Part IV Magnetic Properties of Materials

Introduction

- **The phenomenon of magnetism**
  - **- The mutual attraction of two pieces of iron ore was surely known to the antique world.**
  - **- *Magnetism* is said to be derived from the name of *Magnesia*, a region in Turkey which had plenty of iron ore.**

- **Contribution of magnetic materials to the mankind**
  - **- Compass**
  - **- Magnetic tapes or disks (computers), television, motors, generators, telephones, and transformers**

- **Types of magnetism**
  - **- Diamagnetism**
  - **- Paramagnetism**
  - **- Antiferromagnetism**
  - **- Ferromagnetism**
  - **- Ferrimagnetism**

Magnetic Field, *H*

- ***H*** **represents a magnetic force generated in a volume of the space due to a change in magnetic energy of that of the space.**

  Examples of the magnetic force
  - A force on a current-carrying conductor
  - A torque on a magnetic dipole
  - A reorientation of spins on electrons within atoms

- ***H*** **is fundamentally generated by an electrical charge in motion.**
  Earth (~ 0.7Oe)
  Bulk magnets (~ 5,000Oe)
  Current-carrying conductors (~30,000Oe)
  Superconductors (>100,000Oe)
  (ref. Table 1.1 in David Jiles)

- **Unit**
  mks or SI(Systeme Interrnational): [A/m]
  cgs: [Oe]

$$1\ \mathrm{Oe} = \frac{1000}{4\pi}\ \mathrm{A/m}\ (\sim 79.6\ \mathrm{A/m})$$

Magnetic Induction, *B*

- ***B*** **is the response of a medium to an applied magnetic field *H***
- ***B*** **is defined by** $B = \frac{\Phi}{A}$

B is the magnetic flux, Φ [Wb] passing through a unit cross-sectional area.

Magnetic flux, Φ?
- Generated by the presence of a magnetic field in a medium.

- By Lentz law, the voltage V is induced as Φ changes

$$V = -N\frac{d\Phi}{dt} = -NA\frac{di}{dt}$$ **called, "electromagnetic induction"**

$$1 \text{ volt} = -(1)\frac{\Phi_i - \Phi_f}{1\sec}$$ **If $\Phi_f = 0$, $\Phi_i = 1$ Wb**
(1Wb = 1 volt•sec)

Relation between B and H

$B = \mu H$, where μ is permeability(투자율)

$\mu = \mu_o$ in free space

$= 4\pi \times 10^{-7}$ H/m (or Wb/A)

Relative permeability $\mu_r = \mu/\mu_o$

$\mu_r = 1$ in a perfect vacuum(free space)

Maxwell's equation (Gauss's law)
$\nabla \bullet B = 0$: Always form a closed path!

- **Unit: [G], [T]**
  1 Tesla = 1 Wb/m^2 (= 1 volt•sec/m^2)
  A force of 1 N/m on a conductor carrying 1 A perpendicular to the direction of *B*

Magnetic Moment, *m*

- **Definitions :**
  $m = pl$ in a bar magnet
  $m = iA$ in a conductor loop

- **Unit**

	SI	cgs
	[Am^2]	[emu]
	[Wbm]	[erg/Oe]

$$1\ \text{Wbm} = \frac{1}{4\pi}\, 10^{10}\,\text{Gcm}^3$$

- **Measurements of *m***
  i) Torque measurement: $\tau = \mu_o m \times H = m \times B$
  $\tau = \tau_{max}$ if m is perpendicupar to H (or B), and then
  $m = \tau_{max}/\mu_o H$
  Since $m = pl$ and $p = \Phi/\mu_o$ in the Sommerfeld conversion
  $m = \Phi l/\mu_o$
  ii) Magnetization measurement
  $m = MV$

Magnetization, M

▶ M is the magnetic moment m per unit volume (cf. m per unit mass = specific magnetization σ)

$M =$ (cf. $\sigma = M/\rho$ [emu/g]), where ρ is density

Since $m = \Phi l/\mu_o$, $V = Al$

$M = \Phi/\mu_o A = B/\mu_o$

Therefore, $B = \mu_o M$ when $H = 0$

▶ Saturation Magnetization

M_o : complete saturation, where all atomic moments are aligned parallel to H_a

M_s : technical saturation, where multiple-domains become single domain

▶ Relation between M and H

$M = \chi H$, where χ is susceptibility(자화율) ↔ $B = \mu H$ (μ is permeability(투자율))

μ and χ are not useful for ferromagnets.

Need differential values: $\mu' = dB/dH$, $\chi' = dM/dH$

▶ Relationship between H, B, and M

A universal relationship

$B = \mu_0(H + M)$: SI(Sommerfeld)

$= \mu_0 H + I$: SI(Kennelly)

$= H + 4\pi M$: cgs(Gaussian)

$B = \mu_0(H + M) = \mu_0(H + \chi H) = \mu_0(1 + \chi)H$

Since $B = \mu H = \mu_0\mu_r H$, $\mu_r = 1 + \chi$

μ_r and χ are different ways of describing the response of a material to magnetic fields.

Magnetization Curve

Magnetic Curve : ***χ(T) and M(T) curves***

Diamagnets χ(T)

Normal diamagnets (see Fig. 3.4-5 in O'Handley)

Superconductors(perfect diamagnetism)

Paramagnets χ(T)

Typical paramagnets (Curie & Curie-Weiss law)

Pauli paramagnets

cf) superparamagnetism

Ferromagnets

$T > T_c$: *Typical paramagnetic χ(T) (Curie-Weiss law)*

$T < T_c$: $M_s(T)$

Antiferromagnets χ(T)

$T > T_N$: *Typical paramagnetic χ(T) (Curie-Weiss law)*

$T < T_N$: $\chi(T)$

Ferrimagnets

$T > T_c$: *Peculiar paramagnetic χ(T) (not following Curie-Weiss law)*

$T < T_c$: $M_s(T)$

Basic Concepts in Magnetism

- (See Fig. 14.1) Diamagnetic materials are expelled from the field, whereas para-, ferro-, antiferro-, and ferrimagnetic materials are attracted in different degrees.

$$F = V\chi\mu_0 H \frac{dH}{dx} \quad \textbf{(14.1)}$$

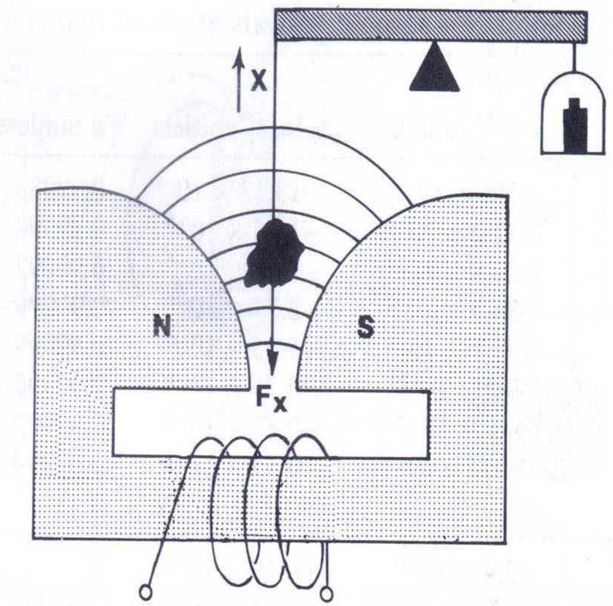

Figure 14.1. Measurement of the magnetic susceptibility in an inhomogeneous magnetic field. The electromagnet is driven by an electric current, which flows through the helical windings of a long insulated wire called a solenoid. The magnetic flux lines (dashed) follow the iron core.

