Chapter 2 Turbulent Jet and Plumes

2.1 Introduction

2.2 Jets and Plumes

2.3 Environmental parameters

2.4 Buoyant Jet Problem and the Entrainment Hypothesis

2.5 Boundary Effects on Turbulent Buoyant Jets

Objectives:

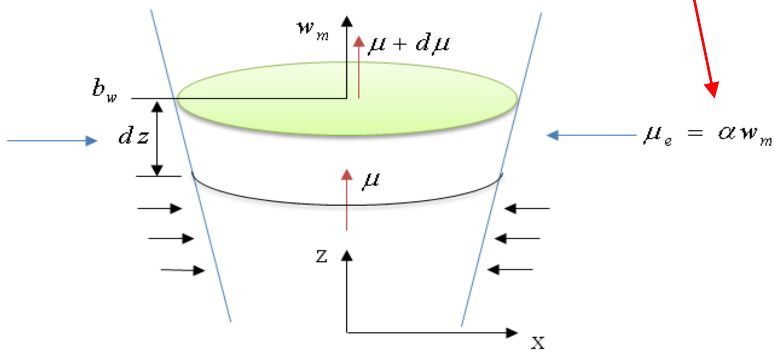
- Study buoyant jets and plumes, strong man-induced flow patterns used to achieve rapid initial dilutions for water quality control
- Understand the theory of jets and plumes before considering the special type of discharge structure for diluted wastes
- Give the design engineer a firm background in the fundamentals of the theory essential to the prediction of how a given discharge system will perform

2.4 Buoyant Jet Problem and the Entrainment Hypothesis

The basis of the entrainment hypothesis method is to relate the rate of inflow of diluting water to the local properties of the jet, especially its local mean velocity.

Taylor's entrainment hypothesis (Morton et al., 1956):

~ The velocity of inflow of diluting water into jet is proportional to the maximum mean velocity in the jet at the level of inflow.



For round jets,

$$\frac{d\mu}{dz} = 2\pi b_w \alpha w_m \tag{2.107}$$

where μ = specific volume flux; b_w = radius of the jet; α = entrainment coefficient

$\boxed{u_e = \alpha w_m = \text{entrainment velocity at } b_w}$

[Re] For axisymmetric jet

Flow in $= \mu + entrainment = \mu + \alpha w_m (2\pi b_w) dz$

Flow out $= \mu + d\mu$

$$\mu + \alpha w_m (2\pi b_W) dz = \mu + d\mu$$

$$\boxed{\frac{d\mu}{dz} = \alpha w_m 2\pi b_W}$$

2.4.1 Equation of Motion

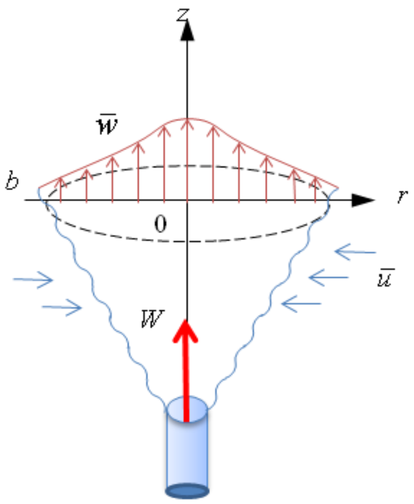
1) Mass conservation equation

Time-averaged volume conservation equation is given as

$$\frac{1}{r}\frac{\partial(\bar{u}r)}{\partial r} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad (2.108)$$

Multiply (2.108) by $2\pi r$

$$2\pi r \frac{\partial \bar{w}}{\partial z} = -2\pi \frac{\partial}{\partial r}(r\bar{u}) \qquad (1)$$



Integrate (1) over the jet cross section across some radius $b(z) \ll b_W$

$$\int_0^{b(z)} 2\pi r \frac{\partial \bar{w}}{\partial z} dr = -2\pi \int_0^{b(z)} \frac{\partial}{\partial r}(r\bar{u})\, dr = -2\pi \left[r\bar{u} \right]_0^{b(z)} = -2\pi b \bar{u}_b \qquad (2)$$

Apply Leibnitz rule:

$$\frac{d}{d\alpha}\int_{u_o(\alpha)}^{u_1(\alpha)} f(x,\alpha)\,dx \;=\; f(u_1,\alpha)\frac{du_1}{d\alpha} - f(u_0,\alpha)\frac{du_0}{d\alpha} + \int_{u_0}^{u_1}\underbrace{f_\alpha(x,\alpha)}_{\frac{\partial f}{\partial\alpha}}\,dx$$

$$\frac{d}{dz}\int_0^{b(z)}\underbrace{2\pi r\,\bar{w}}_{f(x,\alpha)}\,dr \;=\; 2\pi b\,\bar{w}(b(\bar{z}),z)\frac{db}{dz} - 0 + \int_0^{b(z)} 2\pi r\frac{\partial\bar{w}}{\partial z}\,dr$$

$$\int_0^{b(z)} 2\pi r\frac{\partial\bar{w}}{\partial z}\,dr = \frac{d}{dz}\int_0^{b(z)}\underbrace{2\pi r\,\bar{w}}_{f(x,\alpha)}\,dr \;-\; 2\pi b\,\bar{w}(b(\bar{z}),z)\frac{db}{dz} \tag{3}$$

Substituting (3) into (2) gives

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\bar{w}\,dr \;=\; -2\pi b\bar{u}_b + 2\pi b\,\bar{w}(b(\bar{z}),z)\frac{db}{dz} \tag{2.109}$$

As $b(z)$ becomes large, then $\bar{w}(b(z),z) \;\to\; 0$

$$\therefore\quad \underbrace{\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\bar{w}\,dr}_{=\,\mu} \;=\; -\lim_{r\to b(z)}\left[2\pi r\bar{u}\right] \;=\; -2\pi b\bar{u}_b$$

$$\frac{d\mu}{dz} \;=\; -\lim_{r\to b(z)}\left[2\pi r\bar{u}\right] \;=\; -2\pi b\bar{u}_b \;=\; 2\pi b_W(\alpha w_m) \tag{2.110}$$

$$\boxed{u_b \;=\; \alpha\, w_m}$$

→ This is the same result with Eq. (2.107).

2) Momentum equation

Integrate vertical momentum equation in cylindrical polar coordinates across the jet

$$\text{z-eq.:}\quad \underbrace{\frac{d}{dz}\int_0^{b(z)}\left[\bar{w}^2+w'^2+\frac{\bar{p}-p(\infty)}{\rho_o}-\frac{\tau_{zz}}{\rho_o}\right]2\pi r\,dr}_{\mathrm{I}}$$

$$=\underbrace{-\lim_{r\to b(z)}\left[2\pi r\left(\bar{u}\,\bar{w}+\overline{u'w'}-\frac{\tau_{rz}}{\rho_o}\right)\right]}_{\mathrm{II}}$$

$$\underbrace{+\frac{db(z)}{dz}2\pi b(z)\left[\bar{w}^2+\overline{w'}^2+\frac{\bar{p}-p(\infty)}{\rho_o}-\frac{\tau_{zz}}{\rho_o}\right]_{b(z)}}_{\mathrm{III}}\;\underbrace{-\int_0^{b(z)}2\pi r\,g\left(\frac{\bar{\rho}-\rho_a}{\rho_o}\right)dr}_{\mathrm{IV}}\qquad(2.111)$$

where τ_{zz} , τ_{rz} = viscous stresses ~ neglect

$\bar{p}-p(\infty)$ =dynamic pressure distribution

$w' = w - \bar{w}$

ρ_o =reference density

ρ_a =ambient density distribution

$\bar{\rho}$ =time-averaged density in the jet

I = rate of change of flow force of the jet

II = flux of momentum into jet by radial flow

III = flux of axial momentum through the sloping sides of the jet

IV = accelerating force per unit mass of fluid in the jet arising from the density distribution across the jet

Assume II and III are negligible.

Assume net advective flux of axial momentum of (I) is much greater than the transfer by w', τ_{zz}, and pressure gradient force,

$$\underbrace{\int_0^{b(z)} 2\pi r \bar{w}^2 \, dr}_{= m(z)} \gg \left| \int_0^{b(z)} 2\pi r \left(\overline{w'}^2 + \frac{\bar{p} - p(\infty)}{\rho_o} - \frac{\tau_{zz}}{\rho_o} \right) dr \right|$$

Then, simple vertical momentum equation is

$$\frac{dm(z)}{dz} = -\int_0^{b(z)} 2\pi r g \underbrace{\left(\frac{\bar{\rho} - \rho_a}{\rho_o} \right)}_{\theta} dr = -\int 2\pi r g \theta \, dr \qquad (2.113)$$

→ This equation is similar to Eq. (2.70).

