457.562 Special Issue on River Mechanics (Sediment Transport)
.09 Mechanics of Bedload Transport (II)



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1. Dynamics Friction Coefficient

- Each time a saltating particle strikes the bed, it transfers a portion of its forward momentum to the bed.
- The net result of successive collisions is a mean stream wise shear force exerted on the bed by obliquely-striking particles.
- This force now computed.
- Because the collision is both inelastic and oblique, the fo rward momentum of the particle just after collision is less than that value just before collision. That is, the paramet er $\Delta u_p > 0,$

$$\Delta u_p = u_p\big|_{in} - u_p\big|_{out}$$

1. Dynamics Friction Coefficient

- The momentum transferred from the particle to the bed per collision is thus given by $\rho_s V_p \Delta u_p$
- On the average, there is a collision with bed each $\lambda_s / \bar{u}_p$ seconds
- The streamwise shear force F_{gr} (grain) exerted on the bed due to collision is thus given by the momentum transfer per unit time

$$F_{gr} = \rho_s V_p \Delta u_p \frac{\bar{u}_p}{\lambda_s}$$

- This forward shear force of the saltating particles on the bed corresponding to a resistive force of the bed on the moving grains of equal magnitude and opposite direction.

1. Dynamics Friction Coefficient

- It is now possible to define a coefficient of dynamic Coul omb friction in analogy to the coefficient of static Coulom b friction introduced before.

$$\mu_d = \frac{\text{tangential resistive force of collision}}{\text{submerged particle weight}}$$

$$= \frac{F_{gr}}{(\rho_s - \rho) g V_p} = \frac{R+1}{R} \frac{\Delta u_p \overline{u}_p}{g \lambda_s}$$

- In analogy to relations (non-dimensional numbers),

$$\mu_d = \mu_d \left(\tau^*, R_{ep} R \right)$$

2. Some continuity relations for saltating grains

- A collection of grains moving over, and exchanging with, an immobile bed of similar grains.
  - For simplicity, all the saltation trajectories are taken to be i dentical although the analysis readily generalizes to the st ochastic case.
- E_s : the volume upward flux of grains from the bed, due t o erosion of previously immobile particles or ejection of c olliding particles.
- $C_u(z)$: the corresponding concentration of upward movin g volume concentration
- $C_d(z)$ be downward-moving particles
- $C_s(z)$ denote the total volume concentration of saltating b edload particles.

2. Some continuity relations for saltating grains

- The velocities $u_{pu}(z)$ and $u_{pd}(z)$ now correspond to the Eu lerian streamwis grain velocities for upward-and downwa rd- moving particles at elevation z; the corresponding up ward normal Eulerian grain velocities are w_{pu} and w_{pd}.
- The following conditions hold for continuity:

$$C_u w_{pu} = -C_d w_{pd}$$

$$C_s = C_u + C_d$$

- Phase-average Eulerian streamwise velocity of saltating grains

$$u_{se} = \frac{C_u u_{pu} + C_d u_{pd}}{C_s}$$

2. Some continuity relations for saltating grains

- The volume streamwise bedload transport rate q is given as

$$q = \int_{bed}^{h_s} C_s(z) u_{se}\, dz \equiv \xi \bar{u}_{se} \qquad \xi = \int_{bed}^{h_s} C_s(z)\, dz$$

- ξ denotes the volume of moving grains per unit bed and $\bar{u}_{se}$ a flux-averaged value of u_{se} given by

$$\bar{u}_{se} = \frac{\int C_s u_{se}\, dz}{\int C_s\, dz}$$

- Finally, the following continuity relation holds

$$\boxed{q = E_s \lambda_s} \quad (E_s \text{ was the volume upward flux of grains from the bed})$$

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- It is now possible to compute the net upward flux of strea mwise momentum F_{gmsz} due to saltating grains: this qua ntity is given by

$$F_{gmsz} = \rho_s \left(C_u u_{pu} w_{pu} + C_d u_{pd} w_{pd} \right)$$

- In analogy to the Reynolds stress, the shear grain stress, or the shear stress associated with the upward normal tr ansfer of streamwise momentum by saltating grains is gi ven by

$$\tau_g = -F_{gmsz} = -\rho_s \left(C_u u_{pu} w_{pu} + C_d u_{pd} w_{pd} \right)$$