F: force

V : the volume of the sample

χ : susceptibility

H : magnetic field

$\frac{dH}{dx}$: the change of the magnetic field strength *H* in the *x*-direction

Basic Concepts in Magnetism

$$H = \frac{In}{L}(A/m) \quad (14.2)$$

H : the field strength

I : current

n : the number of the windings

L : the length of the solenoid

$$B = \mu\mu_0 H \quad (14.3)$$

B : magnetic induction or ***magnetic flux density*** (tesla; ***T***)

μ : permeability or relative permeability(μ_r)

(unitless)

Basic Concepts in Magnetism

$$\mu = 1 + \chi \qquad (14.4)$$

(See Table 14.1)

- For empty space and, for all practical purpose, also for air,

  $\chi = 0$ and thus $\mu = 1$

- For diamagnetic materials,

  χ is small and negative. $\rightarrow$ μ is slightly less than 1

- For para- and antiferromagnetic materials,

  χ is small and positive. $\rightarrow$ μ is slightly larger than 1

- For ferro- and ferrimagnetic materials,

  χ and μ are large and positive.

$\Rightarrow$ The magnetic constants are temperature-dependent, expect diamagnetic materials. The susceptibility for ferromagnetic materials depends on the field strength, *H*.

Basic Concepts in Magnetism

Table 14.1. Magnetic constants of some materials at room temperature

Material	χ (SI) unitless	χ (cgs) unitless	μ unitless	Type of magnetism
Bi	-165×10^{-6}	-13.13×10^{-6}	0.99983	
Be	-23.2×10^{-6}	-1.85×10^{-6}	0.99998	
Ag	-23.8×10^{-6}	-1.90×10^{-6}	0.99997	
Au	-34.4×10^{-6}	-2.74×10^{-6}	0.99996	Diamagnetic
Ge	-71.1×10^{-6}	-5.66×10^{-6}	0.99999	
Cu	-9.7×10^{-6}	-0.77×10^{-6}	0.99999	
Si	-4.1×10^{-6}	-0.32×10^{-6}	0.99999	
Water	-9.14×10^{-6}	-0.73×10^{-6}	0.99999	
Superconductors[a]	-1.0	$\sim -8 \times 10^{-2}$	0	
β-Sn	$+2.4 \times 10^{-6}$	$+0.19 \times 10^{-6}$	1	
W	$+77.7 \times 10^{-6}$	$+6.18 \times 10^{-6}$	1.00008	
Al	$+20.7 \times 10^{-6}$	$+1.65 \times 10^{-6}$	1.00002	Paramagnetic
Pt	$+264.4 \times 10^{-6}$	$+21.04 \times 10^{-6}$	1.00026	
Low carbon steel	$\approx 5 \times 10^{3}$	3.98×10^{2}	5×10^{3}	
Fe–3%Si (grain-oriented)	4×10^{4}	3.18×10^{3}	4×10^{4}	Ferromagnetic
Ni–Fe–Mo (supermalloy)	10^{6}	7.96×10^{4}	10^{6}	

[a] See Section 7.6.

Note: The table lists the unitless susceptibility, χ, in SI and cgs units. (The difference is a factor of 4π, see Appendix 4.) Other sources may provide mass, atomic, molar, volume, or gram equivalent susceptibilities in cgs or SI units. μ has the same value in both unit systems, see Section 14.3.

Source: Landolt-Börnstein, *Zahlenwerte der Physik*, Vol. 11/9, 6th Edition, Springer-Verlag, Berlin (1962).

Basic Concepts in Magnetism

$$B = \mu_0 H + \mu_0 M$$ in free space (14.5)

$$M = \chi H$$ ***M*** : magnetization (14.6)

$$\phi = BA$$ (14.7)

ϕ : magnetic flux

B : magnetic flux density

$$\phi = \mu_0 HA$$ (14.7a)

in free space (**M=0**)

$$M = \frac{\mu_m}{V}$$ (14.8)

μ_m : magnetic moment

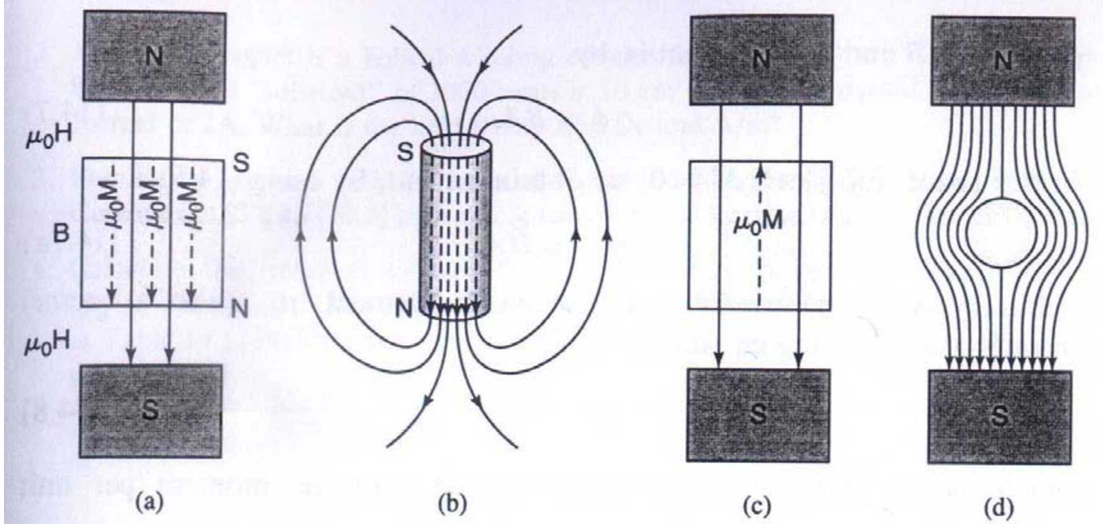

Figure 14.2. Schematic representation of magnetic field lines in and around different types of materials. (a) *Para- or ferromagnetics.* The magnetic induction (**B**) inside the material consists of the free-space component ($\mu_0\mathbf{H}$) plus a contribution by the material ($\mu_0\mathbf{M}$); see Eq. (14.5). (b) The magnetic field lines outside a material point from the north to the south poles, whereas inside of para- or ferromagnetics, **B** and $\mu_0\mathbf{M}$ point from south to north in order to maintain continuity. (c) In *diamagnetics*, the response of the material counteracts (weakens) the external magnetic field. (d) In a thin surface layer of a *superconductor*, a supercurrent is created (below its transition temperature) which causes a magnetic field that opposes the external field. As a consequence, the magnetic flux lines are expelled from the interior of the material. Compare to Figure 9.18.

