Engineering Economic Analysis

2019 SPRING

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Chap. 19

TECHNOLOGY

Describing Technological Constraints

- Technology constraints on a firm's behavior
  - Certain combinations of inputs are feasible ways to produce a given amount of outputs

Inputs $\longrightarrow$ Technology $\longrightarrow$ Outputs

- Production (possibilities) set: $Y \subset R^n$
  - the set of all combinations of inputs and outputs that comprise a technologically feasible way to produce
  - Production plan: $\tilde{y} \in Y$, where $y_j \geq (\leq) 0$ if j is output (input)

Describing Technological Constraints

- A production plan $\tilde{y} \in Y$ is *technologically efficient* if there is no $\tilde{y}' \in Y$ such that $\tilde{y}' \geq \tilde{y}$ and $\tilde{y}' \neq \tilde{y}$
  - there is no way to produce more output with same inputs or
  - there is no way to produce same output with less inputs

Describing Technological Constraints

- Single output case

Inputs $\longrightarrow$ Technology $\longrightarrow$ Single output

$$(y, \tilde{x}), \quad \text{where } y \geq 0 : \text{single output}, \;\; \tilde{x} \geq \mathbf{0} : \text{input bundle vector}$$

- Production function
  - the maximum possible amount of output that can be produced from a given amount of inputs

$$f(\tilde{x}) = \left\{ y \in R \,\middle|\, \max.\, y \text{ such that } (y, -\tilde{x}) \in Y \right\}$$

  - description of efficient production plan
  - the boundary(frontier) of (transformed) production set
  - similar concept with *utility function*

Describing Technological Constraints

- Input requirement set for a single output
  - The set of all input bundles that produce at least *y*

$$V(y) = \left\{ \tilde{x} \in R_+^n \,\middle|\, (y, -\tilde{x}) \in Y \right\}$$

  - The *upper contour set* of production function

$$Q(y) = \left\{ \tilde{x} \in R_+^n \,\middle|\, f(\tilde{x}) \geq y \right\}$$

- Isoquant
  - The set of all input bundles to produce exactly *y*

$$Q(y) = \left\{ \tilde{x} \in R_+^n \,\middle|\, \tilde{x} \in V(y) \text{ and } \tilde{x} \notin V(y') \text{ for } y' > y \right\}$$

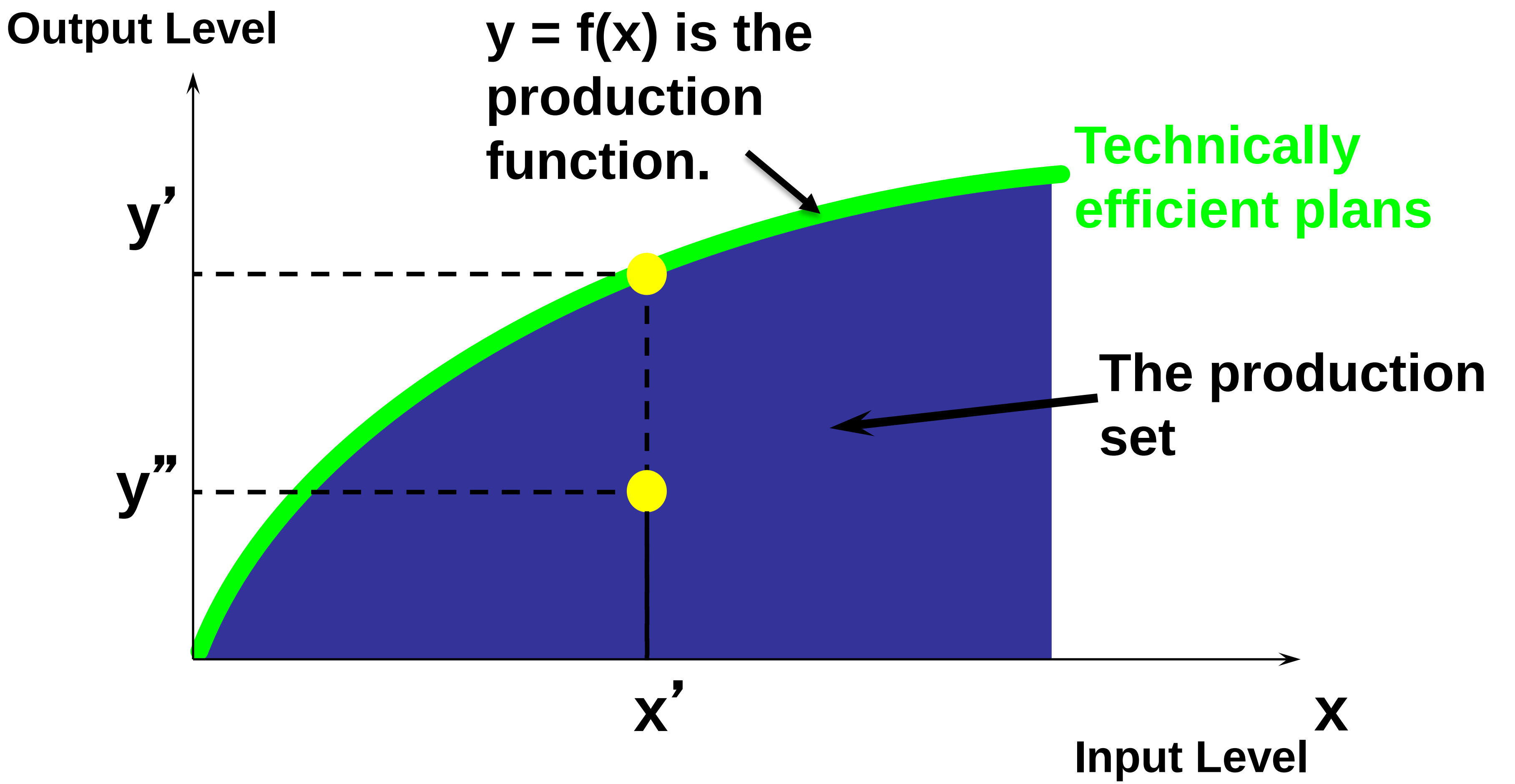
  - The *level set* of production function

