[2008][03-2]

Computer aided ship design

Part 1. Curve & Surface

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Prof. Kyu-Yeul Lee

Department of Naval Architecture and Ocean Engineering,
Seoul National University of College of Engineering

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Geometrical Meaning of the de Boor Algorithm

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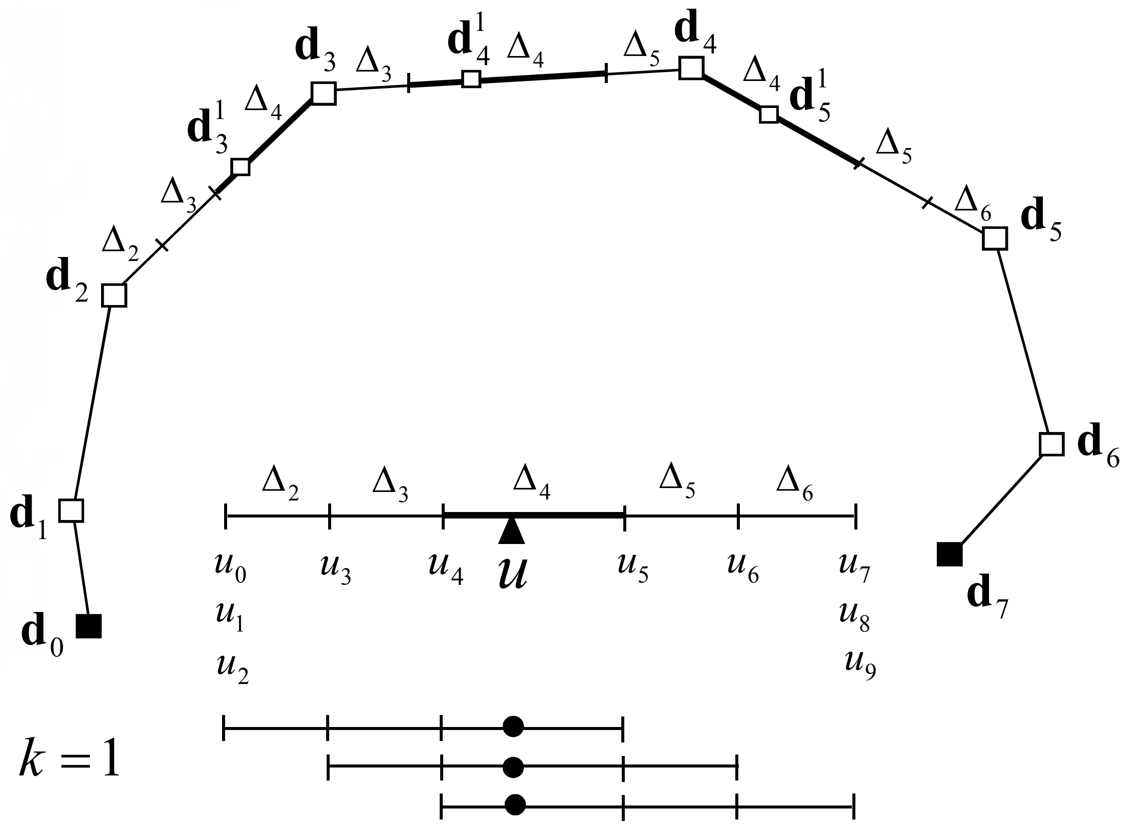
$$\mathbf{d}_i^k(u) = \frac{u_{i+n-k} - u}{u_{i+n-k} - u_{i-1}} \mathbf{d}_{i-1}^{k-1}(u) + \frac{u - u_{i-1}}{u_{i+n-k} - u_{i-1}} \mathbf{d}_i^{k-1}(u)$$

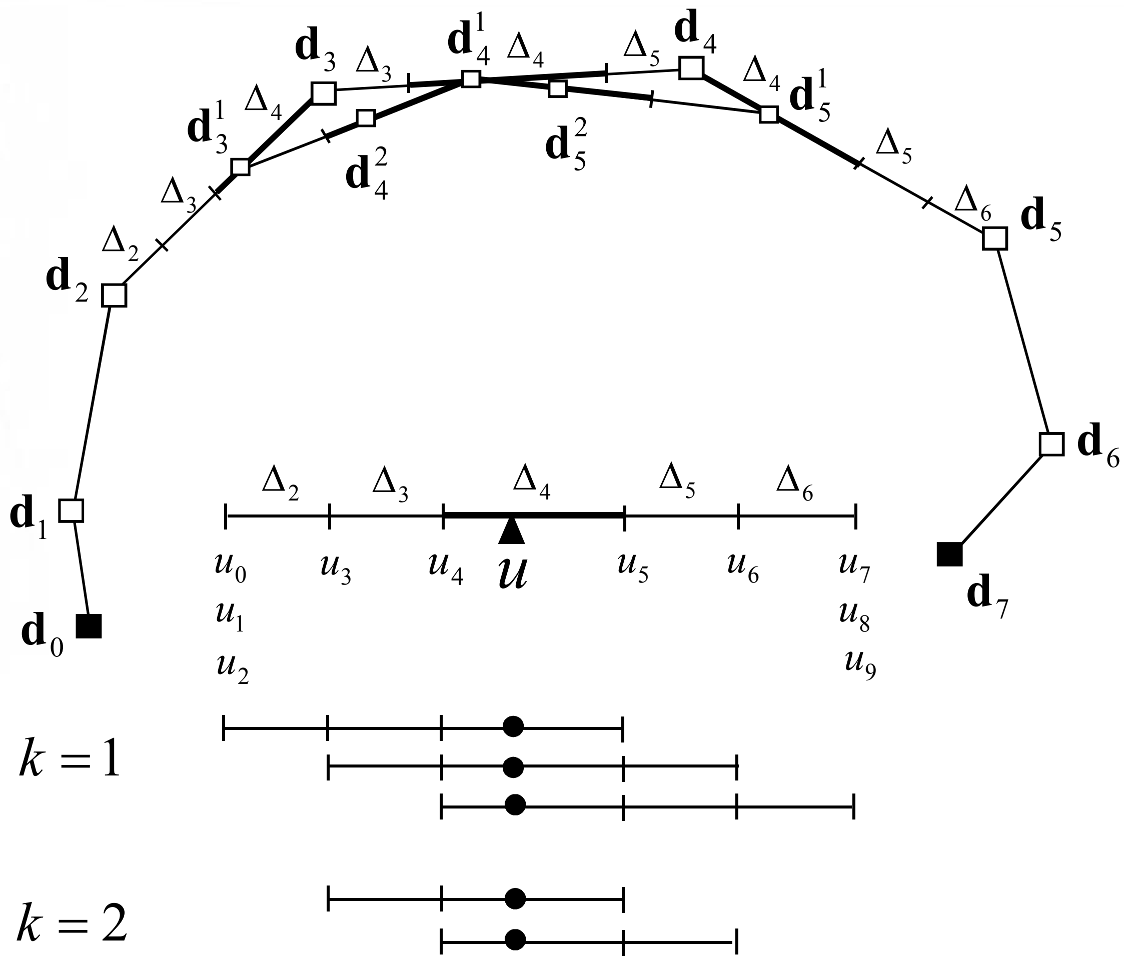
Geometrical Meaning of the de Boor Algorithm(1)



- Linear Interpolation **비율이** t:(1-t)**로 일정했던** de Casteljau algorithm**에 비하여** de Boor algorithm**에서는** Linear Interpolation **비율이 변한다**
- **이는** B-spline curve **를 구성하는** Bezier curve segment**의 매개변수 간격이 서로 다르기 때문이다**

Geometrical Meaning of the de Boor Algorithm(2)

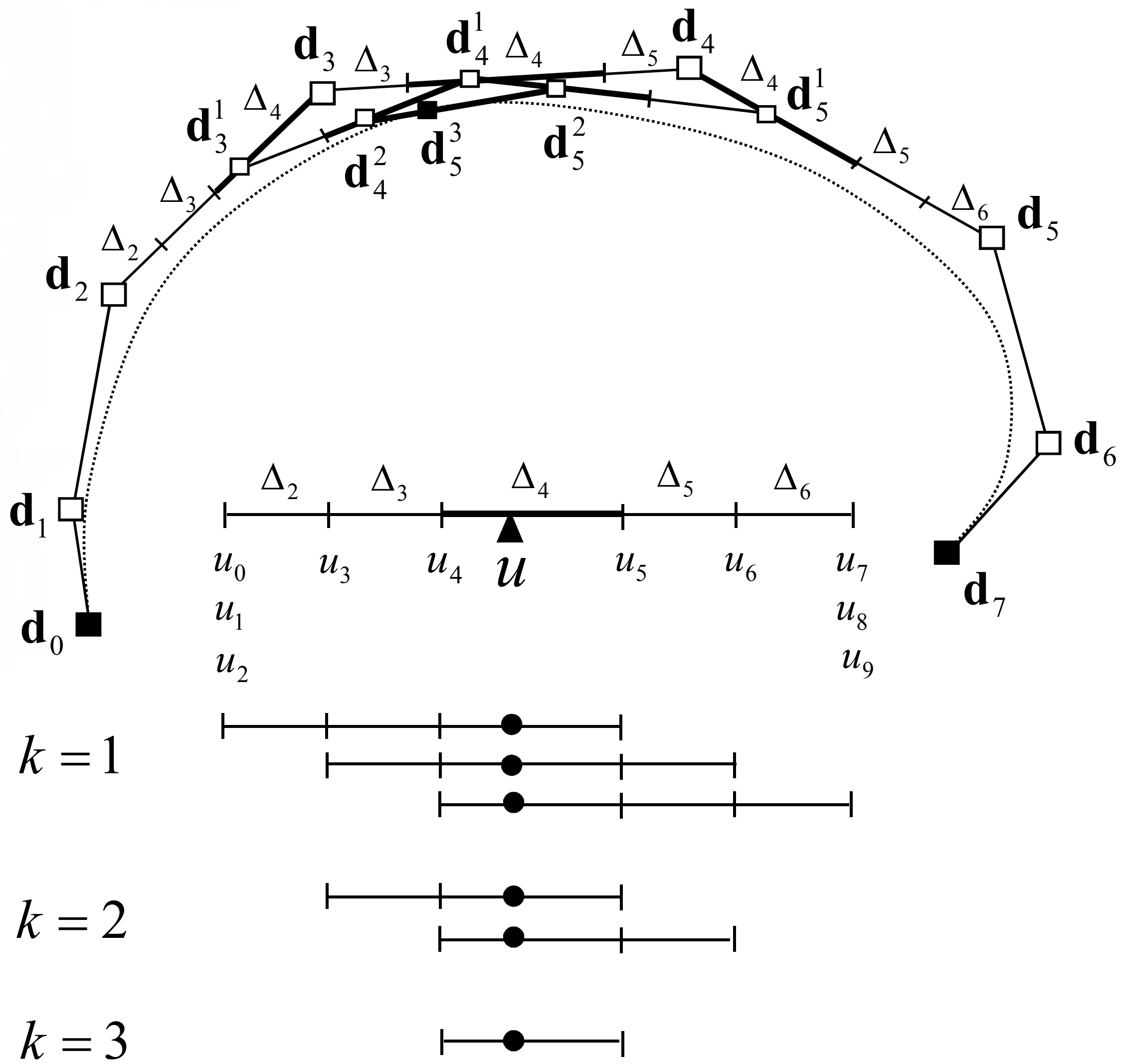
$$\mathbf{d}_i^k(u)=\frac{u_{i+n-k}-u}{u_{i+n-k}-u_{i-1}}\mathbf{d}_{i-1}^{k-1}(u)+\frac{u-u_{i-1}}{u_{i+n-k}-u_{i-1}}\mathbf{d}_i^{k-1}(u)$$



$$\mathbf{r}(u)=\mathbf{d}_0N_0^3(u)+\mathbf{d}_1N_1^3(u)+\mathbf{d}_2N_2^3(u)+\cdots+\mathbf{d}_nN_n^3(u)$$

Geometrical Meaning of the de Boor Algorithm(3)

$$\mathbf{d}_i^k(u) = \frac{u_{i+n-k} - u}{u_{i+n-k} - u_{i-1}}\mathbf{d}_{i-1}^{k-1}(u) + \frac{u - u_{i-1}}{u_{i+n-k} - u_{i-1}}\mathbf{d}_i^{k-1}(u)$$



$$\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_n N_n^3(u)$$

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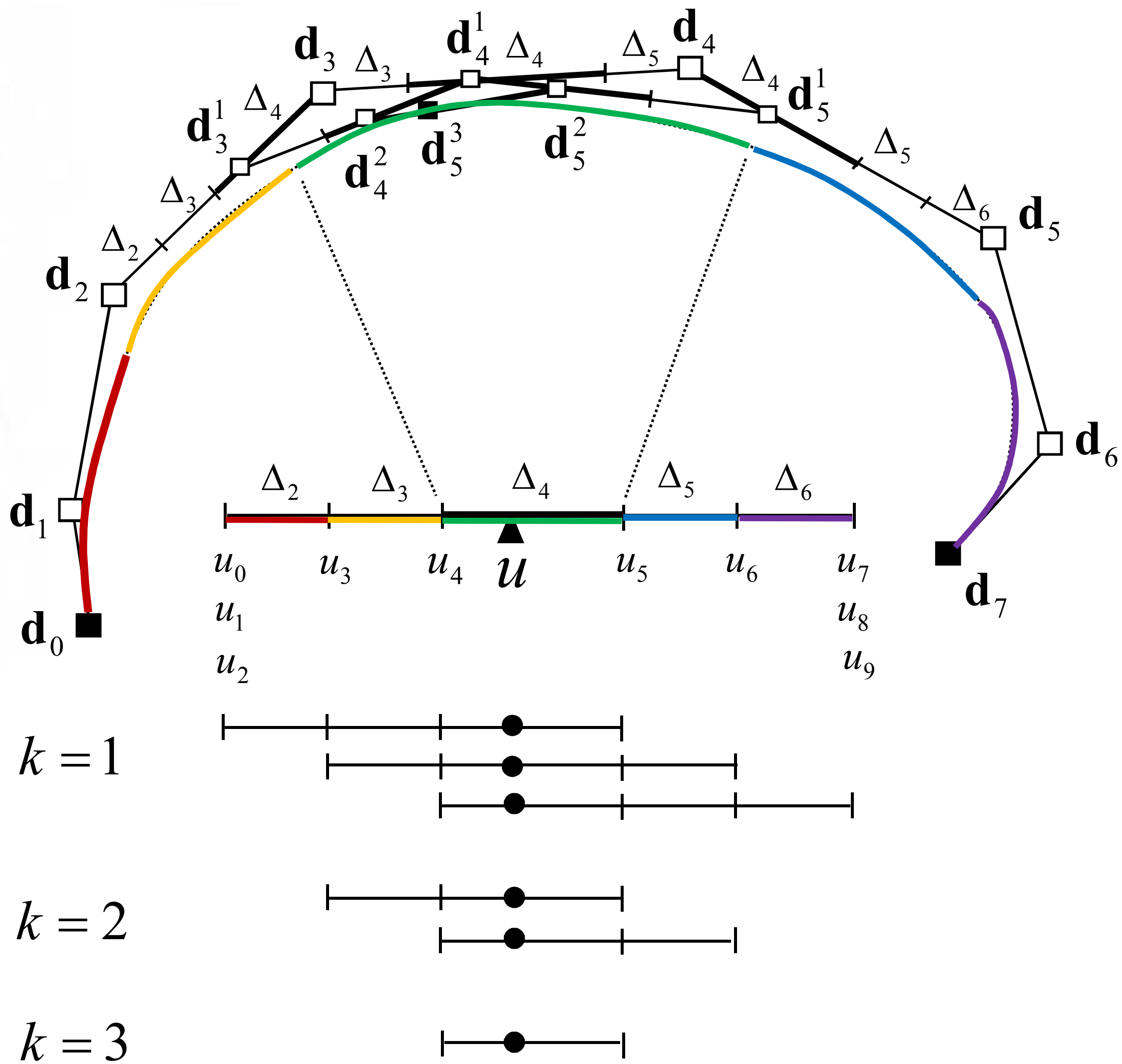
Geometrical Meaning of the de Boor Algorithm(3)

$$\mathbf{d}_i^k(u) = \frac{u_{i+n-k} - u}{u_{i+n-k} - u_{i-1}} \mathbf{d}_{i-1}^{k-1}(u) + \frac{u - u_{i-1}}{u_{i+n-k} - u_{i-1}} \mathbf{d}_i^{k-1}(u)$$



$$\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_n N_n^3(u)$$

Relationship between de Boor algorithm & B-spline curves

- de Boor 알고리즘 : “Constructive Approach”

Input: $\mathbf{d}_i$ (de Boor Points)

Processor: 구간별로 $\mathbf{d}_i$를 n번 순차적 ‘linear interpolation’

