[2008][04-2]

Computer aided ship design

Part 1. Curve & Surface

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Ch 3. 곡면(Surfaces)

3.1 Parametric Surfaces

3.2 Bezier Surfaces

3.3 B-spline surfaces

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3.1 Parametric Surfaces

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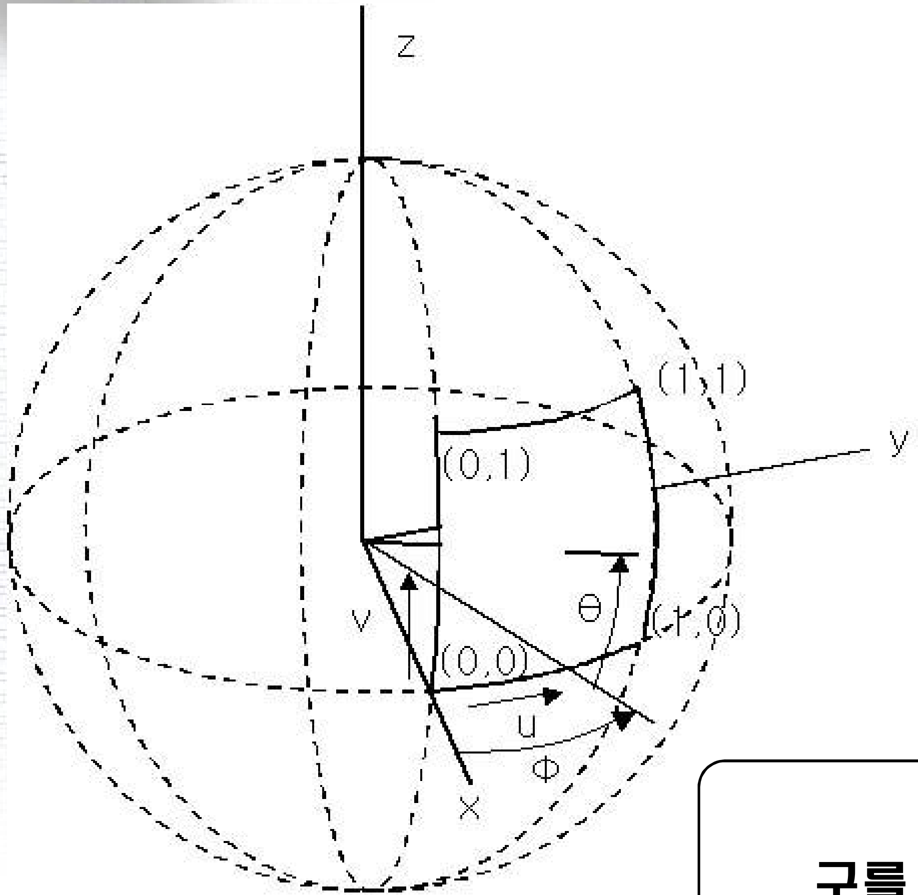
2.1 Parametric Surfaces



	곡면
양함수	$z = \pm\sqrt{d^2 - x^2 - y^2}$
음함수	$x^2 + y^2 + z^2 = d^2$
매개변수	$x = d\cos\phi\cos\theta$ $y = d\sin\phi\cos\theta$ $z = d\sin\theta$
	$\mathbf{r} = r(x(\phi,\theta), y(\phi,\theta), z(\phi,\theta))$

구를 2개의 매개변수 (ϕ, θ) 로 표현하면 간단히 수식화 할 수 있다.

3.2 Bezier surfaces

3.2.1 Generation of Bezier surfaces
by de Casteljau algorithm

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3.2.1 Generation of Bezier surfaces by de Casteljau algorithm

3.2.1.1 Bi-linear Bezier Surface Patch

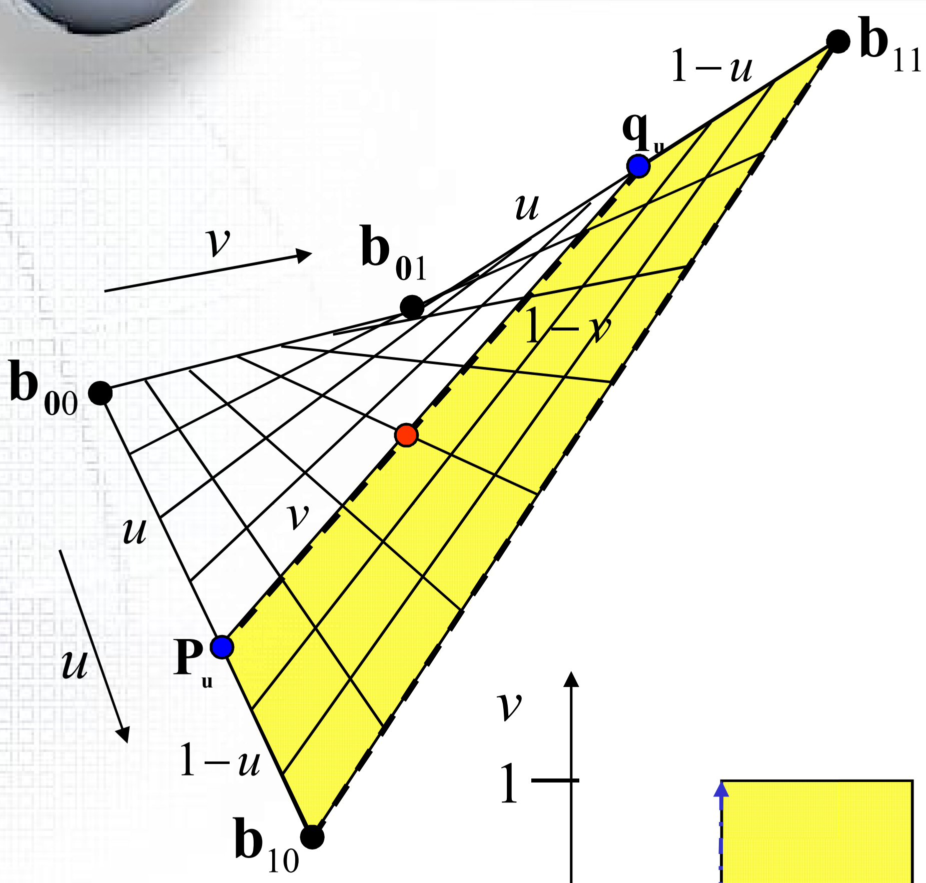
3.2.1.2 Bi-quadratic Bezier Surface Patch

3.2.1.3 Bi-Cubic Bezier Surface Patch

3.2.1.1 Bi-linear Bezier Surface Patch

- Given: 2x2 Bezier control point
- Find: Points on bi-linear Bezier Surface Patch

방법: u, v **방향으로** 'de Casteljau algorithm' **사용**



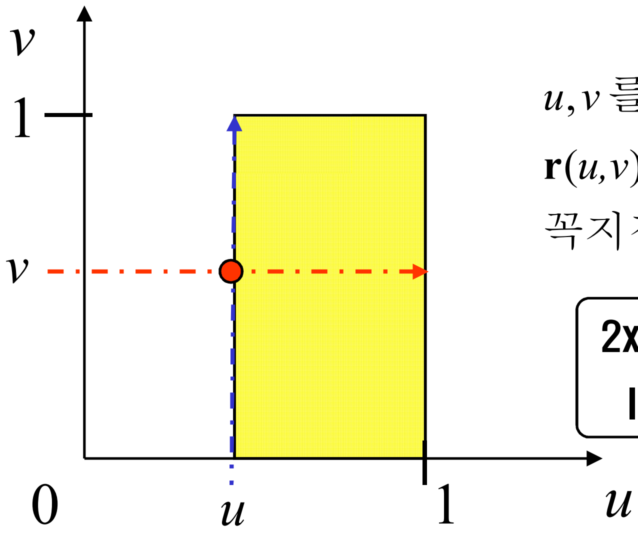
$$\mathbf{P}_u = (1-u)\mathbf{b}_{00} + u\,\mathbf{b}_{10}$$

$$\mathbf{q}_u = (1-u)\mathbf{b}_{01} + u\,\mathbf{b}_{11}$$

$$\mathbf{r}(u,v) = (1-v)\,\mathbf{P}_u + v\,\mathbf{q}_u$$

$$\mathbf{r}(u,v) = (1-v)(1-u)\mathbf{b}_{00} + (1-v)\,u\,\mathbf{b}_{10} + v\,(1-u)\mathbf{b}_{01} + v\,u\,\mathbf{b}_{11}$$

$$r(u,v) = \begin{bmatrix}(1-v) & v\end{bmatrix}\begin{bmatrix}\mathbf{b}_{00} & \mathbf{b}_{10}\\ \mathbf{b}_{01} & \mathbf{b}_{11}\end{bmatrix}\begin{bmatrix}(1-u)\\ u\end{bmatrix}$$

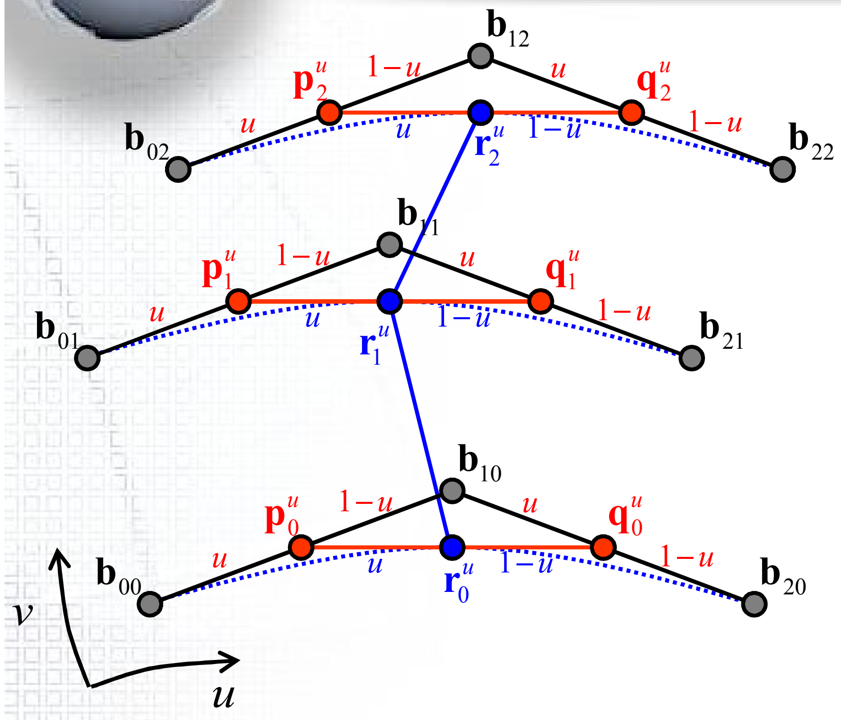


u, v를 0에서 1까지 증가하며 $\mathbf{r}(u,v)$를 계산하면 점 $\mathbf{b}_{00}, \mathbf{b}_{10}, \mathbf{b}_{01}, \mathbf{b}_{11}$ 을 꼭지점으로 하는 곡면을 얻을 수 있다.