$$Q(y) = \left\{ \tilde{x} \in R_+^n \,\middle|\, f(\tilde{x}) = y \right\}$$

  - similar concept with indifference curve

Describing Technological Constraints

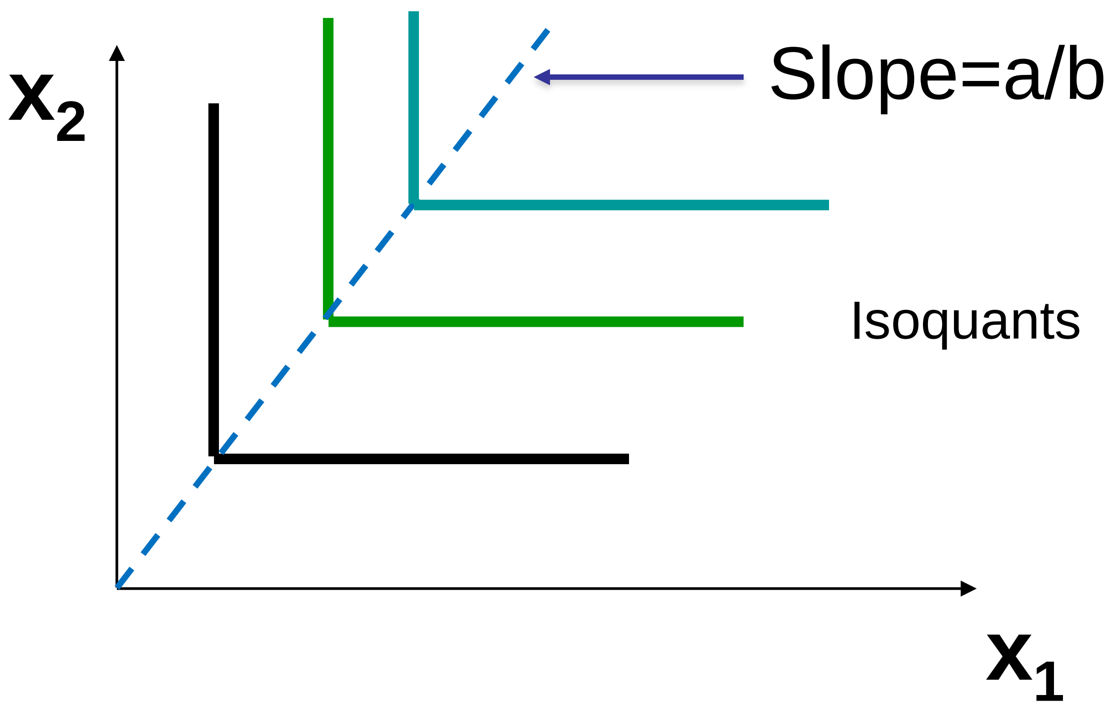
- 1-input & 1-output case



Examples of Technology

- Fixed proportions (Leontief technology)
  - Inputs: Cheese (x_1) & Flour (x_2)
  - Output: Pizza (y)

$$y = f(x_1, x_2) = \min\{ax_1, bx_2\}$$

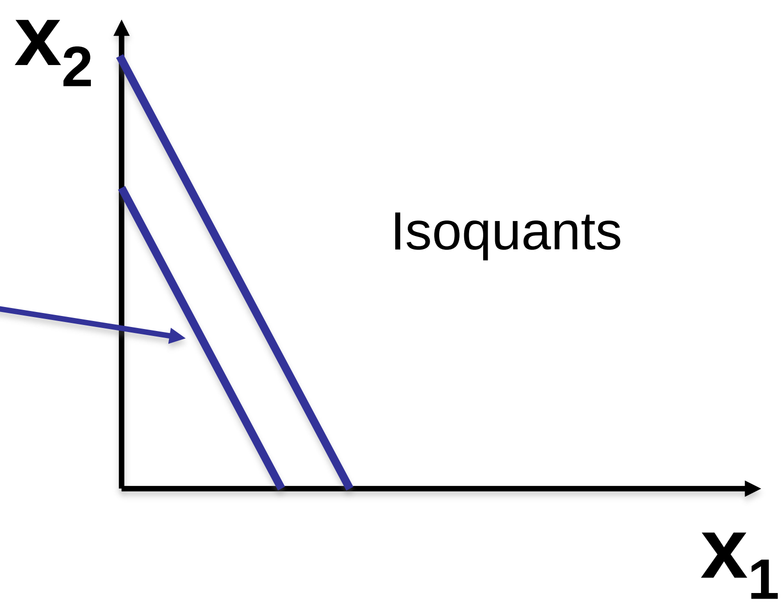


Examples of Technology

- Perfect substitutes (Linear technology)
  - Inputs: Fossil fuel (x_1) & Renewable energy (x_2)
  - Output: Electricity (y)