Output : n차 곡선상의 점

→ ‘B-spline function’(Cox-de Boor recurrence formula) 형태로 표현 됨

$$\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_n N_n^3(u)$$

Relationship between de Boor algorithm & B-spline curves

- de Boor **알고리즘** : “Constructive Approach”

  Input: $\mathbf{d}_i$ (de Boor Points)

  Processor: **구간별로** $\mathbf{d}_i$**를** n**번 순차적** ‘linear interpolation’

  Output : n**차 곡선상의 점**

  → ‘B-spline function’(Cox-de Boor recurrence formula) **형태로 표현 됨**

- B-spline **곡선식**: “B-spline function evaluation Approach”

  Input: $\mathbf{d}_i$ (de Boor Points)

  Processor: **공간 상의 점** $\mathbf{d}_i$**와** B-spline function**을** “blending”**하여 함수 값을 계산하면 곡선상의 점을 구할 수 있음**

  Output: B-spline function**과** $\mathbf{d}_i$**의 혼합 함수 형태로 표현**

Bezier Curve and B-Spline Curve

항목		Bezier Curve	B-Spline Curve
Make Curve	Given	Bezier Control Point $\mathbf{b}_i$ Parameter t Bernstein Polynomial Func. $B_i^n(t)$	B-Spline Control Point $\mathbf{d}_i$ Parameter u B-Spline Basis Func. $N_i^n(u)$
	Find	Bezier Curve $\mathbf{r}(t)$ $\mathbf{r}(t) = \mathbf{b}_0 B_0^n(t) + \mathbf{b}_1 B_1^n(t) + \ldots\ldots + \mathbf{b}_n B_n^n(t).$	B-Spline Curve $\mathbf{r}(u)$ $\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_{D-1} N_{D-1}^3(u)$
-		Bernstein Polynomial Function $B_i^n(t) = \begin{pmatrix} n \\ i \end{pmatrix} t^i (1-t)^{n-i},$ $\begin{pmatrix} n \\ i \end{pmatrix} = {}_nC_i = \begin{cases} \frac{n!}{i!(n-i)!} & \textbf{if } 0 \le i \le n \\ \mathbf{0} & \textbf{else} \end{cases}$	B-Spline Basis Function (Cox-de boor Recursive Formula) $N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$ $N_i^0(u) = \begin{cases} 1 & \textbf{if } u_{i-1} \le u < u_i \\ 0 & \textbf{else} \end{cases}, \sum_{i=0}^{D-1} N_i^n(u) = 1$
Constructive Approach		de Casteljau Algorithm $\mathbf{b}_i^k(t) = (1-t)\mathbf{b}_i^{k-1} + t\mathbf{b}_{i+1}^{k-1}$	de Boor Algorithm $\mathbf{d}_i^k(u) = \frac{u_{i+n-k} - u}{u_{i+n-k} - u_{i-1}} \mathbf{d}_{i-1}^{k-1}(u) + \frac{u - u_{i-1}}{u_{i+n-k} - u_{i-1}} \mathbf{d}_i^{k-1}(u)$
Inter-Polation	Given	Points on Curve: $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$	Points on Curve: $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$
	Find	Bezier Control Point $\mathbf{b}_i$	B-Spline Control Point $\mathbf{d}_i$

Example of de Boor Algorithm

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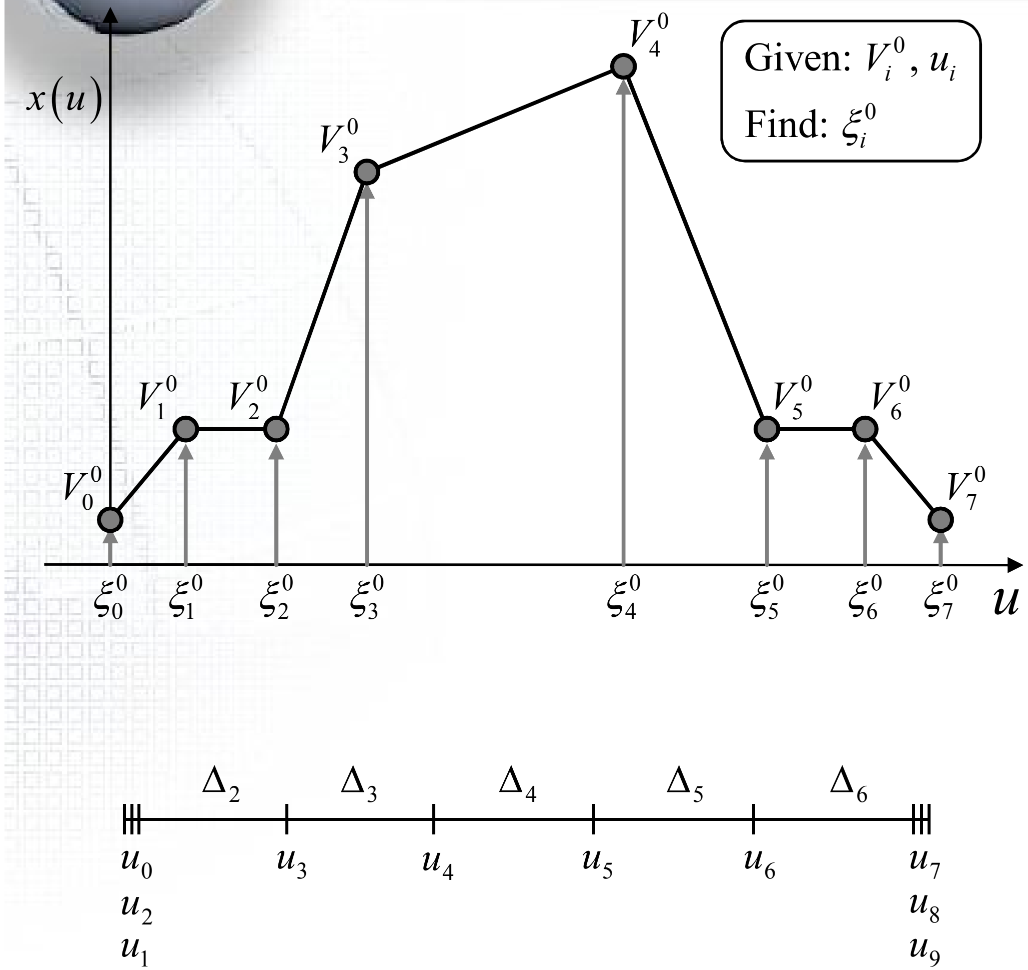
Example : de Boor Algorithm

- Given problem



$$\xi_0^0 = \frac{u_0+u_1+u_2}{3} = \frac{0+0+0}{3} = 0$$

$$\xi_1^0 = \frac{u_1+u_2+u_3}{3} = \frac{0+0+\Delta_2}{3} = \frac{\Delta_2}{3}$$

$$\xi_2^0 = \frac{u_2+u_3+u_4}{3} = \frac{0+\Delta_2+(\Delta_2+\Delta_3)}{3} = \frac{2\times\Delta_2+\Delta_3}{3}$$

$$\xi_3^0 = \frac{u_3+u_4+u_5}{3} = \frac{3\times\Delta_2+2\times\Delta_3+\Delta_4}{3}$$

$$\xi_4^0 = \frac{u_4+u_5+u_6}{3} = \frac{3\times\Delta_2+3\times\Delta_3+2\times\Delta_4+\Delta_5}{3}$$

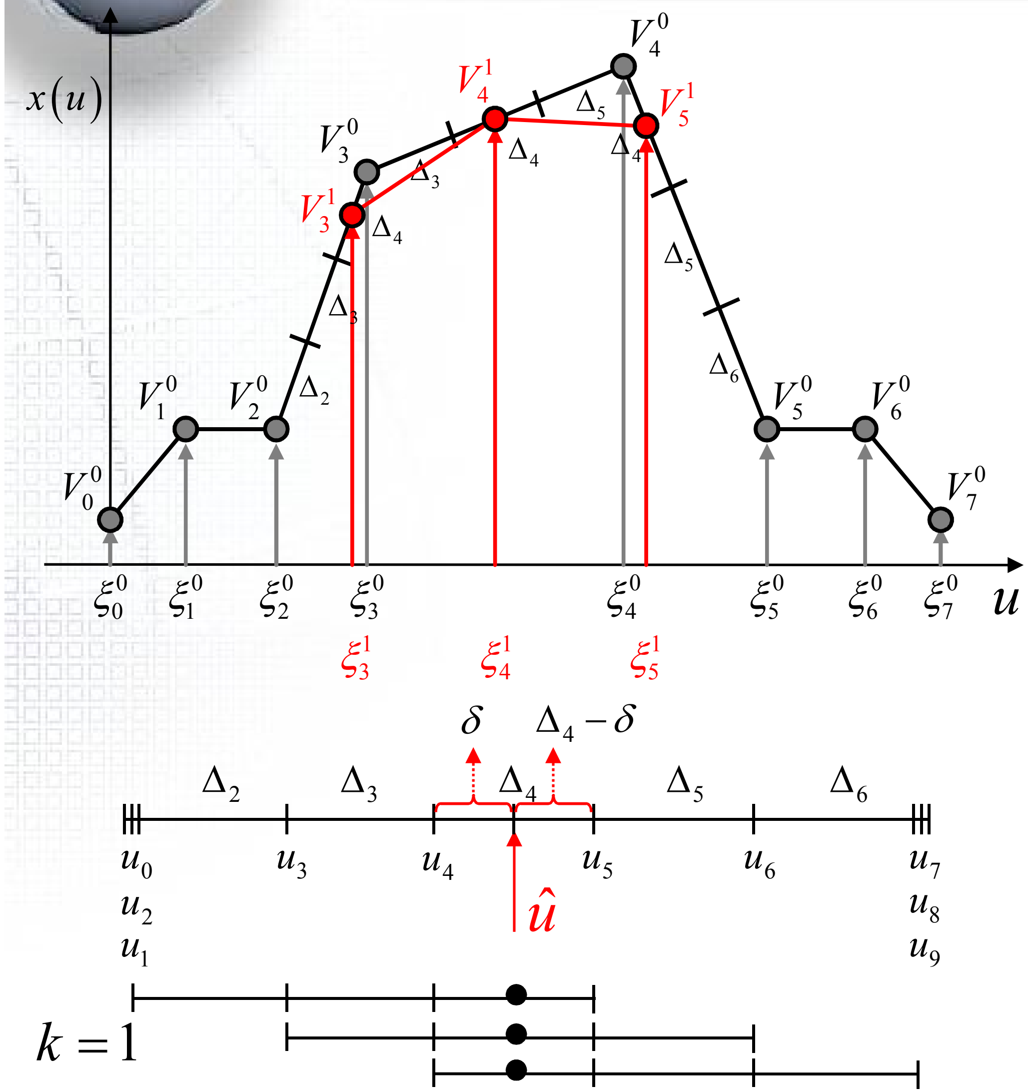
$$\xi_5^0 = \frac{u_5+u_6+u_7}{3} = \frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+2\times\Delta_5+\Delta_6}{3}$$

$$\xi_6^0 = \frac{u_6+u_7+u_8}{3} = \frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+3\times\Delta_5+2\times\Delta_6}{3}$$

$$\xi_7^0 = \frac{u_7+u_8+u_9}{3} = \Delta_2+\Delta_3+\Delta_4+\Delta_5+\Delta_6$$

Example : de Boor Algorithm

– 1st Knot Insertion



$$\xi_0^1 = \frac{u_0 + u_1 + u_2}{3} = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^1 = \frac{u_1 + u_2 + u_3}{3} = \frac{0+0+\Delta_2}{3} = \frac{\Delta_2}{3} = \xi_1^0$$

$$\xi_2^1 = \frac{u_2 + u_3 + u_4}{3} = \frac{0+\Delta_2+(\Delta_2+\Delta_3)}{3} = \frac{2\times\Delta_2+\Delta_3}{3} = \xi_2^0$$

$$\xi_3^1 = \frac{u_3 + u_4 + \hat{u}}{3} = \frac{3\times\Delta_2+2\times\Delta_3+\delta}{3}$$

$$\xi_4^1 = \frac{u_4 + \hat{u} + u_5}{3} = \frac{3\times\Delta_2+3\times\Delta_3+\Delta_4+\delta}{3}$$

$$\xi_5^1 = \frac{\hat{u} + u_5 + u_6}{3} = \frac{3\times\Delta_2+3\times\Delta_3+2\times\Delta_4+\Delta_5+\delta}{3}$$

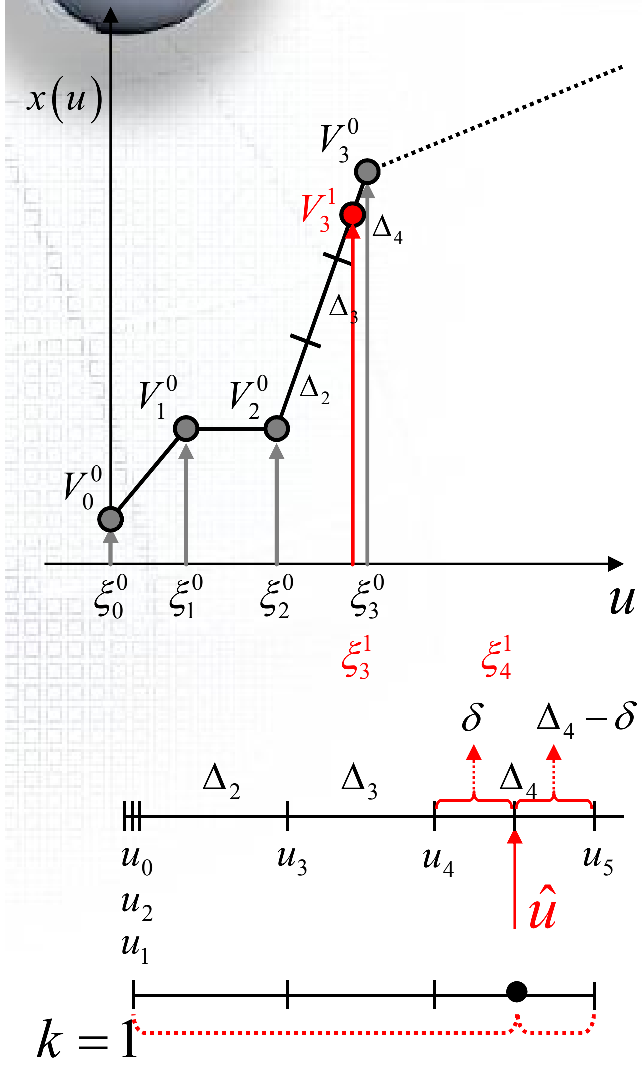
$$\xi_6^1 = \frac{u_5 + u_6 + u_7}{3} = \frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+2\times\Delta_5+\Delta_6}{3} = \xi_5^0$$

$$\xi_7^1 = \frac{u_6 + u_7 + u_8}{3} = \frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+3\times\Delta_5+2\times\Delta_6}{3} = \xi_6^0$$

$$\xi_8^1 = \frac{u_7 + u_8 + u_9}{3} = \Delta_2+\Delta_3+\Delta_4+\Delta_5+\Delta_6 = \xi_7^0$$

Example : de Boor Algorithm

- 1st Knot Insertion



$$\overline{V_2^0 V_3^1} : \overline{V_3^1 V_3^0} = \overline{\xi_2^0 \xi_3^1} : \overline{\xi_3^1 \xi_3^0}$$

$$V_3^1 = \frac{\xi_3^0 - \xi_3^1}{\xi_3^0 - \xi_2^0} V_2^0 + \frac{\xi_3^1 - \xi_1^0}{\xi_3^0 - \xi_2^0} V_3^0$$

$$\begin{cases} \xi_2^0 = \dfrac{u_2 + u_3 + u_4}{3} \\ \xi_3^0 = \dfrac{u_3 + u_4 + u_5}{3} \\ \xi_3^1 = \dfrac{u_2 + u_3 + \hat{u}}{3} \end{cases}$$

$$V_3^1 = \frac{u_5 - \hat{u}}{u_5 - u_2} V_2^0 + \frac{\hat{u} - u_2}{u_5 - u_2} V_3^0$$