Units

- **SI unit** ***VS*** **cgs unit**

$$B = \mu_0 H + \mu_0 M$$ **(14.5)**

$$B = \mu\mu_0 H$$ **(14.3)**

$$\mu = 1 + \chi$$ **(14.4)**

magnetic field strength, *H* (Oersted)
magnetic induction, *B* (Gauss)

In some European countries, and in many international scientific journals

$$B = H + 4\pi M$$ **(14.9)**

$$B = \mu H$$ **(14.10)**

$$\mu = 1 + 4\pi\chi$$ **(14.11)**

magnetic field strength, *H* (A/m)
magnetic induction, *B* (Tesla)

The scientific and technical literature on magnetism, particularly in the USA

Part IV Magnetic Properties of Materials

Chap. 14. Foundations of Magnetism

Chap. 15. Magnetic Phenomena and Their Interpretation- Classical Approach

Chap. 16. Quantum Mechanical Considerations

Chap. 17 Applications

Overview –Types of Magnetism

➢ Diamagnetism

- **Lenz's Law** : a current is induced in a wire loop whenever a bar magnet is moved toward (or from) the loop. The current induce a magnetic moment opposite to the bar magnet (Fig.15.1(a)

- The external field (H_{ex}) accelerates or decelerates the orbiting electrons, in order that their magnetic moment is in opposite direction from H_{ex}

- **Lamor precession**: Precessions of electron orbits about the magnetic field direction (Fig.15 1(b))

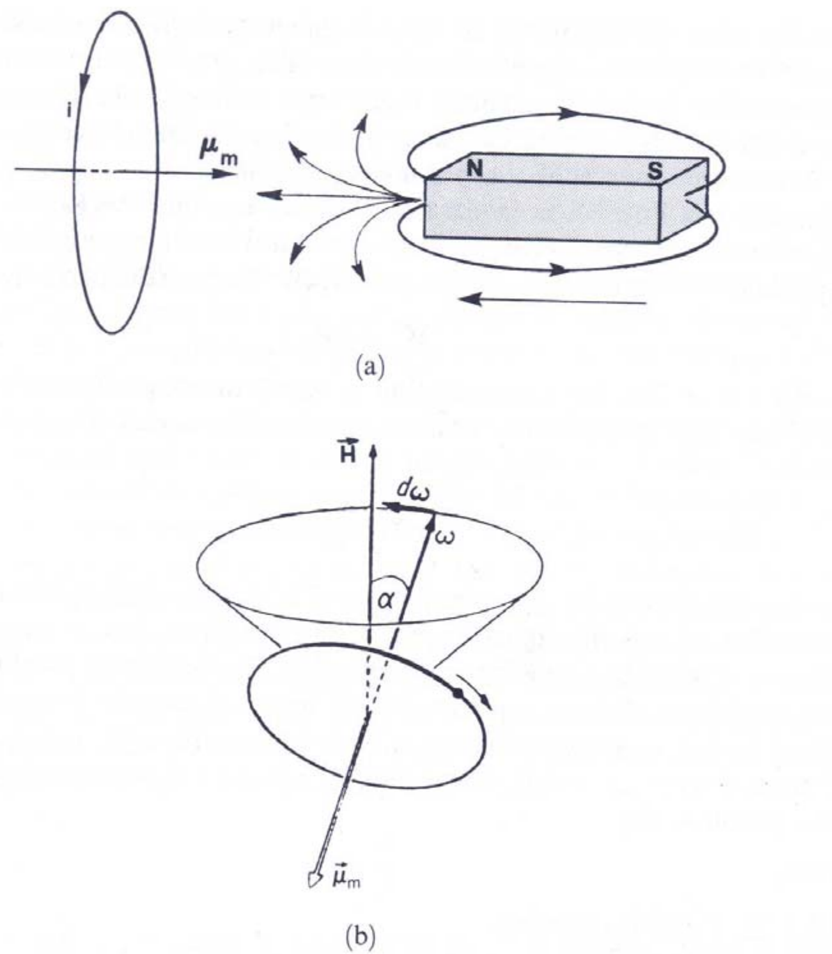

Figure 15.1. Explanation of diamagnetism. (a) Induction of a current in a loop-shaped piece of wire by moving a bar magnet toward the wire loop. The current in the loop causes a magnetic field that is directed opposite to the magnetic field of the bar magnet (Lenz's law). (b) Precession of an orbiting electron in an external magnetic field. Precession is the motion which arises as a result of external torque acting on a spinning body (such as a spinning top) or, as here, on an orbiting electron.

Overview –Types of Magnetism

➢ Diamagnetism

❑ Diamagnetism in Superconducting materials (Sec. 7.6)

- Meissner effect : Superconductors expel the magnetic flux lines in the superconducting state. Inside superconductor B is zero. (Fig. 14.2(d))

$$H = -M$$

- Perfect diamagnetism: Magnetization is equal and opposite to the external magnetic field.

Susceptibility, $\chi = M/H = -1$

❑ Usage of strong diamagnetism of superconductor

- Frictionless bearing: support of loads by a repelling magnetic field
- Levitation: magnet hovers above a superconducting materials
- Suspension effect: a chip of superconducting material hangs beneath a magnet

Overview –Types of Magnetism

➢ Paramagnetism

Largely due to electron spin motion. An additional source stems from orbiting motion.

- An external field turns randomly oriented magnetic moments into the field direction

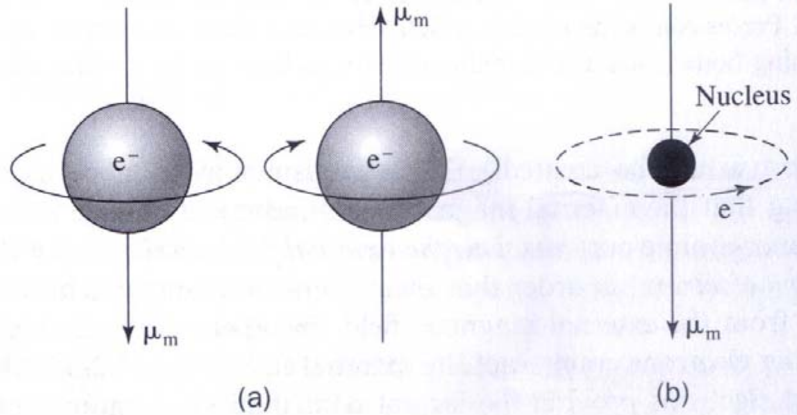

Figure 15.2. (a) Schematic representation of electrons which spin around their own axes. A (para)magnetic moment μ_m results; its direction depends on the mode of rotation. Only two spin directions are shown (called "spin up" and "spin down"). (b) An orbiting electron is the source of *electron-orbit paramagnetism*.