→ rate of change of vertical momentum flux = vertical buoyancy force acting per unit height of jet

For discharges with no buoyancy force, $\bar{\rho} = \rho_a$

Then Eq. (2.113) becomes

$$\frac{dm(z)}{dz} = 0$$

→ The momentum flux is conserved.

→ $m(z) = M$

3) Conservation of tracer equation

For steady, 2-D flow in polar coordinates, tracer conservation equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u\,C)+\frac{\partial}{\partial z}(wC) = 0 \qquad (1)$$

Decompose velocity and concentration

$$u = \bar{u} + u'\text{, and } C = \bar{C} + C' \qquad (2)$$

Substitute (2) into (1), then average over time, and ignore turbulent transport terms

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,\bar{u}\,\bar{C})+\frac{\partial}{\partial z}(\bar{w}\bar{C}) = 0 \qquad (3)$$

Integrate (3) over $b(z)$ (same as with Eq. (2.108)), and apply Leibnitz rule

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\bar{w}\bar{C}\,dr = -\lim_{r\to b(z)}(2\pi r\,\bar{u}\bar{C}) + \frac{db(z)}{dz}2\pi b(z)\left[\bar{w}\bar{C}\right]_{b(z)} \qquad (2.114)$$

Recall Eq. (2.109)

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\bar{w}\,d\,r = -2\pi b\bar{u}_b + 2\pi b\,\bar{w}\,(b(\bar{z}), z)\frac{db}{dz}$$

Multiplying (2.109) by C_a (ambient concentration of tracer) gives

$$C_a\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\bar{w}\,dr = -2\pi b\bar{u}_b C_a + 2\pi b\,\bar{w}(\,b(\bar{z}),\ z\,)C_a\frac{db}{dz} \qquad (4)$$

Add $\dfrac{dC_a}{dz}\displaystyle\int_0^{b(z)} 2\pi r \overline{w}\, dr$ to each side of (4)

$$\text{L.H.S.} = \frac{d}{dz}\left\{C_a \int_0^{b(x)} 2\pi r \overline{w}\, dr\right\} = \frac{d}{dz}\int_0^{b(x)} 2\pi r \overline{w}\, C_a\, dr$$

Then, Eq. (4) becomes

$$\frac{d}{dz}\int_0^{b(x)} 2\pi r \overline{w}\, C_a\, dr = -\lim_{r\to b(z)}(2\pi r \overline{u}\, C_a) + \frac{db(z)}{dz} 2\pi b(z)[\overline{w}\overline{C}]_{b(z)}$$

$$+\frac{dC_a}{dz}\int_0^{b(z)} 2\pi r \overline{w}\, dr \qquad (2.115)$$

$\overline{C} \approx C_a$ as $r \to b$

Now, subtract Eq. (2.115) from Eq. (2.114)

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r \overline{w}\overline{C}\, dr = -\cancel{\lim_{r\to b(z)}(2\pi r \overline{u}\overline{C})} + \cancel{\frac{db(z)}{dz} 2\pi b(z)[\overline{w}\overline{C}]_{b(z)}} \quad (2.114)$$

$$-\frac{d}{dz}\int_0^{b(z)} 2\pi r \overline{w} C_a\, dr = -\cancel{\lim_{r\to b(z)}(2\pi r \overline{u}\, C_a)} + \cancel{\frac{db(z)}{dz} 2\pi b(z)[\overline{w}\overline{C}]_{b(z)}}$$

$$+\frac{dC_a}{dz}\int_0^{b(z)} 2\pi r \overline{w}\, dr \qquad (2.115)$$

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r \overline{w}(\overline{C} - C_a)\, dr = -\frac{dC_a}{dz}\int_0^{b(z)} 2\pi r \overline{w}\, dr \qquad (2.116)$$

Now, relate tracer concentrations to density variations as

$$\frac{\overline{\rho} - \rho_o}{\rho_o} = \gamma\left(\frac{\overline{C} - C_o}{C_o}\right) \qquad (2.117)$$

where ρ_o = reference density

C_o = reference concentration

γ = coefficient of proportionality

For density stratified ambient,

$$\bar{\rho} = \rho_a - \rho_o \theta(r, z, t) \tag{2.118}$$

where

$$\rho_a = \rho_o (1 - \varepsilon(z)) \tag{2.119}$$

$\theta(r, z)$ =time-averaged density anomaly caused by jet $= \dfrac{\bar{\rho} - \rho_a}{\rho_o}$

Differentiating Eq. (2.119) gives

$$-\frac{d\varepsilon}{dz} = \frac{d\rho_a}{dz}\frac{1}{\rho_o} \tag{1}$$

Eq. (2.117) becomes

$$\frac{\rho_a - \rho_o}{\rho_o} = \gamma\left(\frac{C_a - C_o}{C_o}\right) \tag{2}$$

Differentiate (2) w.r.t. z

$$\frac{d\rho_a}{dz}\frac{1}{\rho_o} = \gamma\frac{dC_a}{dz}\frac{1}{C_o}$$

$$\frac{dC_a}{dz} = \frac{C_0}{\rho_0}\frac{1}{\gamma}\frac{d\rho_a}{dz} \qquad (3)$$

Substitute (1) into (3)

$$\frac{dC_a}{dz} = -\frac{d\varepsilon}{dz}\frac{C_0}{\gamma} \qquad (4)$$

Rearrange (2.117)

$$\overline{C} - \overline{C}_a = \frac{C_0}{\gamma}\left(\frac{\overline{\rho} - \rho_a}{\rho_o}\right) \qquad (5)$$

Substitute (2.118) into (5)

$$\overline{C} - \overline{C}_a = \frac{C_0}{\gamma}\left(-\theta\right) \qquad (6)$$

Substitute (4) and (6) into Eq. (2.116)

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\overline{w}\,\cancel{\frac{C_0}{\gamma}}(-\theta) = -\left(-\frac{d\varepsilon}{dz}\cancel{\frac{C_0}{\gamma}}\right)\int_0^{b(z)} 2\pi r\,\overline{w}\,dr$$

$$\frac{d}{dz}\int_0^{b(z)} 2\pi r\,\overline{w}\,\theta\,dr = -\frac{d\varepsilon}{dz}\underbrace{\int_0^{b(z)} 2\pi r\,\overline{w}\,dr}_{\mu}$$

$$\frac{d}{dz}\underbrace{\int_0^{b(z)} 2\pi r\,\overline{w}\,\theta\,dr}_{\beta/g} = -\frac{d\varepsilon}{dz}\,\mu(z) \qquad (2.120)$$

Define specific buoyancy flux of jet as

$$\beta = \int_0^{b(z)} 2\pi r\, g\, \overline{w}\theta \, dr \tag{2.121}$$

Then Eq. (2.120) becomes

$$\boxed{\frac{d\beta}{dz} = -g\frac{d\varepsilon}{dz}\mu} \tag{2.122}$$

For stable density gradient, $\varepsilon' > 0$

→ Buoyancy flux decreases as the volume flux in the jet increases.

• Summary

1. Governing equation of motion for a vertical buoyant jet in a density-stratified ambient

1) volume flux: $$\frac{d\mu}{dz} = -\lim_{r \to b(z)} (2\pi r \bar{u}) \tag{2.123}$$

2) momentum flux: $$\frac{dm}{dz} = \int_0^{b(z)} 2\pi r\, g\, \theta\, dr \tag{2.124}$$

3) buoyancy flux: $$\frac{d\beta}{dz} = -g \frac{d\varepsilon}{dz} \mu \tag{2.125}$$

where

$$\mu = \int_0^{b(z)} 2\pi r \bar{w}\, dr \tag{2.126}$$

$$m = \int_0^{b(z)} 2\pi r \bar{w}^2\, dr \tag{2.127}$$

$$\beta = \int_0^{b(z)} 2\pi r\, g\, \bar{w} \theta\, dr \tag{2.128}$$

2. Unknowns

$$\bar{u}\,,\quad \bar{w}\,,\quad \theta\left(= \frac{\bar{\rho} - \rho_a}{\rho_o} \right)$$

b ← spreading eq. / entrainment eq.

x, y, z ← geometric equations

→ Integration of (2.123)~(2.128) leads to Jet Integral Model.