(아래의 식에서 i=3, n=3, k=1 **인 경우)**

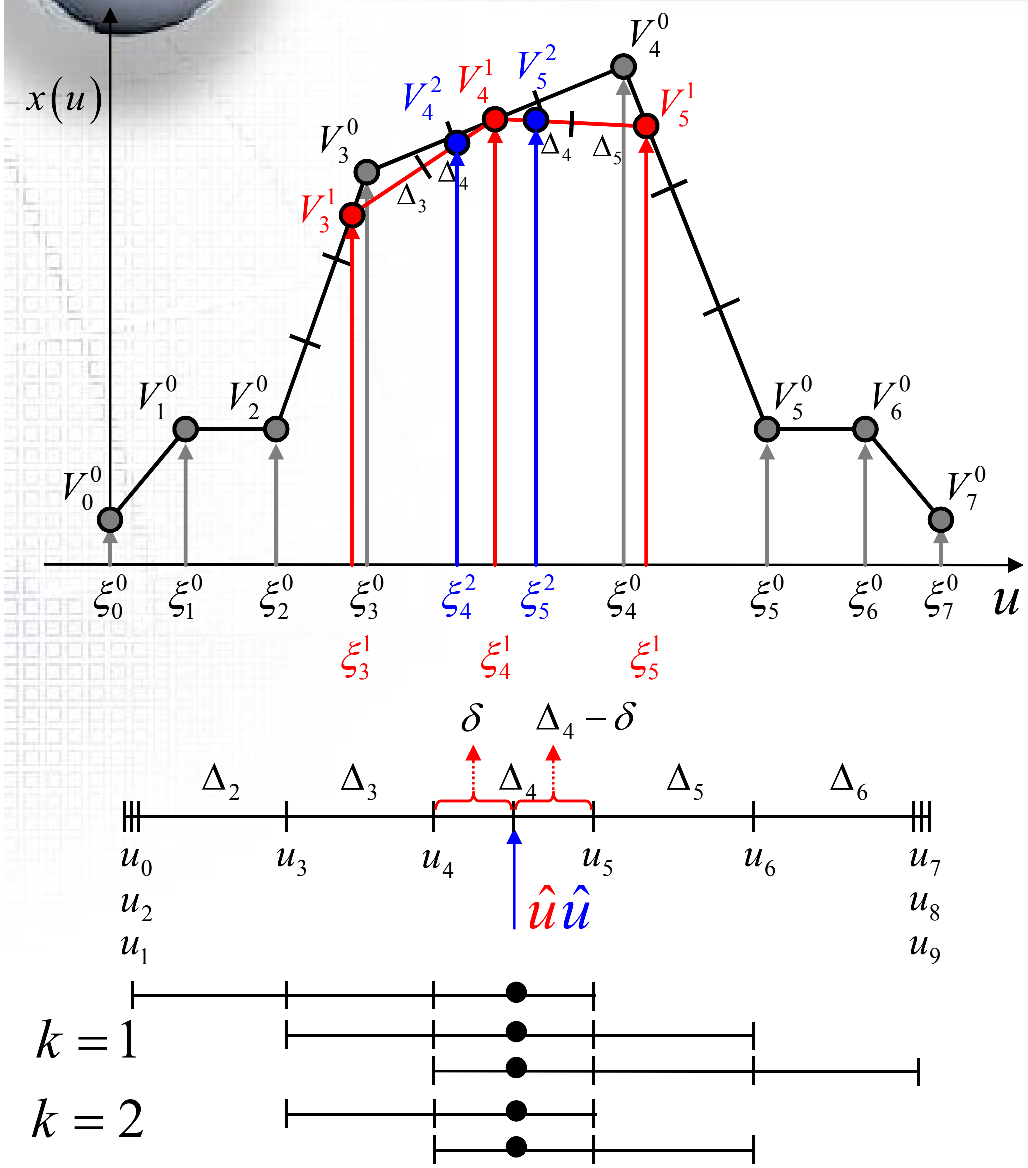
de Boor algorithm

$$V_i^k(u) = \frac{u_{i+n-k} - u}{u_{i+n-k} - u_{i-1}} V_{i-1}^{k-1}(u) + \frac{u - u_{i-1}}{u_{i+n-k} - u_{i-1}} V_i^{k-1}(u)$$

V_2^0, V_3^0 를 잇는 선분을 내분한 점은 Δ2: Δ3: Δ4로 내분한 선분상에 Δ4구간에 위치한다

Example : de Boor Algorithm

- 2nd Knot Insertion



$$\xi_0^2=\frac{u_0+u_1+u_2}{3}=\frac{0+0+0}{3}=0=\xi_0^0$$

$$\xi_1^2=\frac{u_1+u_2+u_3}{3}=\frac{0+0+\Delta_2}{3}=\frac{\Delta_2}{3}=\xi_1^0$$

$$\xi_2^2=\frac{u_2+u_3+u_4}{3}=\frac{0+\Delta_2+(\Delta_2+\Delta_3)}{3}=\frac{2\times\Delta_2+\Delta_3}{3}=\xi_2^0$$

$$\xi_3^2=\frac{u_3+u_4+\hat{u}}{3}=\frac{3\times\Delta_2+2\times\Delta_3+\delta}{3}=\xi_3^1$$

$$\xi_4^2=\frac{u_4+\hat{u}+\hat{u}}{3}=\frac{3\times\Delta_2+3\times\Delta_3+2\times\delta}{3}$$

$$\xi_5^2=\frac{\hat{u}+\hat{u}+u_5}{3}=\frac{3\times\Delta_2+3\times\Delta_3+\Delta_4+2\times\delta}{3}$$

$$\xi_6^2=\frac{\hat{u}+u_5+u_6}{3}=\frac{3\times\Delta_2+3\times\Delta_3+2\times\Delta_4+\Delta_5+\delta}{3}=\xi_5^1$$

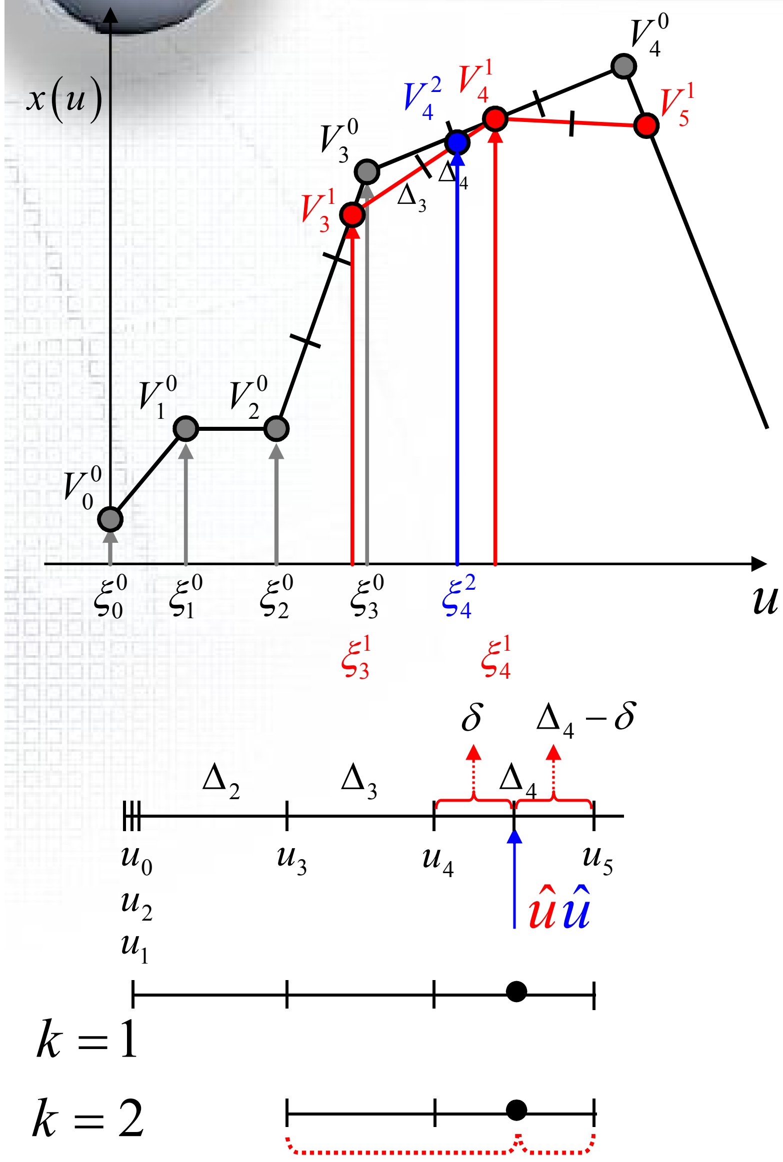
$$\xi_7^2=\frac{u_5+u_6+u_7}{3}=\frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+2\times\Delta_5+\Delta_6}{3}=\xi_5^0$$

$$\xi_8^2=\frac{u_6+u_7+u_8}{3}=\frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+3\times\Delta_5+2\times\Delta_6}{3}=\xi_6^0$$

$$\xi_9^2=\frac{u_7+u_8+u_9}{3}=\Delta_2+\Delta_3+\Delta_4+\Delta_5+\Delta_6=\xi_7^0$$

Example : de Boor Algorithm

- 2nd Knot Insertion



$$\overline{V_3^1V_4^2} : \overline{V_4^2V_4^1} = \overline{\xi_3^1\xi_4^2} : \overline{\xi_4^2\xi_4^1}$$

$$V_4^2 = \frac{\xi_4^1-\xi_4^2}{\xi_4^1-\xi_3^1}V_3^1 + \frac{\xi_4^2-\xi_3^1}{\xi_4^1-\xi_3^1}V_4^1$$

$$\begin{cases} \xi_3^1 = \dfrac{u_3+u_4+\hat{u}}{3} \\ \xi_4^1 = \dfrac{u_4+\hat{u}+u_5}{3} \\ \xi_4^2 = \dfrac{u_4+\hat{u}+\hat{u}}{3} \end{cases}$$

$$V_4^2 = \frac{u_5-\hat{u}}{u_5-u_3}V_3^1 + \frac{\hat{u}-u_3}{u_5-u_3}V_4^1$$

(**아래의 식에서** i=4, n=3, k=2 **인 경우**)

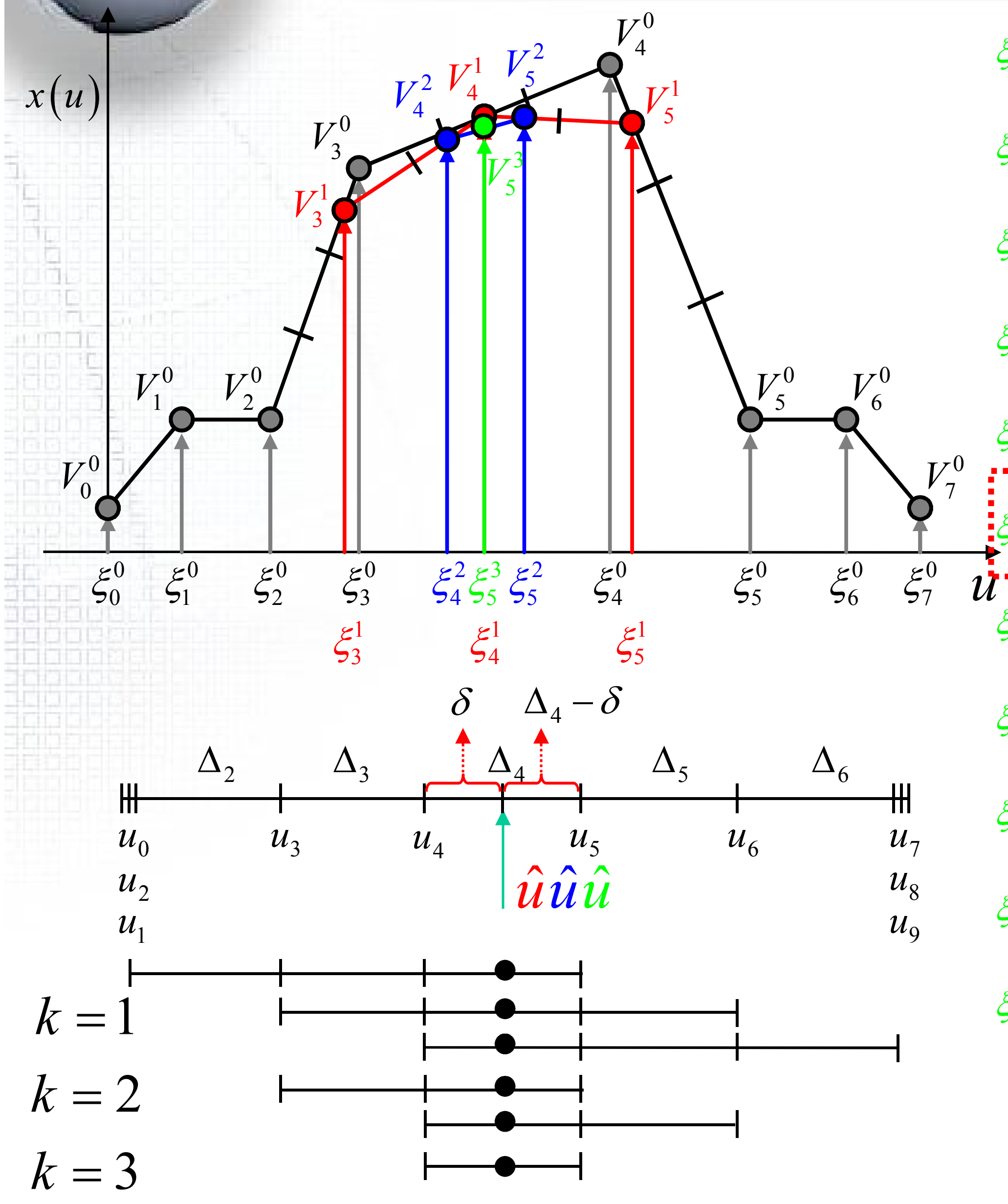
de Boor algorithm

$$V_i^k(u) = \frac{u_{i+n-k}-u}{u_{i+n-k}-u_{i-1}}V_{i-1}^{k\text{-}1}(u) + \frac{u-u_{i-1}}{u_{i+n-k}-u_{i-1}}V_i^{k\text{-}1}(u)$$

V_3^1, V_4^1 를 잇는 선분을 내분한 점은 Δ2: Δ3: Δ4로 내분한 선분상에 Δ4구간에 위치한다

Example : de Boor Algorithm

- 2nd Knot Insertion



$$\xi_0^3=\frac{u_0+u_1+u_2}{3}=\frac{0+0+0}{3}=0=\xi_0^0$$

$$\xi_1^3=\frac{u_1+u_2+u_3}{3}=\frac{0+0+\Delta_2}{3}=\frac{\Delta_2}{3}=\xi_1^0$$

$$\xi_2^3=\frac{u_2+u_3+u_4}{3}=\frac{0+\Delta_2+(\Delta_2+\Delta_3)}{3}=\frac{2\times\Delta_2+\Delta_3}{3}=\xi_2^0$$

$$\xi_3^3=\frac{u_3+u_4+\hat{u}}{3}=\frac{3\times\Delta_2+2\times\Delta_3+\delta}{3}=\xi_3^1$$

$$\xi_4^3=\frac{u_4+\hat{u}+\hat{u}}{3}=\frac{3\times\Delta_2+3\times\Delta_3+2\times\delta}{3}=\xi_4^2$$

$$\boxed{\xi_5^3=\frac{\hat{u}+\hat{u}+\hat{u}}{3}=\frac{3\times\Delta_2+3\times\Delta_3+3\times\delta}{3}}$$

$$\xi_6^3=\frac{\hat{u}+\hat{u}+u_5}{3}=\frac{3\times\Delta_2+3\times\Delta_3+\Delta_4+2\times\delta}{3}=\xi_5^2$$

$$\xi_7^3=\frac{\hat{u}+u_5+u_6}{3}=\frac{3\times\Delta_2+3\times\Delta_3+2\times\Delta_4+\Delta_5+\delta}{3}=\xi_5^1$$

$$\xi_8^3=\frac{u_5+u_6+u_7}{3}=\frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+2\times\Delta_5+\Delta_6}{3}=\xi_5^0$$