2x2개의 Bezier 조정점을 이용하여 Bi-linear Interpolation 으로 곡면식을 구할 수 있다.

3.2.1.2 Bi-quadratic Bezier Surface Patch

- Given: 3x3 Bezier control point
- Find: Points on bi-quadratic Bezier Surface Patch   방법: u, v 방향으로 'de Casteljau algorithm' 사용



$$\mathbf{p}_0^u = (1-u)\mathbf{b}_{00} + u\mathbf{b}_{10}$$
$$\mathbf{q}_0^u = (1-u)\mathbf{b}_{10} + u\mathbf{b}_{20}$$
$$\mathbf{p}_1^u = (1-u)\mathbf{b}_{01} + u\mathbf{b}_{11}$$
$$\mathbf{q}_1^u = (1-u)\mathbf{b}_{11} + u\mathbf{b}_{21}$$
$$\mathbf{p}_2^u = (1-u)\mathbf{b}_{02} + u\mathbf{b}_{12}$$
$$\mathbf{q}_2^u = (1-u)\mathbf{b}_{12} + u\mathbf{b}_{22}$$
$$\mathbf{r}_0^u = (1-u)\mathbf{p}_0^u + u\mathbf{q}_0^u$$
$$\mathbf{r}_1^u = (1-u)\mathbf{p}_1^u + u\mathbf{q}_1^u$$
$$\mathbf{r}_2^u = (1-u)\mathbf{p}_2^u + u\mathbf{q}_2^u$$

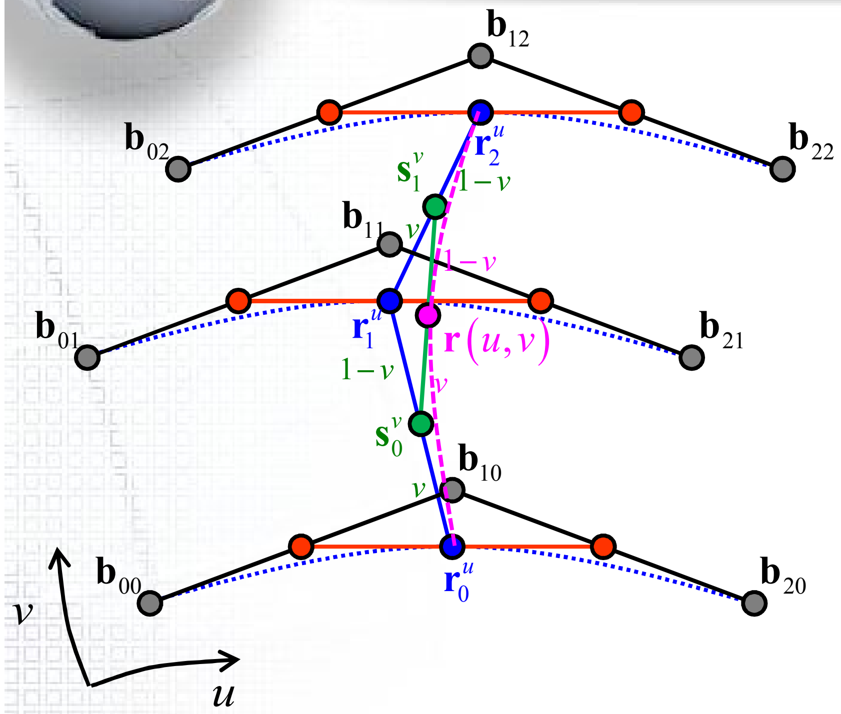
$$\begin{bmatrix} \mathbf{r}_0^u \\ \mathbf{r}_1^u \\ \mathbf{r}_2^u \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{10} & \mathbf{b}_{20} \\ \mathbf{b}_{01} & \mathbf{b}_{11} & \mathbf{b}_{21} \\ \mathbf{b}_{02} & \mathbf{b}_{12} & \mathbf{b}_{22} \end{bmatrix} \begin{bmatrix} (1-u)^2 \\ 2u(1-u) \\ u^2 \end{bmatrix}$$

$$\mathbf{r}_0^u = (1-u)^2\mathbf{b}_{00} + 2u(1-u)\mathbf{b}_{10} + u^2\mathbf{b}_{20}$$
$$\mathbf{r}_1^u = (1-u)^2\mathbf{b}_{01} + 2u(1-u)\mathbf{b}_{11} + u^2\mathbf{b}_{21}$$
$$\mathbf{r}_2^u = (1-u)^2\mathbf{b}_{02} + 2u(1-u)\mathbf{b}_{12} + u^2\mathbf{b}_{22}$$

3.2.1.2 Bi-quadratic Bezier Surface Patch

- Given: 3x3 Bezier control point
- Find: Points on bi-quadratic Bezier Surface Patch

방법: u, v 방향으로 'de Casteljau algorithm' 사용



$$\mathbf{s}_0^v = (1-v)\mathbf{r}_0^u + v\mathbf{r}_1^u$$

$$\mathbf{s}_1^v = (1-v)\mathbf{r}_1^u + v\mathbf{r}_2^u$$

$$\mathbf{r}(u,v) = (1-v)\mathbf{s}_0^v + v\mathbf{s}_1^v$$

$$\mathbf{r}(u,v) = (1-v)^2\mathbf{r}_0^u + 2v(1-v)\mathbf{r}_1^u + v^2\mathbf{r}_2^u$$

$$\mathbf{r}(u,v) = \begin{bmatrix}(1-v)^2 & 2v(1-v) & v^2\end{bmatrix}\begin{bmatrix}\mathbf{r}_0^u \\ \mathbf{r}_1^u \\ \mathbf{r}_2^u\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{r}_0^u \\ \mathbf{r}_1^u \\ \mathbf{r}_2^u\end{bmatrix} = \begin{bmatrix}\mathbf{b}_{00} & \mathbf{b}_{10} & \mathbf{b}_{20} \\ \mathbf{b}_{01} & \mathbf{b}_{11} & \mathbf{b}_{21} \\ \mathbf{b}_{02} & \mathbf{b}_{12} & \mathbf{b}_{22}\end{bmatrix}\begin{bmatrix}(1-u)^2 \\ 2u(1-u) \\ u^2\end{bmatrix}$$

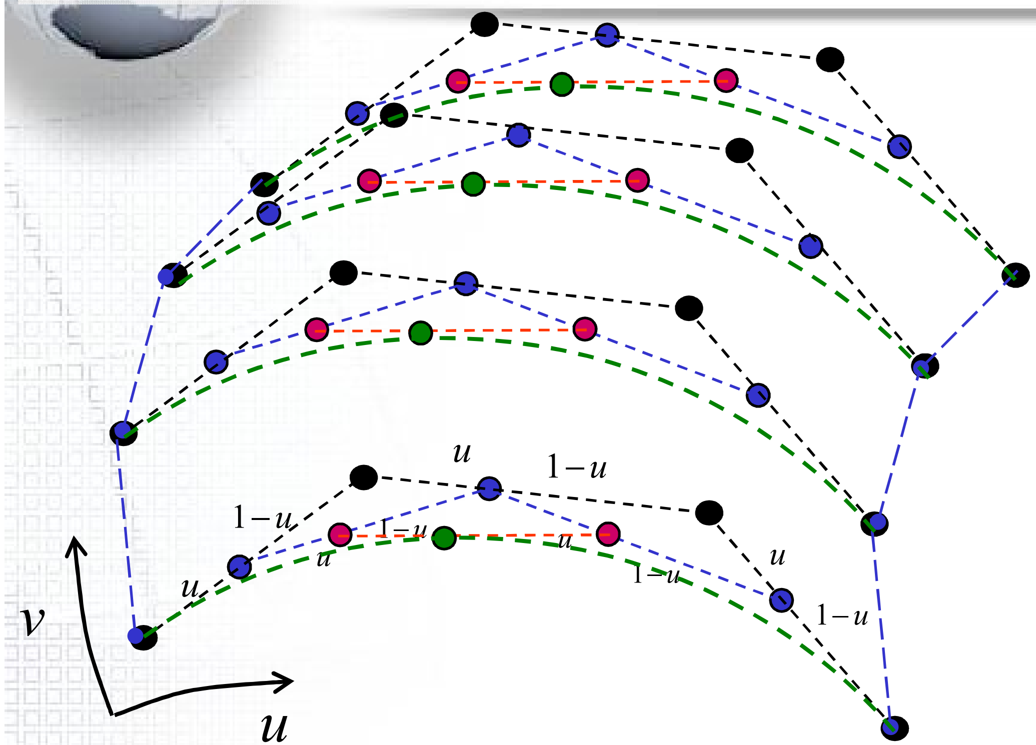
$$\mathbf{r}(u,v) = \begin{bmatrix}(1-v)^2 & 2v(1-v) & v^2\end{bmatrix}\begin{bmatrix}\mathbf{b}_{00} & \mathbf{b}_{10} & \mathbf{b}_{20} \\ \mathbf{b}_{01} & \mathbf{b}_{11} & \mathbf{b}_{21} \\ \mathbf{b}_{02} & \mathbf{b}_{12} & \mathbf{b}_{22}\end{bmatrix}\begin{bmatrix}(1-u)^2 \\ 2u(1-u) \\ u^2\end{bmatrix}$$

3x3개의 조정점을 이용하여
Bi-Quadratic Bezier Patch를 구할 수 있다.

3.2.1.3 Bi-cubic Bezier Surface Patch

- Given: 4x4 Bezier control point
- Find: Points on bi-cubic Bezier Surface Patch