2.4.2 Application to Density-stratified Environment

Entrainment hypothesis: $\bar{u} = \alpha w_m$

Combine Eq. (2.107) and Eq. (2.123)

$$-\lim_{r \to b(z)} 2\pi r \bar{u} = 2\pi \alpha b_W w_m \tag{2.129}$$

Adopt Gaussian distributions for $\bar{w}$ and θ ← similarity assumption

$$\bar{w} = w_m \exp\left[-\left(\frac{r}{b_w}\right)^2\right] \tag{2.130}$$

$$\theta = \theta_m \exp\left[-\left(\frac{r}{b_T}\right)^2\right] \tag{2.131}$$

Adopt constant value for ratio of half-width, b_T / b_W

$$b_T / b_W = \lambda = 1.2 \tag{2.132}$$

Substituting Eqs. (2.129) ~ (2.132) into Eqs. (2.123) ~ (2.128), and assuming $b(z) \to \infty$

gives a set of <u>3 ordinary differential equations</u> for w_m, θ_m, b_W

$$\frac{d}{dz}(\pi b_W^2 w_m) = 2\pi \alpha b_W w_m \tag{2.133}$$

$$\frac{d}{dz}(\frac{\pi}{2} b_W^2 w_m^2) = \pi g \lambda^2 b_W^2 \theta_m \tag{2.134}$$

$$\frac{d}{dz}(\frac{\pi g \lambda^2 b_W^2 w_m \theta_m}{1+\lambda^2}) = -g \frac{d\varepsilon}{dz} \pi b_W^2 w_m \tag{2.135}$$

Initial condition for w_m , θ_m , b_W are given as

$$\left[\pi b_W{}^2 w_m \right]_0 = Q \tag{2.136}$$

$$\left[\frac{\pi}{2} b_W{}^2 w_m{}^2 \right]_0 = M \tag{2.137}$$

$$\left[\pi g \frac{\lambda^2}{1+\lambda^2} w_m b_W{}^2 \theta_m \right]_0 = B \tag{2.138}$$

[Re] Derivation

Substitute (2.130) into (2.126), and then integrate

$$\mu = \int_0^\infty 2\pi r w_m e^{-\left(\frac{r}{b_w}\right)^2} dr$$

$$= -\pi w_m b_W{}^2 \int_0^\infty \left(-\frac{2r}{b_W{}^2} \right) e^{-\left(\frac{r}{b_w}\right)^2} dr$$

$$= -\pi w_m b_W{}^2 \int_0^\infty \left[\frac{d}{dr} \left\{ e^{-\left(\frac{r}{b_w}\right)^2} \right\} \right] dr$$

$$= -\pi w_m b_W{}^2 \left[e^{-\left(\frac{r}{b_w}\right)^2} \right]_0^\infty$$

$$= -\pi w_m b_W{}^2 \left[0-1 \right] = \pi w_m b_W{}^2$$

$$\therefore \mu = \pi w_m b_W{}^2$$

$$\frac{d\mu}{dz} = \frac{d}{dz}(\pi\, w_m\, b_W^{\,2}) \qquad (1)$$

Substitute (1) into Eq. (2.123)

$$\frac{d}{dz}(\pi\, b_W^{\,2} w_m) = 2\pi\alpha\, b_W\, w_m \qquad (2.133)$$

[Re] Jet integral model

Solutions of Eqs. (2.133) ~ (2.135) are the same as the asymptotic solution obtained by the dimensional analysis and experiments, given in Table 2.2 and 2.3.

- Entrainment coefficient

(i) For jets,

Eq. (2.12): $$w_m \frac{Q}{M} = 7.0\ \frac{l_Q}{z} \qquad (1)$$

From Table 2.2: $$b_W = 0.107\, z \qquad (2)$$

Substitute (1) ~ (2) into Eq. (2.133)

$$\frac{d}{dz}\left\{\pi (0.107)^2\, z^2 \left(7.0\, W\, \frac{l_Q}{z}\right)\right\} = 2\pi\alpha(0.107\, z)\left(7.0\, W\, \frac{l_Q}{z}\right)$$

$$(0.107)^2 = 2\alpha(0.107)$$

$$\therefore\ \alpha_j = \frac{0.107}{2} = 0.0535 \pm 0.0025 \qquad (2.139)$$

→ entrainment coefficient for jet

(ii) For plumes, use Table 2.3

$$w_m = (4.7 \pm 0.2)\, B^{1/3} z^{-1/3} \qquad (3)$$

$$b_W = (0.100 \pm 0.005)\, z \qquad (4)$$

Substitute (3) ~ (4) into Eq. (2.133)

Then,

$$\pi b_W{}^2 w_m = \pi \left((0.100 \pm 0.005)^2 z^2\right)\left((4.7 \pm 0.2)\frac{B^{1/3}}{z^{1/3}}\right) = \pi (0.100 \pm 0.005)^2 (4.7 \pm 0.2) B^{1/3} z^{5/3}$$

L.H.S. $= \dfrac{d}{dz}(\pi b_W{}^2 w_m) = \dfrac{5}{3}\pi (0.100 \pm 0.005)^2\, z^{2/3} (4.7 \pm 0.2)\, B^{1/3}$

R.H.S. $= 2\pi \alpha_p (0.100 \pm 0.005)\, z\, (4.7 \pm 0.2)\dfrac{B^{1/3}}{z^{1/3}}$

Thus, combining two yields

$$\alpha_p = \frac{5}{6}(0.100 \pm 0.005) = 0.0833 \pm 0.0042$$

$$\alpha_p = 0.0833 \;>\; \alpha_j \;(= 0.0535) \qquad (2.140)$$

→ Dilution rate of a plume is higher than that for a jet even if the local momentum fluxes are equal.

▪ Entrainment function

The entrainment coefficient, α is proportional to the square of the local Richardson number of the jet (Priestley and Ball, 1955).

$$\alpha = \alpha_j - (\alpha_j - \alpha_p)\left(\frac{R}{R_p}\right)^2 \tag{2.141}$$

where R_p = plume Richardson number = 0.557

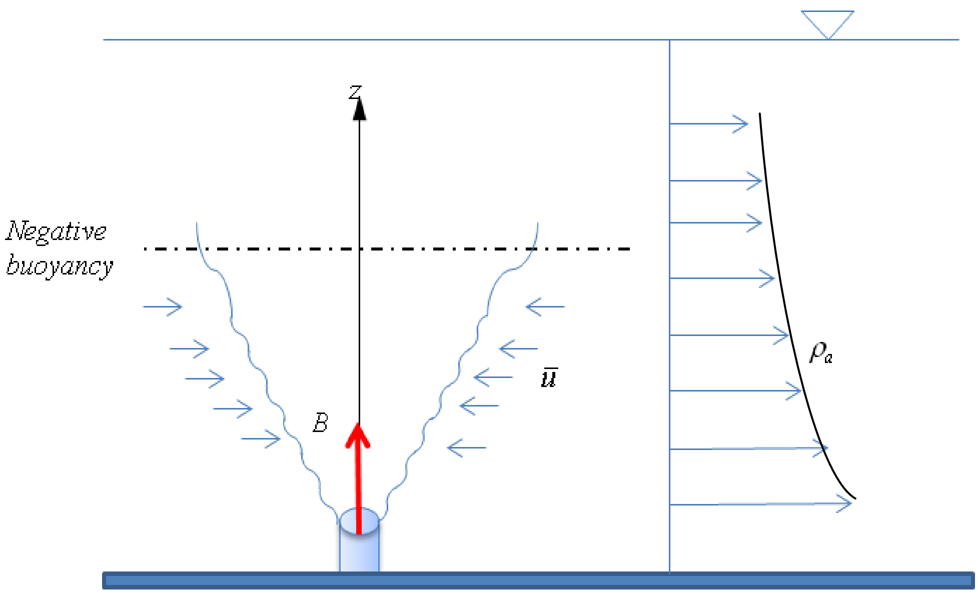
$$R = \text{local jet Richardson number} = \mu \beta^{1/2} / m^{5/4} \tag{2.142}$$

Rewrite R in terms of w_m, θ_m, b_W

[Substitute Eq. (2.126) ~ Eq. (2.128) into Eq. (2.142)]

$$R = \left[\frac{4\sqrt{2\pi}\lambda^2}{(1+\lambda^2)} \left(\frac{g b_W \theta_m}{w_m^2} \right) \right]^{1/2} \tag{2.143}$$

• Entrainment function for density-stratified flow



When a turbulent buoyant jet is rising in a density-stratified environment, the fluid entrained at a given level is denser that that at a higher level.