☐ **Spin paramagnetism** : net magnetic moment results from electrons which spin around their own axis (Fig.15.2(a))

- Observed in some metal and salts of transition elements

☐ **Electron-orbit paramagnetism** : net magnetic moment stems from magnetic moments of orbiting electrons (Fig.15.2(b))

- Free atoms (dilute gases), rare earth elements and their salts and oxides

Overview –Types of Magnetism

➢ Paramagnetism

Temperature dependence of paramagnetism

- **Curie law** : susceptibility, χ, inversely proportional to the absolute temperature T

$$\chi = C/T \qquad (15.1)$$

where, C is Curie constant

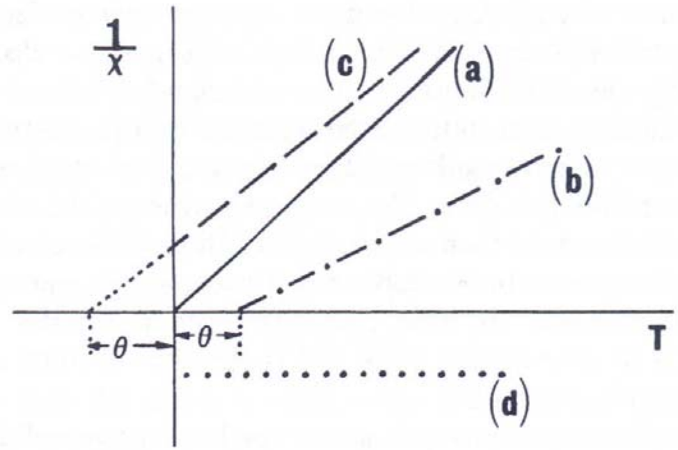

Figure 15.3. Schematic representation of (a) the Curie law and (b) and (c) the Curie–Weiss law. (d) The diamagnetic behavior is also shown for comparison.

- **Curie-Weiss law** : a more general relationship

$$\chi = C/(T-\theta) \qquad (15.2)$$

where θ is another constant that has same unit as the T

• Ni (above curie Temperature), Fe and β-Co, rare earth elements, salts of transition elements (e.g., the carbonate, chlorides, and sulfates of Fe, Co, Cr, Mn obey Curie-Weiss law)

Overview –Types of Magnetism

➢ Paramagnetism

Figure 15.4. Schematic representation of the spin alignment in a *d*-band which is partially filled with eight electrons (Hund's rule).

- Why only spin paramagnetism is observed in most solids?

In crystals, the electron orbits are essentially coupled to the lattice, which prevents the orbital magnetic moments from turning into the field direction *("orbital quenched")*.

- Exception of *"orbital quenched"* elements: Rear earth elements and their derivatives having "deep-lying 4*f*-electrons" The latter ones are shielded by the outer electrons from the crystalline field of the neighboring ions , and thus orbital magnetic moments of the *f*-electrons may turn into the external magnetic field and contributed to electron-orbit paramagnetism

- **The *g*-factor** : the friction of total magnetic moment contributed by orbital motion versus by spin motion
- **Hund's rule** and **Pauli principle**
- **Bohr magneton** : the smallest unit (or quantum) of the magnetic moment

$$\mu_B = eh/(4\pi m) = 9.274 \times 10^{-24}\ J/T \equiv (A \cdot m^2) \qquad (15.3)$$

Overview –Types of Magnetism

➢ Ferromagnetism

❑ A ring shaped solenoid (Fig.15.5)

By increasing current external field is increased, then the magnetization, *M*, rises showing a hysteresis loop (Fig 15.6)

- M_s : saturation magnetization
- M_r : remanance
- H_c : coercive field

❑ Hard (soft) magnetic materials:

a large (small) M_r and H_C

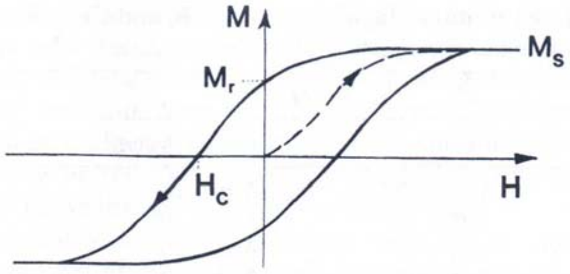

Figure 15.6. Schematic representation of a hysteresis loop of a ferromagnetic material. The dashed curve is for virgin material.

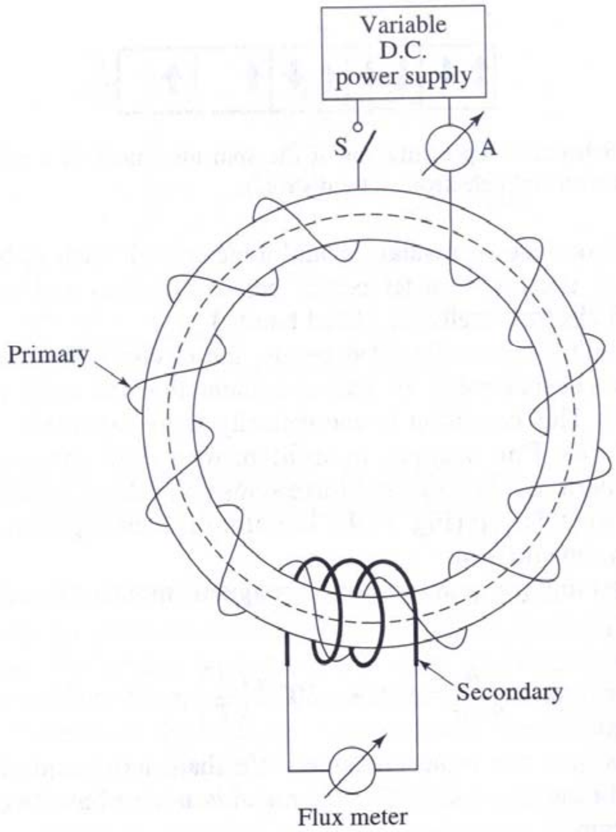

Figure 15.5. A ring-shaped solenoid with primary and secondary windings. The magnetic flux lines are indicated by a dashed circle. Note, that a current can flow in the secondary circuit only if the current (and therefore the magnetic flux) in the primary winding changes with time. An on–off switch in the primary circuit may serve for this purpose.

Overview –Types of Magnetism

➢ Ferromagnetism

❑ *T* dependence of M_s (Fig.15.7(a))

Above the Curie Temperature, T_C ferromagnetics become paramagnetic.

❑ A small difference between T_C and θ *(in Curie-Weiss law)* is due to a gradual transition from ferromagnetism to paramagnetism (Fig. 15.7(b))

- Magnetic short-range transition: Small clusters of spins are still aligned slightly above T_C → gradual transition (Fig. 15.7(b))

Table 15.1. Saturation Magnetization at 0 K and Curie Temperature (T_C) for Some Ferromagnetic Materials.

Metal	M_{S0} (A/m)	M_{S0} (Maxwells/cm^2)	T_C (K)
Fe	1.75×10^6	2.20×10^4	1043
Co	1.45×10^6	1.82×10^4	1404
Ni	0.51×10^6	0.64×10^4	631
Gd	5.66×10^6	7.11×10^4	289

