

## Chap 8. Hydraulic Model Tests with Irregular Waves

### 8.1 Similarity Laws and Scale Effects

- Geometric similitude (length)
- kinematic similitude (velocity and acceleration)
- dynamic similitude (all forces)

inertia, gravity, viscous force, surface tension

↘ ✓

important for sea waves → Froude similitude

For Froude law (see Table 8.1),

horizontal and vertical length scale =  $l_r = l_m / l_p$

horizontal and vertical velocity scale =  $V_r = V_m / V_p$

time scale =  $t_r = l_r / V_r$

Froude law requires

$$\frac{V_m}{\sqrt{g l_m}} = \frac{V_p}{\sqrt{g l_p}} \rightarrow V_r = \sqrt{l_r} \quad \text{and} \quad t_r = \sqrt{l_r}$$

Wave pressure (force per unit area):  $p_r = \rho_r V_r^2 = \rho_r g_r l_r = l_r$  for  $\rho_r = 1$ ,  $g_r = 1$

Force per unit length:  $p_r \times l_r = l_r^2$

Weight per unit length:  $w_r = S_r \rho_r g_r l_r^3 / l_r = l_r^2$  if  $S_r = 1$

Weight of armor unit:  $W_r = S_r \rho_r g_r l_r^3 = l_r^3$  if  $S_r = 1$

⋮

Typically,  $l_r = \frac{1}{50} \sim \frac{1}{150}$  for harbor model tests

$l_r = \frac{1}{10} \sim \frac{1}{50}$  for coastal structure model tests

Require wave period  $> 0.5$  s in laboratory to minimize scale effects,  $T_s \geq 1.0$  s,  $H_s \geq 10$  cm or so (at least several cm).

## 8.2 Necessity of Hydraulic Model Test with Random Waves

Development of computers → Many problems are solved by numerical models (especially, when the computational area is large, like storm surge generation and tsunami propagation, for which field survey or laboratory experiment is impossible).

However, hydraulic model tests are widely used for very complicated problems, for which analytical or numerical approaches cannot give accurate solutions.

See Goda's book for problems for which hydraulic model tests are used.

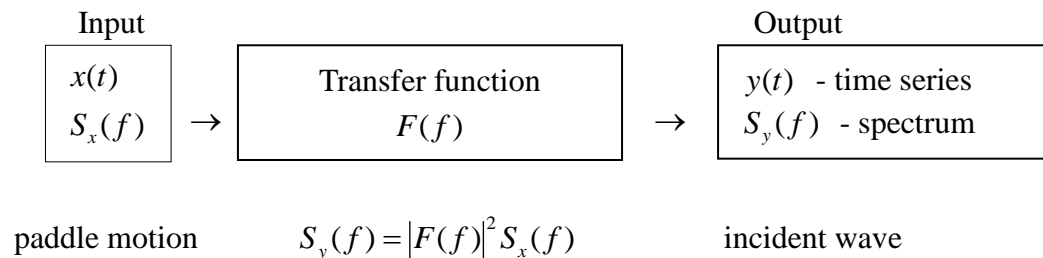
## 8.3 Generation of Random Waves in Test Basins

### 8.3.1 Random Wave Generator (Read text)

### 8.3.2 Preparation of Input Signal to the Generator

- Specification of incident wave spectrum (target):  $S_w(f)$
- Specify transfer function:  $F(f, h)$
- Compute spectrum of wave paddle motion:  $S_G(f) = S_w(f) / F^2(f, h)$
- Input signal (time series) of paddle motion
- Generate waves in a tank and measure incident waves
- Compare measured and target spectra until specified design waves are reproduced.