→ Reduces the buoyancy flux.

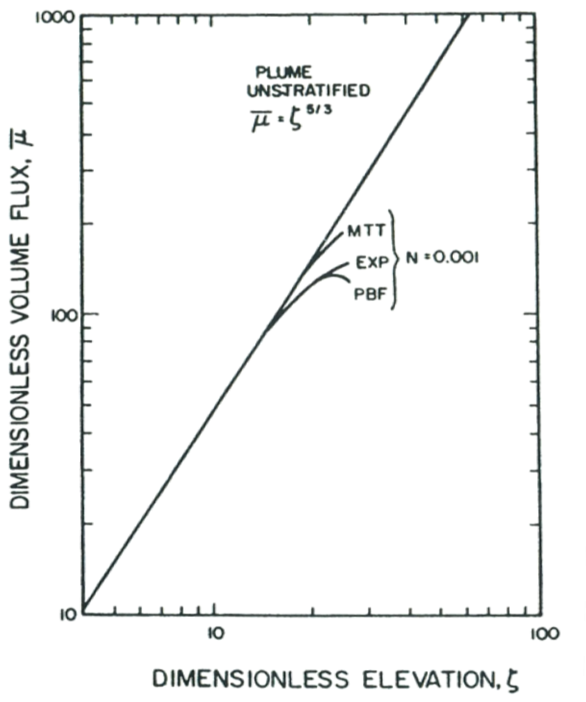
→ At some point, the buoyancy flux becomes negative.

→ Thus if Eq. (2.141) is used, the entainment actually stops when R^2 is - 0.56.

An alternative entrainment function for density-stratified flows is

$$\alpha = \alpha_j \exp\left[\ln\left(\frac{\alpha_p}{\alpha_j}\right)\left(\frac{R}{R_p}\right)^2 \right] \tag{2.144}$$

• Dilution in a linear density stratification



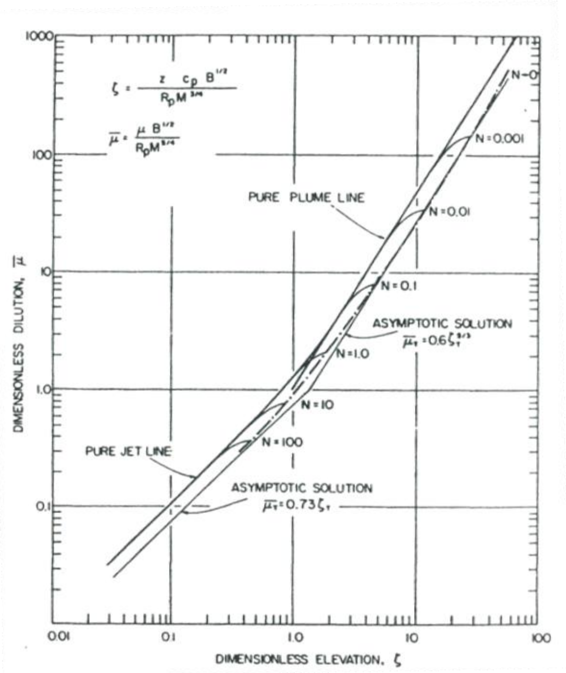


Fig. 2.25:

Use entrainment functions by (2.140), (2.141), and (2.144)

$$\overline{\mu} = \xi^{5/3}$$

Fig. 2.26:

Use entrainment function by (2.144)

For plume, $\overline{\mu} = 0.6\,\xi_T^{5/3}$ ← Eq. (2.61)

For jet, $\overline{\mu} = 0.73\,\xi_T$ ← Eq. (2.61)

[Cf] Compare with Fig. 2.7 for pure jet and plume solutions

2.4.3 Other Application

- Extend the analysis to include curved buoyant jets resulting from a jet discharge other than vertical
- Application of entrainment theory to horizontal jets: Eq. (2.141) & Eq. (2.144)
- Influence of various factors on entrainment function: buoyancy, flow curvature, crossflows, and ambient turbulence