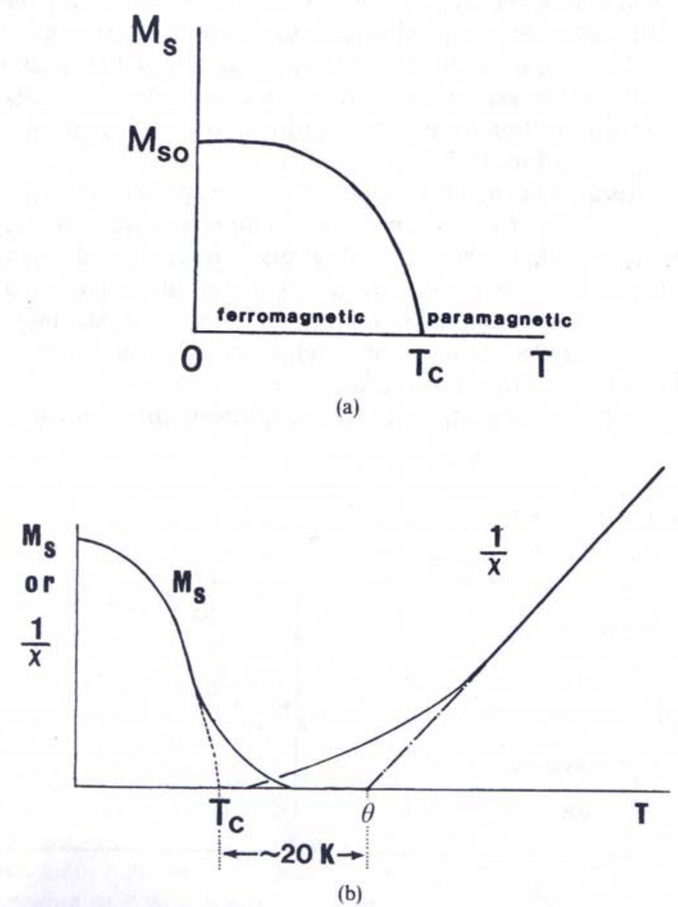

Figure 15.7. (a) Temperature dependence of the saturation magnetization of ferromagnetic materials. (b) Enlarged area near the Curie temperature showing the paramagnetic Curie point θ (see Fig. 15.3) and the ferromagnetic Curie temperature T_C.

Overview –Types of Magnetism

➢ Ferromagnetism

- **Piezomagnetism :** the magnetization of ferromagnetics is stress dependent (Fig 15.8)

  Ex) a compressive stress increases *M* for Ni, while tensile stress reduces *M*.

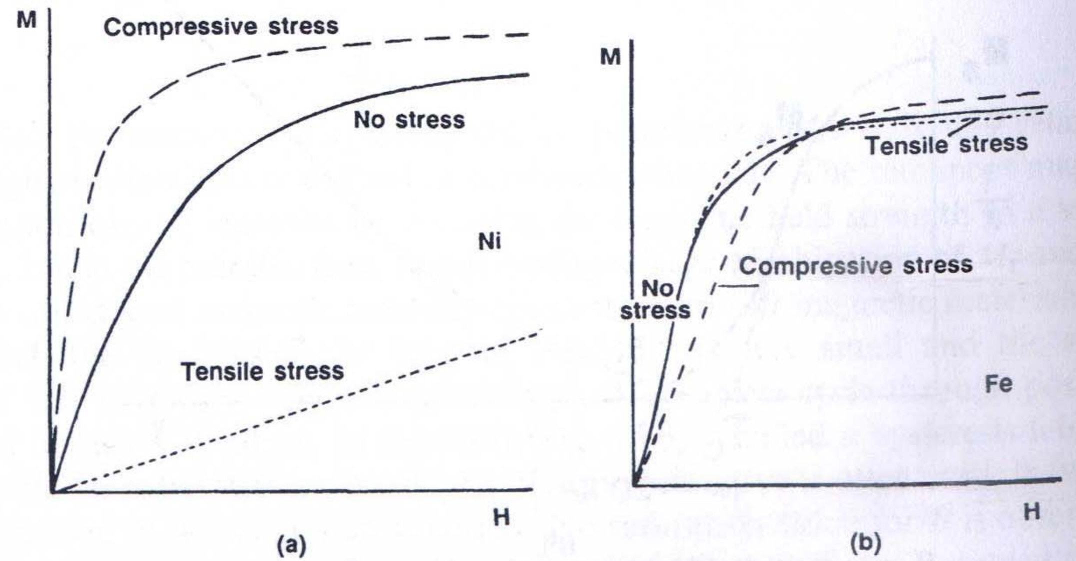

Figure 15.8. Schematic representation of the effect of tensile and compressive stresses on the magnetization behavior of (a) nickel and (b) iron. (Piezomagnetism.)

- **Magnetostriction :** inverse of piezomagnetism
  - magnetic field causes a change in dimension of a ferromagnetic substance
  - also observed in ferrimagnetic or antiferromagetic materials
  - terbium-dysprosium-iron display magnetostriction about 3 orders of magnitude larger than iron-nickel alloys

Overview –Types of Magnetism

➢ Ferromagnetism

❑ Explanation of Ferromagnetism

- **Spontaneous magnetization:**

• the spins of unfilled *d*-band spontaneously aligned parallel to each other below T_C within magnetic domains without the presence of external magnetic field (Fig 15.9)

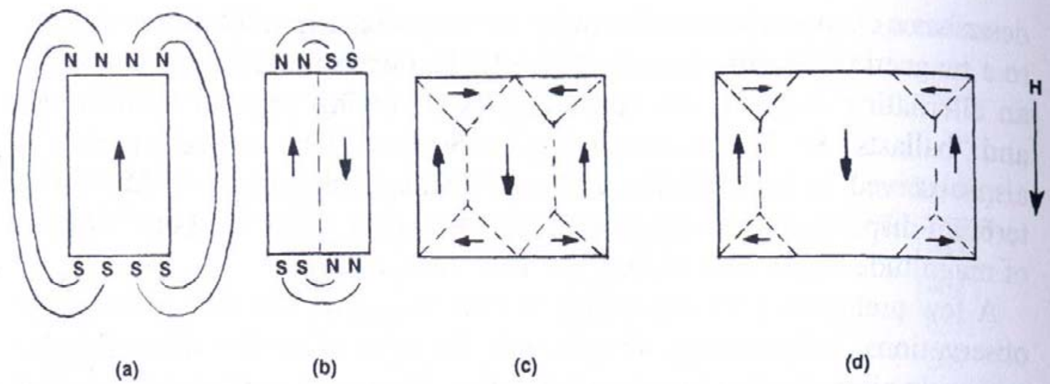

Figure 15.9. (a) Spontaneous alignment of all spins in a single direction. (b) Division into two magnetic domains having opposite spin directions. (c) Closure domains in a cubic crystal. (d) Growth of a domain whose spins are parallel to an external magnetic field. (The domain walls are *not* identical with the grain boundaries.)

• **exchange energy** causes adjacent spins to align parallel to each other

❑ Magnetic Domain structure

• Energetically favorable by a reduction in **magnetostatic energy**

→ Spontaneous division into many individual domains in which all spins are aligned in the same direction

• Closure domain structure: most favorable in the point of magnetostatic energy Fig. 15.9 (c)

Overview –Types of Magnetism

➢ Ferromagnetism

❑ Magnetic Domain

- The individual domains are magnetized to saturation.
- The spin direction in each domain is different so that as a whole it cancels each other and thus the net magnetization is zero.
- An external magnetic field causes to grow the domain whose spins are parallel or nearly parallel to the external field.
- At the technical saturation magnetization, M_S, the entire crystal contains one single domain, having all spins aligned parallel to external field.
- Domain wall: the region between individual domains in which the spins rotate from one direction into the next.
- Barkhausen effect : a discontinuous domain wall movement by external field

Overview –Types of Magnetism

➢ Antiferromagnetism

- Spontaneous alignment of moment below critical Temp. (Néel Temp.)
- Aligned in antiparallel (Fig 15.10)
- No net magnetism
- Néel Temperature, T_N
- Modified Curie-Weiss law for antiferromangtics

