpha_r} - \frac{\exp\left[\frac{u}{2E}\left(1+\alpha_a\right)x\right]}{\alpha_a}\right\}, \quad x \ge 0$$

in which

$$\alpha_a = \sqrt{1 + \frac{4k_a E}{u^2}}$$

Thomann & Mueller (1987)



6.3.2 Streeter-Phelps Equation

◆ Streeter-Phelps Equation (1925)

- no dispersion (river) $E = 0$

- steady state

BOD: $0 = -u\dfrac{\partial L}{\partial x} - k_1 L$

DOD: $0 = -u\dfrac{\partial D}{\partial x} + k_1 L - k_2 D$

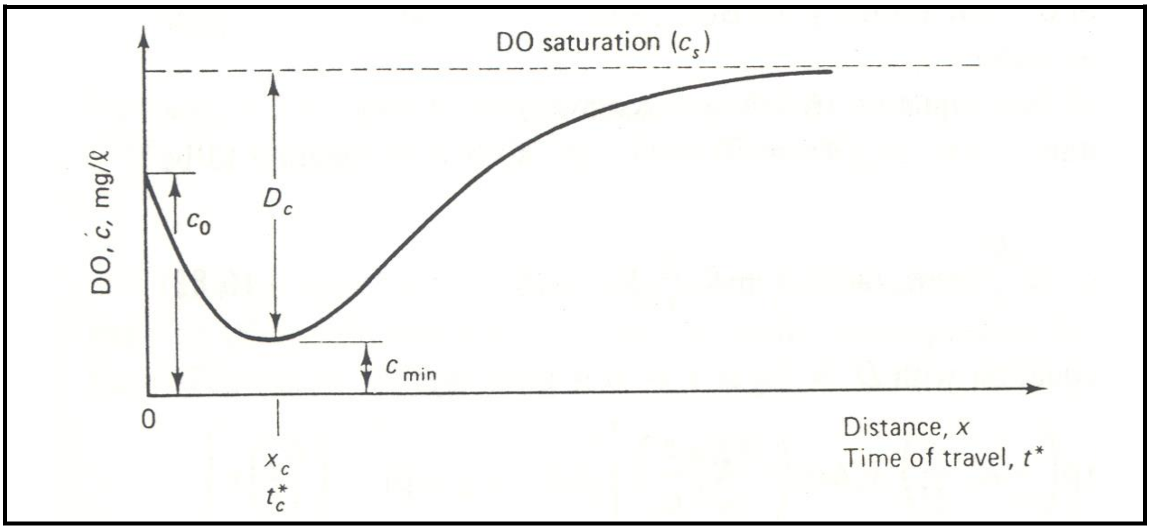
For BOD we have solution (Case 3)

$$\therefore L = L_0 \exp(-\frac{k_1}{u}x) = \frac{W}{Q}\exp(-\frac{k_1}{u}x)$$

B.C.: $D(0) = D_0 = C_s - C_0$

Solution:

$$D(x) = \frac{k_1}{k_2 - k_1} L_0 \left[e^{-\left(\frac{k_1}{u}\right)x} - e^{-\left(\frac{k_2}{u}\right)x} \right] + D_0 e^{-\left(\frac{k_2}{u}\right)x}, \quad x \ge 0 \qquad (1)$$



◆ Critical deficit D_c (@ t_c uptake of oxygen by BOD is just balanced by the input of oxygen from atmosphere)

let $x/u = t$ (= time of flow) time of travel

→ Then Eq. (1) becomes

$$D(t) = \frac{k_1}{k_2 - k_1} L_0 \left[e^{-k_1 t} - e^{-k_2 t} \right] + D_0 e^{-k_2 t}, \quad x \geq 0 \tag{2}$$

→ t_c may be found as

$$\frac{\partial D}{\partial t} = 0; \quad t_c = \frac{1}{k_2 - k_1} \ln \left\{ \frac{k_2}{k_1} \left[1 - \frac{(k_2 - k_1)}{k_1} \frac{D_0}{L_0} \right] \right\}$$

$$D_c = \frac{k_1}{k_2} L_0 e^{-k_1 t_c} \qquad \leftarrow \quad \text{from Eq.(2)}$$

◆ Modified Streeter-Phelps equation

- account for other processes

(1) BOD removal due to sedimentation

- non-oxygen-demanding

- reduces BOD w/o changing oxygen concentration

$$\frac{\partial L}{\partial t} = -k_3 L$$

(2) BOD addition due to scour from the bottom and surface runoff from the land

- rate of addition is assumed to be constant

(3) Oxygen use other than by aerobic biochemical oxygen demand in the water, and oxygen addition other than through reaeration.

- net of these processes is constant rate A

◆ Solution :

$$D_t = \frac{k_1}{k_2 - (k_1 + k_3)}\left(L_0 - \frac{P}{k_1 + k_2}\right)\left\{e^{-(k_1+k_2)t} - e^{-k_2 t}\right\}$$

$$+\frac{k_1}{k_2}\left(\frac{P}{k_1 + k_2} - \frac{A}{k_1}\right)\left(1 - e^{-k_2 t}\right)$$

Consider only k_3 neglect A & P

Let $k_r = k_1 + k_3$, $k_d = k_1$, $k_s = k_3$, $k_a = k_2$

Then

$$D_t = \frac{k_d}{k_a - k_r} L_0 \left\{e^{-k_r t} - e^{-k_a t}\right\} + D_0 e^{-k_a t}$$

[Remark]

G.E.: $0 = -u\dfrac{\partial L}{\partial x} - k_r L$ BOD

$0 = -u\dfrac{\partial D}{\partial x} + k_d L - k_a D$ DOD