• Trajectory and dilution of round horizontal turbulent buoyant jets

→ Figs. 2.27~2.28

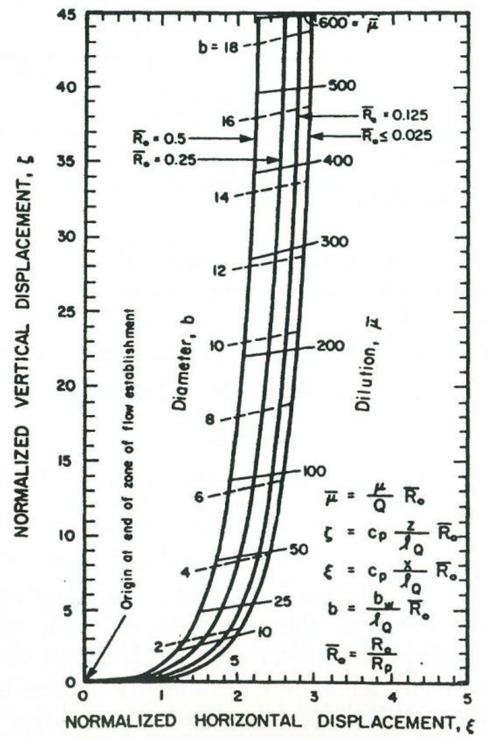


Fig. 2.27 Jet trajectory and diameter vs dimensionless distance

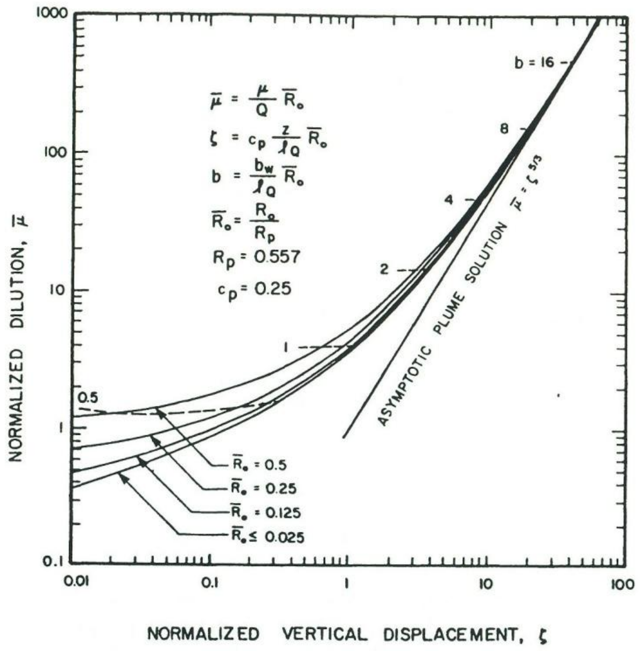


Fig. 2.28 Normalized dilution vs normalized vertical distance

$$\xi = C_p \frac{x}{l_Q}\bar{R}_o = C_p \frac{x}{l_Q}\frac{R_o}{R_p}$$

$$\zeta = C_p \frac{z}{l_Q}\bar{R}_o = C_p \frac{z}{l_Q}\frac{R_o}{R_p}$$

$$\bar{\mu} = \frac{\mu}{Q}\bar{R}_o = \frac{\mu}{Q}\frac{R_o}{R_p}$$

$$b = \frac{b_W}{l_Q}\frac{R_o}{R_p}$$

$$\bar{R}_o = \frac{R_o}{R_p}$$

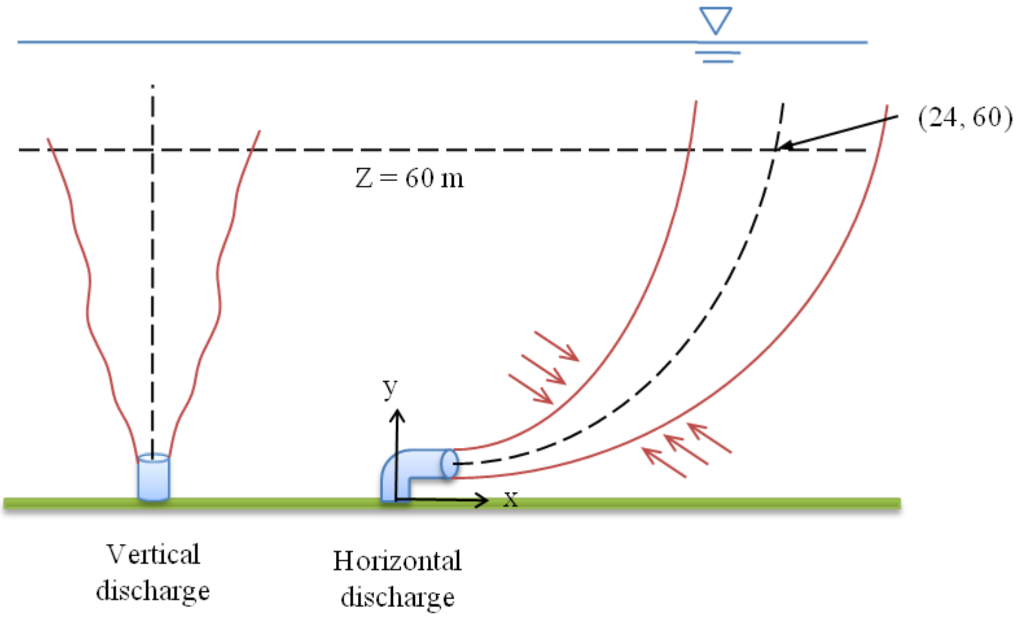
$$R_p = 0.557$$

$$C_p = 0.25$$

[Example 2.8] Comparison of dilution of vertical and horizontal jets at z = 60m

Given: $Q = 1 \text{ m}^3/\text{s}$, $B = 0.257 \text{ m}^4/\text{s}^3$, $W = 4 \text{ m/s}$, $M = 4 \text{ m}^4/\text{s}^2$

Assume ambient with no stratification and no crossflow.



$$l_Q = \frac{Q}{\sqrt{M}} = \frac{1}{\sqrt{4}} = 0.5\,\text{m}$$

$$l_M = \frac{M^{3/4}}{B^{1/2}} = \frac{(4)^{0.75}}{(0.257)^{1/2}} = 5.58\,\text{m}$$

$$R_o = \frac{l_Q}{l_M} = \frac{0.5}{5.58} = 0.0896$$

$$\overline{R}_o = \frac{R_o}{R_p} = \frac{0.0896}{0.557} = 0.16$$

$$b = \frac{b_W}{l_Q}\overline{R}_o = \frac{0.107\,(60)}{0.5}(0.16) = 2.05$$

i) Horizontal jet

$$\zeta_{60} = C_p \frac{z}{l_Q} \overline{R}_o = 0.25 \left(\frac{60}{0.5} \right) (0.16) = 4.8$$

From Fig. 2.27: $\overline{R}_o = 0.16; \ \zeta_{60} = 4.8 \rightarrow \xi_{60} \cong 1.9$

By the way, $\xi = C_p \frac{x}{l_Q} \overline{R}_o \qquad \therefore \ 1.9 = 0.25 \frac{x}{0.5} \ (0 \qquad \therefore \ x \cong 24$ m

From Fig. 2.28: $\overline{R}_o = 0.16 \ ; \ \zeta_{60} = 4.8 \rightarrow \overline{\mu}_{60} \cong 22$

By the way, $\overline{\mu} = \frac{\mu}{Q} \overline{R}_o \qquad \therefore \ \left(\frac{\mu}{Q} \right)_{60} = \frac{\overline{\mu}_{60}}{\overline{R}_o} = \frac{22}{0.16} \cong \boxed{138}$

ii) Vertical jet

$$\zeta_{60} = C_p \left(\frac{z}{l_Q} \right) \overline{R}_o = 0.25 \left(\frac{60}{0.5} \right) (0.16) = 4.8$$

From Fig. 2.7 $\rightarrow \ \overline{\mu}_{60} \approx 13.6$

$$\left(\frac{\mu}{Q} \right)_{60} = \frac{\overline{\mu}_{60}}{\overline{R}_o} = \frac{13.6}{0.16} = \boxed{85}$$

→ Dilution of horizontal discharge is higher than that for vertical discharge.

2.5 Boundary Effects on Turbulent Buoyant Jets

When a rising jet reaches the water surface, or when a sinking jet contacts the bottom, the flow must turn and flow horizontally as a buoyant layer.

→ Boundary effects:

River: water surface, bottom, side wall
Ocean: water surface, bottom

• Effect of fluid boundaries

1) Surface layer: rising jet reaching the water surface (flow horizontally as a <u>buoyant layer)</u>

~ open channel discharge of heated water into ocean- Fig. 2.30

~ turbid rivers flow into the coastal ocean - Fig 2.29

Figure 9.29 Two small turbid rivers flow into the coastal ocean. Note the sharp boundary between turbid river and clear ocean water.

Fig. 2.29 Two small turbid rivers flow into the coastal ocean

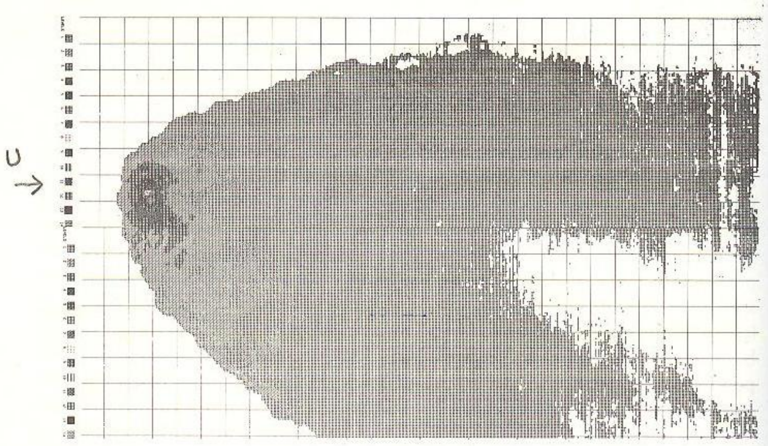


Figure 9.30 Thermal discharge ($Q = 21\ m^3/sec$) from a coastal power plant made visible with a scanning infrared radiometer. Vertical discharge is 5 m below the surface and the peak temperature difference is 11.1°C. There is a coastal current of 0.27 knot (0.14 m/sec) and grid spacing is 30.5 m (100 ft). Note bifurcation of the plume.

Fig 2.30 Thermal discharge ($Q = 21\ m^3/s$) from a costal power plant

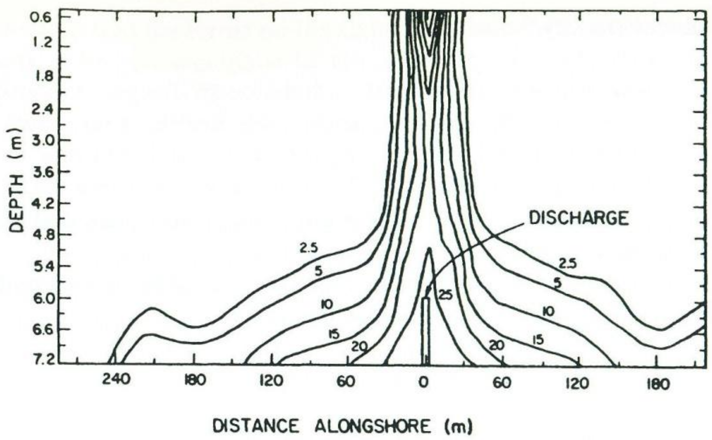
2) Bottom spreading layer: cooled water discharge from LNG vaporization plant

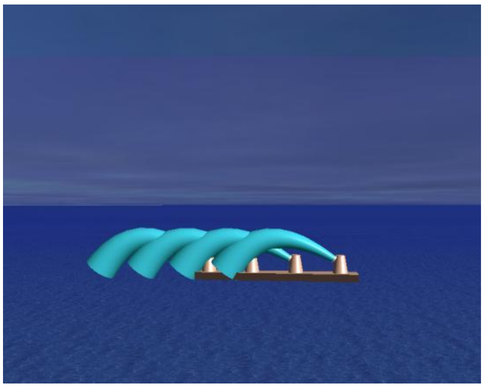
brine plume from desalination plant

[Re] LNG vaporization plant

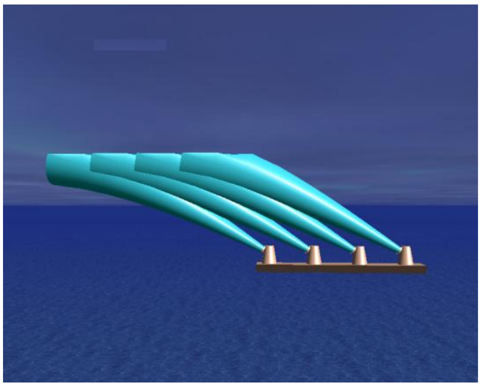
~ use seawater for heat required to revaporize liquefied natural gas

→ cooled sea water discharges to the ocean is denser than ambient water and sinks forming a bottom spreading layer of cool water





Bottom spreading layer



Surface layer

• Flow interface:

fluid interface at water surface → surface heat transfer, wind stress

internal density interface → interfacial friction

→ If the spreading layer becomes thin, then interfacial friction between the spreading layer and the liquid beneath may become an important feature of the flow.

