Second-Order Active Filters Based on Inductor Replacement

- Obtained by replacing the inductor $L$ in the $LCR$ resonator with an op amp-$RC$ circuit that has an inductive input impedance.

2GHz LNA

transistor
The Antoniou Inductance-Simulation Circuit

- Invented by A. Antoniou.
- If the circuit is fed at its input (node 1) with a voltage source $V_1$ and the input current is denoted $I_1$, (for ideal op amps)

$$Z_{in} \equiv \frac{V_1}{I_1} = sC_4R_1R_3R_5/R_2$$

$$L = \frac{C_4R_1R_3R_5}{R_2}$$
$i = v / sa$
The Antoniou Inductance-Simulation Circuit

1. Assuming ideal op amps.
2. Analysis begins at node 1, which is assumed to be fed by a voltage source $V_1$.
3. Analysis proceeds step by step, with the order of the steps indicated by the circled numbers.

- The design of this circuit is usually based on selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C \rightarrow L = CR^2$
- Convenient values are selected for $C$ and $R$ to yield the desired inductance value $L$. 

\[ Z_{in} = \frac{V_1}{I_1} = \frac{sC_4R_1R_3R_5R_2}{1} \]
The Op Amp-RC Resonator

- Replacing the inductor $L$ with a simulated inductance realized by the Antoniou circuit → Second-order resonator.

- Pole frequency

$$\omega_o = \frac{1}{\sqrt{LC_6}} = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}}$$
The Op Amp-RC Resonator

- Pole Q factor

\[ Q = \omega_o C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4 R_1 R_3 R_5}} \frac{R_2}{R} \]

- Usually selects \( C_4 = C_6 = C \)
  and \( R_1 = R_2 = R_3 = R_5 = R \),
which results in

\[ \omega_o = \frac{1}{CR} \]

\[ Q = \frac{R_6}{R} \]

- Select a practically convenient value for \( C \)
  → Determine the value of \( R \) to realize a given \( \omega_o \)
  → Determine the value of \( R_6 \) to realize a given \( Q \)
- Op-amp buffer amplifier is used at the output to eliminate loading effect.
Realization of the Various Filter Types

- Bandpass function.
  → disconnect node \( z \) from ground
  and connect it to the signal source \( V_i \)
Realization of the Various Filter Types

- High-pass function
  \( \rightarrow \) inject \( V_i \) to node \( y \)
Realization of the Various Filter Types

- Low-pass function
  - inject $V_i$ to node $x$
Realization of the Various Filter Types

- Regular Notch function \((\omega_n = \omega_o)\).
  - Feed \(V_i\) to node \(x\) and \(y\)
The All – Pass Circuit

- An all-pass function with a flat gain of unity
  \[ \text{AP} = 1 - \left( \text{BP with a center-frequency gain of 2} \right) \] 
  \[ T = \frac{V_o}{V_{\text{in}}} = 1 - T \] 
  \[ \Rightarrow \text{complementary} \]

- All-pass circuit with unity flat gain is the complement of the bandpass circuit with a center-frequency gain of 2.

- A simple procedure for obtaining the complement of a given linear circuit.
  - Interchanging input and ground in a linear circuit generates a circuit whose transfer function is the complement of that of the original circuit
The All – Pass Circuit

- All pass filter implementation.
  ① Use the circuit of Fig. 12.22(c) to realize a BP with a gain of 2 by simply selecting \( K = 2 \) and implementing the buffer amplifier with the circuit of Fig. 12.21(c) with \( r_1 = r_2 \).
  ② Then interchange input and ground and thus obtain the all-pass circuit of Fig. 12.22(g)
Derivation of the Two-Integrator-Loop Biquad

- To derive the two-integrator-loop biquadratic circuit, start from the second-order high-pass transfer function

\[
\frac{V_{hp}}{V_i} = \frac{K s^2}{s^2 + s(\omega_o/Q) + \omega_o^2}
\]

\[\downarrow\text{HP}\]

\[K\text{ is the high-frequency gain}\]

\[
V_{hp} + \frac{1}{Q} \left( \frac{\omega_o}{s} V_{hp} \right) + \left( \frac{\omega_o^2}{s^2} V_{hp} \right) = KV_i
\]

- To express \(V_{hp}\),

\[
V_{hp} = KV_i - \frac{1}{Q} \frac{\omega_o}{s} V_{hp} - \frac{\omega_o^2}{s^2} V_{hp}
\]
Derivation of the Two-Integrator-Loop Biquad

- A complete block diagram realization

- From the output of the summer, obtained high-pass transfer function
  \[ T_{hp} = \frac{V_{hp}}{V_i} = \frac{C s^2}{s^2 + s \omega_o/Q + \omega_o^2} \]

- From the output of the first integrator, obtained bandpass function
  \[ T_{bp}(s) = \frac{(-\omega_o/s)V_{hp}}{V_i} = \frac{K \omega_o s}{s^2 + s(\omega_o/Q) + \omega_o^2} \]