- Reduction with the previous several equations

$$\tau_g(z) = \rho_s \xi \overline{u}_{se} \frac{u_{pd} - u_{pu}}{\lambda_s}$$

3. Fluid and Grain Shear Stress: Bagnold Hyphthesis

- Now compare the equations of slide 2,

$$F_{gr} = \rho_s V_p \Delta u_p \frac{\bar{u}_p}{\lambda_s} \qquad \tau_g(z) = \rho_s \xi \bar{u}_{se} \frac{u_{pd} - u_{pu}}{\lambda_s}$$

- Those equations are similar but the new (second) one has the distance above the bed.
- Bagnold (1957) used some of the above idea to gain a picture of the interaction of the flow and saltating grains.
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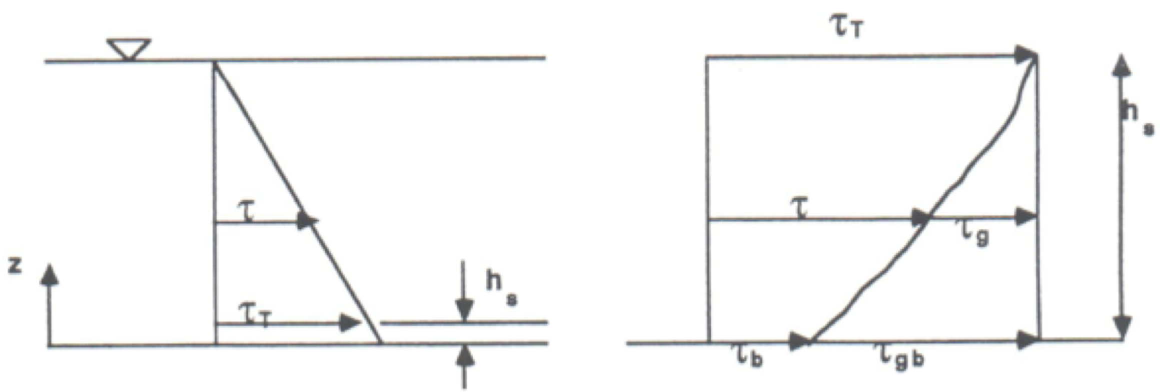
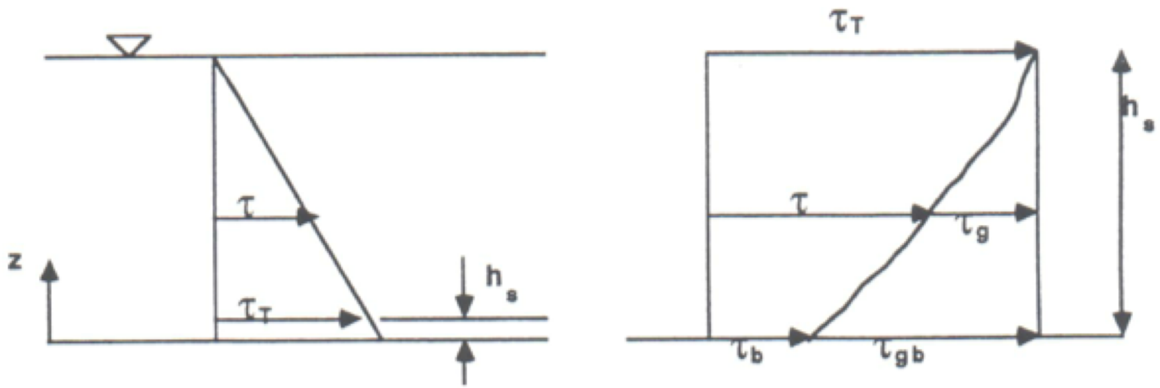