$$y = f(x_1, x_2) = ax_1 + bx_2$$

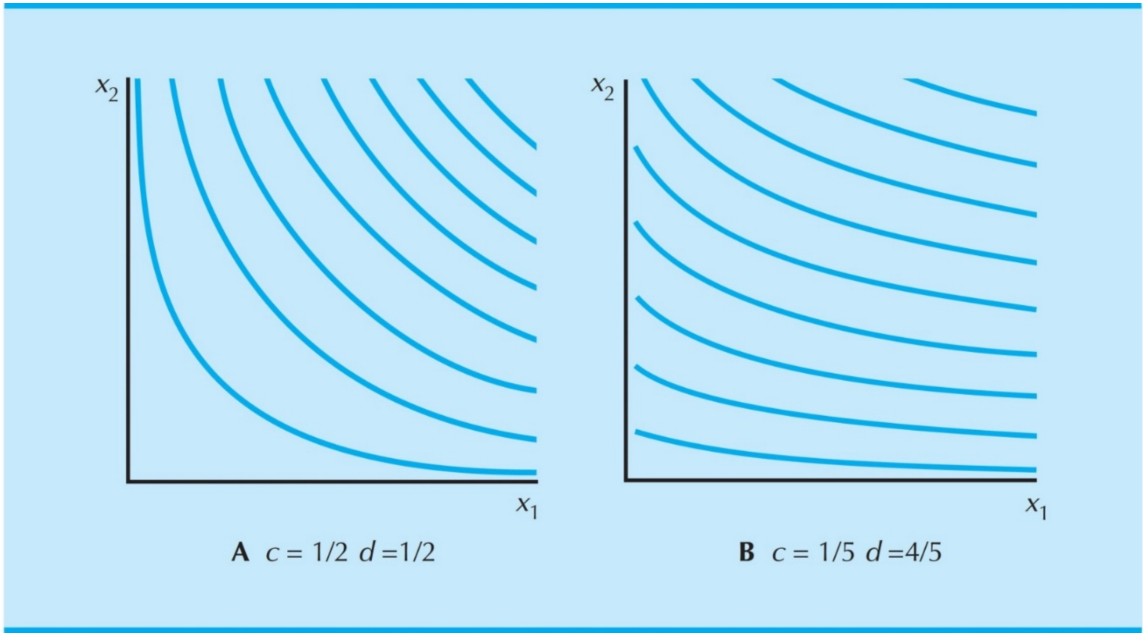
$$x_2 = -\frac{a}{b}x_1 + \frac{k}{b}$$



Examples of Technology

- Cobb-Douglas technology
  - Well-behaved technology

$$f\left(x_1, x_2\right) = A x_1^a x_2^b$$



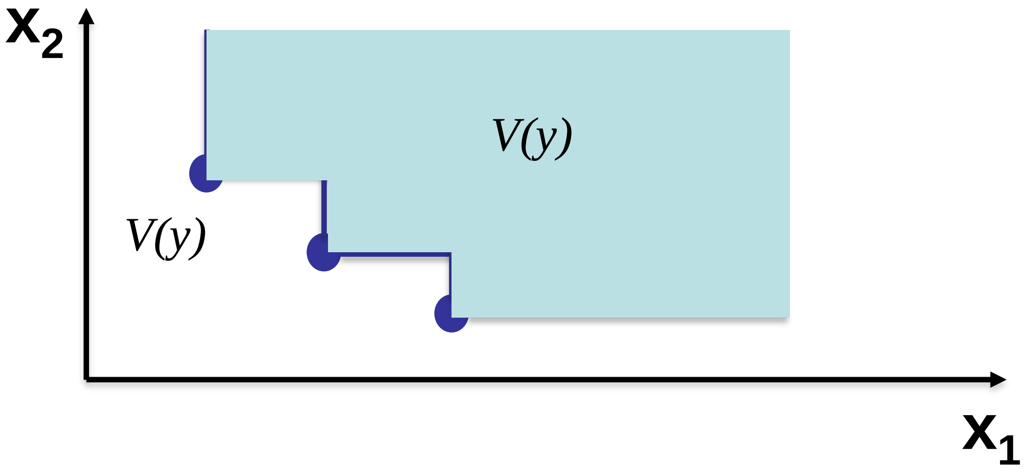
Isoquants

Properties of Technology

- Monotonicity

  If $\tilde{x} \in V(y)$ and $\tilde{x}' \geq \tilde{x}$, then $\tilde{x}' \in V(y)$

  - if you increase the amount of at least one of the inputs, it should be possible to produce at least as much outputs as you were producing originally
  - Free disposal: if the firm can costlessly dispose of any inputs, having extra inputs around can't hurt it.

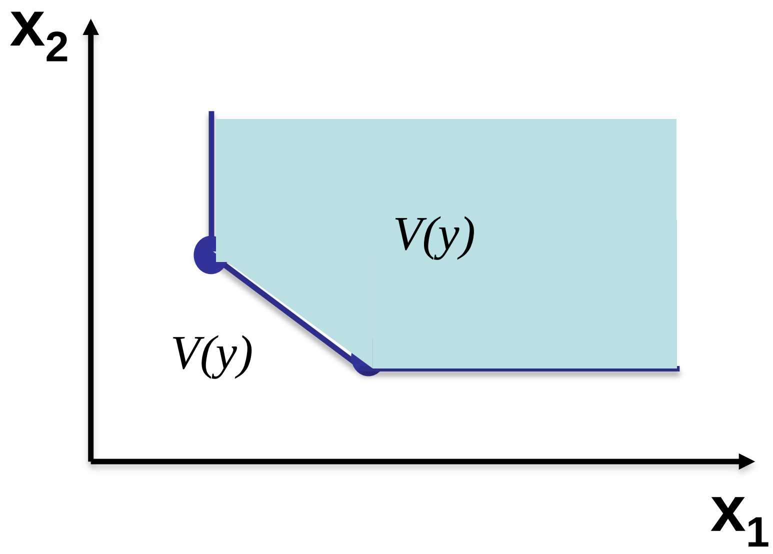


Properties of Technology

- Convexity

If $\tilde{x} \in V(y)$ and $\tilde{x}' \in V(y)$, then $t\tilde{x} + (1-t)\tilde{x}' \in V(y)$ for all $0 \le t \le 1$

  - if you have two ways to produce *y* unit of output, then their weighted average will produce at least *y* units of output
  - Input requirement set, *V(y)*, is a convex set



Properties of Technology

- [Def] *A function f defined on a convex subset of* R^n *is quasi-concave if every upper contour set is convex*
- *V(y) is a convex set if and only if the production function is a quasi-concave function*
- *A technology is convex if and only if the corresponding production function is a quasi-concave*

Technical Rate of Substitution

- Marginal product
  - How much more output will we get if we use a little bit more of input *i* while the other inputs are fixed?

$$MP_1 = \frac{\partial y}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f\left(x_1 + \Delta x_1, x_2\right) - f\left(x_1, x_2\right)}{\Delta x_1}$$

$$MP_i = \frac{\partial f(\tilde{x})}{\partial x_i} \text{ in more generally}$$

  - similar concept of marginal utility

Technical Rate of Substitution

- Technical rate of Substitution (TRS)
  - measures the rate at which the firm will have to substitute one input for another in order to keep output constant

Given a production function $f(x_1, x_2) = y$

By total differnetiation, $\dfrac{\partial f}{\partial x_1} dx_1 + \dfrac{\partial f}{\partial x_2} dx_2 = dy$

TRS implies, $\dfrac{dx_2}{dx_1}$ when $dy = 0$

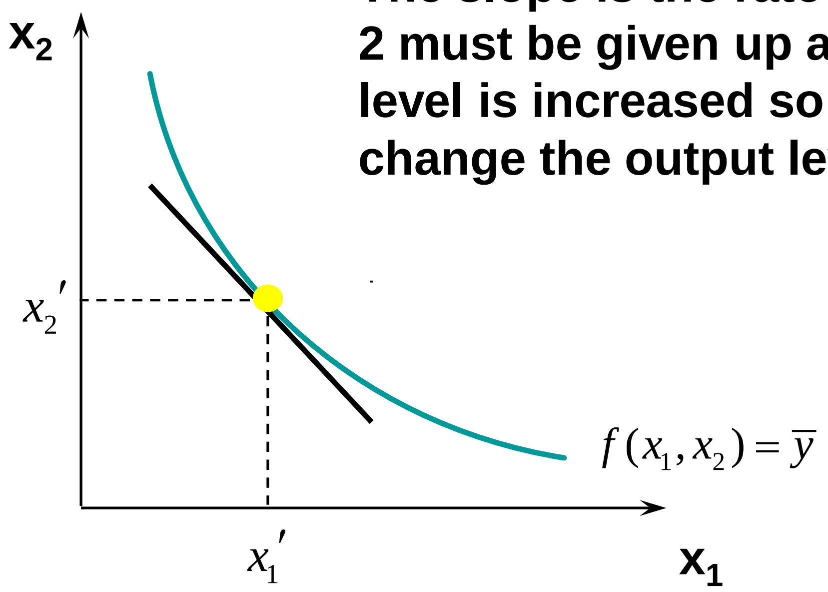
Thus, from $\dfrac{\partial f}{\partial x_1} dx_1 + \dfrac{\partial f}{\partial x_2} dx_2 = 0$

$$TRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{\partial f / \partial x_1}{\partial f / \partial x_2} = -\frac{MP_1}{MP_2}$$

Technical Rate of Substitution

- Technical rate of Substitution (TRS)
  - measures how one of the inputs must adjust in order to keep output unchanged when another input changes
  - Tradeoff between two inputs in production
  - The slope of isoquant

The slope is the rate at which input 2 must be given up as input 1's level is increased so as not to change the output level.