방법: u, v 방향으로 'de Casteljau algorithm' 사용



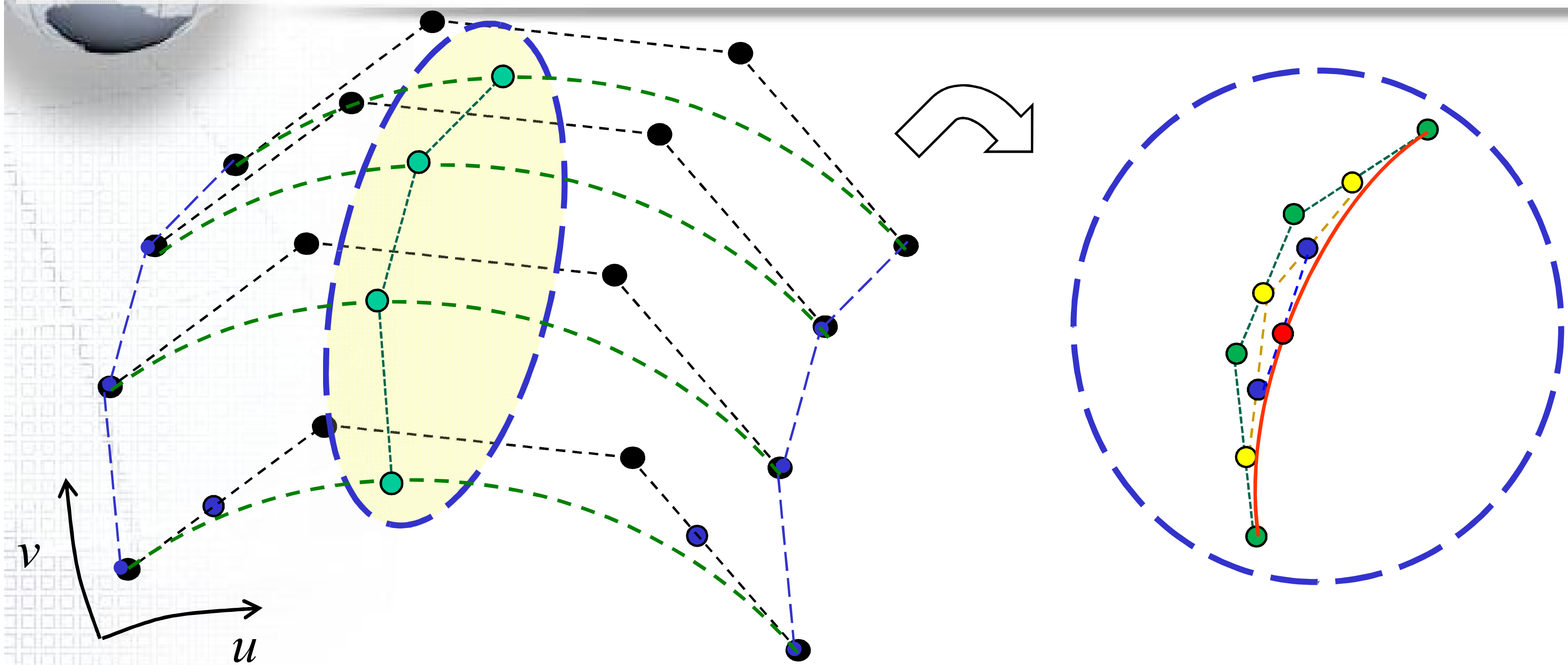
$$\mathbf{b}(u,v) = \begin{bmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_{0,0} & \mathbf{b}_{0,1} & \mathbf{b}_{0,2} & \mathbf{b}_{0,3} \\ \mathbf{b}_{1,0} & \mathbf{b}_{1,1} & \mathbf{b}_{1,2} & \mathbf{b}_{1,3} \\ \mathbf{b}_{2,0} & \mathbf{b}_{2,1} & \mathbf{b}_{2,2} & \mathbf{b}_{2,3} \\ \mathbf{b}_{3,0} & \mathbf{b}_{3,1} & \mathbf{b}_{3,2} & \mathbf{b}_{3,3} \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

4x4개의 조정점을 이용하여 Bi-Cubic Bezier Patch를 구할 수 있다.

3.2.1.3 Bi-cubic Bezier Surface Patch

- Given: 4x4 Bezier control point
- Find: Points on bi-cubic Bezier Surface Patch

방법: u, v 방향으로 'de Casteljau algorithm' 사용



$$\mathbf{b}(u,v)=\begin{bmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_{0,0} & \mathbf{b}_{0,1} & \mathbf{b}_{0,2} & \mathbf{b}_{0,3} \\ \mathbf{b}_{1,0} & \mathbf{b}_{1,1} & \mathbf{b}_{1,2} & \mathbf{b}_{1,3} \\ \mathbf{b}_{2,0} & \mathbf{b}_{2,1} & \mathbf{b}_{2,2} & \mathbf{b}_{2,3} \\ \mathbf{b}_{3,0} & \mathbf{b}_{3,1} & \mathbf{b}_{3,2} & \mathbf{b}_{3,3} \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

4x4개의 조정점을 이용하여 Bi-Cubic Bezier Patch를 구할 수 있다.

3.3 B-spline surfaces

3.3.1 Generation of B-spline surfaces by tensor-product approach

3.3.2 B-spline surface Interpolation

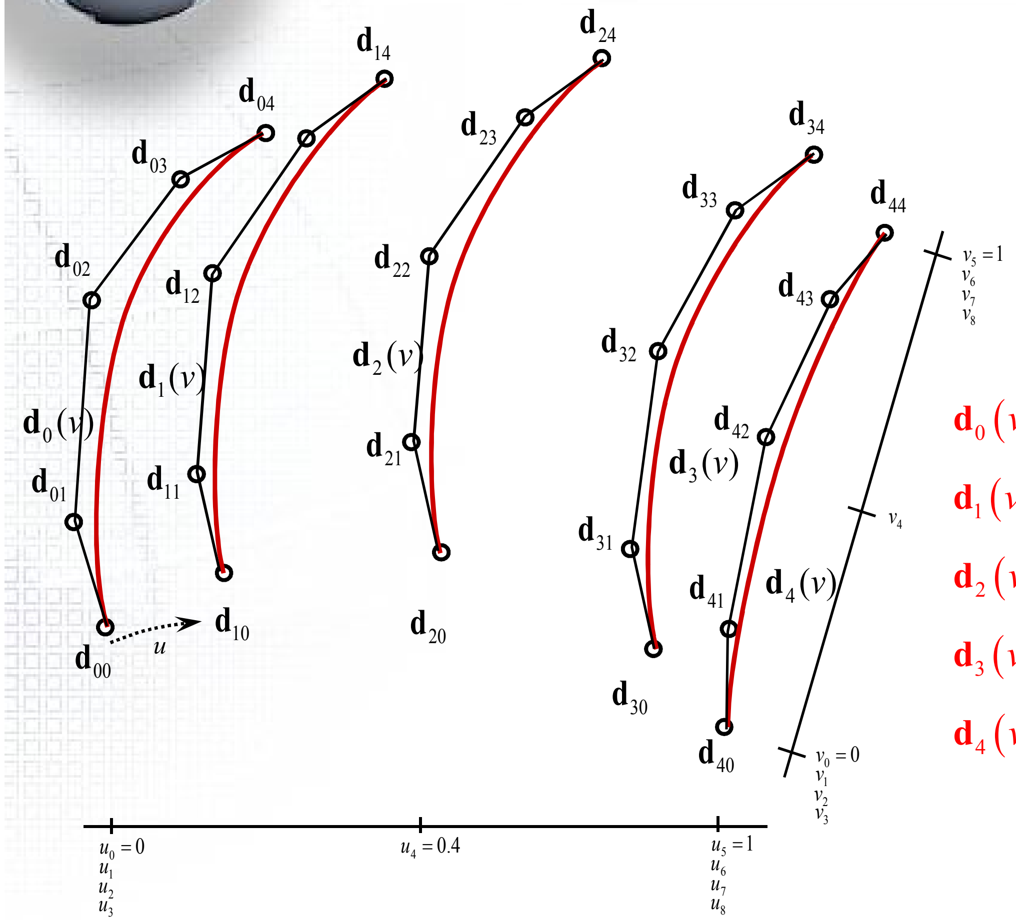
3.3.1 Generation of B-spline surfaces by tensor-product approach

3.3.1.1 Tensor-product bicubic B-spline surface

3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface

3.3.1.1 Tensor-product bicubic B-spline surface (1)

- Given: Control Points of bicubic B-spline surface
- Find: Points on bicubic B-spline surface



- Given 5x5 Control Points $\mathbf{d}_{ij}$, u-knots, v-knots, u-degree(=3), v-degree(=3),
- Generate start/end moving curves and directional curves in cubic B-spline form:

$$\mathbf{d}_0(v) = \mathbf{d}_{00}N_0^3(v) + \mathbf{d}_{01}N_1^3(v) + \mathbf{d}_{02}N_2^3(v) + \mathbf{d}_{03}N_3^3(v) + \mathbf{d}_{04}N_4^3(v)$$

$$\mathbf{d}_1(v) = \mathbf{d}_{10}N_0^3(v) + \mathbf{d}_{11}N_1^3(v) + \mathbf{d}_{12}N_2^3(v) + \mathbf{d}_{13}N_3^3(v) + \mathbf{d}_{14}N_4^3(v)$$

$$\mathbf{d}_2(v) = \mathbf{d}_{20}N_0^3(v) + \mathbf{d}_{21}N_1^3(v) + \mathbf{d}_{22}N_2^3(v) + \mathbf{d}_{23}N_3^3(v) + \mathbf{d}_{24}N_4^3(v)$$

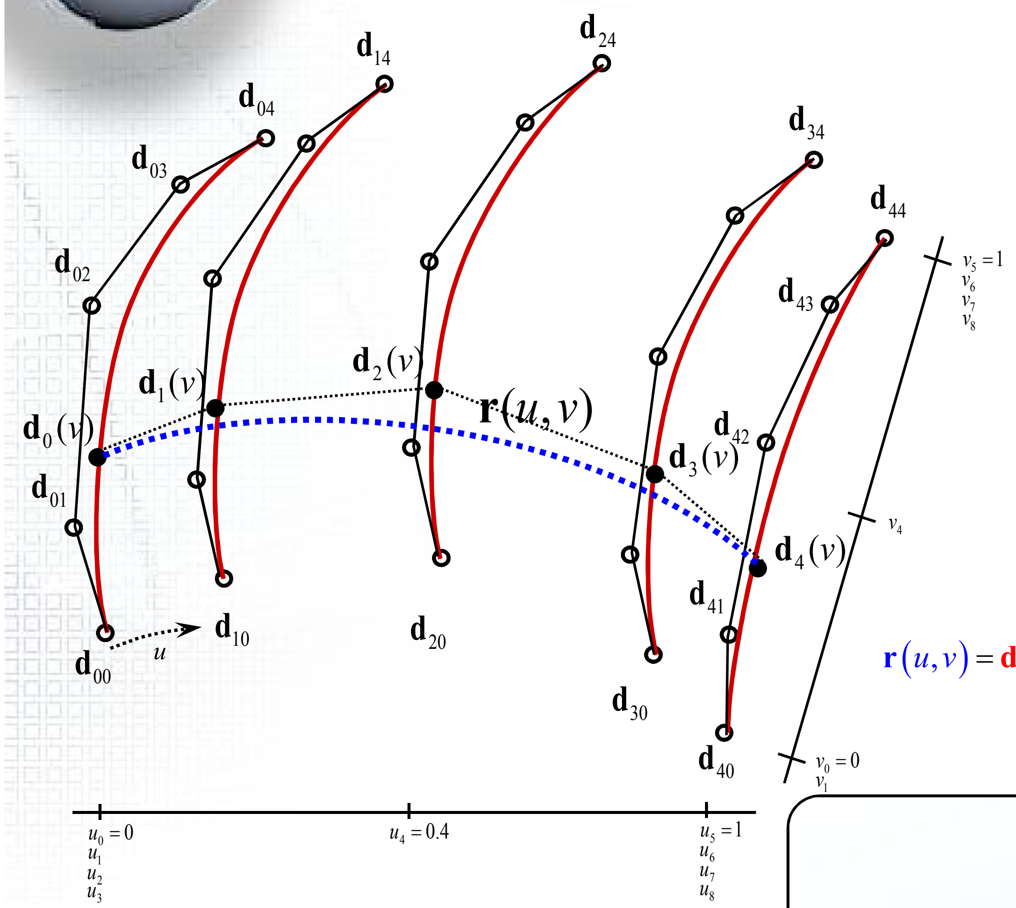
$$\mathbf{d}_3(v) = \mathbf{d}_{30}N_0^3(v) + \mathbf{d}_{31}N_1^3(v) + \mathbf{d}_{32}N_2^3(v) + \mathbf{d}_{33}N_3^3(v) + \mathbf{d}_{34}N_4^3(v)$$

$$\mathbf{d}_4(v) = \mathbf{d}_{40}N_0^3(v) + \mathbf{d}_{41}N_1^3(v) + \mathbf{d}_{42}N_2^3(v) + \mathbf{d}_{43}N_3^3(v) + \mathbf{d}_{44}N_4^3(v)$$

$$\begin{bmatrix} \mathbf{d}_0(v) \\ \mathbf{d}_1(v) \\ \mathbf{d}_2(v) \\ \mathbf{d}_3(v) \\ \mathbf{d}_4(v) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