• Difference between steady state and starting motion

Unsteady – instantaneous or pulse discharge

Steady – continuous discharge

~ Front formed by the release of a finite volume of buoyant or dense fluid is quite different from the flow established by a continuously operating discharge.

• General models of surface jets (<u>asymptotic limiting solutions</u>) to describe the interactions of important influences such as friction, density stratification, momentum, buoyancy, entrainment, loss of buoyancy, wind stress, crossflows, and ambient turbulence do not exist.

→ use high-accuracy hydraulic model

[Cf] general mathematical model of submerged jets

→ asymptotic solutions are available.

2.5.1 Momentum Effects

Consider a horizontal momentum jet located in the free surface

- deep water condition → half of regular 3-D jet flow
- shallow water condition → jet becomes attached to the bottom (Fig. 2.31)

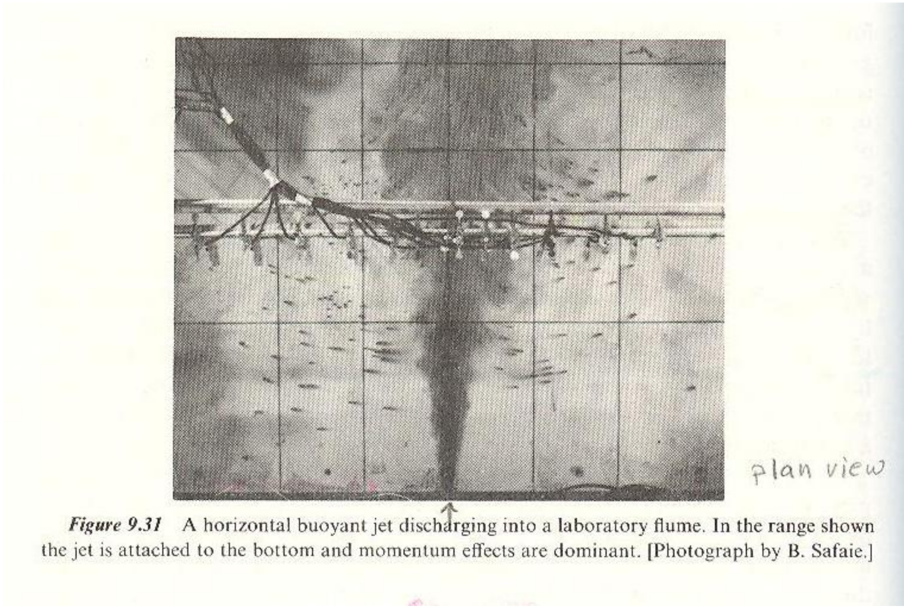
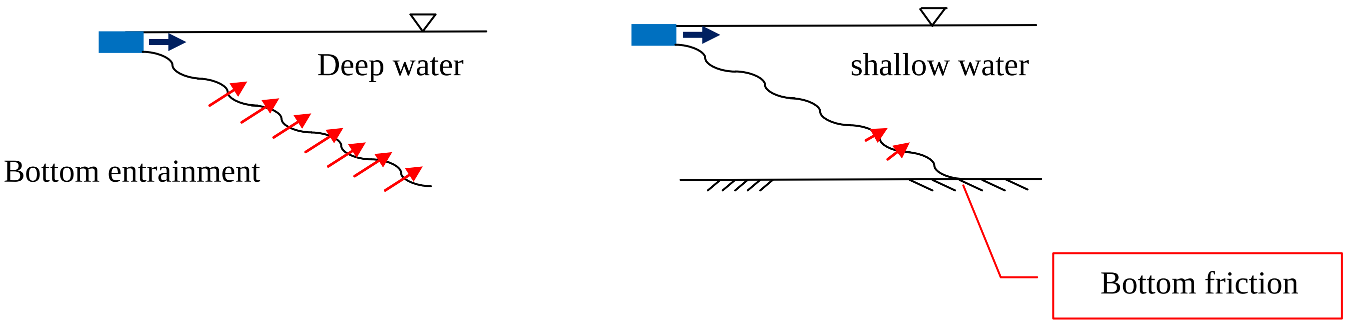


Fig. 2.31 A horizontal buoyant jet discharging into a laboratory flume



The limited depth has two major influences:

1)It restricts entrainment from below.

→ The entrainment demand of the jet is not to be met.

→ The jet becomes attached to the bottom.

→ All of the entrainment flow enters from the sides.

→ The jet flow becomes a complex three-dimensional motion.

2) It imposes a frictional effect.

- It is difficult to analyze these two effects.

When jet reaches the bottom, then lower entrainment ceases.

$$\mu \sim x^{1/2} \quad \text{for a plane jet}$$

where x = longitudinal distance

[Cf] In the absence of bottom effects

$$u_m \sim M^{1/2} x^{-1} \tag{2.145}$$

$$\mu \sim M^{1/2} x^{1} \tag{2.146}$$

2.5.2 Buoyancy Effects

Surface jet with buoyancy:

~ The discharge has a different temperature or salinity from the ambient liquid.

$$B = g_o{}' Q$$

• Characteristic length scales:

$$l_Q = \frac{Q}{M^{1/2}}$$

→ distance at which effect of initial geometry is important

$$l_M = \frac{M^{3/4}}{B^{1/2}}$$

→ distance at which effects of buoyancy become evident

• Buoyancy effect:

1) For vertical jet: buoyancy acts in the same direction as the transport flow, and buoyancy serves to increase the momentum flux of the jet if the buoyancy is positive.

2) Horizontal jet: buoyancy forces still act vertically but are translated into a horizontal radial pressure gradient which tends to spread out the buoyant liquid at the surface.

→ spreading surface layer

[Ex] Horizontal buoyant jet in a density-stratified environment: Internal spreading layer

- The initial horizontal momentum is conserved.

→ The buoyancy provides the vertical momentum.

→ The jet keeps rising until a level of <u>neutral buoyancy</u> is attained.

→ A radial pressure gradient is built up because the pool has a finite thickness.

→ The initial horizontal momentum causes the pool to drift to the right but the radial pressure gradient is strong enough to provide some flow to the left as well.

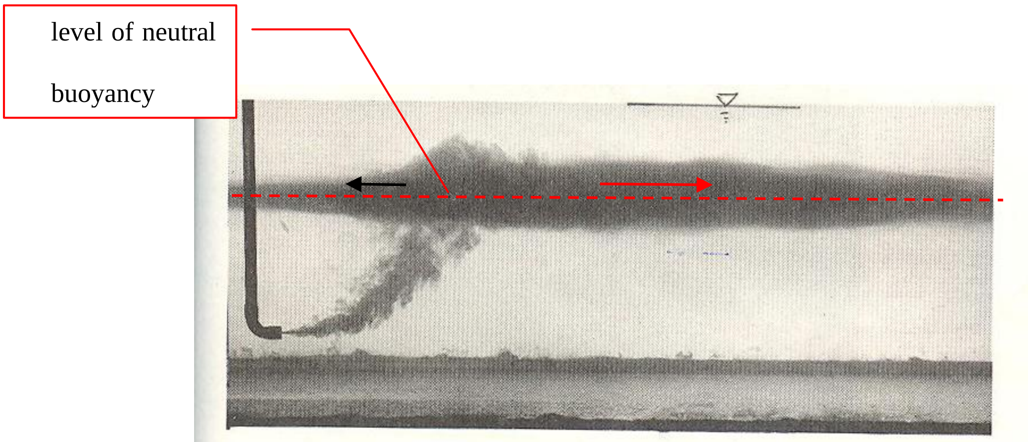
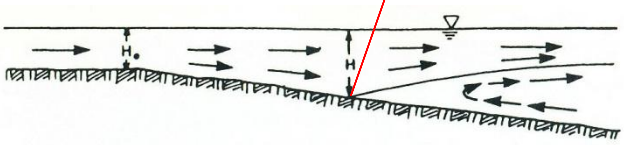


Figure 9.32 Horizontal turbulent buoyant jet in a density-stratified environment. $R_0 = 0.036$, $N = 0.56$. [From Fan (1967).]