Technical Rate of Substitution

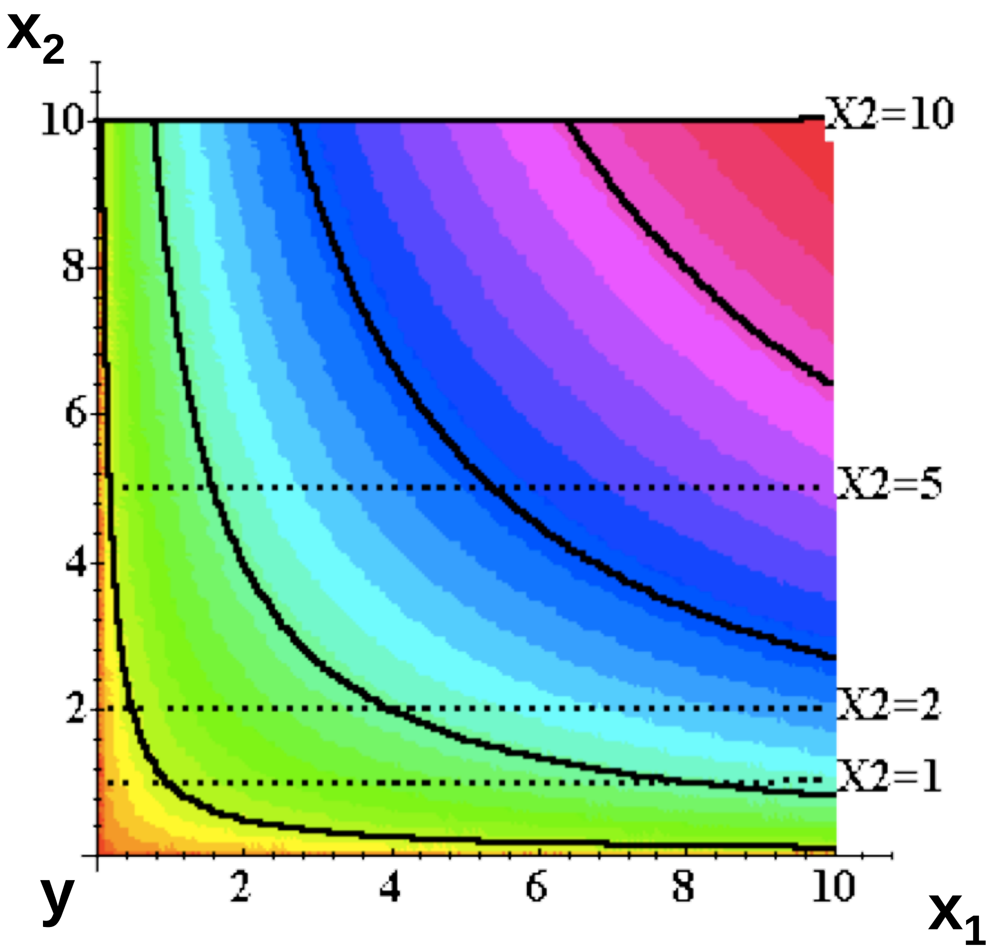
- Example: Cobb-Douglas technology

The Long-Run and the Short-Runs

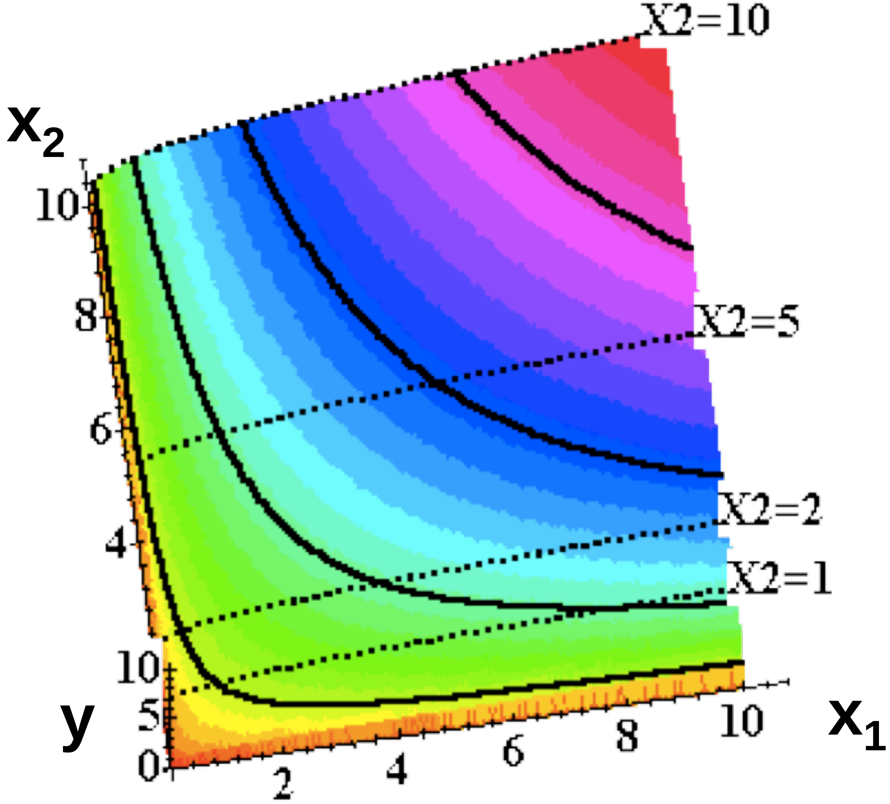
- The long-run is the circumstance in which a firm is unrestricted in its choice of all input levels.
- A short-run is a circumstance in which a firm is restricted in some way in its choice of at least one input level.
- A useful way to think of the long-run is that the firm can choose as it is best among short-run circumstances

The Long-Run and the Short-Runs

- When input 2 is fixed input in the short-run.



The Long-Run and the Short-Runs



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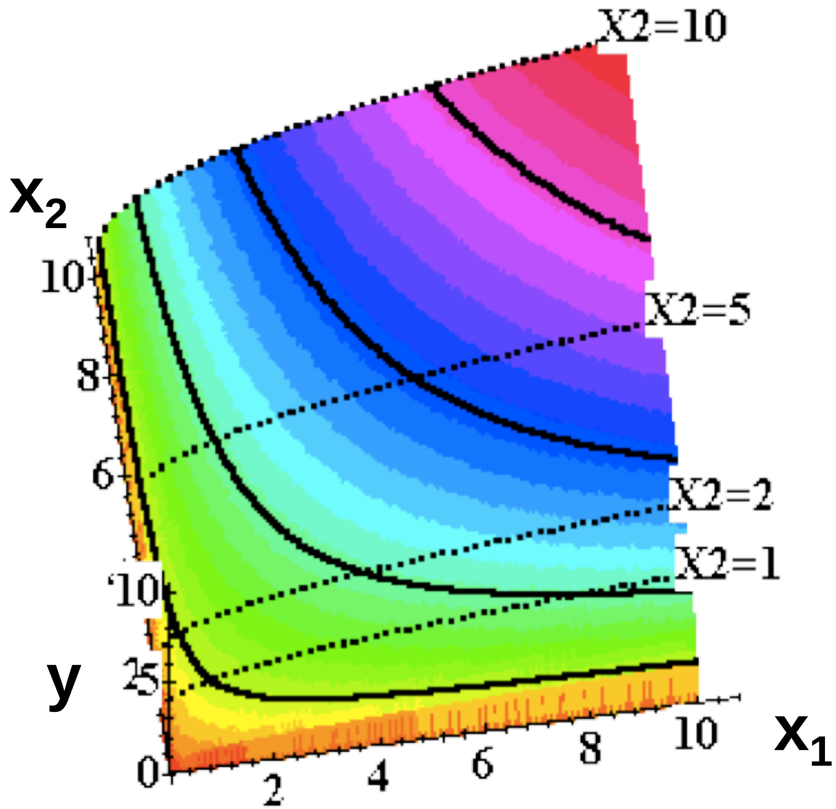