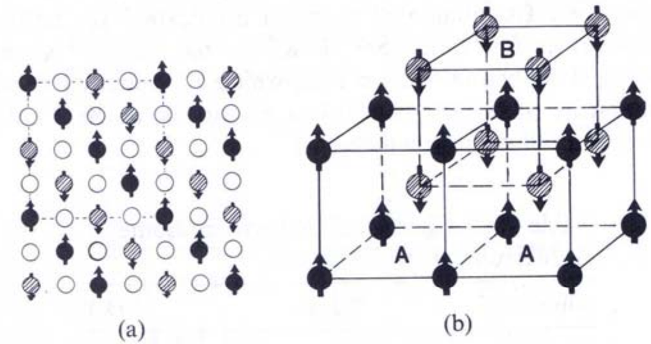

Figure 15.10. Schematic representation of spin alignments for antiferromagnetics at 0 K. (a) Display of a (100) plane of MnO. The gray (spin down) and black (spin up) circles represent the Mn ions. The oxygen ions (open circles) do not contribute to the antiferromagnetic behavior. MnO has a NaCl structure. (b) Three-dimensional representation of the spin alignment of manganese ions in MnF_2. (The fluorine ions are not shown.) This figure demonstrates the interpenetration of two manganese sublattices, A and B, having antiparallel aligned moments.

$$\chi = C/(T-(-\Theta)) = C/(T+\Theta) \quad (15.4)$$

the extrapolation of paramagetic (above T_N) line to $1/\chi = 0$ yield a negative Θ

Table 15.2. Characteristic Data for Some Antiferromagnetic Materials.

Substance	T_N (K)	$-\theta$ (K)
MnO	116	610
MnF_2	67	82
α-Mn	100	?
FeO	198	570
NiO	523	~2000
CoO	293	330
Cr	310	?

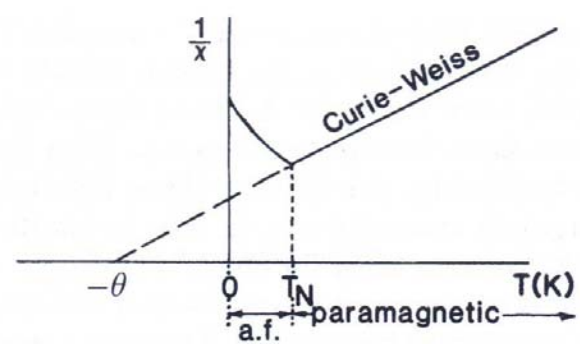

Figure 15.11. Schematic representation of the temperature dependence of a polycrystalline antiferromagnetic (a.f.) material.

Overview –Types of Magnetism

➢ Ferrimagnetism

- Exhibit spontaneous magnetic moment and hysteresis below a Curie temperature, similarly as ferromagntics
- Aligned in antiparallel, but magnetic moment remain uncancelled.
- Ceramic (oxide) materials, poor electrical conductor

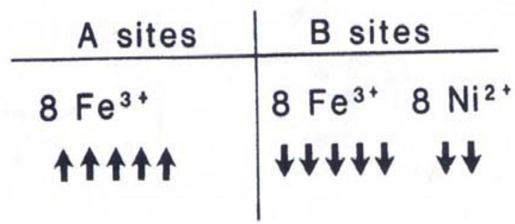

Figure 15.12. Distribution of spins upon A and B sites for the inverse spinel $NiO \cdot Fe_2O_3$. The spins within one site are arranged considering Hund's rule (Fig. 15.4). The iron ions are equally distributed among the A and B sites. The nickel ions are only situated on B sites. The relevance of the number of ions per unit cell is explained later on in the text.

Table 15.3. Calculated and Measured Number of Bohr Magnetons for Some Ferrites.

Ferrite	Mn	Fe	Co	Ni	Cu
Calculated μ_B	5	4	3	2	1
Measured μ_B	4.6	4.1	3.7	2.3	1.3

- Nickel ferrite $NiO \cdot Fe_2O_3$ (Fig 15.12)

• Two uncancelled spins, $2\mu_B$ per formula unit

- The small discrepancy between experiment and calculation (Table 15.3) is caused by some contribution of orbital effects to the overall magnetic moment.

Overview –Types of Magnetism

➢ Ferrimagnetism

❑ Cubic ferrite (Spinel structure) (Fig.15.13)

- $MO \cdot Fe_2O_3$, where M = Mn, Ni, Fe, Co, Mg, etc.
- In the unit cell, total 56 ions (8 M^{2+} ions, 16 Fe^{3+} ions, 32 O_2^- ions)
  64 tetrahedral *A* site / 8 = 8
  32 octahedral *B* site / 2 = 16
- Normal Spinel :
  8 M^{2+} in A, 16 Fe^{3+} in B
- Inverse Spinel :
  8 Fe^{3+} in A, 8 M^{2+} + 8 Fe^{3+} in B

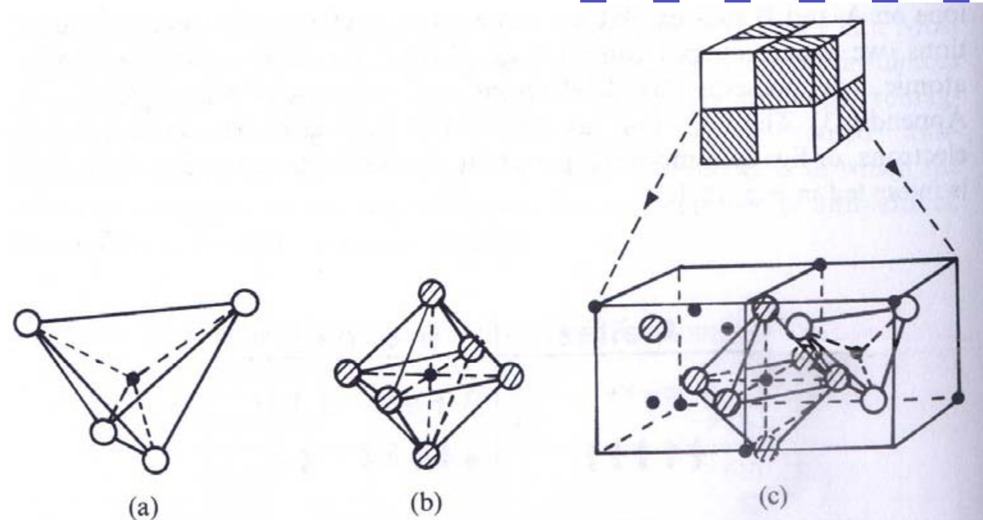

Figure 15.13. Crystal structure of cubic ferrites. The small filled circles represent metal ions, the large open or shaded circles represent oxygen ions: (a) tetrahedral or A sites; (b) octahedral or B sites; and (c) one-fourth of the unit cell of a cubic ferrite. A tetrahedron and an octahedron are marked. Adapted from J. Smit, and H.P.J. Wijn, *Ferrites*, Wiley, New York (1959).