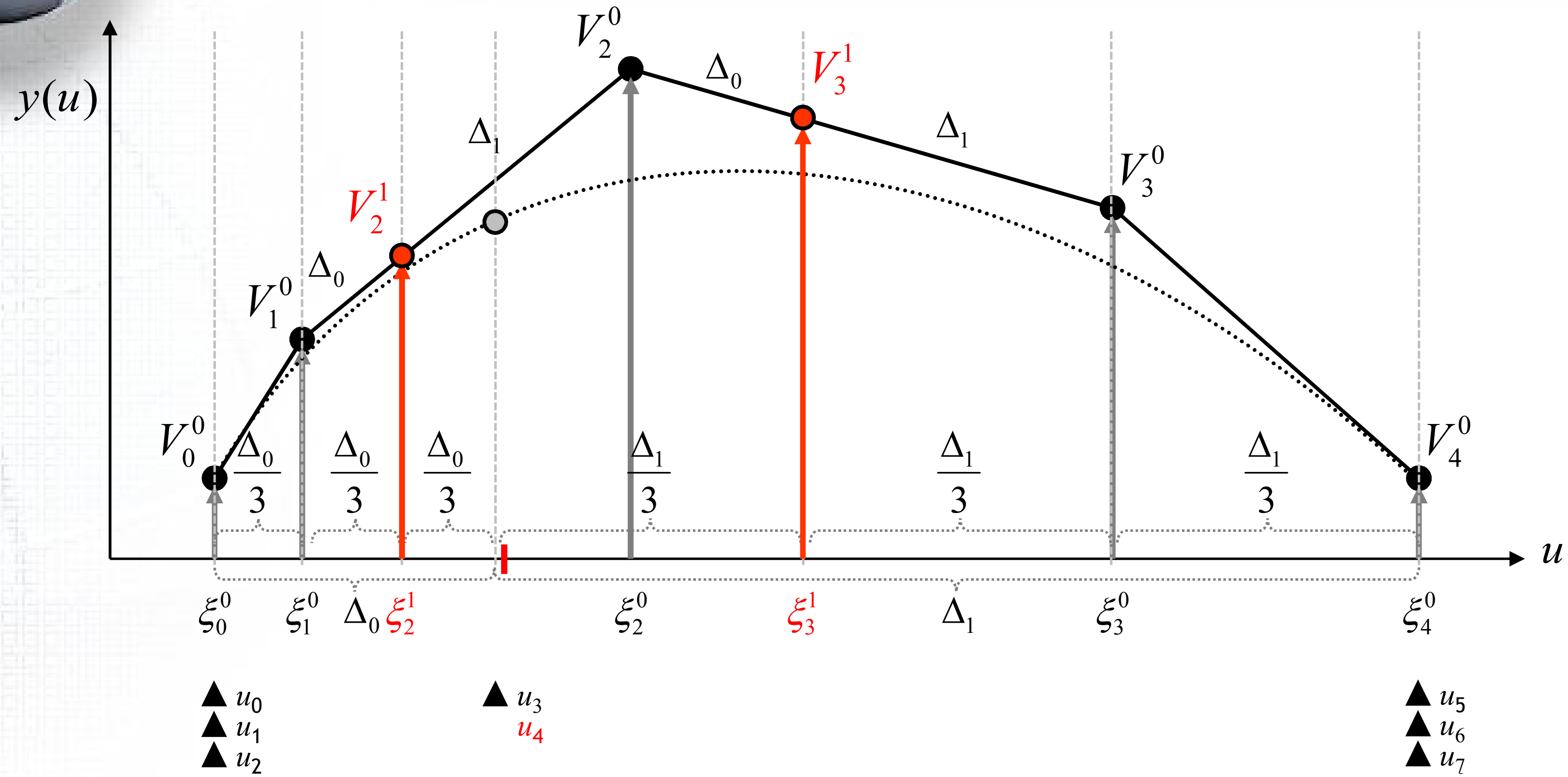
$$\xi_9^3=\frac{u_6+u_7+u_8}{3}=\frac{3\times\Delta_2+3\times\Delta_3+3\times\Delta_4+3\times\Delta_5+2\times\Delta_6}{3}=\xi_6^0$$

$$\xi_{10}^3=\frac{u_7+u_8+u_9}{3}=\Delta_2+\Delta_3+\Delta_4+\Delta_5+\Delta_6=\xi_7^0$$

Example:
n-th B-Spline function으로부터 n-th Bezier function 찾기

n-th B-Spline function으로부터 n-th Bezier function 찾기

- 원래 knot위치에 n번 knot 중첩



$$\xi_0^1 = \frac{u_0 + u_1 + u_2}{3} = u_2 = \xi_0^0$$

$$\xi_1^1 = \frac{u_1 + u_2 + u_3}{3} = u_2 + \frac{u_3 - u_2}{3}$$

$$= u_2 + \frac{\Delta_0}{3} = \xi_1^0$$

$$\xi_2^1 = \frac{u_2 + u_3 + u_4}{3} = \frac{u_2 + 2u_3}{3} =$$

$$= u_3 - \frac{u_3 - u_2}{3} = u_3 - \frac{\Delta_0}{3}$$

$$\xi_3^1 = \frac{u_3 + u_4 + u_5}{3} = u_3 + \frac{u_5 - u_4}{3}$$

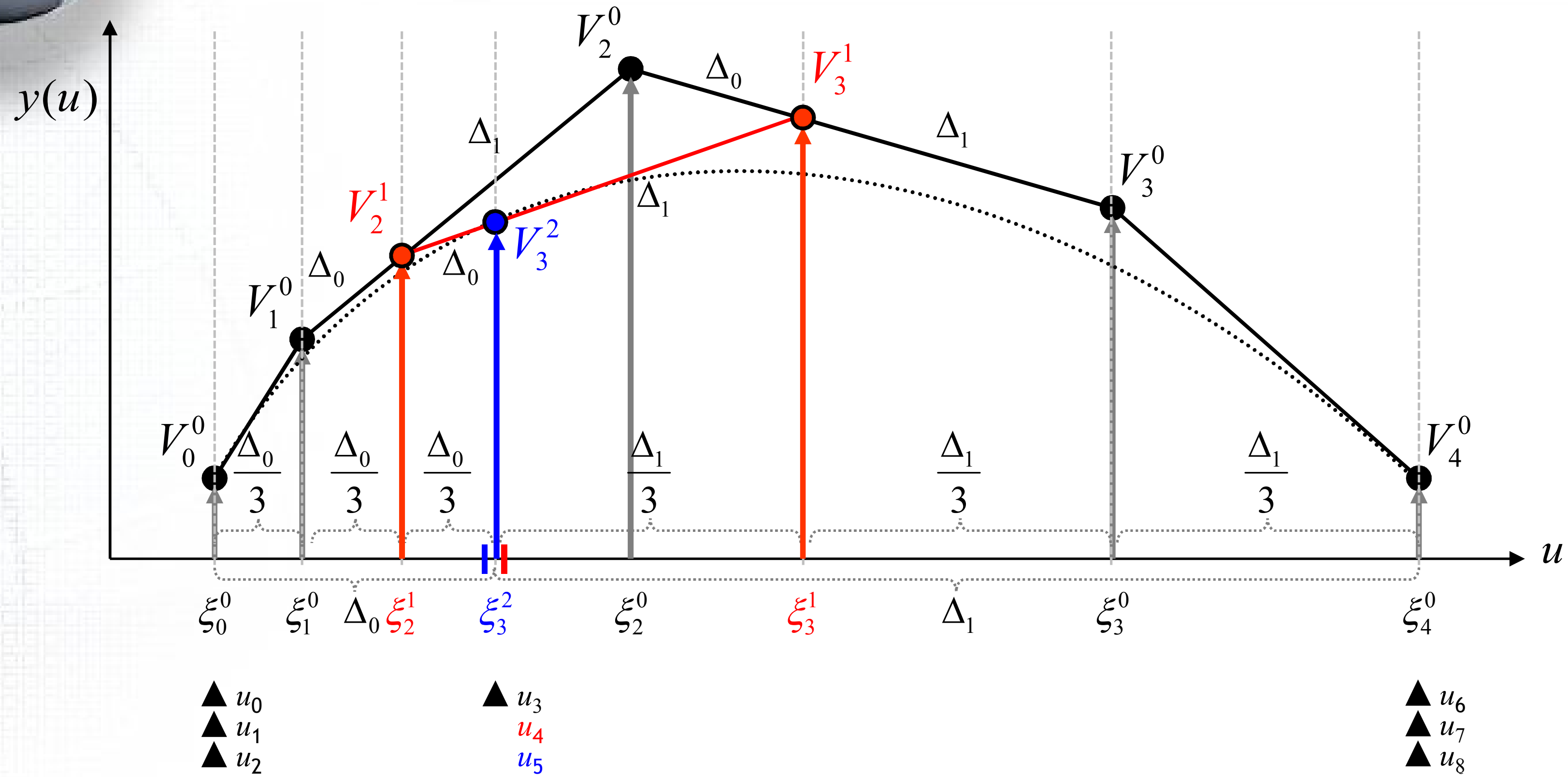
$$= \Delta_0 + \frac{\Delta_1}{3}$$

$$\xi_4^1 = \frac{u_4 + u_5 + u_6}{3} = u_5 - \frac{u_5 - u_4}{3}$$

$$= u_5 - \frac{u_5 - u_4}{3} = u_5 - \frac{\Delta_1}{3} = \xi_3^0$$

$$\xi_5^1 = \frac{u_5 + u_6 + u_7}{3} = u_5 = \xi_4^0$$

n-th B-Spline function으로부터 n-th Bezier function 찾기
- 원래 knot위치에 n번 knot 중첩



$$\xi_0^2 = \frac{u_0 + u_1 + u_2}{3} = u_2 = \xi_0^0$$

$$\xi_1^2 = \frac{u_1 + u_2 + u_3}{3} = u_2 + \frac{u_3 - u_2}{3} = u_2 + \frac{\Delta_0}{3} = \xi_1^0$$

$$\xi_2^2 = \frac{u_2 + u_3 + u_4}{3} = \frac{u_2 + 2u_3}{3} = u_3 - \frac{u_3 - u_2}{3} = u_3 - \frac{\Delta_0}{3} = \xi_2^1$$

$$\xi_3^2 = \frac{u_3 + u_4 + u_5}{3} = u_3$$

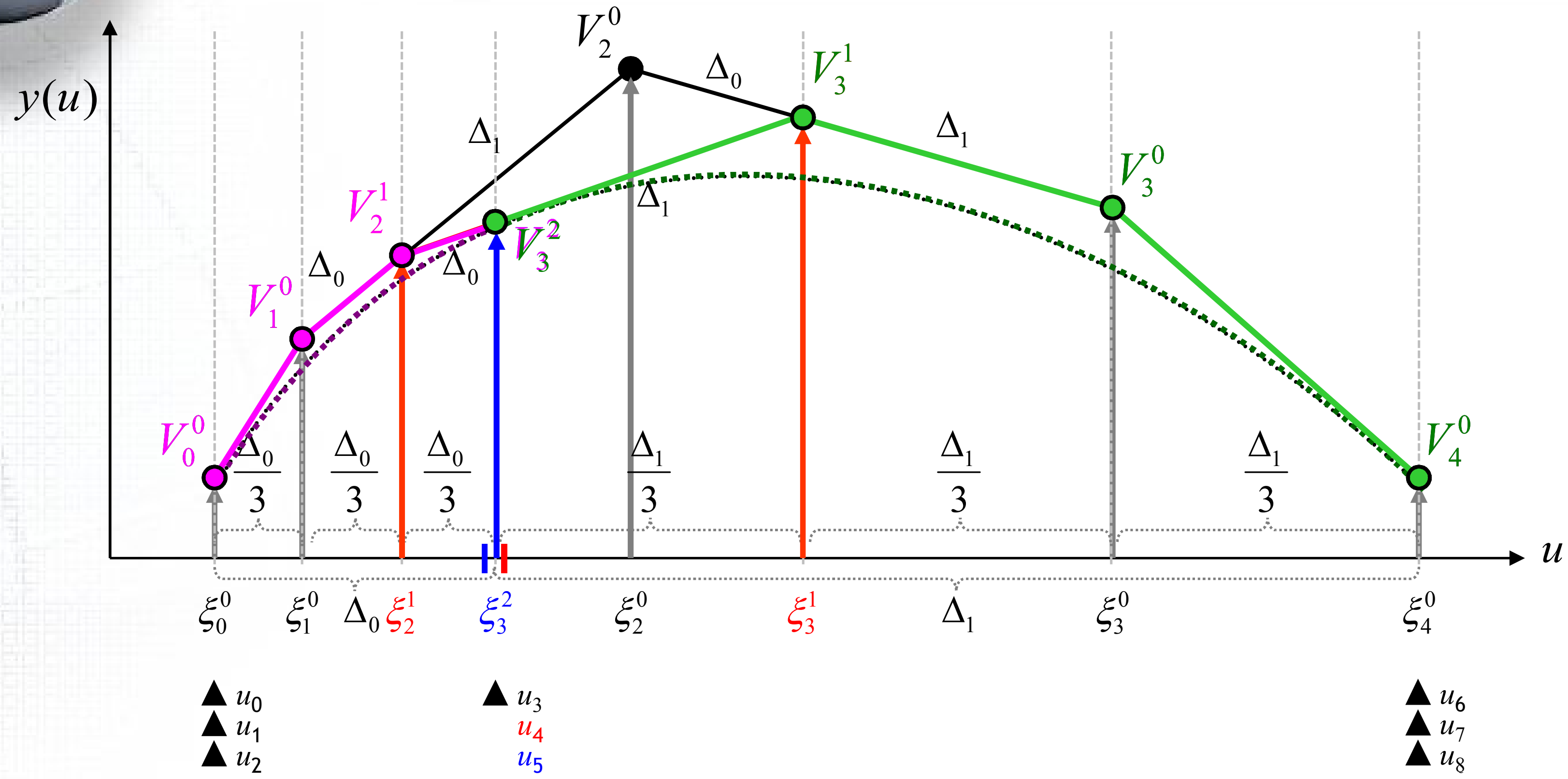
$$\xi_4^2 = \frac{u_4 + u_5 + u_6}{3} = u_4 + \frac{u_6 - u_5}{3} = \Delta_0 + \frac{\Delta_1}{3} = \xi_3^1$$

$$\xi_5^2 = \frac{u_5 + u_6 + u_7}{3} = u_6 - \frac{u_6 - u_5}{3} = u_6 - \frac{\Delta_1}{3} = \xi_3^0$$

$$\xi_6^2 = \frac{u_6 + u_7 + u_8}{3} = \xi_4^0$$

n-th B-Spline function으로부터 n-th Bezier function 찾기

- 원래 knot위치에 n번 knot 중첩



1개의 B-Spline Function

u_3 위치에 knot를 차수(=3)만큼 중첩

2개의 Bezier Function으로 나누어짐

분홍색 function: $V_0^0,\ V_1^0,\ V_2^1,\ V_3^2$ 를 y-control ordinate로 하는 Bezier Function

녹색 function: $V_3^2,\ V_3^1,\ V_3^0,\ V_4^0$ 를 y-control ordinate로 하는 Bezier Function

Example

Knot Insertion of Bezier Curve

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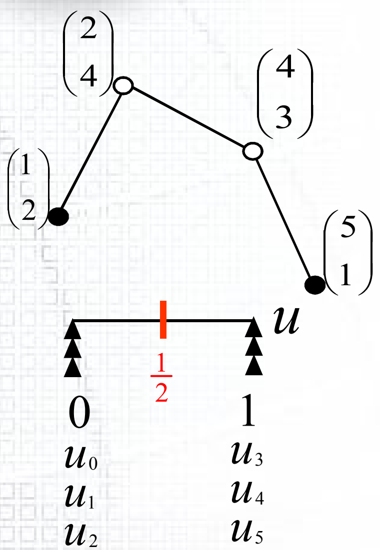
Example: Knot Insertion of Bezier Curve

- Knot Insertion #1

knot Insertion 1



Greville abscissae updated

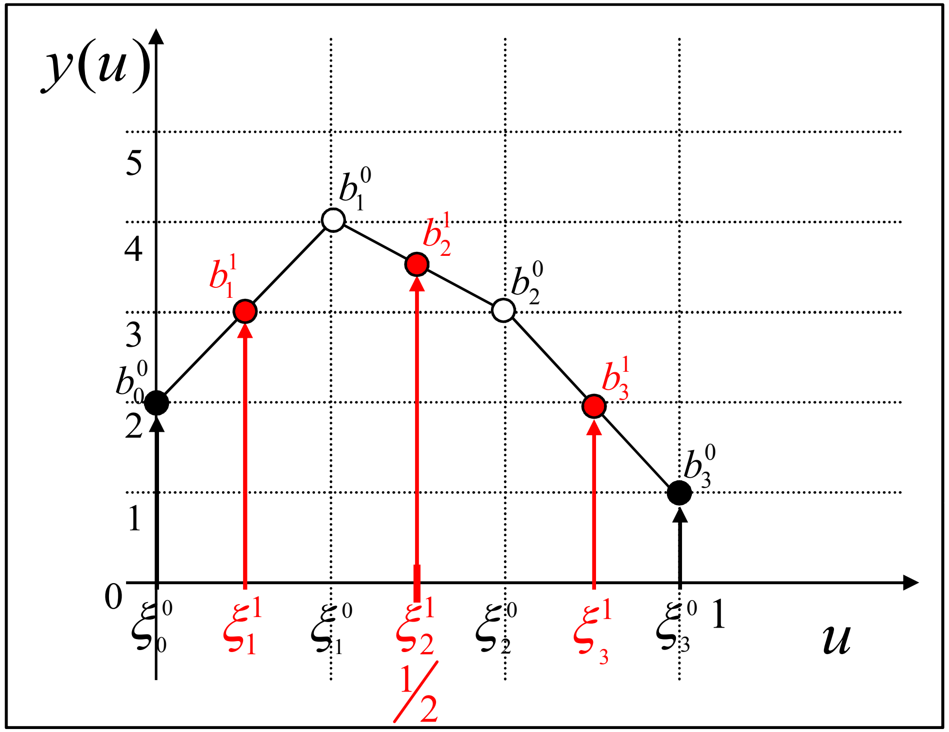
$$\xi_0^1 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^1 = \frac{0+0+\frac{1}{2}}{3} = \frac{1}{6}$$

$$\xi_2^1 = \frac{0+\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{3}$$

$$\xi_3^1 = \frac{\frac{1}{2}+1+1}{3} = \frac{5}{6}$$

$$\xi_4^1 = \frac{1+1+1}{3} = 1 = \xi_3^0$$



Given
- Bezier y-control ordinate $b_0^0, b_1^0, b_2^0, b_3^0$
- Knot Insertion에 의해 계산되는 $\xi_0^0, \xi_1^1, \xi_2^1, \xi_3^1, \xi_3^0$