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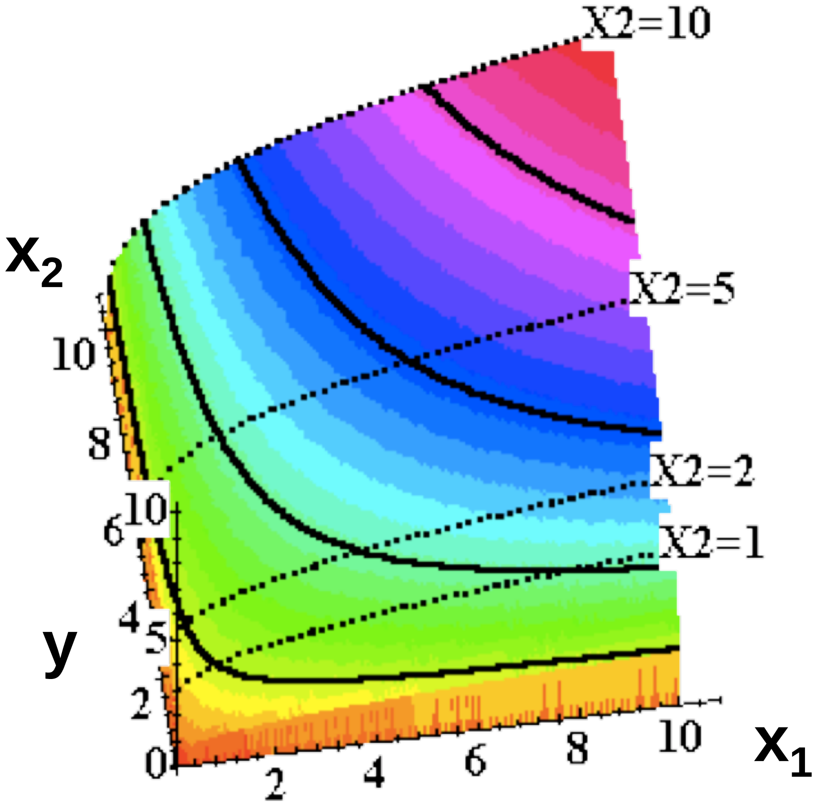

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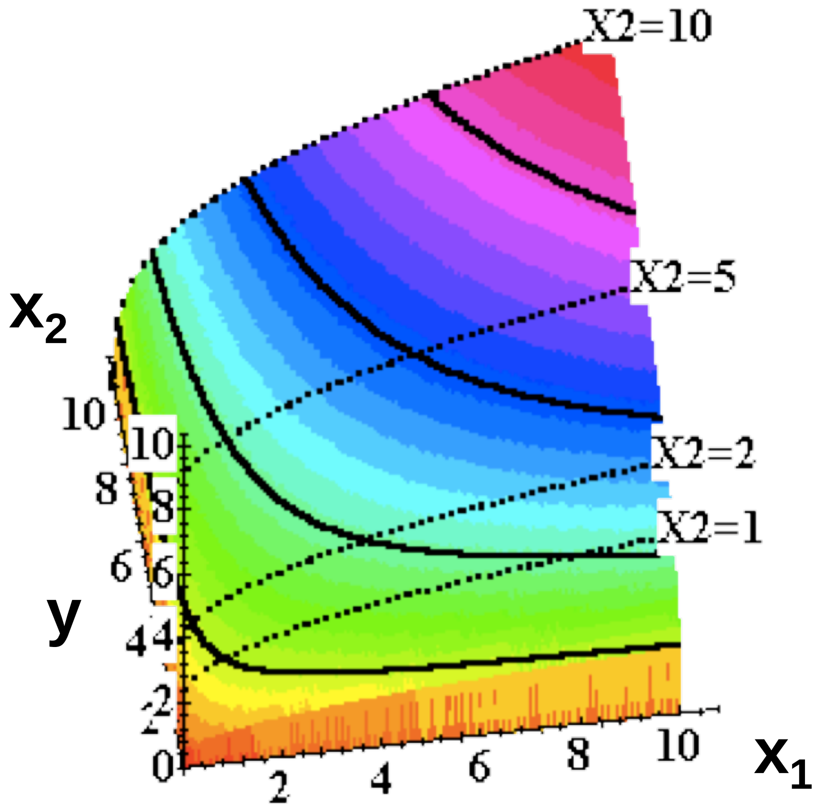

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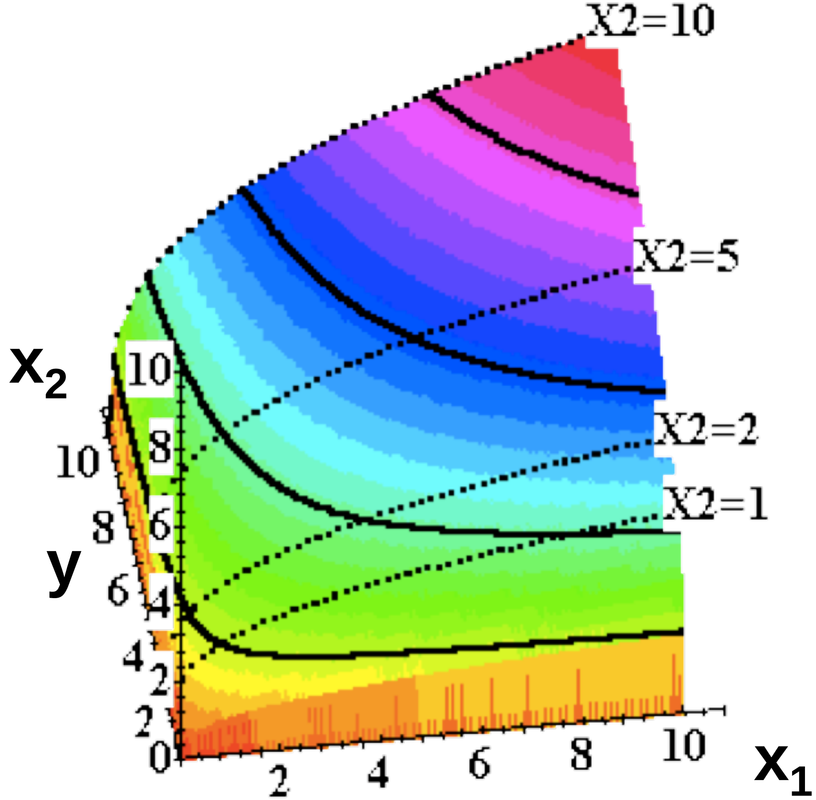