❑ Temperature dependence of ferrimagnetics (Fig.15.14)

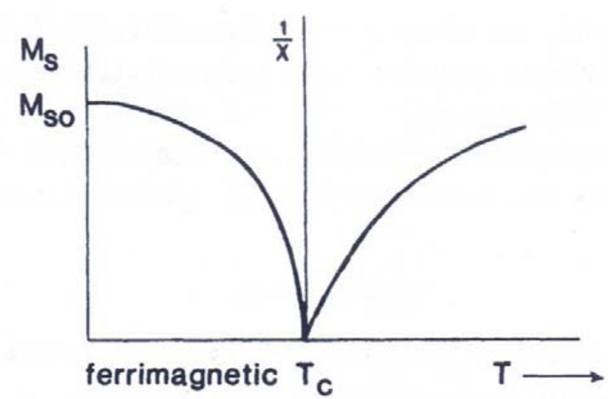

Figure 15.14. Schematic representation of the temperature dependence of the saturation magnetization, M_s, and the reciprocal susceptibility for ferrites.

Langevin Theory of Diamagnetism

Magnetic moment μ_m, created by a current *I*, passing through a loop-shaped wire of area *A*

$$\mu_m = I \cdot A = \frac{e}{t} A = \frac{e}{s / v} A = \frac{ev\pi r^2}{2\pi r} = \frac{evr}{2} \qquad (15.5)$$

Where, *e* = electron charge, *r* = radius of the orbit, $s = 2\pi r$ = length of orbit, *v* = velocity of the orbiting electron, *t* = orbiting time

Electrostatic force |*F*| on the orbiting electron

$$F = ma = Ee \qquad (15.6)$$

where, *E* is the electric field and *m* is mass of the electron

Acceleration of the electron

$$a = dv/dt = Ee/m \qquad (15.7)$$

$$E = V_e / L$$

where, V_e = induced voltage(or emf), *L* = orbit length

Langevin Theory of Diamagnetism

A change in an external magnetic flux, ϕ, induces in loop-shaped wire an emf which opposes, according to Lentz's law, the change in flux:

$$V_e = -d\phi / dt = d(\mu_0 HA) / dt \qquad \textbf{(15.9)}$$

By, combining (15.7) – (15.9)

$$dv/dt = Ee/m = V_e e/Lm = -\frac{eA\mu_0}{Lm} dH/dt = -\frac{e\pi r^2 \mu_0}{2\pi rm} dH/dt = -\frac{er\mu_0}{2m} dH/dt \qquad \textbf{(15.10)}$$

A change in the magnetic field strength from 0 to *H* yields a change in the velocity of the electron

$$\int_{v_1}^{v_2} dv = -\frac{er\mu_0}{2m} \int_0^H dH \qquad \textbf{(15.11)}$$

$$\Delta v = -\frac{er\mu_0 H}{2m} \qquad \textbf{(15.12)}$$

Langevin Theory of Diamagnetism

This change in electron velocity yields in turn a change in magnetic moment as we see by combining (15.5) with (15.12):

$$\Delta\mu_m = \frac{e\Delta v r}{2} = -\frac{e^2 r^2 \mu_0 H}{4m} \qquad \textbf{(15.13)}$$

So far we assumed that magnetic field is perpendicular to the plane of the orbiting electron. In reality the orbit plane varies constantly in direction with respect to the external field. Thus we have to find a average value for $\Delta\mu_m$

$$\overline{\Delta\mu_m} = -\frac{e^2 r^2 \mu_0 H}{6m} \qquad \textbf{(15.14)}$$

If you take all Z electron, Z = atomic number , and $\bar{r}$ is the average radius of all electronic orbits,

$$\overline{\Delta\mu_m} = -\frac{e^2 Z \bar{r}^2 \mu_0 H}{6m} \qquad \textbf{(15.15)}$$

Langevin Theory of Diamagnetism

The magnetization caused by this change of magnetic moment:

$$M = \mu_m / V = -\frac{e^2 Z \overline{r}^2 \mu_0 H}{6mV} \qquad \textbf{(15.16)}$$

This finally yields, together with (14.6), the diamagnetic susceptibility,

$$\chi_{dia} = M / H = -\frac{e^2 Z \overline{r}^2 \mu_0}{6mV} = -\frac{e^2 Z \overline{r}^2 \mu_0}{6m} \frac{N_0 \delta}{W} \qquad \textbf{(15.17)}$$

Where, $N_0\delta/W$ is the number of atoms per unit volume, N_0= Avogadro constant, δ= density, W= atomic mass

The quantities in (15.17) are essentially temperature-independent.

Langevin Theory of (Electron Orbit) Paramagnetism

Langevin postulated that the magnetic moment of the orbiting electron are responsible for paramangetism.

When magnetic moment, μ_m is aligned by an external magnetic field, the potential energy is:

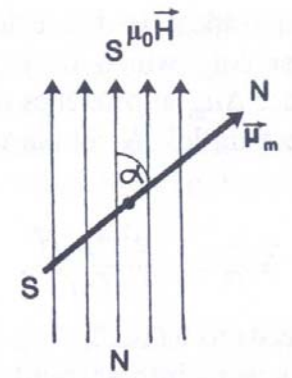

Figure 15.15. Schematic representation of the magnetic moment of an electron that has been partially aligned by an external magnetic field.

$$E_p = -\mu_m \mu_0 H \cos\alpha \qquad \textbf{(15.18)}$$

Where α is the angle between field direction and μ_m

The probability of an electron to have the energy E_p is proportional to *exp(-E_p/k_BT),* where k_B is the Boltzmann constant, T is the absolute temperature.

Langevin Theory of (Electron Orbit) Paramagnetism

Assume the electrons to be situated at the center of a sphere. The vectors, representing their magnetic moment, may point in all possible direction (Fig 15.16)
This infinitesimal number *dn* of magnetic moments per unit vol. which have the energy E_p is:

$$dn = const.dA\ exp(-E_p/k_B T) \qquad (15.19)$$

$$dA = 2\pi R^2 \sin\alpha\, d\alpha \qquad (15.20)$$

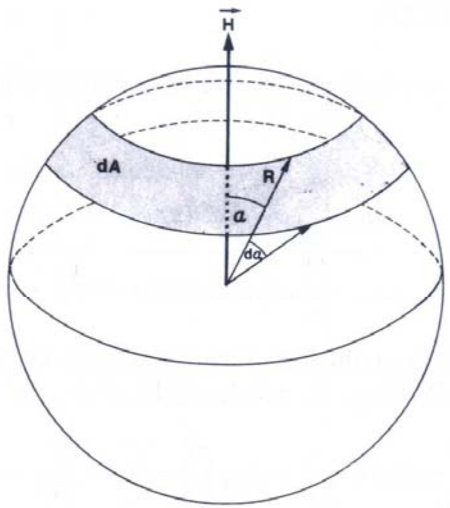

Figure 15.16. Schematic representation of a unit sphere in whose center the electrons are thought to be located.

where R=1 is the radius of the unit sphere. Combining (15.18) –(15.20)

$$dn = const.2\pi \sin\alpha d\alpha \cdot \exp(\frac{\mu_m \mu_0 H}{k_B T}\cos\alpha) \qquad (15.21)$$

For abbreviation $$\zeta = \frac{\mu_m \mu_0 H}{k_B T} \qquad (15.22)$$

Langevin Theory of (Electron Orbit) Paramagnetism

Integrating (15.21)

$$n = 2\pi const. \int_0^{\pi} \sin\alpha \exp(\zeta \cos\alpha) d\alpha \qquad (15.23)$$

$$const. = \frac{n}{2\pi \int_0^{\pi} \sin\alpha \exp(\zeta \cos\alpha) d\alpha} \qquad (15.24)$$