Find
- ξ_i 위에 위치하는 Bezier function ordinates
  (b_1^1, b_2^1, b_3^1)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함

(예) b_1^1 는 직선 $\overline{b_0^0 b_1^0}$ 사이에 위치

$b_1^1 = ?$

$$\overline{b_0^0 b_1^1} : \overline{b_1^1 b_1^0} = \overline{\xi_0^0 \xi_1^1} : \overline{\xi_1^1 \xi_1^0}$$

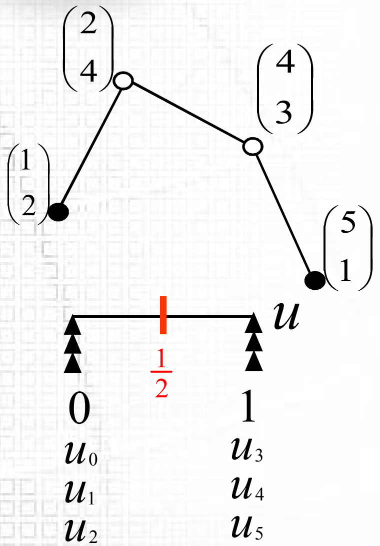
$$\left\{ \begin{array}{l} \overline{\xi_0^0 \xi_1^1} = \frac{1}{6} - 0 = \frac{1}{6} \\ \overline{\xi_1^1 \xi_1^0} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} \\ \overline{\xi_1^0 \xi_0^0} = \frac{1}{3} - 0 = \frac{1}{3} \end{array} \right.$$

$$b_1^1 = \tfrac{1}{2} b_0^0 + \tfrac{1}{2} b_1^0$$

Example: Knot Insertion of Bezier Curve

- Knot Insertion #1

knot Insertion 1



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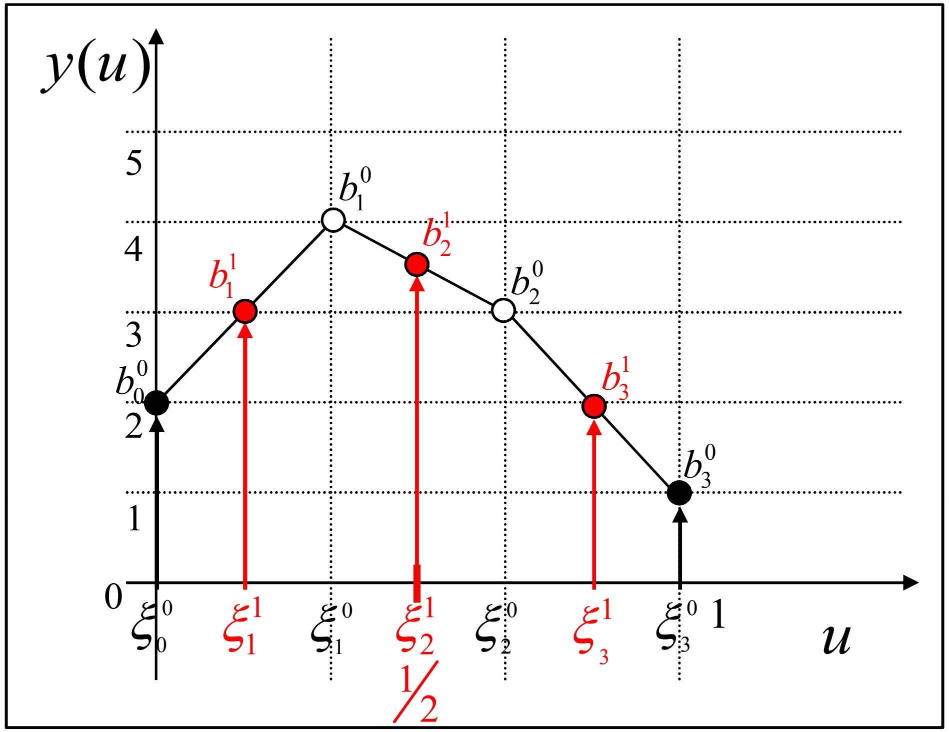

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Find
- ξ_i 위에 위치하는 Bezier function ordinates (b_1^1, b_2^1, b_3^1)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함
(예) b_1^1 는 직선 $\overline{b_0^0 b_1^0}$ 사이에 위치

$b_2^1 = ?$

$$\overline{b_1^0 b_2^1} : \overline{b_2^1 b_2^0} = \overline{\xi_1^0 \xi_2^1} : \overline{\xi_2^1 \xi_2^0}$$

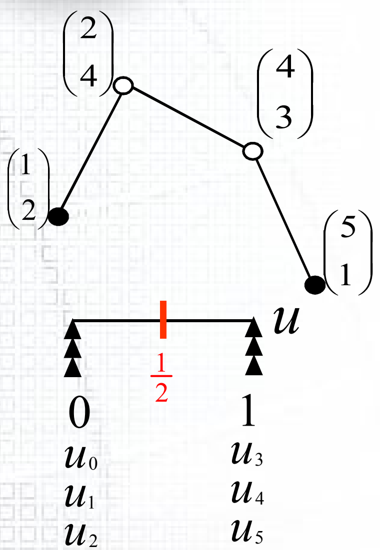
$$\begin{cases} \overline{\xi_1^0 \xi_2^1} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \\ \overline{\xi_2^1 \xi_2^0} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \\ \overline{\xi_2^0 \xi_1^0} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \end{cases}$$

$$b_2^1 = \tfrac{1}{2} b_1^0 + \tfrac{1}{2} b_2^0$$

Example: Knot Insertion of Bezier Curve

- Knot Insertion #1

knot Insertion 1



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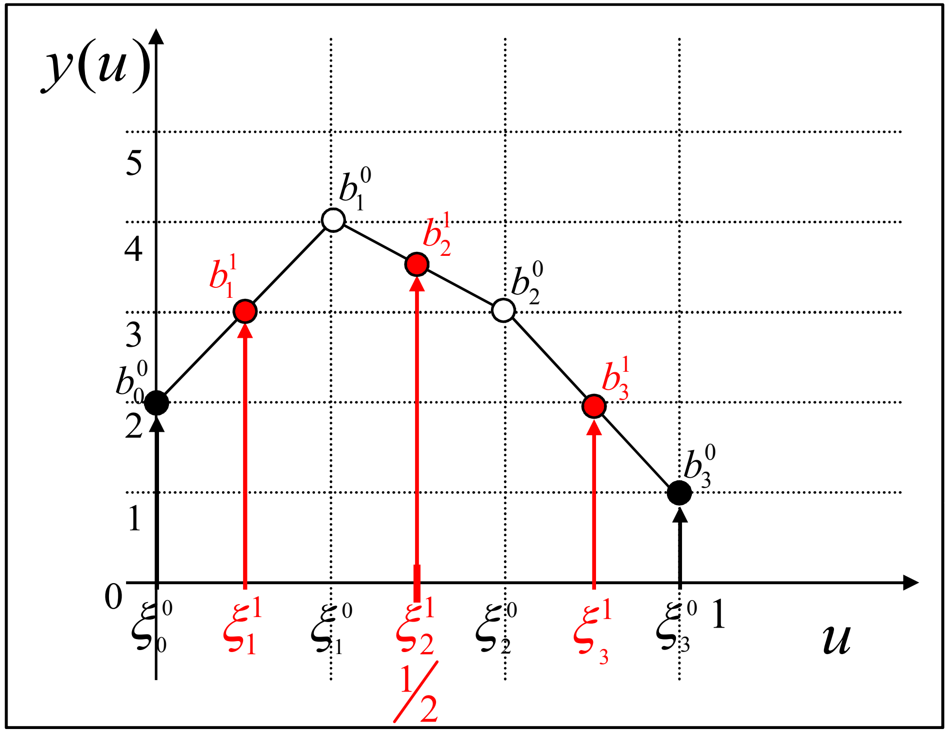

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- Bezier y-control ordinate $b_0^0, b_1^0, b_2^0, b_3^0$
- Knot Insertion에 의해 계산되는 $\xi_0^0, \xi_1^1, \xi_2^1, \xi_3^1, \xi_3^0$

Find

- ξ_i 위에 위치하는 Bezier function ordinates (b_1^1, b_2^1, b_3^1)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함

(예) b_1^1 는 직선 $\overline{b_0^0 b_1^0}$ 사이에 위치

$b_3^1 = ?$

$$\overline{b_2^0 b_3^1} : \overline{b_3^1 b_3^0} = \overline{\xi_2^0 \xi_3^1} : \overline{\xi_3^1 \xi_3^0}$$

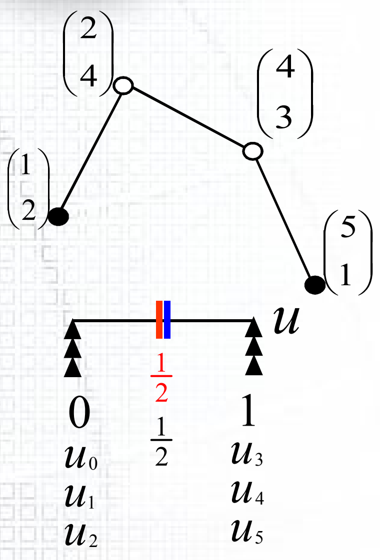
$$\begin{cases} \overline{\xi_2^0 \xi_3^1} = \frac{5}{6} - \frac{2}{3} = \frac{1}{6} \\ \overline{\xi_3^1 \xi_3^0} = 1 - \frac{5}{6} = \frac{1}{6} \\ \overline{\xi_3^0 \xi_2^0} = 1 - \frac{2}{3} = \frac{1}{3} \end{cases}$$

$$b_3^1 = \tfrac{1}{2} b_2^0 + \tfrac{1}{2} b_3^0$$

Example: Knot Insertion of Bezier Curve

- Knot Insertion #2

knot Insertion 2



Greville abscissae updated

$$\xi_0^2 = \frac{0+0+0}{3} = 0 \quad = \xi_0^0$$

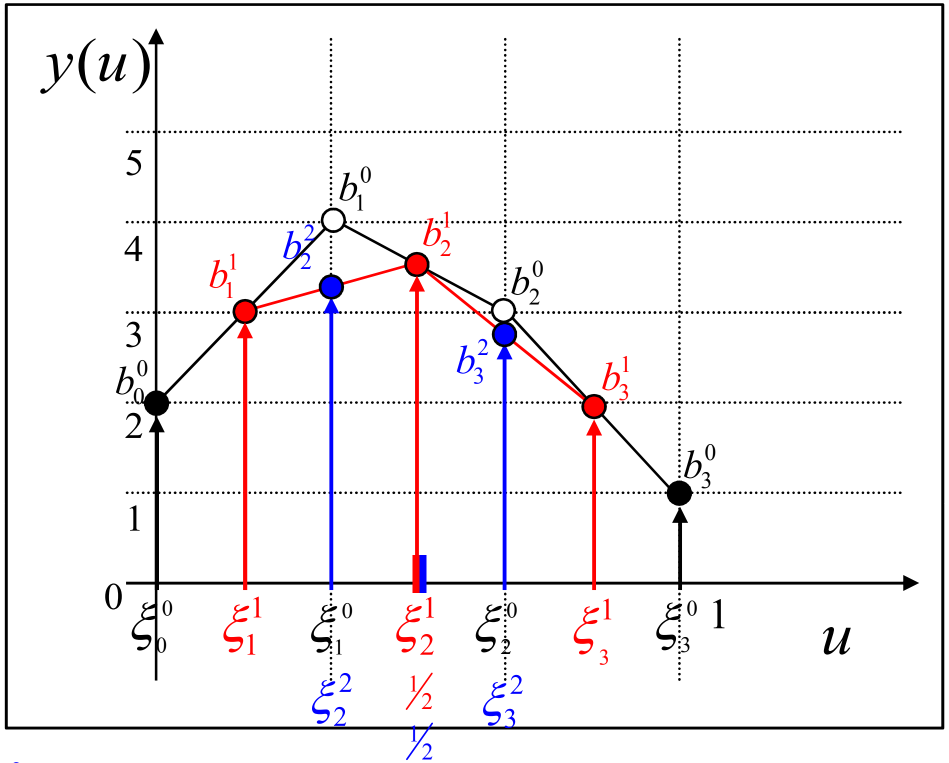
$$\xi_1^2 = \frac{0+0+\frac{1}{2}}{3} = \frac{1}{6} \quad = \xi_1^1$$

$$\xi_2^2 = \frac{0+\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{3}$$

$$\xi_3^2 = \frac{\frac{1}{2}+\frac{1}{2}+1}{3} = \frac{2}{3}$$

$$\xi_4^2 = \frac{\frac{1}{2}+1+1}{3} = \frac{5}{6} \quad = \xi_3^1$$

$$\xi_5^2 = \frac{1+1+1}{3} = 1 \quad = \xi_3^0$$



Given

- Bezier y-control ordinate $b_0^0, b_1^0, b_2^0, b_3^0$
- Knot Insertion에 의해 계산되는 $\xi_0^0, \xi_1^1, \xi_2^2, \xi_3^2, \xi_3^1, \xi_3^0$

Find

- ξ_i 위에 위치하는 Bezier function ordinates
  (b_2^2, b_3^2)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함
(예) b_2^2 는 직선 $\overline{b_1^1 b_2^1}$ 사이에 위치

$b_2^2 = ?$

$$\overline{b_1^1 b_2^2} : \overline{b_2^2 b_2^1} = \overline{\xi_1^1 \xi_2^2} : \overline{\xi_2^2 \xi_2^1}$$

$$\overline{\xi_1^1 \xi_2^2} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$$

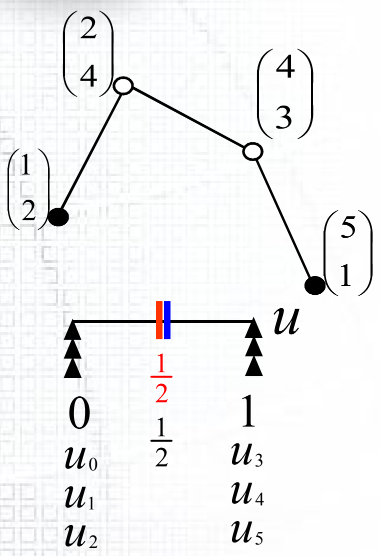
$$\overline{\xi_2^2 \xi_2^1} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\overline{\xi_2^1 \xi_1^1} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

$$b_2^2 = \tfrac{1}{2} b_1^1 + \tfrac{1}{2} b_2^1$$

Example: Knot Insertion of Bezier Curve
- Knot Insertion #2

knot Insertion 2



Greville abscissae updated

$$\xi_0^2 = \frac{0+0+0}{3} = 0 \quad = \xi_0^0$$

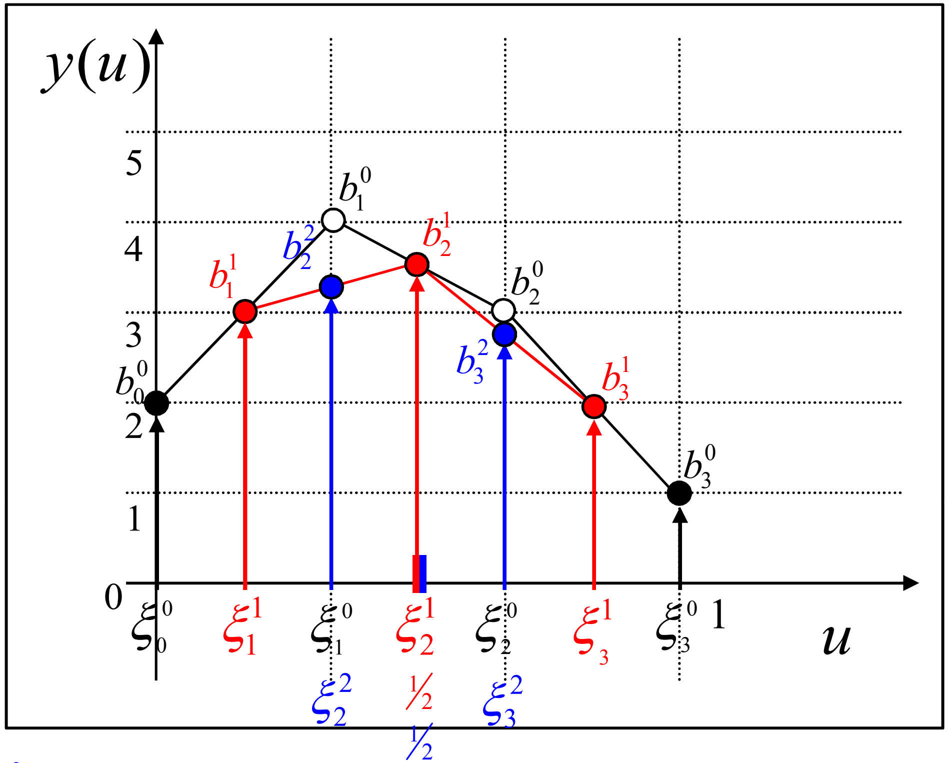
$$\xi_1^2 = \frac{0+0+\frac{1}{2}}{3} = \frac{1}{6} \quad = \xi_1^1$$

$$\xi_2^2 = \frac{0+\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{3}$$

$$\xi_3^2 = \frac{\frac{1}{2}+\frac{1}{2}+1}{3} = \frac{2}{3}$$

$$\xi_4^2 = \frac{\frac{1}{2}+1+1}{3} = \frac{5}{6} \quad = \xi_3^1$$

$$\xi_5^2 = \frac{1+1+1}{3} = 1 \quad = \xi_3^0$$



Given
- Bezier y-control ordinate $b_0^0, b_1^0, b_2^0, b_3^0$
- Knot Insertion에 의해 계산되는 $\xi_0^0, \xi_1^1, \xi_2^2, \xi_3^2, \xi_3^1, \xi_3^0$