- From the output of the second integrator, obtained lowpass function
  \[ T_{lp}(s) = \frac{(\omega_o^2/s^2)V_{hp}}{V_i} = \frac{K \omega_o^2}{s^2 + s(\omega_o/Q) + \omega_o^2} \]
Circuit Implementation using OP-AMP (Miller Integrator)

\[ v_\alpha(t) = \frac{1}{sCR} \int_{t_0}^{t} v_f(t) \, dt \]

\[ \frac{V_o}{V_i} = \frac{1}{sCR} \]

-6 dB/octave

\( \omega \) (log scale)
Circuit Implementation using OP-AMP (Basic)

\[ V_o = v_i \left(1 + \frac{R_2}{R_1}\right) \]

\[ V_o = 0 \]

\[ V_o = \frac{v_i}{R_1} R_2 \]

\[ V_o = -v_i \frac{R_2}{R_1} \]

Virtual ground (virtual short circuit)
The Kerwin-Huelsman-Newcomb circuit (KHN biquad)
- integrator → Miller integrator circuit having $CR=1/\omega_0$
- summer → op-amp summing circuit

Design the circuit

1. Select suitable $C$ and $R$ for $CR=1/\omega_0$.
2. Determine the values of the resistors associated with the summer

$$V_{hp} = \frac{R_3}{R_2 + R_3} \left(1 + \frac{R_f}{R_1}\right) V_i + \frac{R_2}{R_2 + R_3} \left(1 + \frac{R_f}{R_1}\right) \left(-\frac{\omega_o}{s} V_{hp}\right) - \frac{R_f}{R_1} \left(\frac{\omega_o^2}{s^2} V_{hp}\right)$$
Circuit Implementation - Coefficients

\[ V_{hp} = \frac{R_3}{R_2 + R_3} \left( 1 + \frac{R_f}{R_1} \right) V_i + \frac{R_2}{R_2 + R_3} \left( 1 + \frac{R_f}{R_1} \right) \left( -\frac{\omega_o}{s} V_{hp} \right) - \frac{R_f}{R_1} \left( \frac{\omega_o^2}{s^2} V_{hp} \right) \]

\[
\frac{R_f}{R_1} = 1
\]

\[
\frac{R_3}{R_2} = 2Q - 1 \quad (\because R_f = R_1)
\]

\[
K = 2 - \frac{1}{Q} \quad (\because R_f = R_1, \frac{R_3}{R_2} = 2Q - 1)
\]

\[ V_{hp} = KV_i - \frac{1}{Q} \frac{\omega_o}{s} V_{hp} - \frac{\omega_o^2}{s^2} V_{hp} \]

- Different zeros can be obtained by the appropriate selection of the values of the summing resistors

\[ V_o = -\left( \frac{R_F}{R_H} V_{hp} + \frac{R_F}{R_B} V_{bp} + \frac{R_F}{R_L} V_{lp} \right) \]

\[ = -V_i \left( \frac{R_F}{R_H} T_{hp} + \frac{R_F}{R_B} T_{bp} + \frac{R_F}{R_L} T_{lp} \right) \]

\[ \frac{V_o}{V_i} = -K \frac{(R_F / R_H)s^2 - s(R_F / R_B)\omega_o + (R_F / R_L)\omega_o^2}{s^2 + s(\omega_o / Q) + \omega_o^2} \]

- Realizing notch and all-pass functions by summing \( V_{hp}, V_{bp}, \) and \( V_{lp}. \)
An Alternative Two-Integrator-Loop Biquad Circuit

\[ V_{hp} = KV_i - \frac{1}{Q} \omega_o V_{hp} - \frac{1}{S^2} \omega_o^2 V_{hp} \]

\[ -\frac{\omega_o}{S} V_{hp} = -\frac{\omega_o}{S} KV_i - \frac{\omega_o}{S} \frac{1}{Q} V_{hp} + \frac{\omega_o}{S} V_{lp} \]

\[ V_{bp} = -\frac{\omega_o}{S} KV_i - \frac{\omega_o}{S} \frac{1}{Q} V_{bp} - (\frac{\omega_o}{S} V_{lp}) = -\frac{\omega_o}{S} \left\{ KV_i + \frac{1}{Q} V_{bp} + (-V_{lp}) \right\} \]

**Tow-Thomas Biquad Circuit:** all in single-ended mode with the same sign \( \rightarrow \) no HP
An Alternative Two-Integrator-Loop Biquad Circuit

- Feedforward scheme is employed to realize the finite transmission zeros required for the notch and all-pass functions.
- The virtual grounds at the input of each of three amps permits the input signal to be fed to all the op functions.
- Transfer function is (Derive it)

\[
\frac{V_o}{V_i} = -\frac{s^2 \left( \frac{C_1}{C} \right) + s \frac{1}{C} \left( \frac{1}{R} \frac{r}{RR_3} \right) + \frac{1}{C^2 RR_2}}{s^2 + s \frac{1}{QCR} + \frac{1}{C^2 R^2}}
\]
Additional Slides
Realization of the Notch Functions