3. Fluid and Grain Shear Stress: Bagnold Hyphothesis

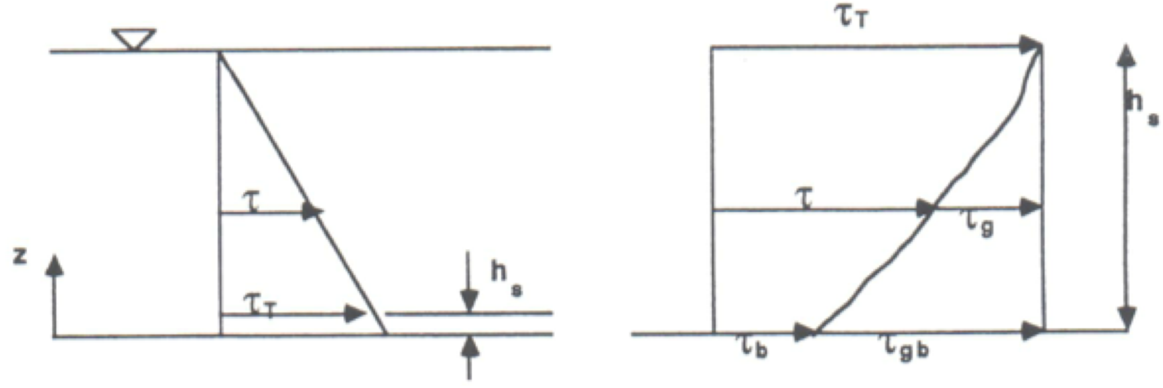
- Consider the diagram below, corresponding to equilibrium flow in a wide rectangular channel with a mobile bed.
- Where H denotes flow depth, it is assumed the D/H <<1.
- The shear stress varies linearly in depth right up to the top of the saltation layer $z=h_s$. As long as saltation is confined to a very thin layer compared to the depth the fluid shear stress at the top of the saltation layer is given by

$$\tau_T = \rho u_*^2 = \rho g H S$$



3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- For distances of the order of the grain size above $z=h_s$, shear stress is very nearly equal to top shear stress in accordance with the constant-stress approximation of the previous shear stress class.



- Now the essential role of saltating grains as regards momentum balance is to effect a transfer of streamwise momentum from the fluid phase to the solid phase.

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- On the average, the saltating grains move more slowly th an at the surrounding fluid. As a result, they extract mo mentum from the fluid via drag.
- This streamwise momentum is then fluxed towards the b ed, given rise to a grain shear stress.
- The value of the grain shear stress at the bed, correspon ds to the force per unit area exerted by the grains on the bed.
- Within the bedload layer, any net gain of momentum by t he grains must correspond to a net loss by the fluid. It th us follows that

$$\tau_T = \tau + \tau_g$$

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- That grain shear stress is indeed positive in the bedload l ayer (corresponding to a net downward flux of streamwis e momentum) can be seen in

$$\tau_g(z) = \rho_s \xi \overline{u}_{se} \frac{u_{pd} - u_{pu}}{\lambda_s}$$

- Since down ward velocity must exceed upward, since pa rticles downward is accelerated more.
- It is thus seen that within the bedload layer, the fluid stre ss is reduced to the value

$$\tau = \tau_T - \tau_g$$

- The larger the number of particles participating in bedloa d motion, the larger is the value of the volume in transpor t per unit bed area xi since above equations larger value of grain shear stress further reduction in fluid stress.

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- Bagnold hypothesized that this process is limited by the critical boundary shear stress.
- That is more and more particles should be entrained into bedload motion until such point as the boundary fluid str ess drops to the value of boundary shear stress.
- Beyond this point, there is no more net entrainment of pa rticles, and an equilibrium state of bedload transport is re ached.
- Thus

$$\tau_{gb} = \rho u_*^2 - \tau_{bc}$$

$$\tau_{gb} = \rho_s \xi \overline{u}_{se} \frac{\Delta u_p}{\lambda_s} \qquad (1)$$

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- Further process can be made by introducing the coefficient of dynamic Coulomb friction. In the context of the present Eularian analysis, this can be defined to be the ratio of grain bottom shear stress to the submerged weight of the moving bedload particles per unit bed area: that is

$$\mu_d = \frac{\tau_{gb}}{\rho R g \xi} \qquad (2)$$

- Reducing with the previous relation

$$\mu_d = \frac{R+1}{R}\frac{\Delta u_p \bar{u}_{se}}{g\lambda_s} = \left(R+1\right)\tau^* \frac{\Delta u_p}{g\lambda_s}\frac{\bar{u}_{se}}{u_*} \qquad (3)$$

- Check the slide 4, this Eulerian equation of dynamic coefficient which is written in Lagrangian form.