Find
- ξ_i 위에 위치하는 Bezier function ordinates (b_2^2, b_3^2)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함

(예) b_2^2 는 직선 $\overline{b_1^1 b_2^1}$ 사이에 위치

$b_3^2 = ?$

$$\overline{b_2^1 b_3^2} : \overline{b_3^2 b_3^1} = \overline{\xi_2^1 \xi_3^2} : \overline{\xi_3^2 \xi_3^1}$$

$$\overline{\xi_2^1 \xi_3^2} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

$$\overline{\xi_3^2 \xi_3^1} = \frac{5}{6} - \frac{2}{3} = \frac{1}{6}$$

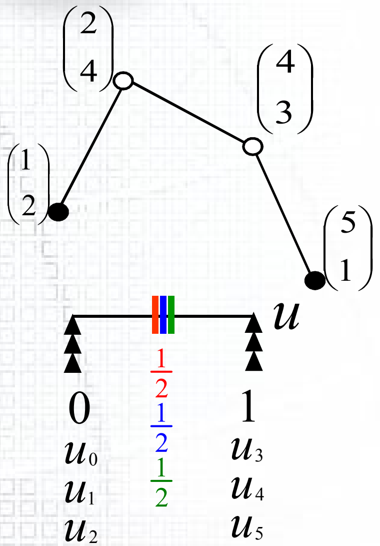
$$\overline{\xi_3^1 \xi_2^1} = \frac{5}{6} - \frac{1}{2} = \frac{1}{3}$$

$$b_3^2 = (1-u) b_2^1 + u b_3^1$$

Example: Knot Insertion of Bezier Curve

- Knot Insertion #3

knot Insertion 3



Greville abscissae updated

$$\xi_0^3 = \frac{0+0+0}{3} = 0 \quad = \xi_0^0$$

$$\xi_1^3 = \frac{0+0+\frac{1}{2}}{3} = \frac{1}{6} \quad = \xi_1^1$$

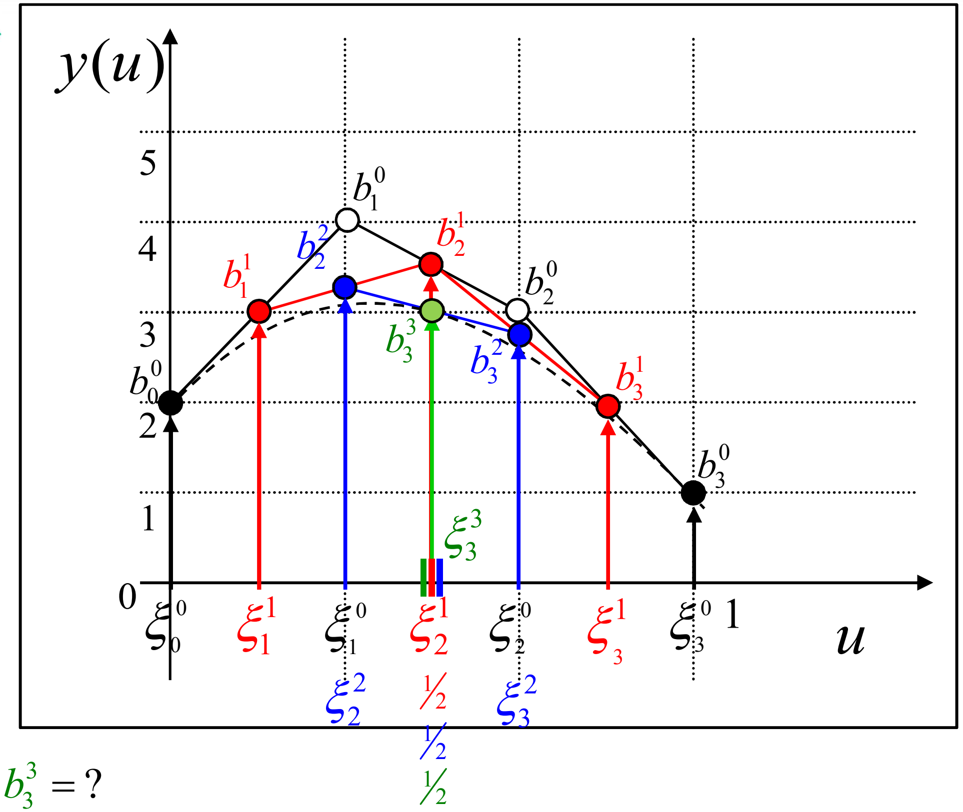
$$\xi_2^3 = \frac{0+\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{3} \quad = \xi_2^2$$

$$\xi_3^3 = \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{2}$$

$$\xi_4^3 = \frac{\frac{1}{2}+\frac{1}{2}+1}{3} = \frac{2}{3} \quad = \xi_3^2$$

$$\xi_5^3 = \frac{\frac{1}{2}+1+1}{3} = \frac{5}{6} \quad = \xi_3^1$$

$$\xi_6^3 = \frac{1+1+1}{3} = 1 \quad = \xi_3^0$$



Given
- Bezier y-control ordinate $b_0^0, b_1^0, b_2^0, b_3^0$
- Knot Insertion에 의해 계산되는 $\xi_0^0, \xi_1^1, \xi_2^2, \xi_3^3, \xi_3^2, \xi_3^1, \xi_3^0$

Find
- ξ_i 위에 위치하는 Bezier function ordinates
  (b_3^3)

두 ordinates의 내분점은 두 ordinates를 잇는 직선 상에 위치함
(예) b_3^3 는 직선 $\overline{b_2^2 b_3^2}$ 사이에 위치

$b_3^3 = ?$

$$\overline{b_2^2 b_3^3} : \overline{b_3^3 b_3^2} = \overline{\xi_2^2 \xi_3^3} : \overline{\xi_3^3 \xi_3^2}$$

$$\begin{cases} \overline{\xi_2^2 \xi_3^3} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \\ \overline{\xi_3^3 \xi_3^2} = \frac{2}{3} - \frac{1}{2} = \frac{1}{2} \\ \overline{\xi_3^2 \xi_2^2} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \end{cases}$$

$$b_3^3 = \tfrac{1}{2} b_2^2 + \tfrac{1}{2} b_3^2$$

Example
Knot Insertion of B-Spline Curve

Advanced
Ship
Design
Automation
Laboratory

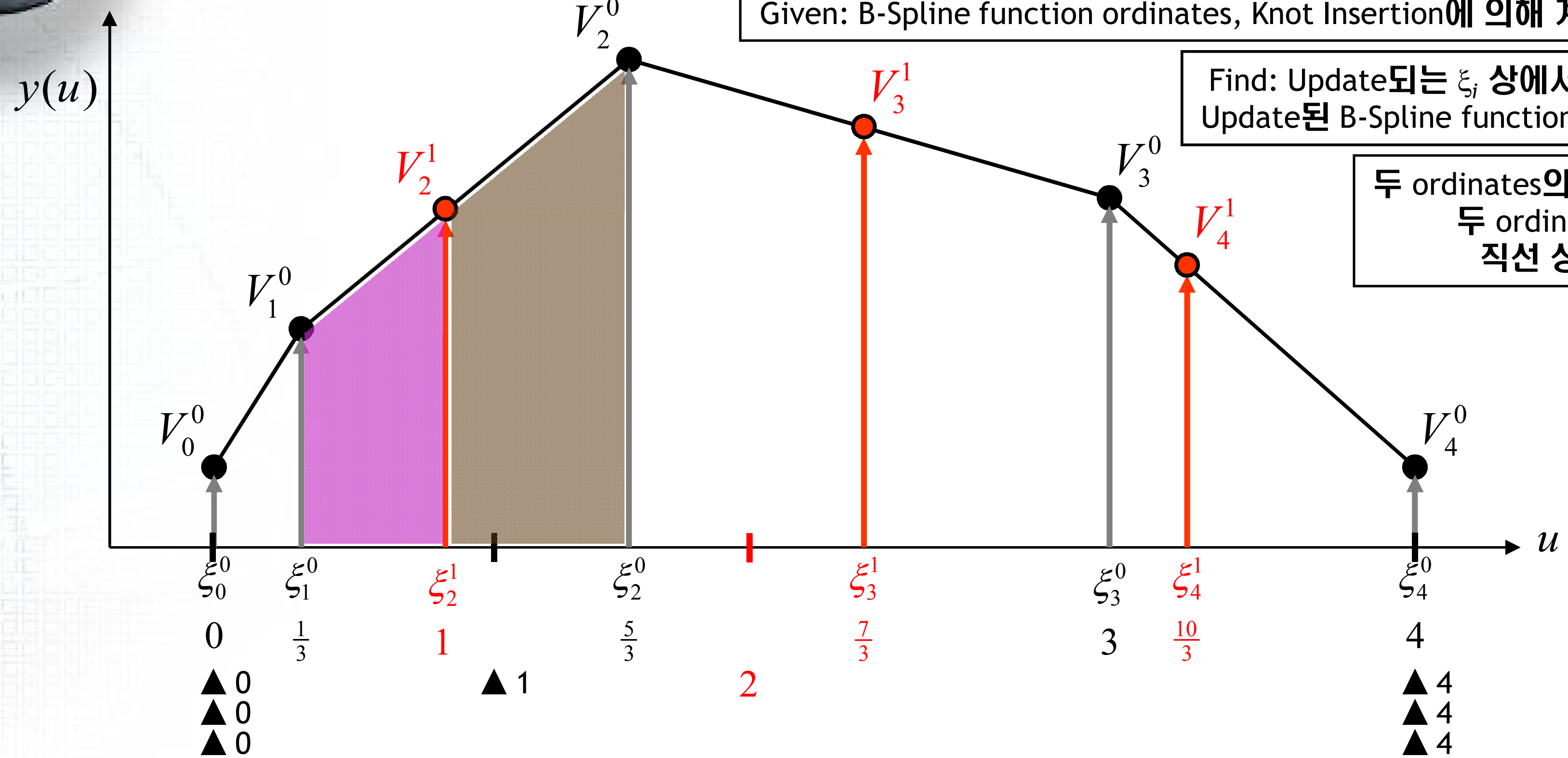
Example: Knot Insertion of B-Spline Curve

- Knot Insertion #1

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

두 ordinates의 내분점은 두 ordinate를 잇는 직선 상에 위치함



$$\xi_0^1 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^1 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

$$\xi_2^1 = \frac{0+1+2}{3} = 1$$

$$\xi_3^1 = \frac{1+2+4}{3} = \frac{7}{3}$$

$$\xi_4^1 = \frac{2+4+4}{3} = \frac{10}{3}$$

$$\xi_5^1 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$V_2^1 = ?$

$$\overline{V_1^0 V_2^1} : \overline{V_2^1 V_2^0} = \overline{\xi_1^0 \xi_2^1} : \overline{\xi_2^1 \xi_2^0}$$

$$\begin{cases} \overline{\xi_1^0 \xi_2^1} = \frac{2+1}{3} - \frac{1}{3} = \frac{2}{3} \\ \overline{\xi_2^1 \xi_2^0} = \frac{5}{3} - \frac{2+1}{3} = \frac{2}{3} \\ \overline{\xi_1^0 \xi_2^0} = \frac{5}{3} - \frac{1}{3} = \frac{4}{3} \end{cases}$$

$$V_2^1 = \frac{1}{2} V_1^0 + \frac{1}{2} V_2^0$$

Example: Knot Insertion of B-Spline Curve

- Knot Insertion #1

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

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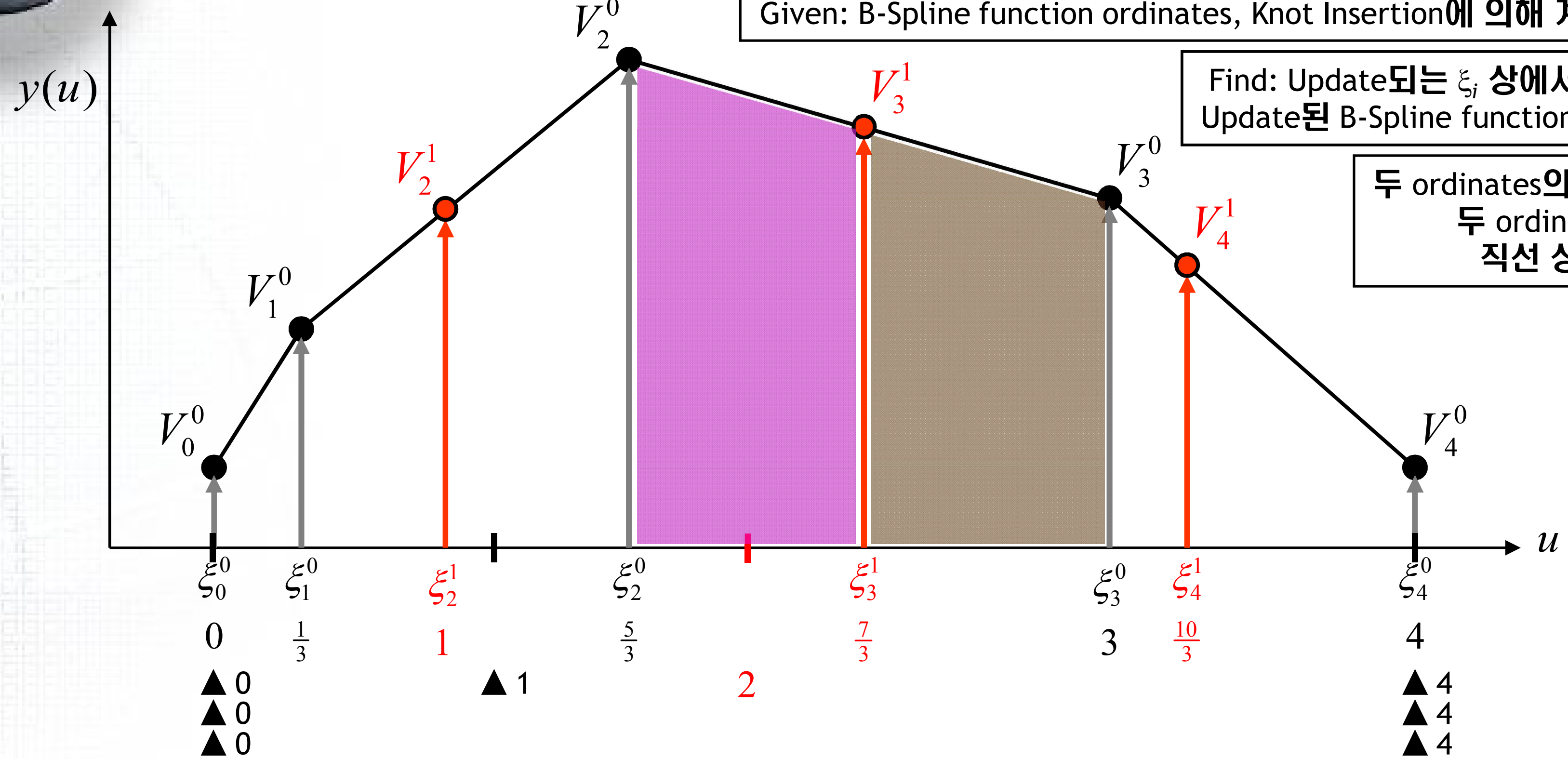