3.3.1.1 Tensor-product bicubic B-spline surface (1)

- Given: Control Points of bicubic B-spline surface
- Find: Points on bicubic B-spline surface



- Given 5x5 Control Points $\mathbf{d}_{ij}$, u-knots, v-knots, u-degree(=3), v-degree(=3),
- Moving curve can be represented in the following form:

$$= \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_0(v) \\ \mathbf{d}_1(v) \\ \mathbf{d}_2(v) \\ \mathbf{d}_3(v) \\ \mathbf{d}_4(v) \end{bmatrix}$$

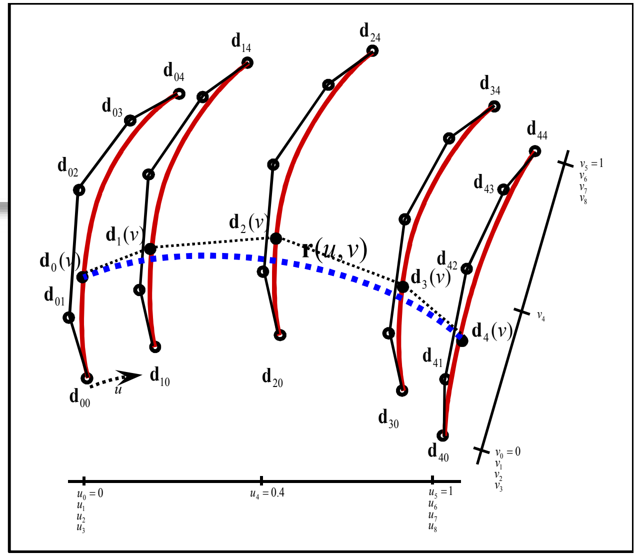
$$\mathbf{r}(u,v) = \mathbf{d}_0(v)N_0^3(u) + \mathbf{d}_1(v)N_1^3(u) + \mathbf{d}_2(v)N_2^3(u) + \mathbf{d}_3(v)N_3^3(u) + \mathbf{d}_4(v)N_4^3(u)$$

$$\begin{bmatrix} \mathbf{d}_0(v) \\ \mathbf{d}_1(v) \\ \mathbf{d}_2(v) \\ \mathbf{d}_3(v) \\ \mathbf{d}_4(v) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

$$= \sum_{j=0}^{5} \sum_{i=0}^{5} \mathbf{d}_{ij} N_i^3(u) N_j^3(v)$$

3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface
- Member Variables of Class




```
class CBsplineSurface
{
public:
        // member variables
        int m_nDegree;

        double* m_pKnot_U;
        int m_nNumOfKnot_U;

        double* m_pKnot_V;
        int m_nNumOfKnot_V;

        Vector** m_pCP;
        int m_nNumOfCP_U;
        int m_nNumOfCP_V;

        // member functions
        …
};
```

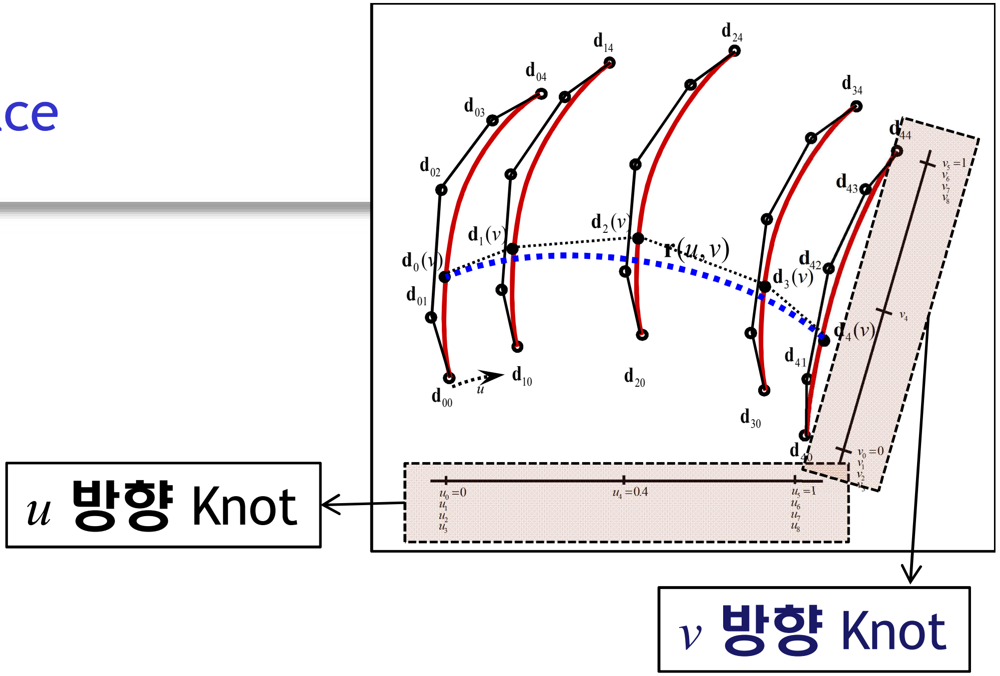
➔ 차수 (3차)

➔ u 방향 Knot와 개수 (9개)

➔ v 방향 Knot와 개수 (9개)

➔ Control Point, u, v 방향 개수 (u 방향 5개, v 방향 5개)

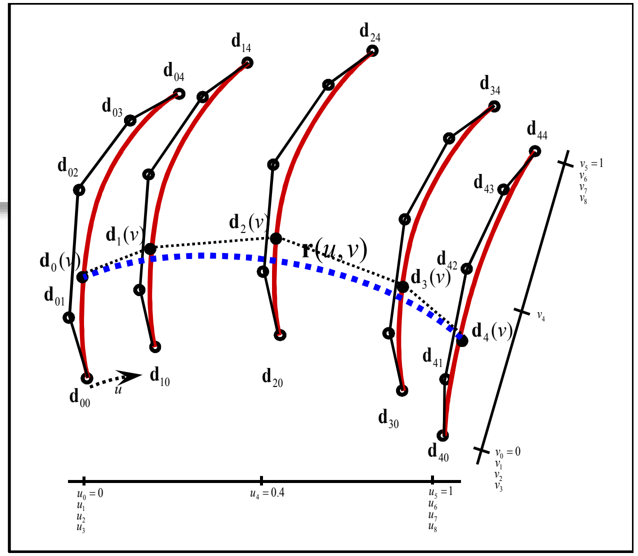
3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface
- Member Functions of Class




```
class CBsplineSurface
{
public:
        // member variables
        …

        // member functions
        …

        void SetKnot(double* pKnot_U, int nNumOfKnot_U, double* pKnot_V, int
nNumOfKnot_V);

        double N(int n, int i, double u, int uv);

        Vector GetPoint(double u, double v);
};
```

➔ Knot 설정

3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface
- Member Functions of Class




```
class CBsplineSurface
{
public:
        // member variables
        …

        // member functions
        …

        void SetKnot(double* pKnot_U, int nNumOfKnot_U, double* pKnot_V, int
nNumOfKnot_V);

        double N(int n, int i, double u, int uv);

        Vector GetPoint(double u, double v);
};
```

u, v 방향을 구분하는 flag

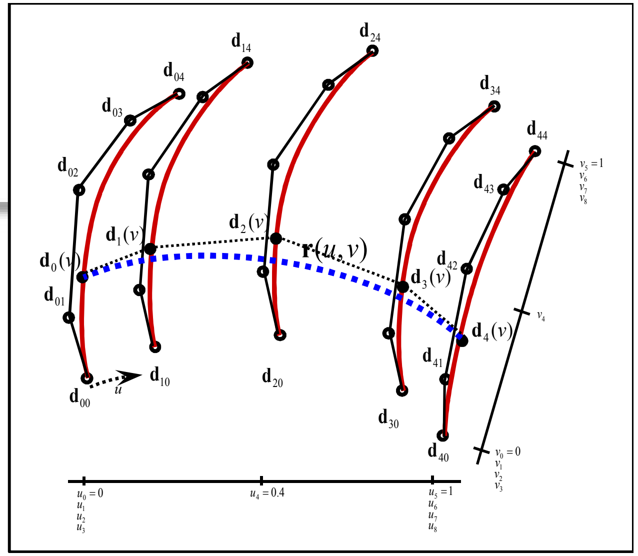
➔ B-spline Basis Function 계산

B-spline Basis Function 계산
(Cox-de Boor Recurrence Formula)

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1 & \text{if } u_{i-1} \le u < u_i \\ 0 & \text{else} \end{cases}$$

3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface - Member Functions of Class




```
class CBsplineSurface
{
public:
        // member variables
        …

        // member functions
        …

        void SetKnot(double* pKnot_U, int nNumOfKnot_U, double* pKnot_V, int
nNumOfKnot_V);

        double N(int n, int i, double u, int uv);

        Vector GetPoint(double u, double v);
};
```

➔ Parameter u, v에 대한 곡면 상의 점 계산

$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

$$= \sum_{j=0}^{5} \sum_{i=0}^{5} \mathbf{d}_{ij} N_i^3(u) N_j^3(v)$$

3.3.1.2 Programming Guide of Tensor-product bicubic B-spline surface
- Member Function Example 'GetPoint'

➔ Parameter u, v에 대한 곡면 상의 점 계산

```
Vector CBsplineSurface::GetPoint(double u, double v)
{
  // return value
  Vector r_u_v(0.0, 0.0, 0.0);

  // get curve
  for (int i=0; i<m_nNumOfCP_U; i++)
  {
    Vector r_v(0.0, 0.0, 0.0);

    for (int j=0; j<m_nNumOfCP_V; j++)
    {
      r_v = r_v + m_pCP[i][j] * N(m_nDegree, j, v, ID_V);
    }

    r_u_v = r_u_v + N(m_nDegree, i, u, ID_U) * r_v;
  }

  return r_u_v;
}
```

$$\mathrm{d}_0(v) = \mathrm{d}_{00}N_0^3(v) + \mathrm{d}_{01}N_1^3(v) + \mathrm{d}_{02}N_2^3(v) + \mathrm{d}_{03}N_3^3(v) + \mathrm{d}_{04}N_4^3(v)$$