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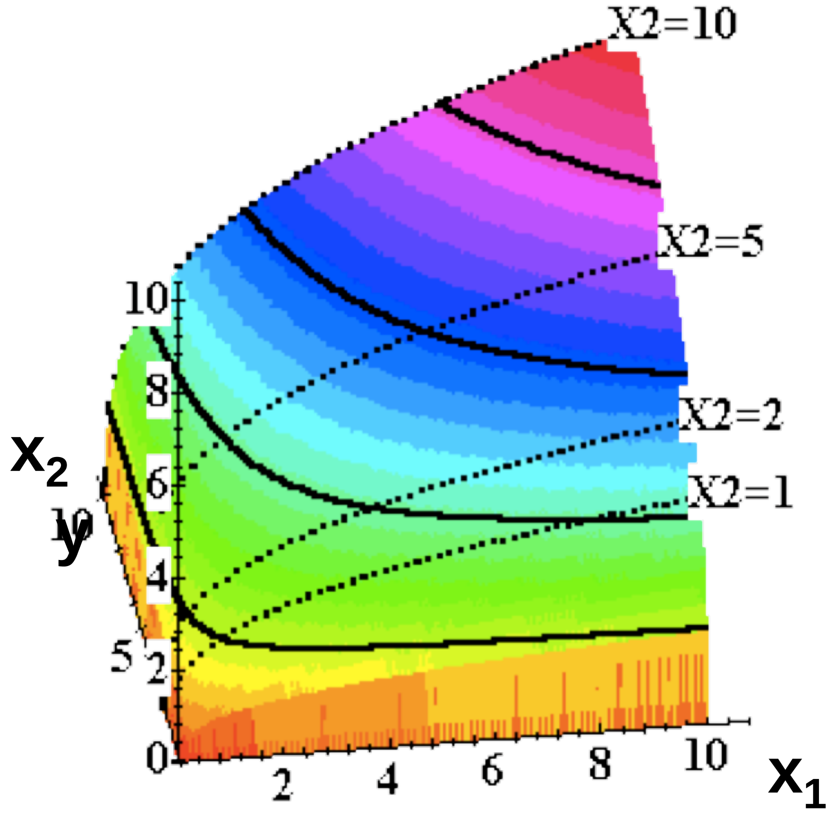

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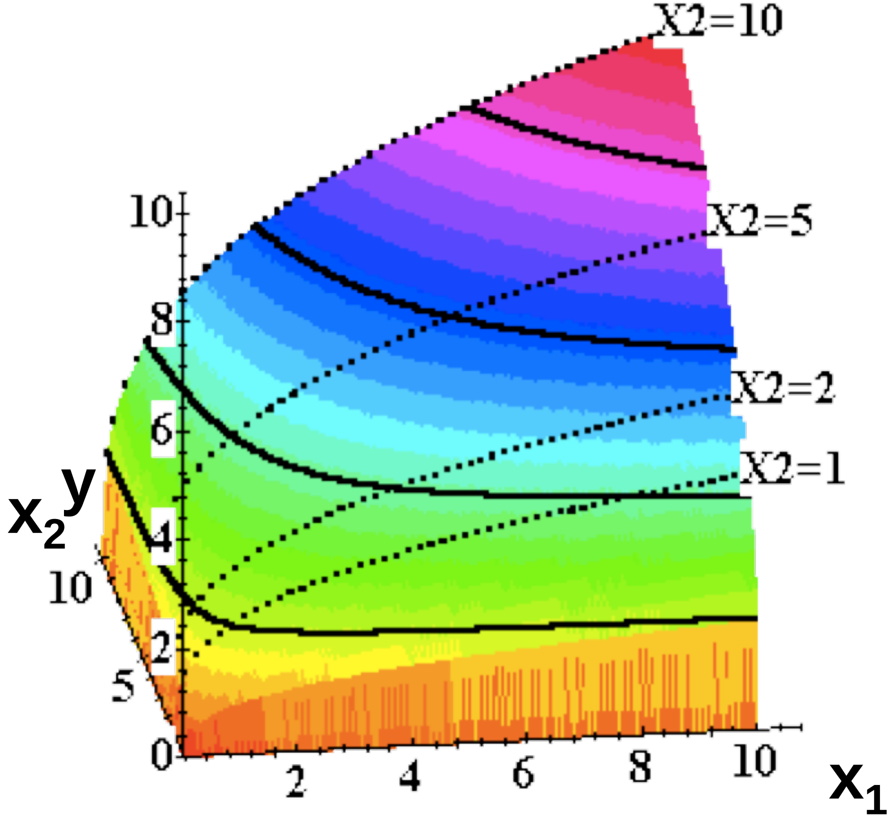
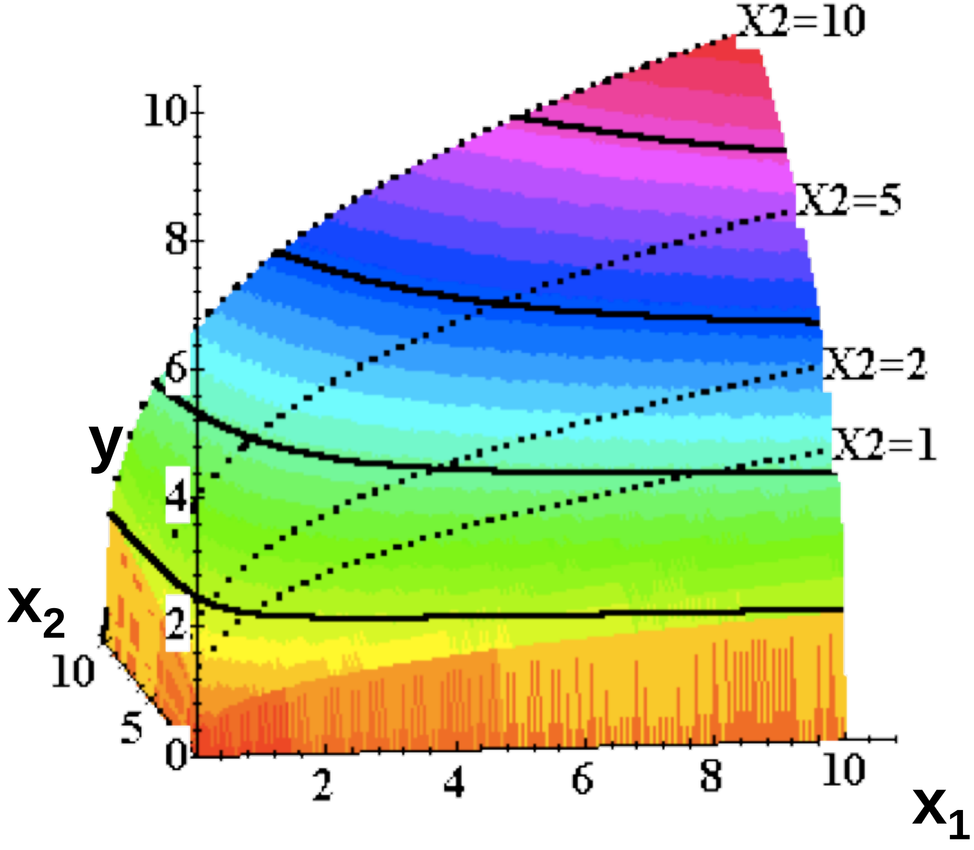
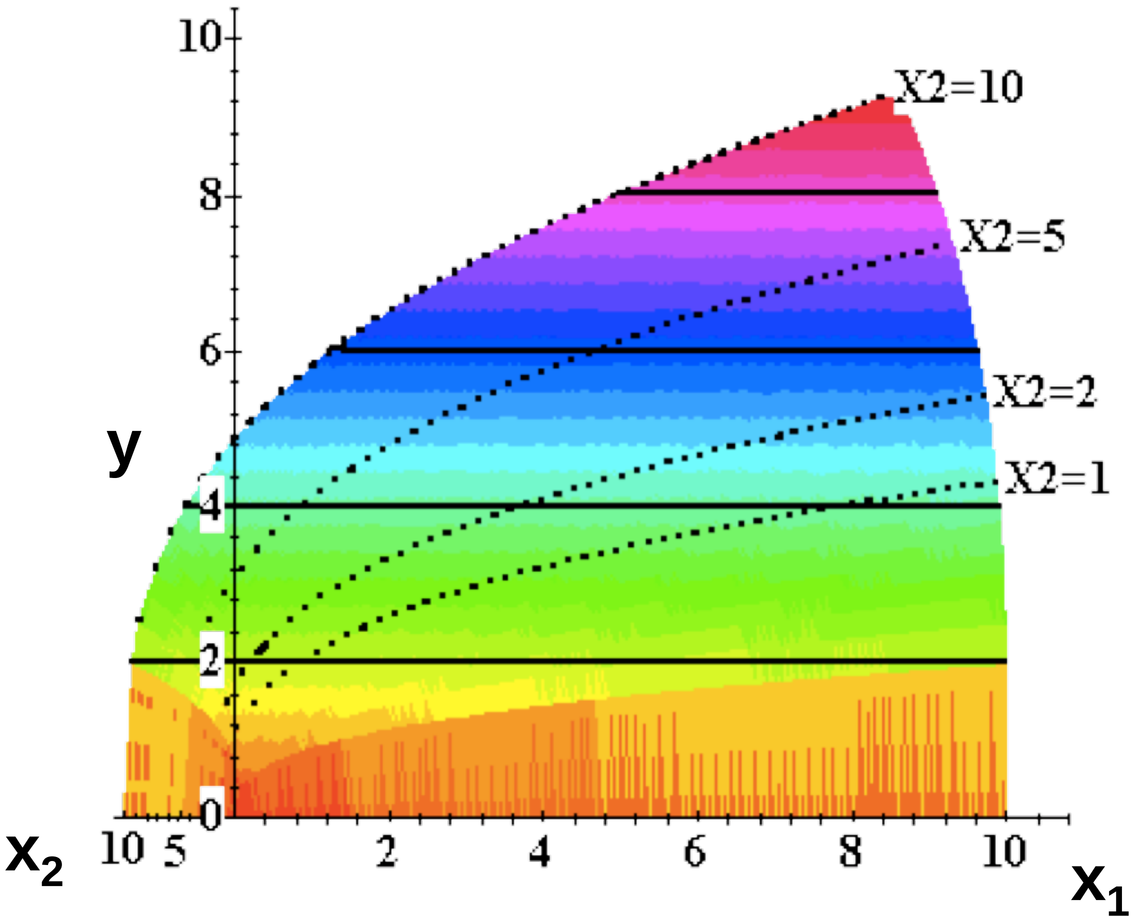