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$$\xi_1^1 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

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$$\xi_5^1 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$V_3^1 = ?$

$$\overline{V_2^0 V_3^1} : \overline{V_3^1 V_3^0} = \overline{\xi_2^0 \xi_3^1} : \overline{\xi_3^1 \xi_3^0}$$

$$\begin{cases} \overline{\xi_2^0 \xi_3^1} = \frac{2+5}{3} - \frac{5}{3} = \frac{2}{3} \\ \overline{\xi_3^1 \xi_3^0} = \frac{9}{3} - \frac{2+5}{3} = \frac{2}{3} \\ \overline{\xi_2^0 \xi_3^0} = \frac{9}{3} - \frac{5}{3} = \frac{4}{3} \end{cases}$$

$$V_3^1 = \frac{1}{2} V_2^0 + \frac{1}{2} V_3^0$$

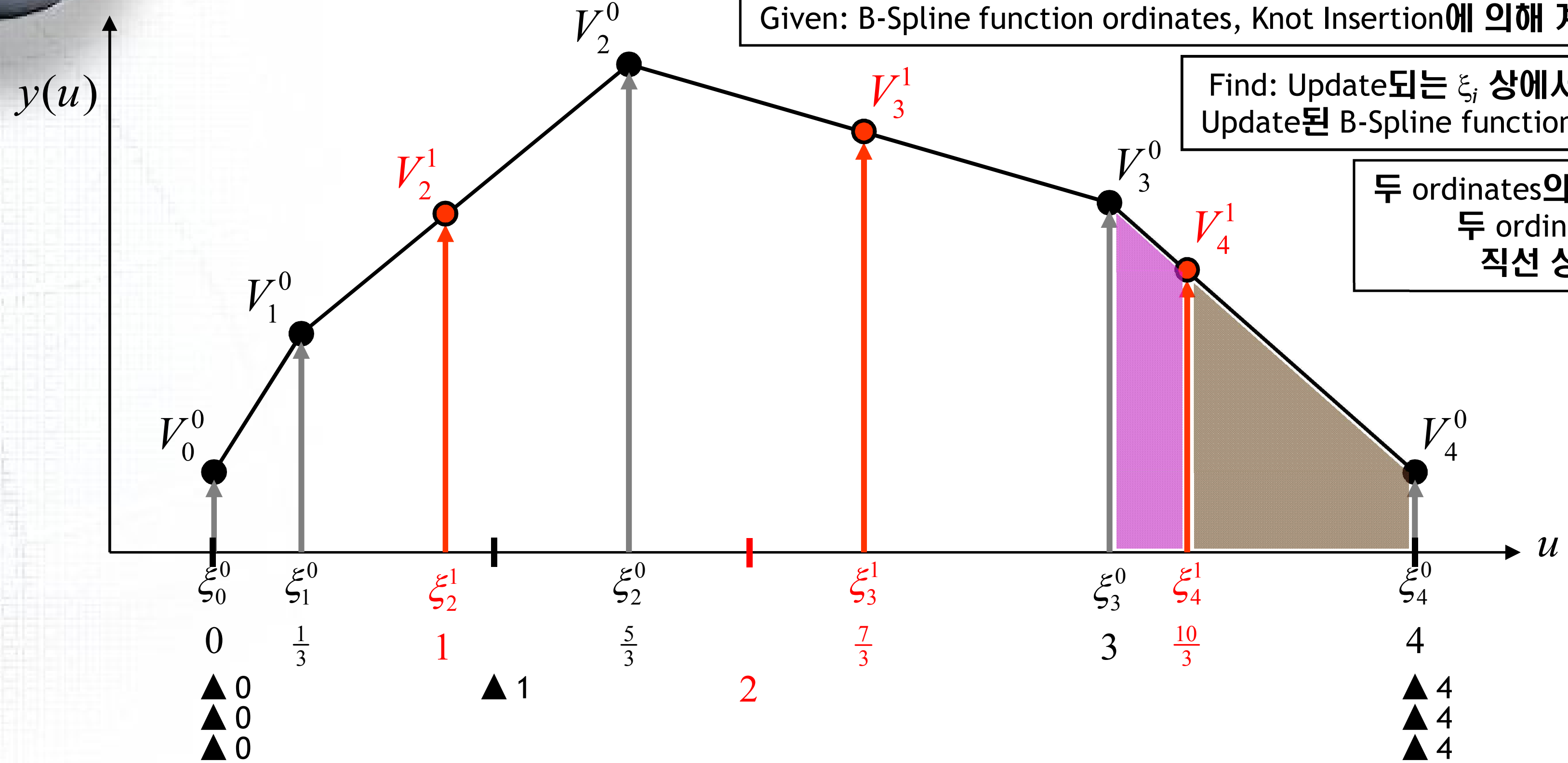
Example: Knot Insertion of B-Spline Curve

- Knot Insertion #1

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

두 ordinates의 내분점은 두 ordinate를 잇는 직선 상에 위치함



$$\xi_0^1 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^1 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

$$\xi_2^1 = \frac{0+1+2}{3} = 1$$

$$\xi_3^1 = \frac{1+2+4}{3} = \frac{7}{3}$$

$$\xi_4^1 = \frac{2+4+4}{3} = \frac{10}{3}$$

$$\xi_5^1 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$V_4^1 = ?$

$$\overline{V_3^0 V_4^1} : \overline{V_4^1 V_4^0} = \overline{\xi_3^0 \xi_4^1} : \overline{\xi_4^1 \xi_4^0}$$

$$\begin{cases} \overline{\xi_3^0 \xi_4^1} = \frac{2+8}{3} - \frac{9}{3} = \frac{1}{3} \\ \overline{\xi_4^1 \xi_4^0} = 4 - \frac{2+8}{3} = \frac{2}{3} \\ \overline{\xi_3^0 \xi_4^0} = 4 - \frac{9}{3} = \frac{3}{3} \end{cases}$$

$$V_4^1 = \frac{2}{3} V_3^0 + \frac{1}{3} V_4^0$$

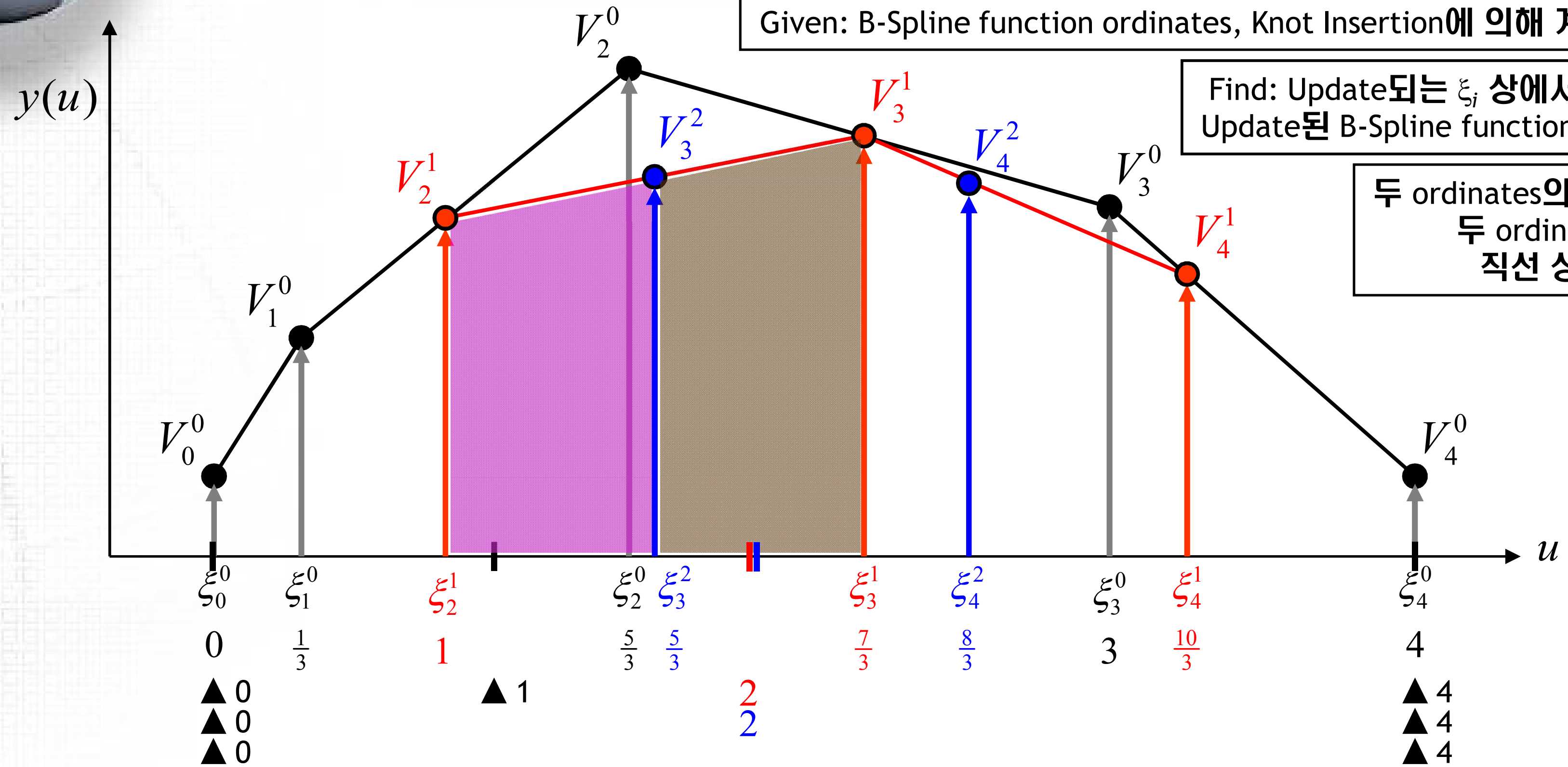
Example: Knot Insertion of B-Spline Curve

- Knot Insertion #2

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

두 ordinates의 내분점은 두 ordinate를 잇는 직선 상에 위치함



$$\xi_0^2 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^2 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

$$\xi_2^2 = \frac{0+1+2}{3} = 1 = \xi_2^1$$

$$\xi_3^2 = \frac{1+2+2}{3} = \frac{5}{3}$$

$$\xi_4^2 = \frac{2+2+4}{3} = \frac{8}{3}$$

$$\xi_5^2 = \frac{2+4+4}{3} = \frac{10}{3} = \xi_4^1$$

$$\xi_6^2 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$V_3^2 = ?$

$$\overline{V_2^1 V_3^2} : \overline{V_3^2 V_3^1} = \overline{\xi_2^1 \xi_3^2} : \overline{\xi_3^2 \xi_3^1}$$

$$\begin{cases} \overline{\xi_2^1 \xi_3^2} = \dfrac{2 \cdot 2+1}{3} - \dfrac{2+1}{3} = \dfrac{2}{3} \\ \overline{\xi_3^2 \xi_3^1} = \dfrac{2+5}{3} - \dfrac{2 \cdot 2+1}{3} = \dfrac{2}{3} \\ \overline{\xi_2^1 \xi_3^1} = \dfrac{2+5}{3} - \dfrac{2+1}{3} = \dfrac{4}{3} \end{cases}$$

$$V_3^2 = \frac{1}{2} V_2^1 + \frac{1}{2} V_3^1$$

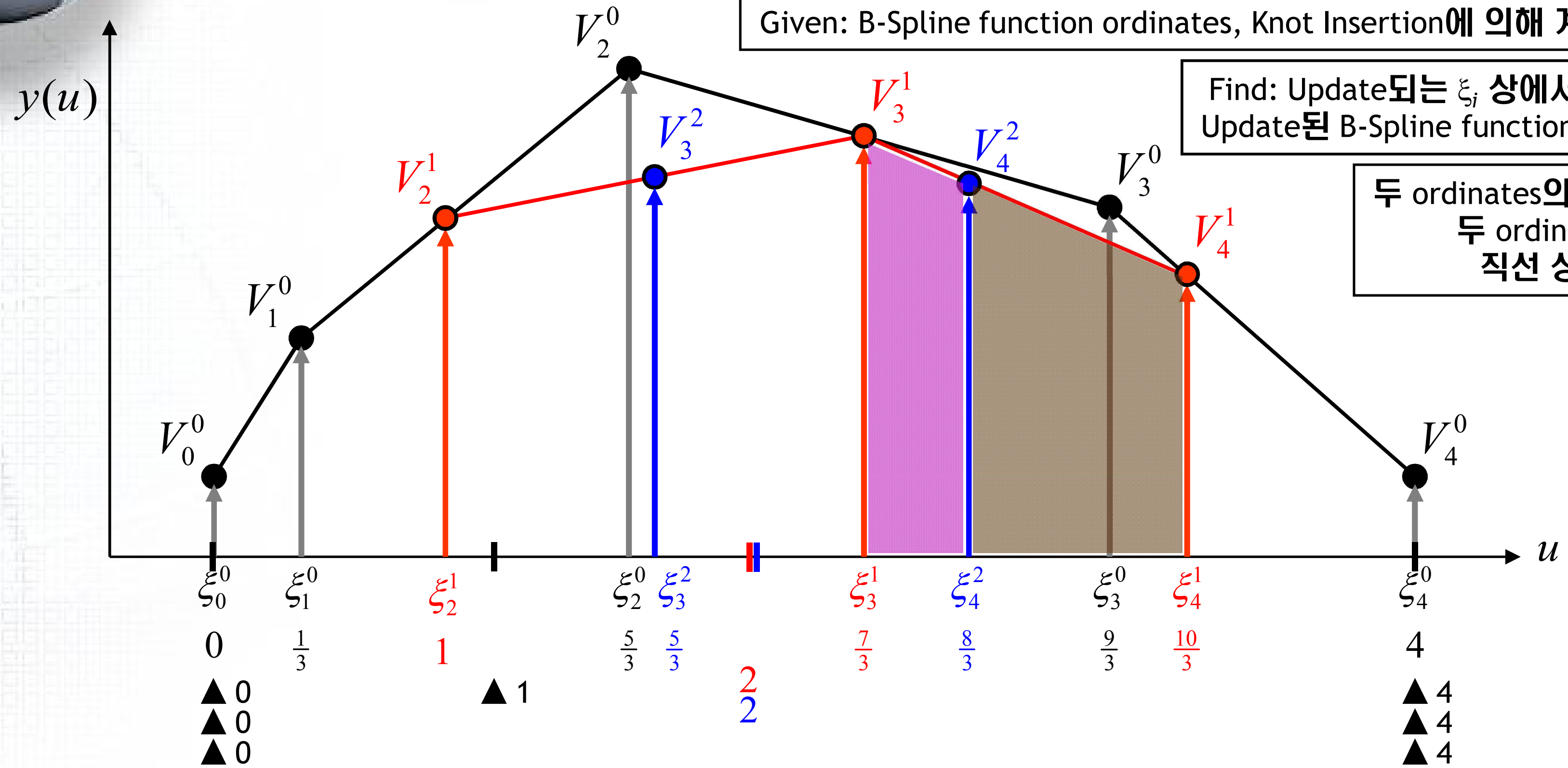
Example: Knot Insertion of B-Spline Curve

- Knot Insertion #2

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

두 ordinates의 내분점은 두 ordinate를 잇는 직선 상에 위치함



$$\xi_0^2 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^2 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

$$\xi_2^2 = \frac{0+1+2}{3} = 1 = \xi_2^1$$

$$\xi_3^2 = \frac{1+2+2}{3} = \frac{5}{3}$$

$$\xi_4^2 = \frac{2+2+4}{3} = \frac{8}{3}$$

$$\xi_5^2 = \frac{2+4+4}{3} = \frac{10}{3} = \xi_4^1$$

$$\xi_6^2 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$$V_4^2 = ?$$

$$\overline{V_3^1 V_4^2} : \overline{V_4^2 V_4^1} = \overline{\xi_3^1 \xi_4^2} : \overline{\xi_4^2 \xi_4^1}$$

$$V_4^2 = \frac{2}{3} V_3^1 + \frac{1}{4} V_4^1$$

$$\begin{cases} \overline{\xi_3^1 \xi_4^2} = \dfrac{2 \cdot 2+4}{3} - \dfrac{2+5}{3} = \dfrac{1}{3} \\ \overline{\xi_4^2 \xi_4^1} = \dfrac{2+8}{3} - \dfrac{2 \cdot 2+4}{3} = \dfrac{2}{3} \\ \overline{\xi_3^1 \xi_4^1} = \dfrac{2+8}{3} - \dfrac{2+5}{3} = \dfrac{3}{3} \end{cases}$$