• Buoyant surface jet



~ is likely to remain attached to the bottom for some distance, and then to spring from the bottom because of buoyancy.

~ lift off depth: $$H = 1.5 H_o \left(\frac{l_M}{l_Q} \right)^{1/2} \qquad (2.147)$$

2.5.3 Crossflows

- Buoyant surface jets into crossflows involve four parameters, Q, M, B, and U.

- Characteristic length scales are

$$l_Q, \quad l_M, \quad z_M \ (= M^{1/2}/U)$$

[Re] At z_m, jet velocity decays to U.

- Numerical analysis based on jet integral model:

~ compute the trajectory and dilution

~ The three-dimensional effects are not adequately represented in the integral analysis.

~ Influences of finite depth and crossflows on the entrainment function are also not unknown.

• Influence of finite depth and crossflows on the entrainment function

1) Vertical thermal discharge → Fig. 2.35 & 2.30

~ An internal hydraulic jump forms upstream of the discharge.

→ Two trailing vortices form horseshoe vortex leading to plume bifurcation.

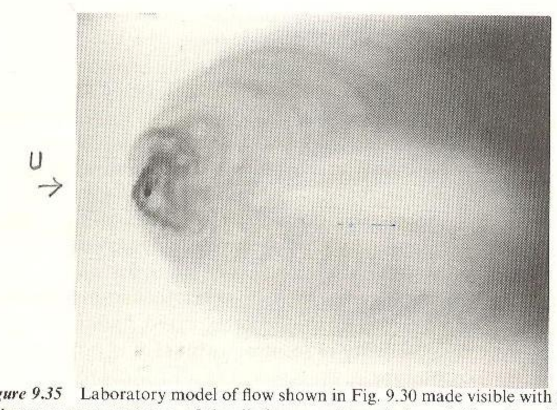


Figure 9.35 Laboratory model of flow shown in Fig. 9.30 made visible with colored dye. Note curved vortex seen upstream of the discharge point and plume bifurcation. Note also that dye intensity is not related to surface temperatures seen in Fig. 9.30.

2) Horizontal buoyant jet into a crossflow → Fig. 2.36

- turbulent buoyant jet discharged perpendicular to the crossflow

~ jet is initially attached to the bottom

~ large trailing eddy induces some of the discharged material is returned and reentrained into the jet.

~ strong interaction between a buoyant jet and a crossflow may cause reentrainment.

→ A hydraulic model study should be performed to analyze the interaction of the various parameters.

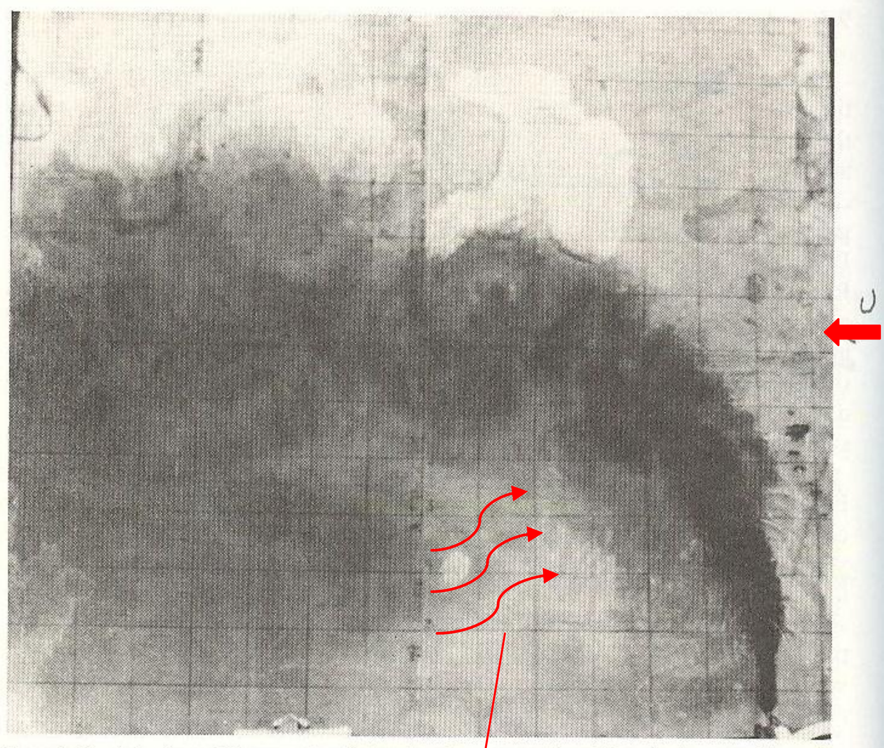


Figure 9.36 A horizontal buoyant jet discharging into a crossflow. Note the large trailing eddy. [Photo by V. Harms.]

2.5.4 Multiple Point Discharges

- The influence of flow boundaries and flow geometry is important for the multiple diffuser in shallow water.

- Multiport diffuser in shallow water:

  - diffuser length: $L = 800 \sim 1000 \ m$

  - depth of submergence: $H = 10 \sim 80 \ m$

• Two important factors for the analysis of multiport diffuser

1) Merging of individual jets:

~ The flow from individual ports merges to form an agglomerated flow.

~ Each jet increases in size with distance from the exit port so that each of the jets will interact, merging one with another.

~ 3D jet will turn to an essentially 2D flow field such as would be produced from a line buoyant jet.

→ Fig. 2.37: Stage diffuser

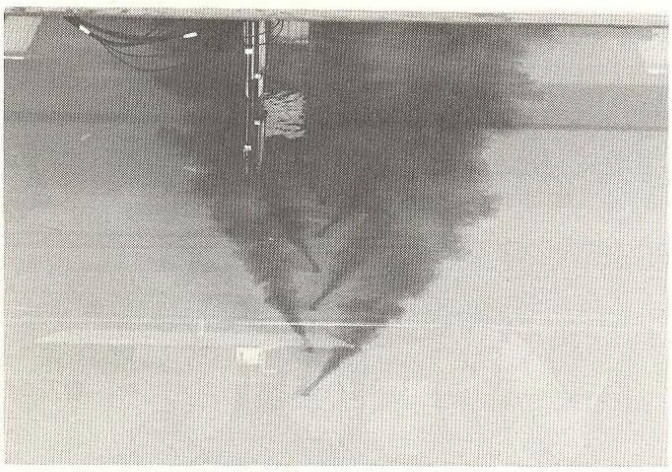


Figure 9.37 A model section of a multiport diffuser. Each discharge port is angled 20° up from horizontal; ports alternate ±25° from diffuser axis. Model scale 49.3 : 1, R_0 = 0.03.

2) Orientation of diffuser to the flow field

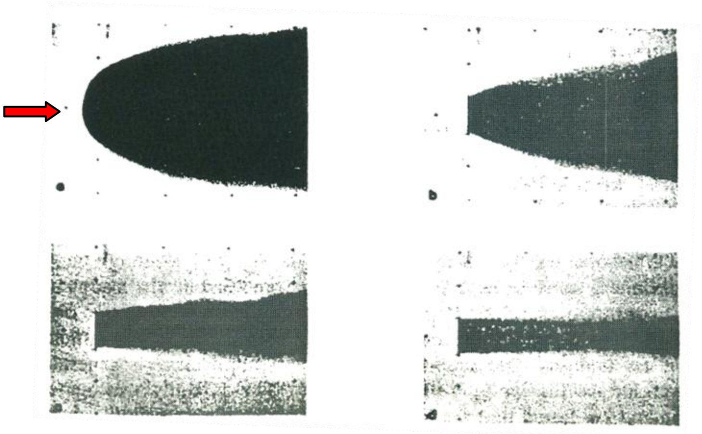
~ The orientation of the diffuser to the ocean current is parallel or perpendicular, or at some angle.

~ As the rising plume reaches a level of neutral buoyancy, or the free surface, the flows will become nearly horizontal.