- The impedance of the LC circuit becomes infinite at $\omega_0 = 1 / \sqrt{LC}$ → zero transmission
- The resistor does not introduce zeros.
- To obtain arbitrary $\omega_n$
  \[ T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0 / Q) + \omega_0^2} \]
- $L_1C_1$ tank will introduce a pair of zeros at $\pm j\omega_n$, provided the $L_2C_2$ tank is not resonant at $\omega_n$.
- The natural modes have not been altered,
  \[ C_1 + C_2 = C \quad ; \quad L_1 \parallel L_2 = L \]
- It is obtained from the original LCR resonator by lifting part of L and part of C off ground.
Switched-Capacitor Filters

The Basic Principle

- A capacitor switched between two circuit nodes is equivalent to a resistor

- The two MOS switches are driven by a non-overlapping two-phase clock

- During $\phi_1$, $C_1$ charges up to $v_i$
  
  $q_{C1} = C_1v_i$

  During $\phi_2$, $C_1$ is connected to the input of the op amp
During each $T_c$, $q_{C1} = C_1 \nu_i$ is extracted from the input source and supplied to $C_2$

\[ i_{av} = \frac{C_1 \nu_i}{T_c} \]

\[ R_{eq} \equiv \frac{\nu_i}{i_{av}} = \frac{T_c}{C_1} \]

Time constant = $C_2 R_{eq} = T_c \frac{C_2}{C_1}$
Tuned Amplifiers

- $\omega_o =$ center frequency
- $B =$ 3-dB bandwidth
- Skirt selectivity =$S/B$
- In many applications, $B < 5\%$ of $\omega_o \rightarrow$ narrow-band
  $\rightarrow$ certain approximations
Tuned Amplifiers

The Basic Principle

- The use of a parallel LCR circuit as the load or at the input
- Single-tuned amplifier
- \( R = R_L \parallel r_o \)
- \( C = C_L + \text{FET output capacitance} \) (usually very small)

\[
V_o = -\frac{g_m V_i}{Y_L} = -\frac{g_m V_i}{sC + 1/R + 1/sL}
\]

\[
\frac{V_o}{V_i} = -\frac{g_m}{C s^2 + s(1/CR) + 1/LC}
\]

\[
\omega_o = 1/\sqrt{LC}, \quad B = 1/CR
\]

\[
Q = \frac{\omega_o}{B} = \omega_o CR
\]

\[
\frac{V_o(j\omega_o)}{V_i(j\omega_o)} = -g_m R
\]

- At resonance the reactance of \( L & C \) cancel out and the impedance of the parallel LCR circuit reduces to \( R \)
Tuned Amplifiers

- Inductor Losses
  
  \[ Q_o \equiv \frac{\omega_o L}{r_s} : 50 \sim 200 \]

  - The analysis of a tuned amplifier is greatly simplified by representing the inductor loss by a parallel resistance \( R_p \)
  
  - \( Y(j\omega_o) = \frac{1}{r_s + j\omega_o L} \)
    
    \[ = \frac{1}{j\omega_o L} \frac{1}{1 - j(1/Q_o)} = \frac{1}{j\omega_o L} \frac{1 + j(1/Q_o)}{1 + (1/Q_o^2)} \]

  \[ Q_o \gg 1 \]
  
  \[ Y(j\omega_o) \cong (1/j\omega_o L)(1 + j(1/Q_o)) \]

  \[ Q_o = \frac{R_p}{\omega_o L} \]

  \[ R_p = \omega_o LQ_o = r_s Q_o^2 \]

  - The coil Q factor poses an upper limit on the value of Q achieved by the tuned circuit
Tuned Amplifiers

- Use of Transformers

\[ n = \frac{n_2}{n_1} \]
Amplifiers with Multiple Tuned Circuits

- To avoid the loading effect of $R_{B1}$ and $R_{B2}$ on the input tuned circuit, a radio-frequency choke (RFC) is inserted.

- The analysis and design is complicated by the Miller effect due to $C_\mu$. The reflected impedance will cause detuning response of the input circuit.

- Method 1: Neutralizing by using additional circuits arranged to feed back a current equal and opposite to that through $C_\mu$.

- Method 2: Using circuits that do not suffer from Miller effect.
The Cascode and the CC-CB Cascade

- No Miller effect: cascode and the common-collector, common-base cascade
Tuned Amplifiers

- Synchronous Tuning
  - A tuned amplifier with multiple tuned circuits
  - Assuming the overall response is the product of the individual responses
  - $N$ identical resonant circuits: synchronously tuned case

\[ B = \frac{W_0}{Q} \sqrt{2^{1/N} - 1} \]

Bandwidth-shrinkage factor