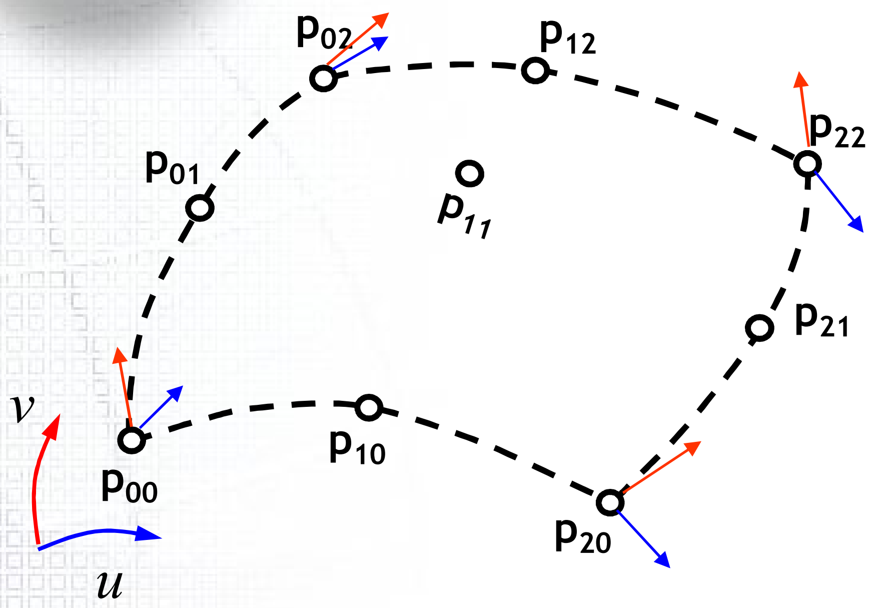
$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

$$= \sum_{j=0}^{5}\sum_{i=0}^{5} \mathbf{d}_{ij} N_i^3(u) N_j^3(v)$$

3.3.2 B-spline surfaces Interpolation

3.3.2.1 bicubic B-spline Surface Interpolation 개요 (1)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface

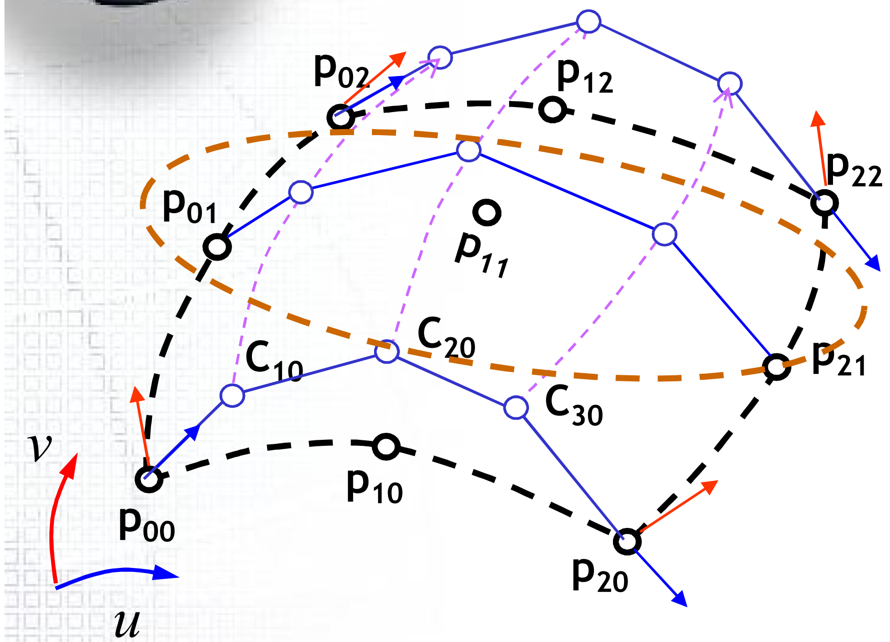


$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{0,0} & \mathbf{d}_{1,0} & \mathbf{d}_{2,0} & \mathbf{d}_{3,0} & \mathbf{d}_{4,0} \\ \mathbf{d}_{0,1} & \mathbf{d}_{1,1} & \mathbf{d}_{2,1} & \mathbf{d}_{3,1} & \mathbf{d}_{4,1} \\ \mathbf{d}_{0,2} & \mathbf{d}_{1,2} & \mathbf{d}_{2,2} & \mathbf{d}_{3,2} & \mathbf{d}_{4,2} \\ \mathbf{d}_{0,3} & \mathbf{d}_{1,3} & \mathbf{d}_{2,3} & \mathbf{d}_{3,3} & \mathbf{d}_{4,3} \\ \mathbf{d}_{0,4} & \mathbf{d}_{1,4} & \mathbf{d}_{2,4} & \mathbf{d}_{3,4} & \mathbf{d}_{4,4} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

5x5 개의 조정점을 구하면 곡면을 생성할 수 있다.

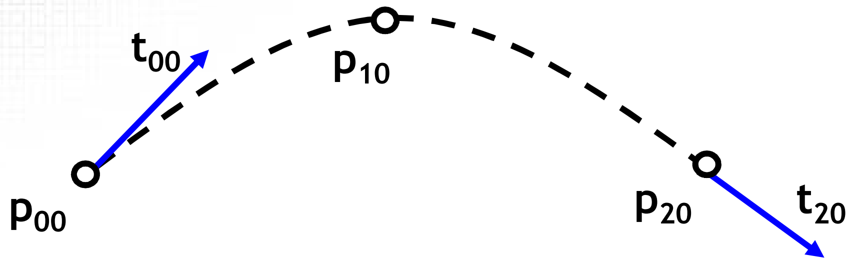
3.3.2.1 bicubic B-spline Surface Interpolation 개요 (2)

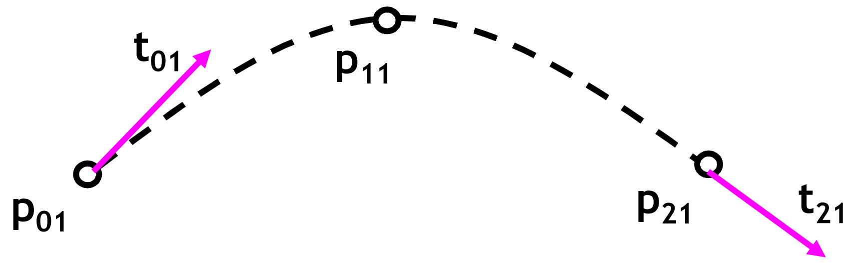
- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



곡선상의 점($P_{i,j}$)과 접선벡터 ($t_{i,j}$) 으로부터 중간 조정점($C_{i,j}$)을 구한다.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{3}{\Delta_s} & \frac{3}{\Delta_s} & 0 & 0 & 0 \\ 0 & \alpha & \beta & \gamma & 0 \\ 0 & 0 & 0 & -\frac{3}{\Delta_E} & \frac{3}{\Delta_E} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \\ C_{40} \end{bmatrix} = \begin{bmatrix} P_{00} \\ t_{00} \\ P_{10} \\ t_{20} \\ P_{20} \end{bmatrix}$$





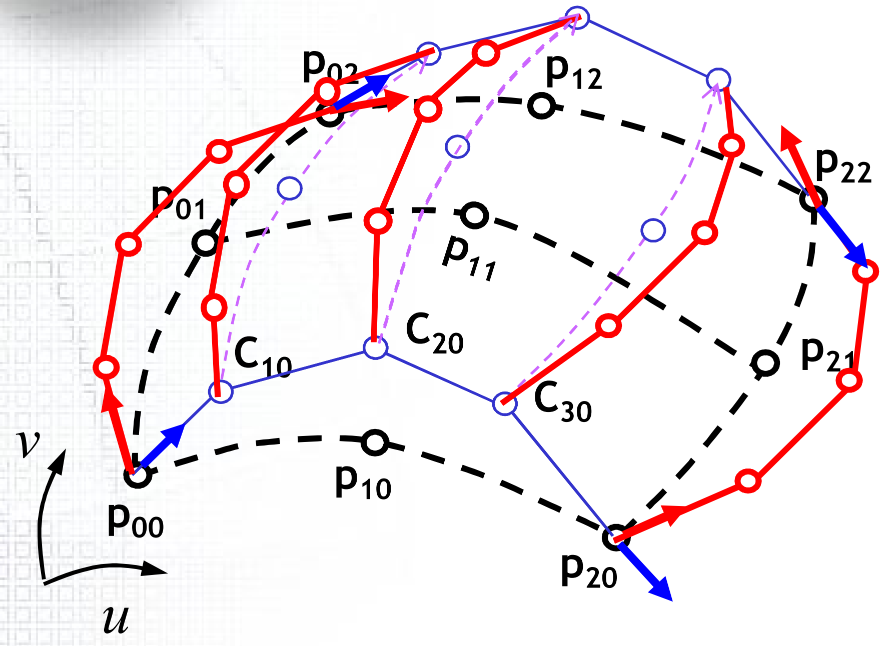
Bessel end condition:

곡선을 지나는 3점을 2차식으로 보간(interpolation)한 후, 곡선의 끝점에서의 1차미분값을 구하는 방법

Bessel end condition으로 접선벡터($t_{i,j}$)를 구한다

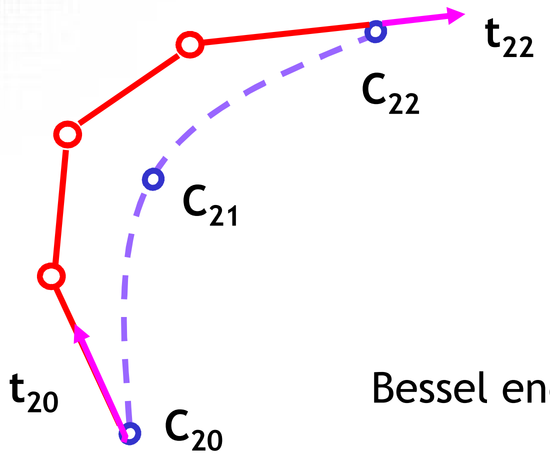
3.3.2.1 bicubic B-spline Surface Interpolation 개요 (3)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



중간 조정점($C_{i,j}$)과 접선 벡터 ($t_{i,j}$)
으로부터 B-spline조정점($d_{i,j}$)을 구한다.