3. Fluid and Grain Shear Stress: Bagnold Hypothesis

- With the several equations (1,2,3), for bedload concentra tion per unit bed area

$$\mu_d \rho R g \xi = \rho u_*^2 - \tau_{bc} \qquad \textit{devide by } \rho R g D$$

$$\frac{\xi}{D} = \frac{\tau^* - \tau_c^*}{\mu_d}$$

4. Bed-load transport relations

- A large number of bed load relations can be expressed i n the general dimensionless form

$$q^* = q^*\left(\tau^*, R_{ep}, R\right)$$

- Here q^* is a dimensionless bed load transport rate know n as the Einstein bed load number and given by

$$q^* = \frac{q_b}{D\sqrt{gRD}}$$

- Einstein's bed load transport model can be expressed in dimensionless form as

$$q^* = E_b^* L_s^* \qquad E_b^* = \frac{E_b}{\sqrt{gRD}} \quad L_s^* = \frac{L_s}{D}$$

4. Bed-load transport relations

- Again Bagnold's approach.

$$q^* = \frac{q_b}{D\sqrt{gRD}} = \frac{c_b \delta_b}{D} \frac{u_b}{\sqrt{gRD}} \qquad (1)$$

- When bed load transport rate is

$$q_b = u_b c_b \delta_b$$

- And the thickness of the bed load layer δ_b
- And as in the previous Bagnold equation

$$\frac{c_b \delta_b (= \xi)}{D} = \frac{\tau^* - \tau_c^*}{\mu_d} \qquad (2)$$

4. Bed-load transport relations

- Ashida and Michiue (1972) presented a macroscopic an alysis that does not account for the complexity of the salt ation process, in particular the treatment of the particle c ollision with the bed.

$$\frac{u_b}{\sqrt{gRD}} = 8.5\left[\left(\tau^*\right)^{1/2} - \left(\tau_c^*\right)^{1/2}\right] \qquad (3)$$

- Put equation (2) and (3) into (1), then (coefficient is 0.5),

$$q^* = 17\left(\tau^* - \tau_c^*\right)\left[\left(\tau^*\right)^{1/2} - \left(\tau_c^*\right)^{1/2}\right]$$

- Ashida and Michiue recommended a value for τ_c^* of 0.05.

4. Other Bed-load transport relations

- Meyer-Peter and Muller (1948)

$$q^* = 8\left(\tau^* - \tau_c^*\right)^{3/2} \qquad \tau_c^* = 0.047$$

  – Coarse for alpine stream but work well costal sediment.

- Wong and Parker reanalyzed and found better fit.

$$q^* = 4.93\left(\tau^* - 0.047\right)^{1.6} \qquad or$$

$$q^* = 3.97\left(\tau^* - 0.0495\right)^{3/2}$$

4. Other Bed-load transport relations

- Einstein

$$1-\frac{1}{\sqrt{\pi}}\int_{-(0.413/\tau^*)-2}^{(0.413/\tau^*)-2} e^{-t^2}\,dt=\frac{43.5q^*}{1+43.5q^*}$$

  - It appropriate the local bed load transport rate needed
  - Large river but not in small rivers and flumes

- Van Rijin (1984)

$$q^*=0.053\frac{T^{2.1}}{D_*^{0.3}}\qquad (mean\ size=0.2\sim 2.0mm)$$

$$D_*=D_{50}\left(\frac{gR}{\nu^2}\right)^{1/3}=R_{ep}^{2/3},\qquad T=\frac{\tau_s^*-\tau_c^*}{\tau_c^*}$$