Total magnetization is the sum of all individual magnetic moments

$$M = \int_0^{n} \mu_m \cos\alpha dn \qquad (15.25)$$

with (15.21)

$$M = const. 2\pi \mu_m \int_0^{\pi} \cos\alpha \sin\alpha \exp(\zeta \cos\alpha) d\alpha \qquad (15.26)$$

with (15.24)

$$M = \frac{n\mu_m \int_0^{\pi} \cos\alpha \sin\alpha \exp(\zeta \cos\alpha) d\alpha}{\int_0^{\pi} \sin\alpha \exp(\zeta \cos\alpha) d\alpha} \qquad (15.27)$$

Langevin Theory of (Electron Orbit) Paramagnetism

This function can be brought into a standard form by setting *x = cos*α, and *dx = - sin*$\alpha d\alpha$

$$M = n\mu_m(\cos\zeta - \frac{1}{\zeta}) = n\mu_m(\frac{\zeta}{3} - \frac{\zeta^3}{45} + \frac{2\zeta^5}{945} - \cdots) \qquad \textbf{(15.28)}$$

Where the expression in parenthesis is called Langevin funtion *L*(ζ).
The term $\zeta = \mu_m \mu_0 H / k_B T$ is usually much smaller than one, so that:

$$M = n\mu_m \frac{\zeta}{3} = \frac{n\mu_m{}^2 \mu_0 H}{3k_B T} \qquad \textbf{(15.29)}$$

Which yield, for the susceptibility (14.6) at not-too-high field strength,

$$\chi_{para}^{orbit} = \frac{M}{H} = \frac{n\mu_m{}^2 \mu_0}{3k_B} \frac{1}{T} \equiv C \frac{1}{T} \qquad \textbf{(15.30)}$$

This is Curie's law (15.1), which express that the susceptibility is inversely proportional to the temperature. The Curie constant is:

$$C = \frac{n\mu_m{}^2 \mu_0}{3k_B} \qquad \textbf{(15.31)}$$

Langevin Theory of (Electron Orbit) Paramagnetism

- **Discussion of the Langevin theory**

- The magnetization, *M* is a linear function of *H* for a given temperature and for small fields (Fig 15.17), (Eq.15.29)

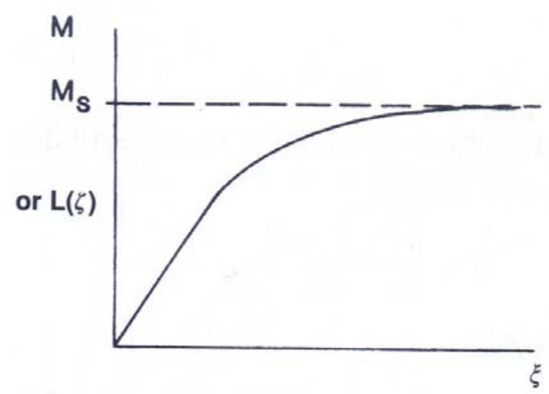

Figure 15.17. Schematic representation of the Langevin function $L(\zeta) = \coth\zeta - 1/\zeta$, where $\xi = \mu_m \mu_0 H / k_B T$.

- For large fields *H*, the magnetization reaches M_s at which all magnetic moment aligned to their maximum value.

- Langevin theory can explain the Curie law.

- Refinement of Langevin function by applying quantum theory
-> Brillouin function

Molecular Field Theory

Weiss postulation: Total magnetic moment H_t is thought to be composed of two parts, external field H_e and *molecular filed* H_m

$$H_t = H_e + H_m \qquad (15.32)$$

where, $H_m = \gamma M$ (γ = *molecular field constant*) **(15.33)**

$$\chi = M / H_t = \frac{M}{H_e + \gamma M} = C / T \qquad (15.34)$$

$$M = \frac{H_e C}{T - \gamma C} \qquad (15.35)$$

Finally, we obtain

$$\chi = \frac{M}{H_e} = \frac{C}{T - \gamma C} = \frac{C}{T - \theta} \qquad (15.36)$$

Weiss postulated that the above-introduced internal or molecular field is responsible for this parallel alignment of spins, and considered ferromagnetics to be essentially paramagnetics having a very large molecular field. In the quantum theory, the H_m is essentially the exchange force (Sec 16.2).

Molecular Field Theory

Let us consider the case for no external magnetic field. Then the spins are only subjected to the molecular filed H_m. This yields for the Langevin variable ζ (see (15.22)) with (15.33)

$$\zeta = \frac{\mu_m \mu_0 H}{k_B T} = \frac{\mu_m \mu_0 \gamma M}{k_B T} \qquad (15.37)$$

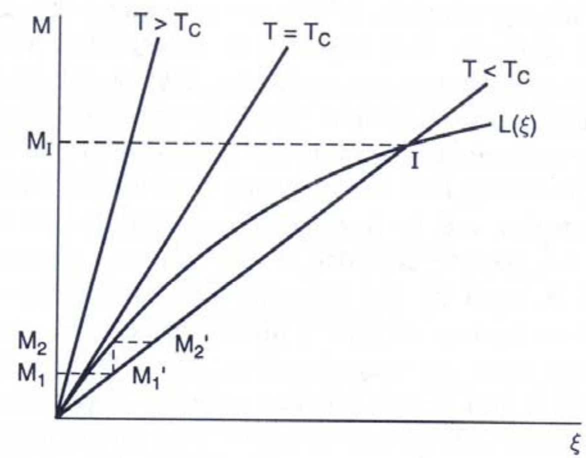

Figure 15.18. Langevin function $L(\zeta)$, i.e., (15.28) and plot of (15.38) for three temperatures.

And provides for the magnetization by rearranging (15.37):

$$M = \frac{k_B T}{\mu_m \mu_0 \gamma} \zeta \qquad (15.38)$$

The magnetization is linear function of ζ with the temperature as a proportionality factor (Fig.15.18)

The intersection *I* of a given temperature line with the Langevin function $L(\zeta)$ represents the finite spontaneous magnetization, M_I, at this temperature

Molecular Field Theory

❑ In Fig.15.18

- $T < T_C$: With increasing temperature, slope is increased, the point of intercept, *I*, is decreased, and therefore the value for the spontaneous magnetization is decreased.

- At Curie temperature, T_C: no intercept, and hence no spontaneous magnetization

The slope $k_B/(\mu_m \mu_0 \gamma)$ in (15.38) is identical to the slope of the $L(\zeta)$ near the origin, which is $n\,\mu_m/3 = M/3$. This yields, for T_C

$$\frac{k_B T}{\mu_m \mu_0 \gamma} = \frac{M}{3} \qquad (15.39)$$

Molecular filed constant, γ, calculated by measuring T_C and inserting T_C into Eq.(15.39)

$$\gamma = \frac{k_B T_C}{\mu_m \mu_0 M} \qquad (15.40)$$

This yield, for the molecular magnetic field strength (15.33)

$$H_m = \gamma M = \frac{3 k_B T_C}{\mu_m \mu_0} \qquad (15.41) \quad (\sim 10^7 \text{ Oe !!!})$$