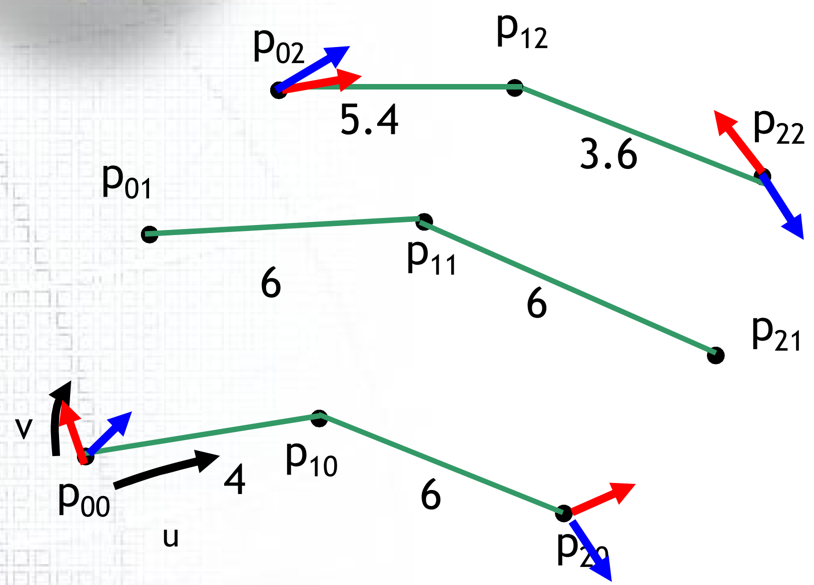
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{3}{\Delta_s} & \frac{3}{\Delta_s} & 0 & 0 & 0 \\ 0 & \alpha & \beta & \gamma & 0 \\ 0 & 0 & 0 & -\frac{3}{\Delta_E} & \frac{3}{\Delta_E} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{d}_{20} \\ \mathbf{d}_{21} \\ \mathbf{d}_{22} \\ \mathbf{d}_{23} \\ \mathbf{d}_{24} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{20} \\ \mathbf{t}_{20} \\ \mathbf{C}_{21} \\ \mathbf{t}_{22} \\ \mathbf{C}_{22} \end{bmatrix}$$



Bessel end condition으로 접선벡터($t_{i,j}$)를 구한다

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (1)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



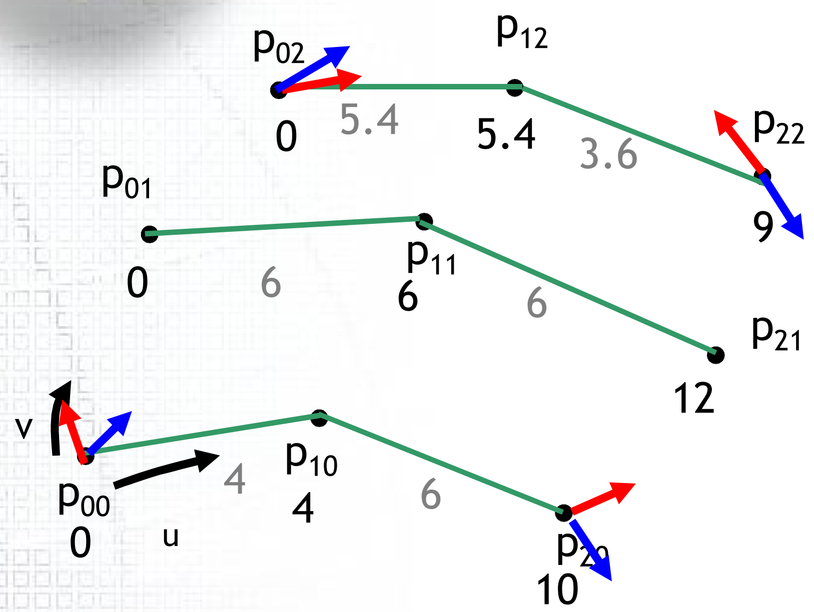
1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다

❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (2)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



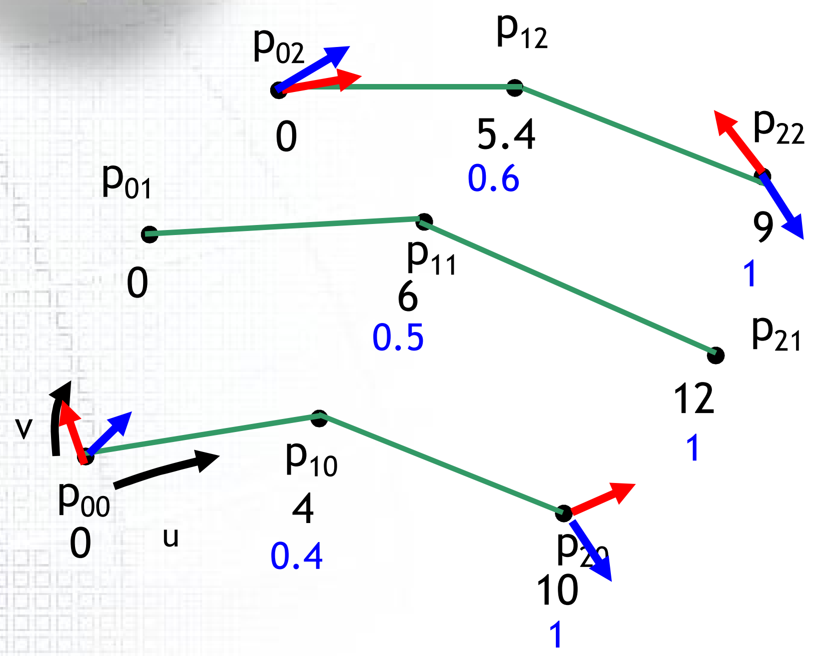
❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다
   - 계산한 거리를 각 점별로 누적한다. 이 거리를 곡면의 u방향 knot라고 부른다

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (3)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



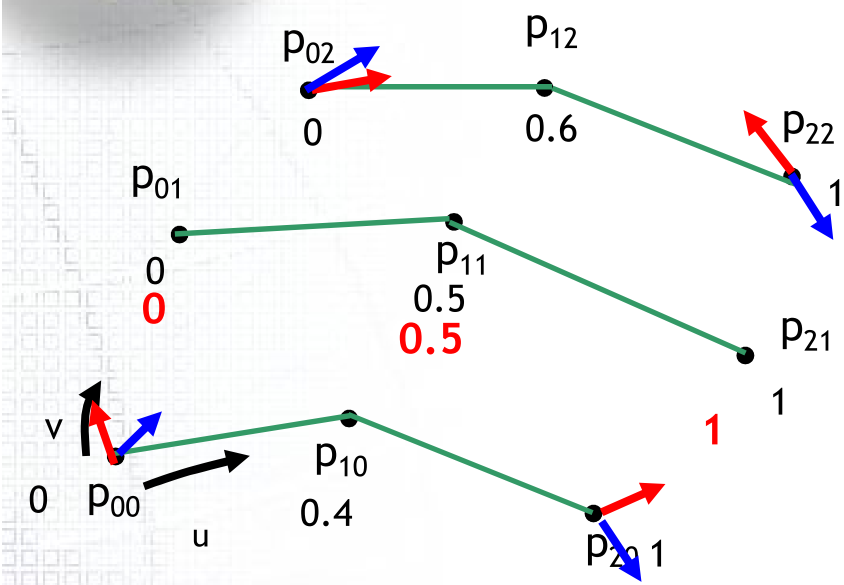
1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다
   - 계산한 거리를 각 점별로 누적한다. 이 거리를 곡면의 u방향 knot라고 부른다
   - 마지막 점의 knot값으로 각 점의 knot값을 나누어 정규화된 knot값을 계산한다

❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (4)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



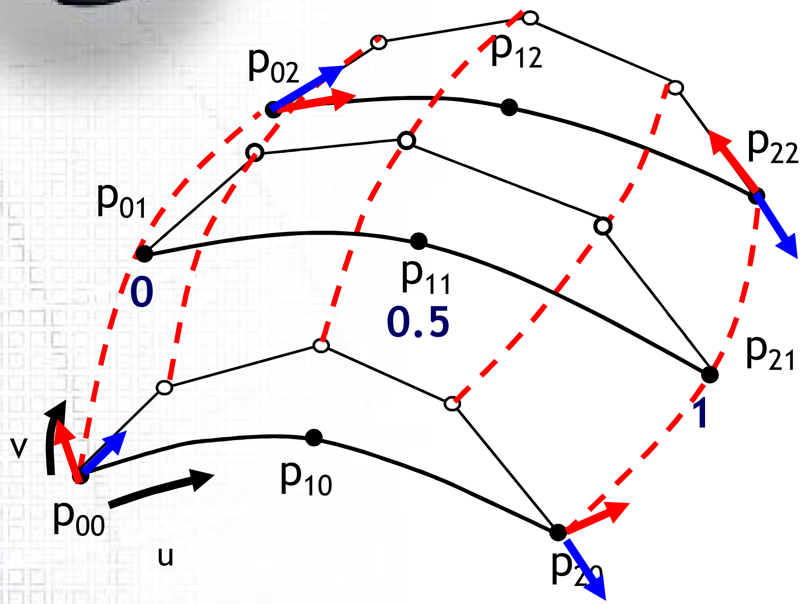
1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다
   - 계산한 거리를 각 점별로 누적한다. 이 거리를 곡면의 u방향 knot라고 부른다
   - 마지막 점의 knot값으로 각 점의 knot값을 나누어 정규화된 knot값을 계산한다
   - 정규화된 knot값들을 v방향으로 평균하여 최종적인 u방향 knot값을 계산한다

❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (5)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



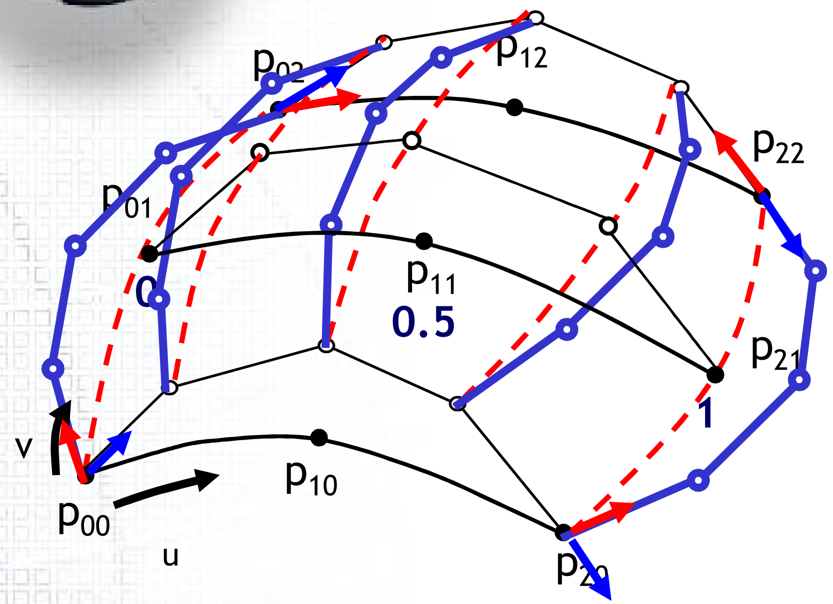
❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다
   - 계산한 거리를 각 점별로 누적한다. 이 거리를 곡면의 u방향 knot라고 부른다
   - 마지막 점의 knot값으로 각 점의 knot값을 나누어 정규화된 knot값을 계산한다
   - 정규화된 knot값들을 v방향으로 평균하여 최종적인 u방향 knot값을 계산한다
2. u 방향 점들을 보간하는 B-spline 곡선을 계산
   - 최종적인 u방향 knot와 점들을 지나는 B-spline 곡선과 그 조정점을 계산한다
3. v 방향 knot 간격을 u뱡향과 동일한 방법으로 구함
4. u 방향 B-spline 곡선의 조정점을 보간하는 v방향 B-spline 곡선을 계산
   - u방향 B-spline 곡선의 조정정에 대해서 1번과 같은 방법으로 v방향 knot를 결정한 후, 이를 이용하여 v방향 B-spline 곡선 (빨간점선) 을 생성한다. 이 곡선의 조정점 (파란점) 이 최종 B-spline 곡면의 조정점이다