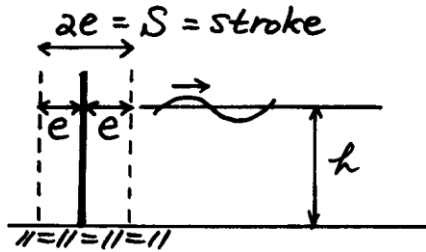
For a linear system response,



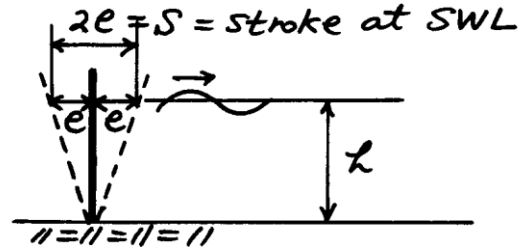
$$F(f) = \frac{\text{height of sinusoidal motion of } y(t) \text{ for given } f}{\text{height of sinusoidal motion of } x(t) \text{ for given } f}$$

on the basis of linear monochromatic wavemaker theory

Piston



Flap



For piston type ( $j = 1$ ),

$$F_1(f) = \frac{H}{2e} = \frac{4 \sinh^2(kh)}{2kh + \sinh(2kh)}$$

For flap type ( $j = 2$ ),

$$F_2(f) = \frac{H}{2e} = \frac{4 \sinh(kh)}{kh} \frac{1 - \cosh(kh) + kh \sinh(kh)}{2kh + \sinh(2kh)}$$

where  $kh \tanh(kh) = \frac{h}{g} (2\pi f)^2$ ;  $kh$  can be found for given  $f$  and  $h$ .

For this case,

$$S_x(f) = S_G(f) = \text{paddle spectrum}$$

$$S_y(f) = S_w(f) = \text{incident wave spectrum}$$

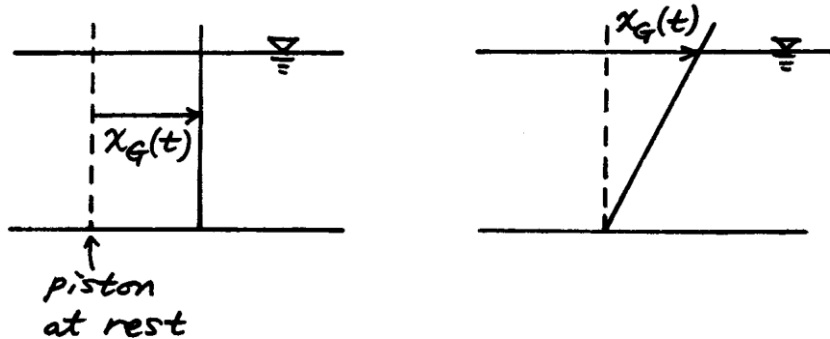
$$S_w(f) = F_j^2(f) S_G(f), \quad j = 1, 2$$

↓

$$S_G(f) = \frac{S_w(f)}{F_j^2(f)}$$

For estimated  $S_G(f)$ , need to generate the corresponding time series of the paddle

displacement  $x_G(t)$ .



Three digital simulation methods are available. But the inverse FFT method is the most common and easiest.

$$x_G(t) = \sum_{n=1}^N a_n \cos(2\pi f_n t + \varepsilon_n) \quad \text{with} \quad f_n = n\Delta f$$

$$a_n = [2S_G(f_n)\Delta f]^{1/2} \quad \text{random phase distributed uniformly over } [0, 2\pi]$$

↑  
use random number generator

Use an inverse FFT rather than summation. After time series  $x_G(t)$  of paddle motion is calculated, compute time series of voltage signal to move paddle, using a calibration curve between paddle displacement and voltage.

Measure incident waves in the tank after steady state is established ( $\because$  low frequency waves travel faster) or use the remedy suggested in Goda (p.342), i.e., phase-delayed signals to wave generators (high-frequency first)

Compare target spectrum  $S_w(f)$  and measured spectrum  $S_m(f)$ . Also check representative wave heights and periods expected from  $S_w(f)$  against measured values. Generally  $S_w(f) \neq S_m(f)$ .

Adjust the transfer function

$$F_{new}(f) = F_{old}(f) \left( \frac{\sqrt{S_m(f)}}{\sqrt{S_w(f)}} \right)^{-1}$$

Repeat the above procedure until  $S_w(f) \cong S_m(f)$ .