X2=10
X2=5
X2=2
X2=1
y
x_2
x_1
10
8
6
4
2
0
5
10
2
4
6
8
10

The Long-Run and the Short-Runs



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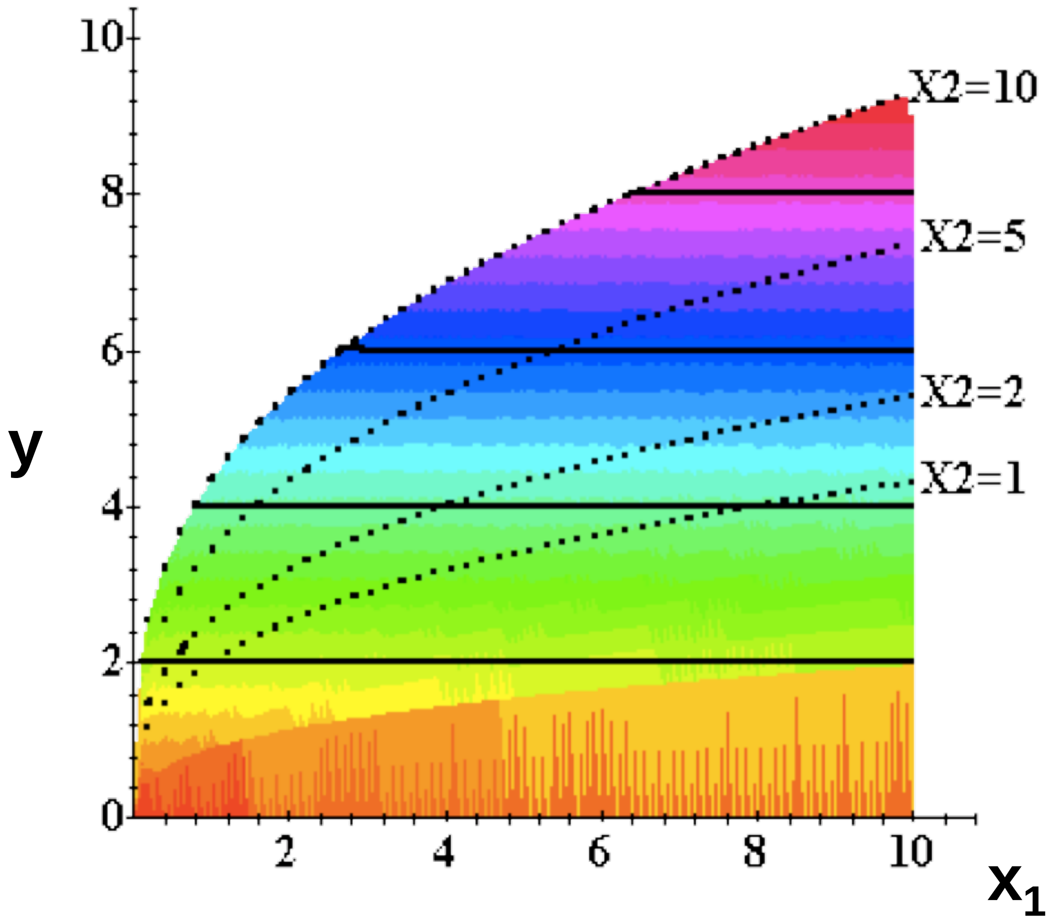