3.3.2.2 bicubic B-spline Surface Interpolation 상세 과정 (6)

- Given: 곡면상의 9개 점과 4 꼭지점에서의 u, v방향의 접선벡터
- Find: Control Points of bicubic B-spline Surface



❑ Given

- 곡면이 지나야할 점들의 좌표
- 점들은 사각형 grid 형태여야 함 (예, 2x2)

1. u 방향 knot를 결정
   - 주어진 점들의 u방향 거리를 계산한다
   - 계산한 거리를 각 점별로 누적한다. 이 거리를 곡면의 u방향 knot라고 부른다
   - 마지막 점의 knot값으로 각 점의 knot값을 나누어 정규화된 knot값을 계산한다
   - 정규화된 knot값들을 v방향으로 평균하여 최종적인 u방향 knot값을 계산한다
2. u 방향 점들을 보간하는 B-spline 곡선을 계산
   - 최종적인 u방향 knot와 점들을 지나는 B-spline 곡선과 그 조정점을 계산한다
3. v 방향 knot 간격을 u방향과 동일한 방법으로 구함
4. u 방향 B-spline 곡선의 조정점을 보간하는 v방향 B-spline 곡선을 계산
   - u방향 B-spline 곡선의 조점정에 대해서 1번과 같은 방법으로 v방향 knot를 결정한 후, 이를 이용하여 v방향 B-spline 곡선 (빨간점선) 을 생성한다. 이 곡선의 조정점 (파란점) 이 최종 B-spline 곡면의 조정점이다

3.3.2.3 Sequences of Finding Knot

B-spline **곡선**에서의
Knot 간격 예측

⇩

주어진 곡선상의 점들로부터 Chord Length를 구함

$$\frac{\Delta_i}{\Delta_{i+1}} = \sqrt{\frac{|p_{i-1} - p_{i-2}|}{|p_i - p_{i-1}|}}, \ (i = 2,3,....,n+2)$$

⇩

곡선상의 점,접선벡터와 위의 Knot 간격을 이용하여 B-spline 조정점을 구할 수 있음

Tensor Product방식으로 정의된
spline **곡면**에서의 Knot 간격예측

⇩

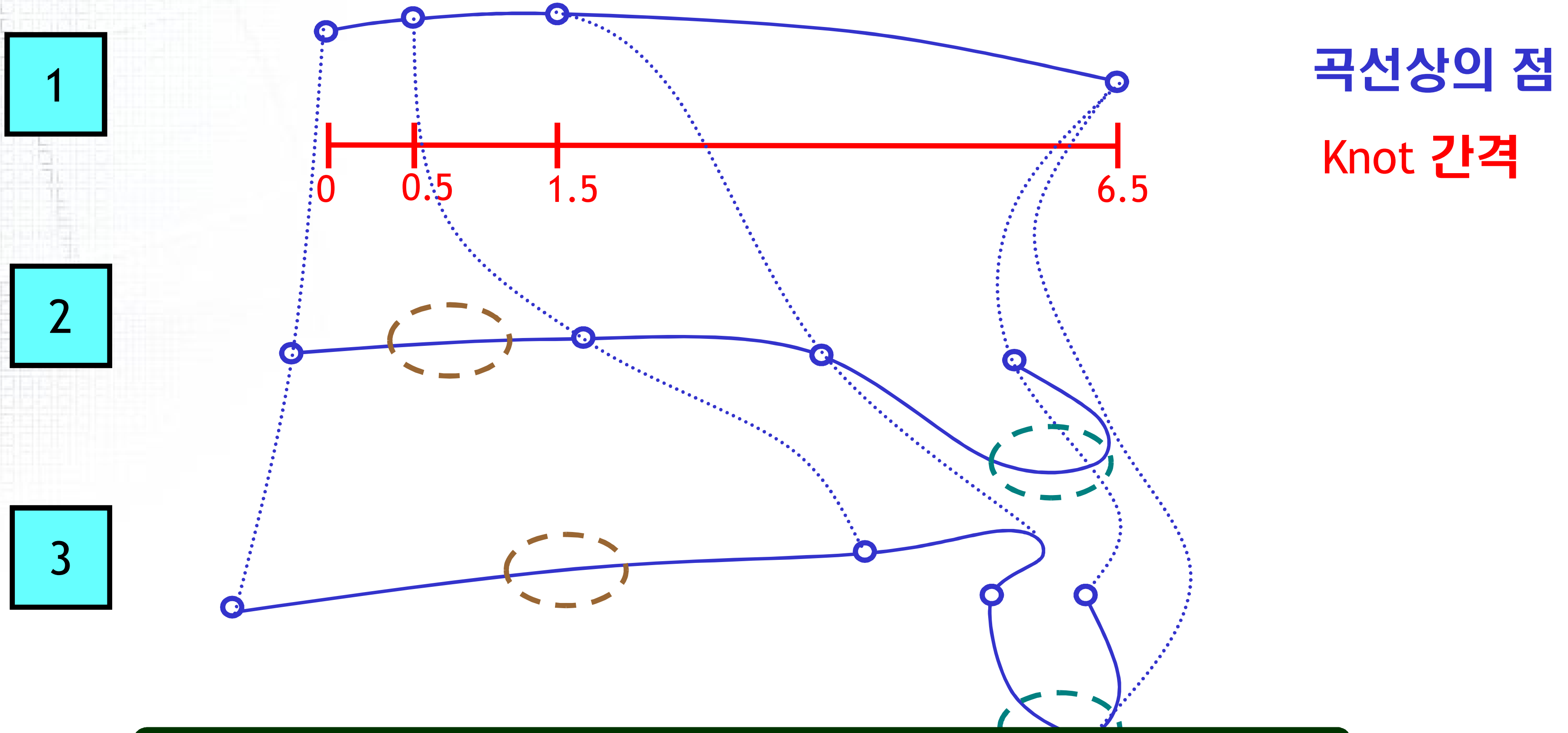
주어지는 곡면상의 점이 GRID와 같이 균일하게 주어져야 이상적인 곡면재현 가능

⇩

즉, 주어진 곡선들의 Knot 간격이 모두 일정해야 함.

3.3.2.4 Knot 간격 차이가 주는 영향

점과 점 사이의 knot 간격은 그 점 사이를 지나가는 데 걸리는 시간과 같은 개념이다.



** CAGD

그러므로 knot간격비가 비슷할 수록 곡면의 왜곡이 적다.

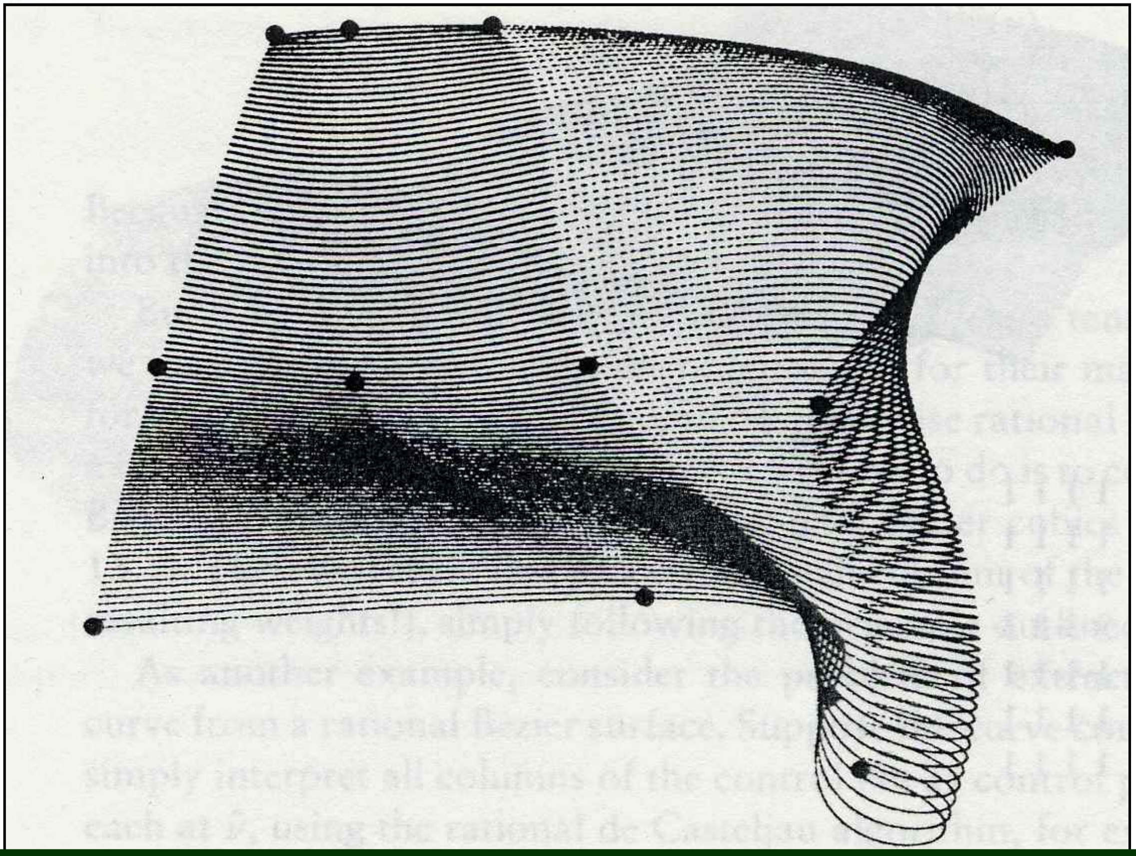
3.3.2.4 Knot 간격 차이가 주는 영향

점과 점 사이의 knot 간격은 그 점 사이를 지나가는 데 걸리는 시간과 같은 개념이다.