### 8.3.3 Input Signals to a Multidirectional Random Wave Generator



$$\eta(x, y, t) = \sum_{m=1}^M \sum_{n=1}^N A_{mn} \cos(k_m x \cos \theta_n + k_m y \sin \theta_n - 2\pi f_m t + \varepsilon_{mn})$$

At  $x = 0$ ,

$$\eta(0, y, t) = \sum_{m=1}^M \sum_{n=1}^N A_{mn} \cos(k_m y \sin \theta_n - 2\pi f_m t + \varepsilon_{mn})$$

$$x_G(y, t) = \sum_{m=1}^M \sum_{n=1}^N a_{mn} \cos(k_m y \sin \theta_n - 2\pi f_m t + \varepsilon_{mn})$$

↑

$$a_{mn} = [2S_G(f_m, \theta_n) \Delta f \Delta \theta]^{1/2}$$

$$S_G(f_m, \theta_n) = \frac{S_w(f_m, \theta_n)}{F_j^2(f_m)}$$

### 8.3.4 Non-Reflective Wave Generator

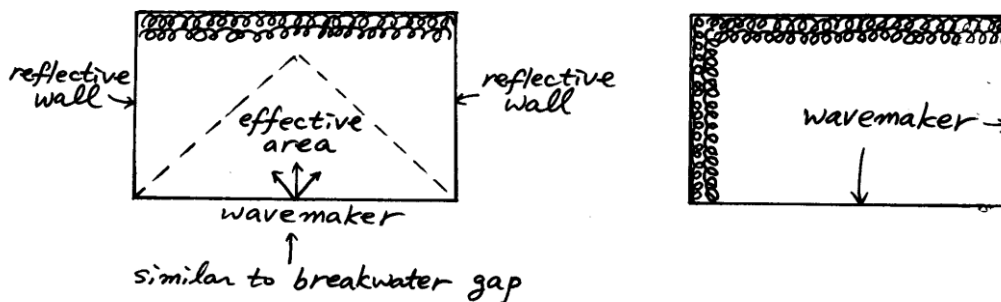
Wave reflection from model structure → Reflected waves towards wave generator →  
 Re-reflection from wave paddle → Re-reflection from model structure → ... →  
 Accumulation of wave energy in wave flume

Detect the reflected wave at the wave generator by either wave gauge or force measurement device → Adjust the input signal to wave generator to absorb the reflected wave

## 8.4 Model Tests Using Multidirectional Wave Generator

Limitations:

- (1) Narrow effective areas



- (2) Absorption of reflected waves from model structures is difficult because the reflected waves come back to the wave paddles at oblique angles, which are difficult to detect.

Problems for which multidirectional wavemaker is desirable: diffraction dominated problems

- harbor tranquility
- 3D stability of breakwater head
- spatial variation of run-up and overtopping of finite-length seawalls

## 8.5 Some Remarks on Execution of Random Wave Tests

### 8.5.1 Number of Test Runs and Their Durations

- A generation of random waves in a test flume is just one sample of many possible realizations of true sea state
- The standard deviation of a measured wave height (e.g. significant wave height) decreases as the duration (i.e. number of generated waves) increases
- In general, it is recommended to make three test runs (with different input seed value) of about 200 waves.

- In the stability tests of armor units, the damage is a function of number of waves. The duration should be 3 to 6 hours in prototype (several thousands of waves)
- In wave overtopping tests, the variation is large. More than 5 test runs are needed.

#### 8.5.2 Calibration of Test Waves

#### 8.5.3 Resolution of Incident and Reflected Waves in a Test Flume

- Check whether the desired incident waves are generated by generating three or more runs of test waves in the absence of model structure
- Save the input seed value and use the same seed value with the test structure
- Separate the incident and reflected waves during the experiment with the structure
- Check whether the resolved incident waves are the same as the calibrated waves

#### 8.5.4 Statistical Variability of Damage Ratio of Armor Units

$$\text{damage ratio } (p) = \frac{\text{number of displaced units } (r)}{\text{total number of units } (n)}$$

↑

stochastic variable with a certain uncertainty

$$\text{standard deviation } \sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

↓

$$90\% \text{ confidence interval} = \pm 1.64\sigma_p$$