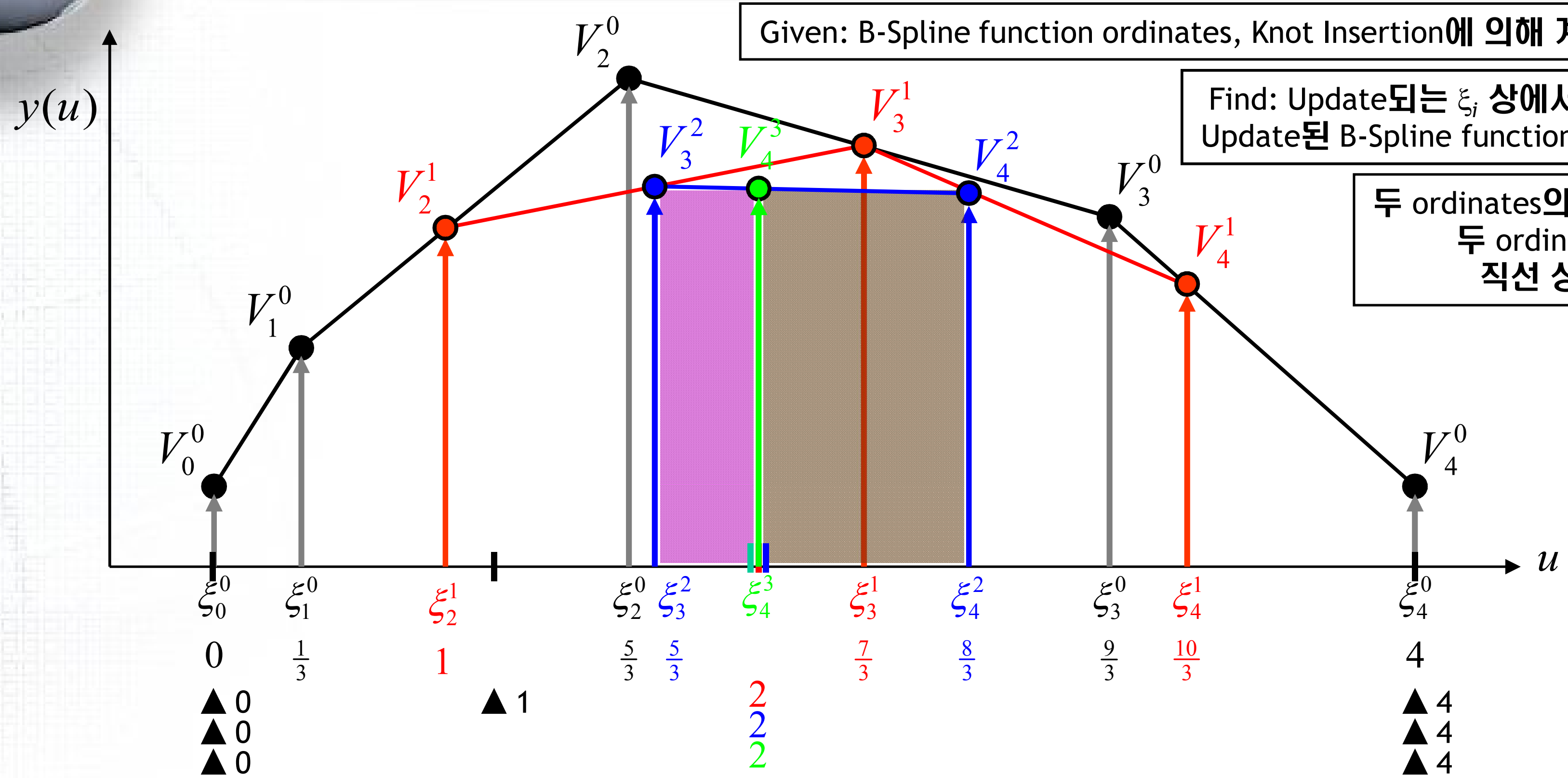
Example: Knot Insertion of B-Spline Curve

- Knot Insertion #3

Given: B-Spline function ordinates, Knot Insertion에 의해 계산되는 ξ_i

Find: Update되는 ξ_i 상에서 위치하는 Update된 B-Spline function ordinates

두 ordinates의 내분점은 두 ordinate를 잇는 직선 상에 위치함



$$\xi_0^3 = \frac{0+0+0}{3} = 0 = \xi_0^0$$

$$\xi_1^3 = \frac{0+0+1}{3} = \frac{1}{3} = \xi_1^0$$

$$\xi_2^2 = \frac{0+1+2}{3} = 1 = \xi_2^1$$

$$\xi_3^3 = \frac{1+2+2}{3} = \frac{5}{3} = \xi_3^2$$

$$\xi_4^3 = \frac{2+2+2}{3} = 2$$

$$\xi_5^3 = \frac{2+2+4}{3} = \frac{8}{3} = \xi_4^2$$

$$\xi_6^3 = \frac{2+4+4}{3} = \frac{10}{3} = \xi_4^1$$

$$\xi_7^3 = \frac{4+4+4}{3} = 4 = \xi_4^0$$

$$V_4^3 = ?$$

$$\overline{V_3^2 V_4^3} : \overline{V_4^3 V_4^2} = \overline{\xi_3^2 \xi_4^3} : \overline{\xi_4^3 \xi_4^2}$$

$$\overline{\xi_3^2 \xi_4^3} = 2 - \frac{2\cdot2+1}{3} = \frac{1}{3}$$

$$\overline{\xi_4^3 \xi_4^2} = \frac{2\cdot2+4}{3} - 2 = \frac{2}{3}$$

$$\overline{\xi_3^2 \xi_4^2} = \frac{2\cdot2+4}{3} - \frac{2\cdot2+1}{3} = \frac{3}{3}$$

$$V_4^3 = \frac{2}{3} V_3^2 + \frac{1}{3} V_4^2$$

참고 자료

Advanced
Ship
Design
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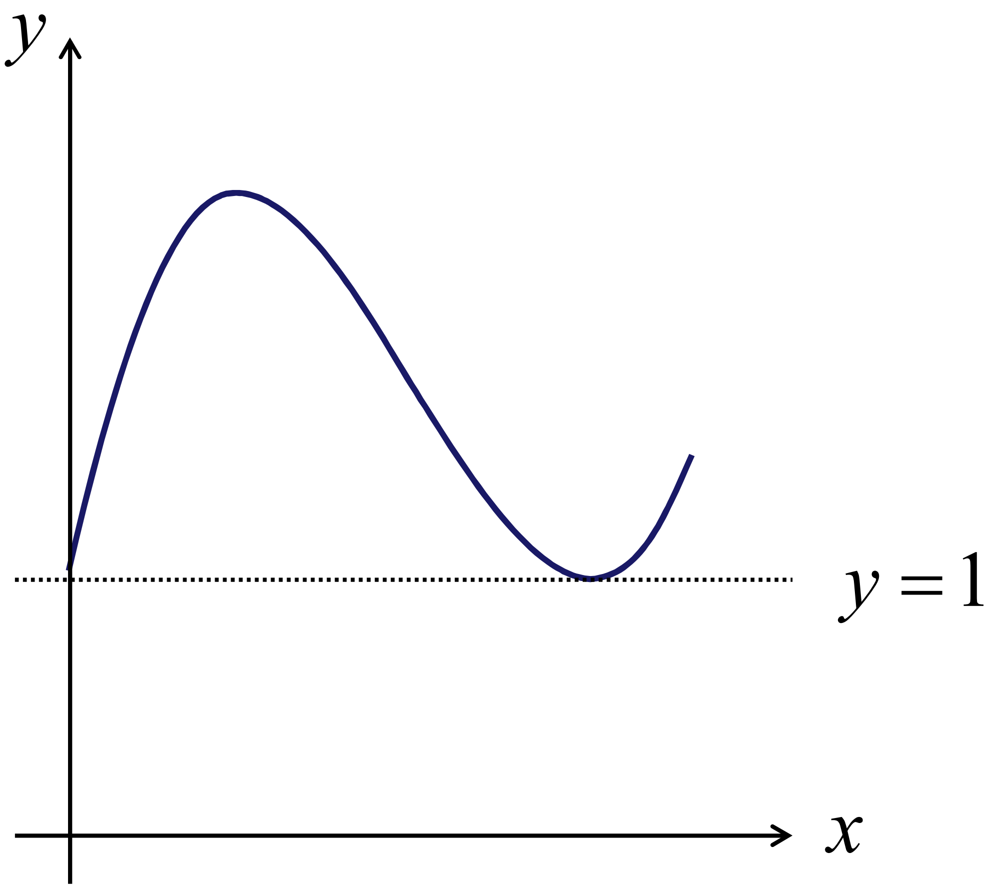
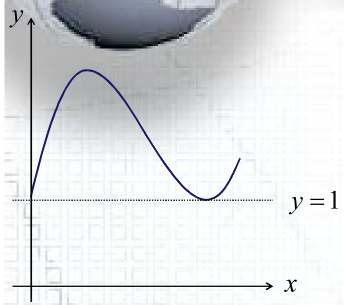
일반 함수의 매개변수 함수 표현 (1)

Given: $y = 2x^3 - 4x^2 + 2x + 1$



일반 함수의 매개변수 함수 표현 (2)



$$y = 2x^3 - 4x^2 + 2x + 1$$

$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t \\ 2t^3 - 4t^2 + 2t + 1 \end{bmatrix}$$

- 이러한 함수식과 계수 2, -4, 2, 1 으로는 그래프의 모양을 "직관적" 으로 예상하기 어려움

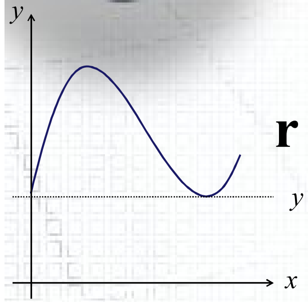
- 함수식을 아래와 같은 형태로 표현한다면

$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t)x_2 + t^3 x_3 \\ (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t)y_2 + t^3 y_3 \end{bmatrix}$$

$$\begin{bmatrix} t \\ 2t^3 - 4t^2 + 2t + 1 \end{bmatrix} = \begin{bmatrix} (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t)x_2 + t^3 x_3 \\ (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t)y_2 + t^3 y_3 \end{bmatrix}$$

일반 함수의 매개변수 함수 표현 (3)

$$y = 2x^3 - 4x^2 + 2x + 1$$



$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t \\ 2t^3 - 4t^2 + 2t + 1 \end{bmatrix}$$

- 함수식을 이와 같은 형태 표현한다면

$$= \begin{bmatrix} (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t)x_2 + t^3 x_3 \\ (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t)y_2 + t^3 y_3 \end{bmatrix}$$

$$t = (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t)x_2 + t^3 x_3$$

$$t = (-x_0 + 3x_1 - 3x_2 + x_3)t^3 + (3x_0 - 6x_1 + 3x_2)t^2 + (-3x_0 + 3x_1)t + x_0$$

상수의 계수: $x_0 = 0$

t의 계수: $-3x_0 + 3x_1 = 1$

t^2의 계수: $3x_0 - 6x_1 + 3x_2 = 0$

t^3의 계수: $-x_0 + 3x_1 - 3x_2 + x_3 = 0$

$\Rightarrow$

$x_0 = 0$

$x_1 = 1/3$

$x_2 = 2/3$

$x_3 = 1$

$$b_{x_i}^{0} = x_i = \frac{i}{n}$$

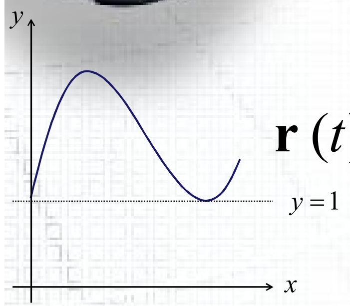
Linear Precision

Gerald E. Farin, The Essentials of CAGD, 2000, p. 29.

- *Linear precision*: If the control points $\mathbf{b}_1$ and $\mathbf{b}_2$ are evenly spaced on the straight line between $\mathbf{b}_0$ and $\mathbf{b}_3$, the cubic Bezier curve is the linear interpolant between $\mathbf{b}_0$ and $\mathbf{b}_3$.

일반 함수의 매개변수 함수 표현 (3)

$$y = 2x^3 - 4x^2 + 2x + 1$$



$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t \\ 2t^3 - 4t^2 + 2t + 1 \end{bmatrix}$$

- 함수식을 이와 같은 형태로
- 표현한다면

$$= \begin{bmatrix} (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t)x_2 + t^3 x_3 \\ (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t)y_2 + t^3 y_3 \end{bmatrix}$$

$$2t^3 - 4t^2 + 2t + 1 = (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t)y_2 + t^3 y_3$$

$$2t^3 - 4t^2 + 2t + 1 = (-y_0 + 3y_1 - 3y_2 + y_3)t^3 + (3y_0 - 6y_1 + 3y_2)t^2 + (-3y_0 + 3y_1)t + y_0$$

상수의 계수: $y_0 = 1$

t의 계수: $-3y_0 + 3y_1 = 2$

t^2의 계수: $3y_0 - 6y_1 + 3y_2 = -4$

t^3의 계수: $-y_0 + 3y_1 - 3y_2 + y_3 = 2$

$\Rightarrow$

$y_0 = 1$

$y_1 = 5/3$

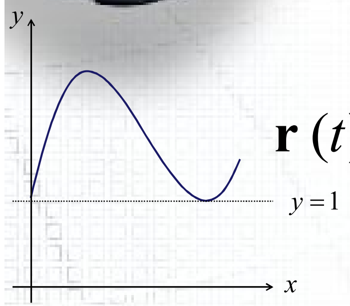
$y_2 = 1$

$y_3 = 1$

Gerald E. Farin, The Essentials of CAGD, 2000, p. 29.

- *Linear precision*: If the control points $\mathbf{b}_1$ and $\mathbf{b}_2$ are evenly spaced on the straight line between $\mathbf{b}_0$ and $\mathbf{b}_3$, the cubic Bezier curve is the linear interpolant between $\mathbf{b}_0$ and $\mathbf{b}_3$.

일반 함수의 매개변수 함수 표현 (3)



$$y = 2x^3 - 4x^2 + 2x + 1$$

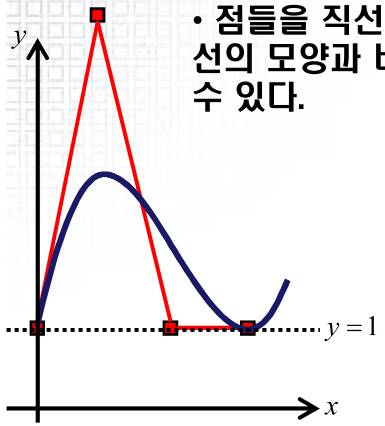
$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t \\ 2t^3 - 4t^2 + 2t + 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t) x_2 + t^3 x_3 \\ (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t) y_2 + t^3 y_3 \end{bmatrix}$$

- 함수식을 이와 같은 형태로
- 표현한다면

$x_0 = 0$	$y_0 = 1$
$x_1 = 1/3$	$y_1 = 5/3$
$x_2 = 2/3$	$y_2 = 1$
$x_3 = 1$	$y_3 = 1$

- 새로운 함수들에 곱해지는 계수들을 점으로 가시화 하면, 처음 점과 마지막 점은 곡선을 지나고,
- 점들을 직선으로 연결하면, 곡선의 모양과 비슷한 형태임을 알 수 있다.



$$= \begin{bmatrix} (1-t)^3 \cdot 0 + 3t(1-t)^2 \cdot \dfrac{1}{3} + 3t^2(1-t) \cdot \dfrac{2}{3} + t^3 \cdot 1 \\ (1-t)^3 \cdot 1 + 3t(1-t)^2 \cdot \dfrac{5}{3} + 3t^2(1-t) \cdot 1 + t^3 \cdot 1 \end{bmatrix}$$

$$= (1-t)^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 3t(1-t)^2 \begin{bmatrix} 1/3 \\ 5/3 \end{bmatrix} + 3t^2(1-t) \begin{bmatrix} 2/3 \\ 1 \end{bmatrix} + t^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= B_0^3(t) \boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} + B_1^3(t) \boxed{\begin{bmatrix} 1/3 \\ 5/3 \end{bmatrix}} + B_2^3(t) \boxed{\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}} + B_3^3(t) \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

Gerald E. Farin, The Essentials of CAGD, 2000, p. 29.

- *Linear precision*: If the control points $\mathbf{b}_1$ and $\mathbf{b}_2$ are evenly spaced on the straight line between $\mathbf{b}_0$ and $\mathbf{b}_3$, the cubic Bezier curve is the linear interpolant between $\mathbf{b}_0$ and $\mathbf{b}_3$.