→ Fig. 2.38~9: Uni-directional diffuser

perpendicular

45˚ current



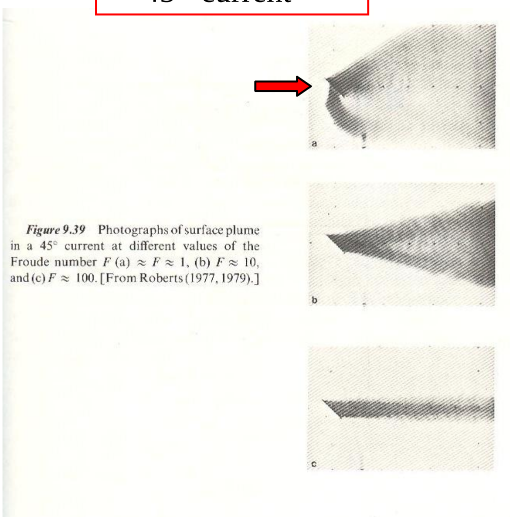


Figure 9.39 Photographs of surface plume in a 45° current at different values of the Froude number F (a) $\approx F \approx 1$, (b) $F \approx 10$, and (c) $F \approx 100$. [From Roberts (1977, 1979).]

• Minimum surface dilution, S_m (Roberts, 1977, 1979)

$$S_m \sim f(U, H, L, b, q)$$

where U = crossflow velocity; H = depth of diffuser; L = length of diffuser;

b = buoyancy flux per unit length of diffuser

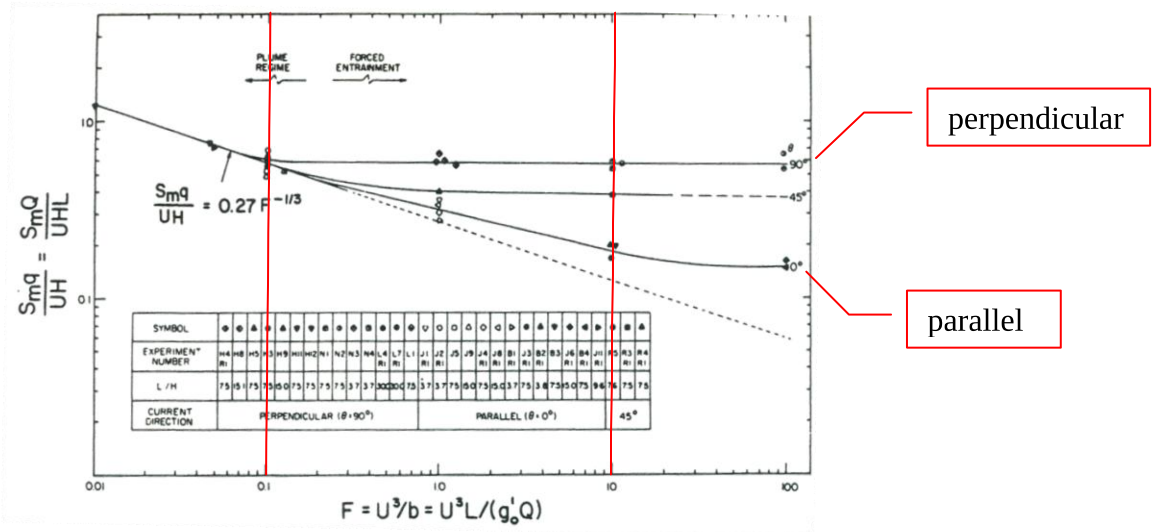
$$= \frac{B}{L} = g_o{}' \frac{Q}{L} = g_o{}' W \frac{A}{L} \quad [L^3/T^3]$$

$q =$ volume flux per unit length of diffuser

$$= \frac{Q}{L} = \frac{A}{L} W \quad [L^2/T]$$

- Froude number is the dominant parameter in determining the shape of flow field and dilution.

$$F_{r_U} = \frac{U^3}{b} = \frac{U^3 L}{g_o{}' W A}$$



i) $F_{r_U} < 0.1$: $\quad \dfrac{S_m q}{U H} = 0.27 F_{r_U}^{-1/3}$ (2.148)

ii) $0.1 < F_{r_U} < 10$: nonlinear

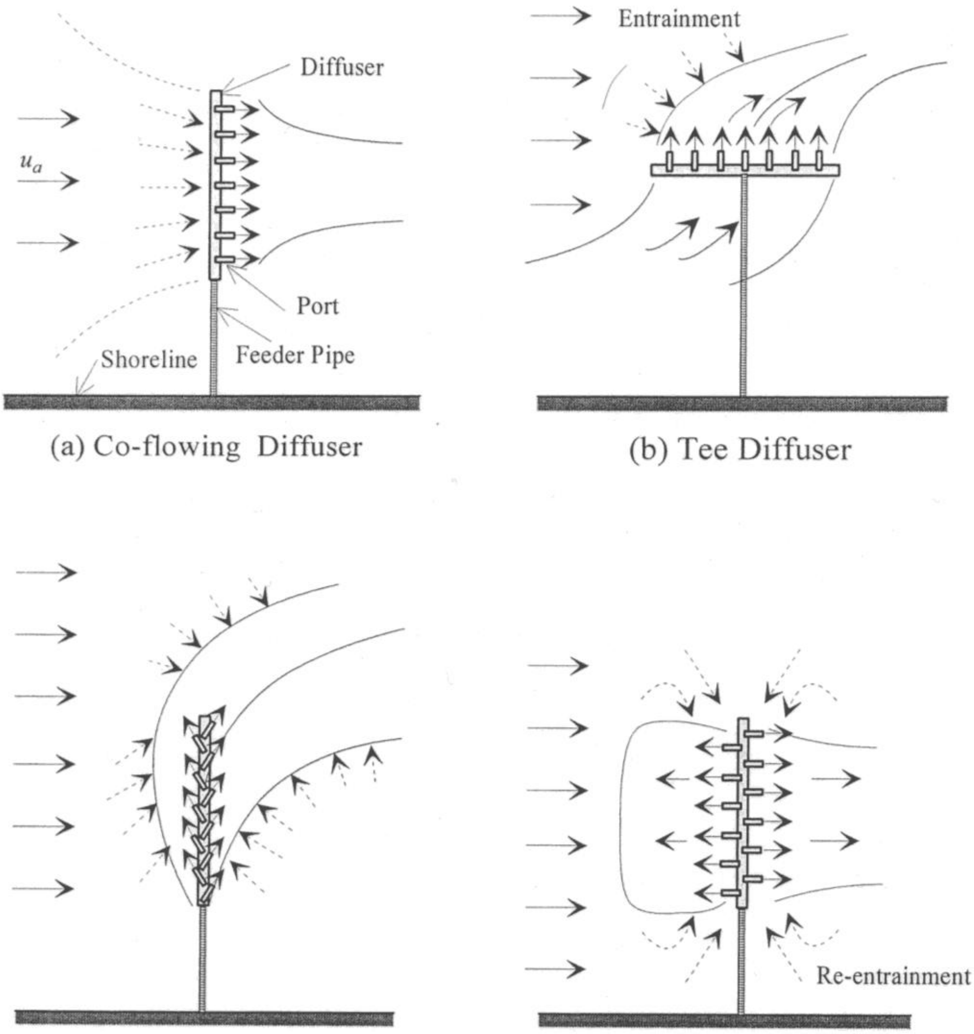
iii) $F_{r_U} > 10$: $\quad \dfrac{S_m q}{U H} = \left[\begin{array}{ll} 0.6 & perpendicular \\ 0.4 & 45^o \\ 0.15 & parallel \end{array}\right.$

Eq. (2.148) is equivalent to

$$\mu = 0.27 B^{1/3} H \qquad (2.149)$$

~ similar to $\mu = 0.34\, B^{1/3} z$ for plane plume (Table 2.3)

~ constant is reduced from 0.34 to 0.27 to account for the non-diluting surface layer of previously discharging fluid. (Blocking effect)



(a) Co-flowing Diffuser (b) Tee Diffuser (c) Staged Diffuser (d) Alternating Diffuser

9.6 Summary

Solving practical dilution problems involving jets is generally difficult matter.

Factors include momentum, buoyancy, density stratification, flow boundaries, loss of buoyancy, crossflows, bottom friction, internal friction, internal hydraulic jump, and ambient turbulence.

→ These factors can be categorized into three groups: jet parameters, environmental parameters, and boundary effects.

We can make order of magnitude analyses for the influences of first two groups, i.e., jet parameters, environmental parameters.

When geometric influences become dominant, we must look to a hydraulic model to provide the results.

Homework Assignment #2-4

Due: 2 weeks from today

Compare plume behavior of the buoyant jets which are discharged vertically and horizontally. Flow conditions are the same as given in Example 2.8.

1. Plot 2-D contour of equi-concentration lines, assuming Gaussian distribution for w, C $(C_o = 1000 \ ppm)$

2. Derive Eqs. (2.133) ~ (2.135)