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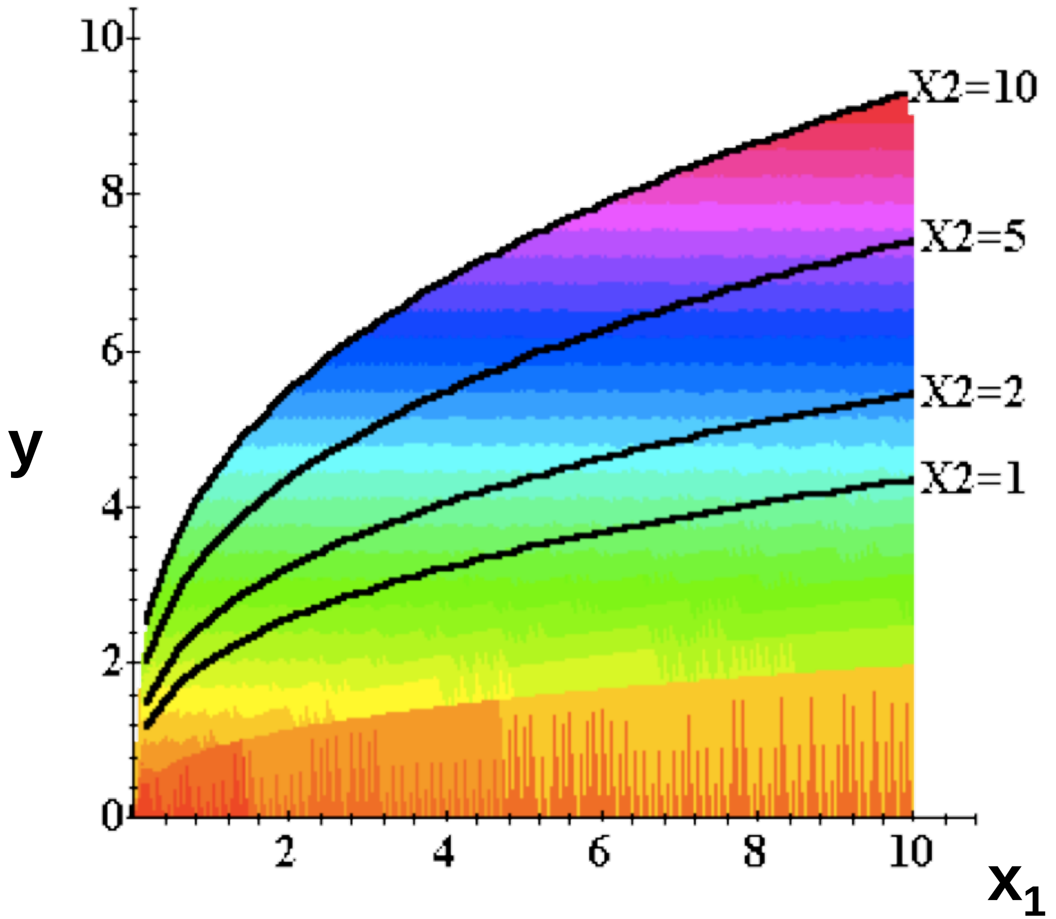

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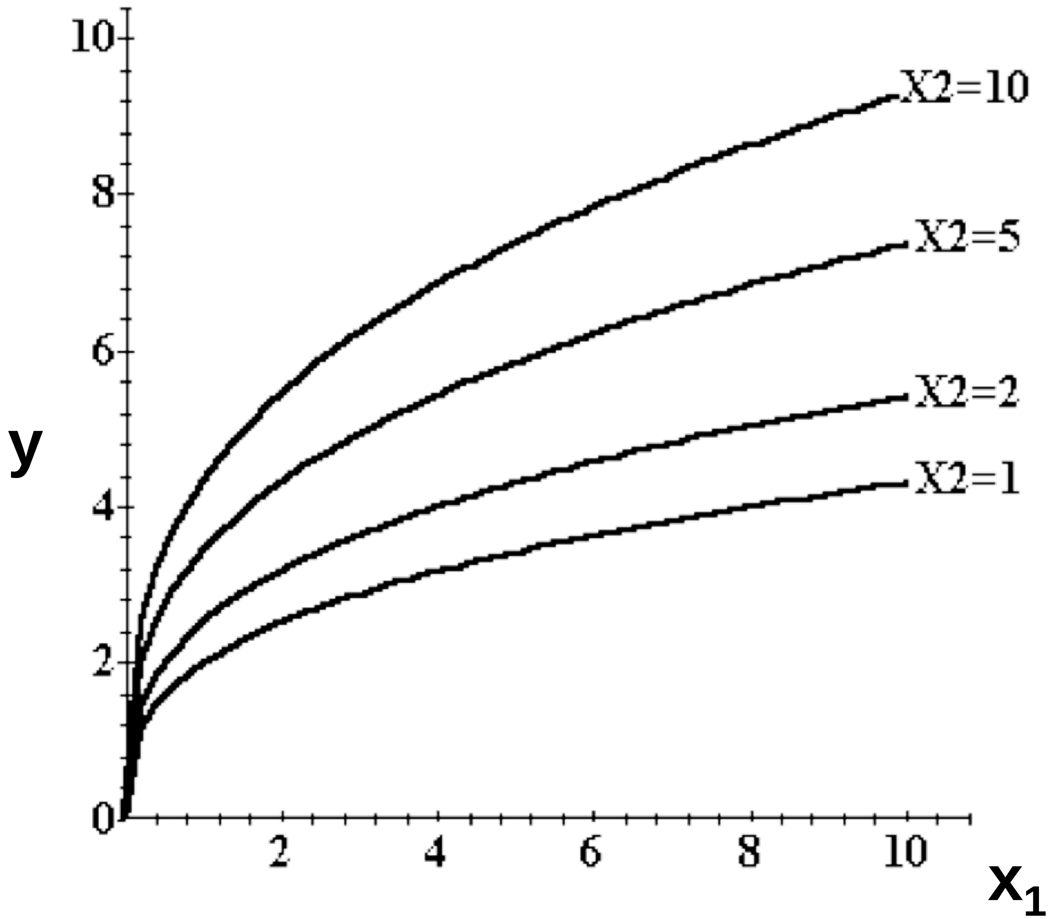

Four short-run production functions.

Returns-to-Scale

- Marginal products describe the change in output level as a single input level changes.
- Returns-to-scale describes how the output level changes as all input levels change in direct proportion (*e.g.* all input levels doubled, or halved)
  - A technology exhibits *constant returns-to-scale (CRS)* if

    $$f(\tilde{x}):\ f(t\tilde{x}) = tf(\tilde{x}) \quad \text{for all } t \geq 0$$

    - Homogeneous of degree 1
  - A technology exhibits *increasing returns-to-scale (IRS)* if

    $$f(\tilde{x}):\ f(t\tilde{x}) > tf(\tilde{x}) \quad \text{for all } t > 1$$

  - A technology exhibits *decreasing returns-to-scale (DRS)* if

    $$f(\tilde{x}):\ f(t\tilde{x}) < tf(\tilde{x}) \quad \text{for all } t > 1$$

Returns to Scale

- Example: Cobb-Douglas technology