1

2

3



곡선상의 점

Knot 간격

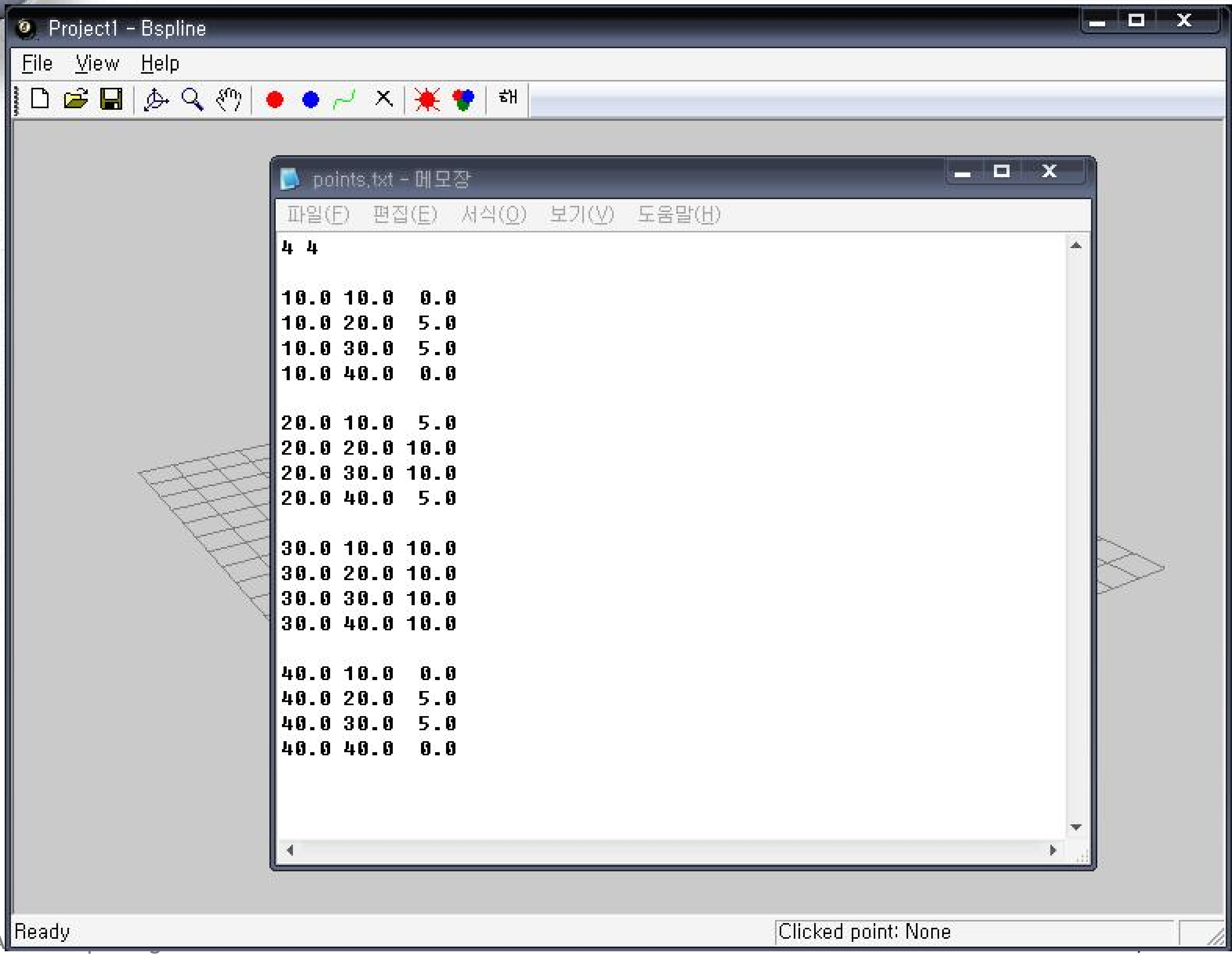
** CAGD

그러므로 knot간격비가 비슷할 수록 곡면의 왜곡이 적다.

3.3.2.5 Example of bicubic B-spline Surface Interpolation (1)

일반 3차원 곡면 형상 예시

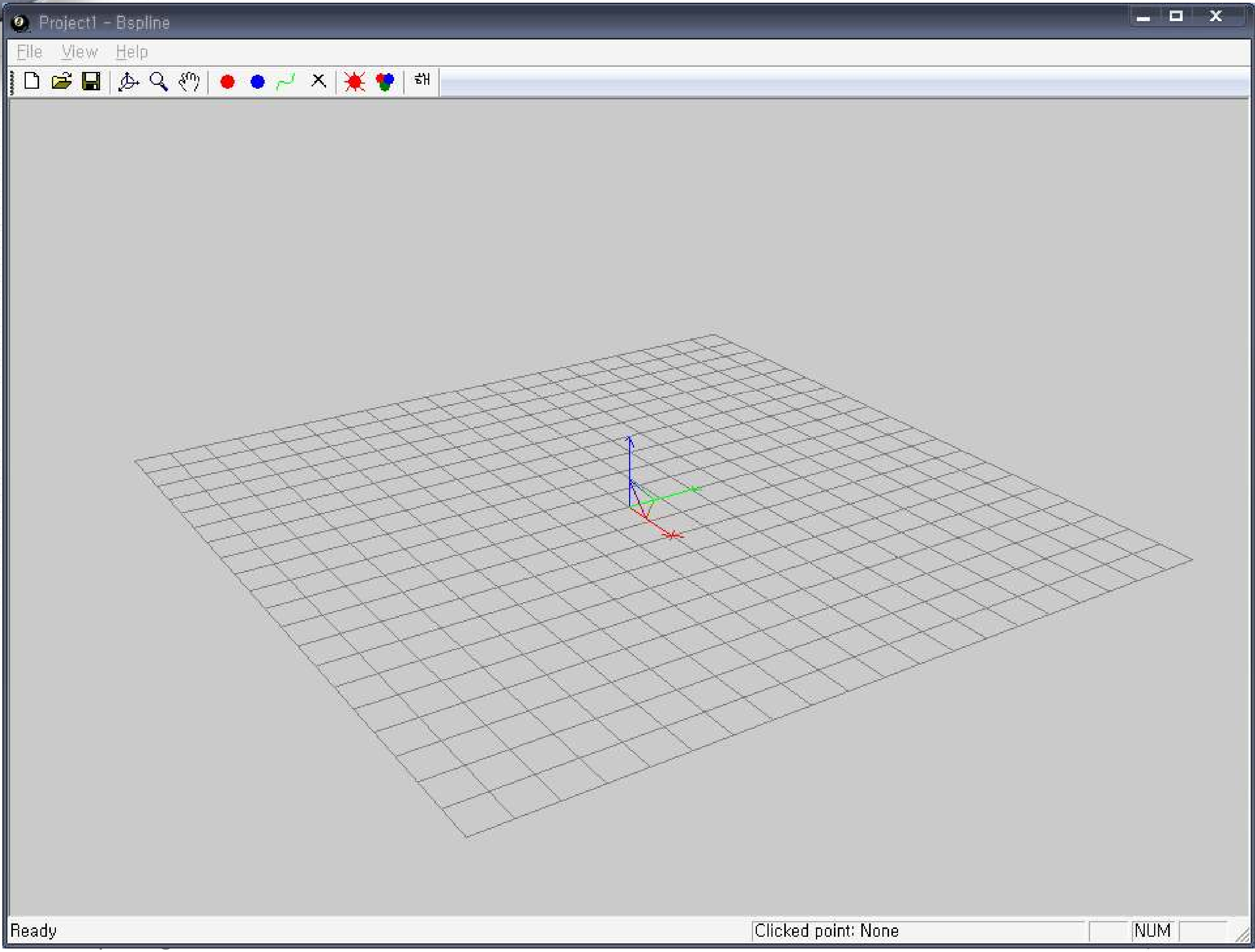
- Given: Points on Surface
- Find: bicubic B-spline Surface (Control Points of bicubic B-spline Surface)



3.3.2.5 Example of bicubic B-spline Surface Interpolation (2)

- Given: Points on Surface
- Find: bicubic B-spline Surface (Control Points of bicubic B-spline Surface)

선박 선형 곡면 예시



3.3.2.5 Sample code of bicubic B-spline Surface Interpolation (1)

```
void BicubicBsplineSurface::Interpolate(Vector **pFittingPoint, int nU, int nV) {
  // Generate u-Knot
    if(m_UKnot) delete[] m_UKnot;
    m_nUKnot = (m_nU - 2) + 2*(3+1);
    m_UKnot = new double [m_nUKnot];

    // Initial u-Knot
    double** tmpUKnots;
    tmpUKnots = new double*[nV];
    for(int j = 0; j < nV; j++){
        tmpUKnots[j] = new double[nU];
        for(int i = 0; i < nU; i++){
            tmpUKnot[j][i] = ...;   // chord length or centripetal
        }
    }

    // generate average u-Knot
   for(int i = 0; i < nU; i++){
        m_UKnot[i] = 0;
        for(int j=0; j<nV; j++) {       m_UKnot[i] += tmpUKnot[j][i];      }
        m_UKnot[i] = m_UKnot[i] / nV;
   }
```

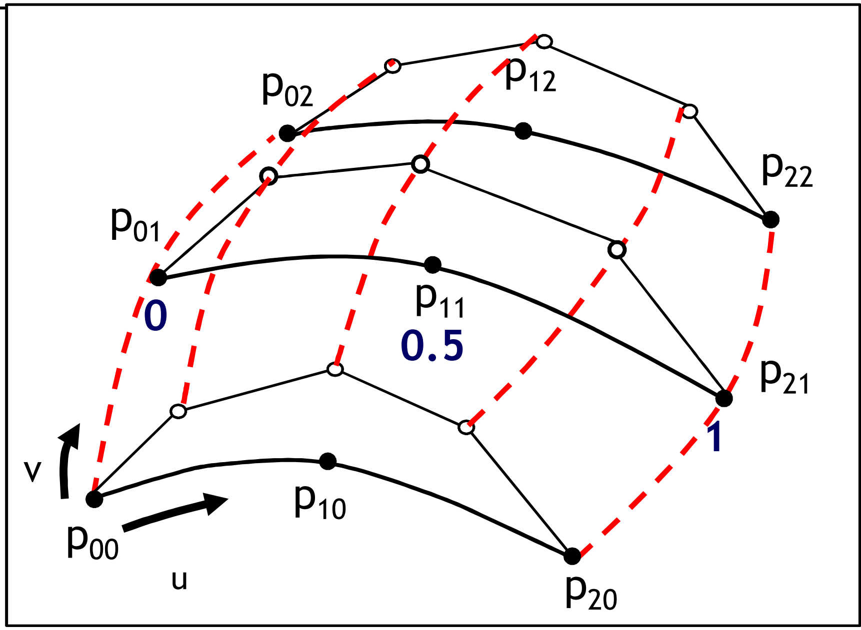
➔ Chord length를 이용하여 u, v 방향 Knot 생성

➔ Knot간 간격의 평균값으로 Knot 재 설정

3.3.2.5 Sample code of bicubic B-spline Surface Interpolation (2)

```
// Interpolate u-directional B-spline curve
CubicBsplineCurve* u_curve = new CubicBsplineCurve[nV];
for(int j = 0; j < nV; j++){
    u_curve[j].SetKnot( m_UKnot );
    u_curve[j].Interpolate(  pFittingPoint[j], nU );
}
```

```
// Generate v-directional Fitting Point
int nvFittingPoint = u_curve[0].m_nControlPoint;
Vector** vFittingPoint = new Vector [ nvFittingPoint ];
for(int j=0; j < nvFittingPoint; j++){
    vFittingPoint[j] = new Vector[ nV ];
    for( int i = 0; i < nV; i++){
        vFittingPoint[j][i] = u_curve[i].m_ControlPoint[j];
    }
}
.........................
```



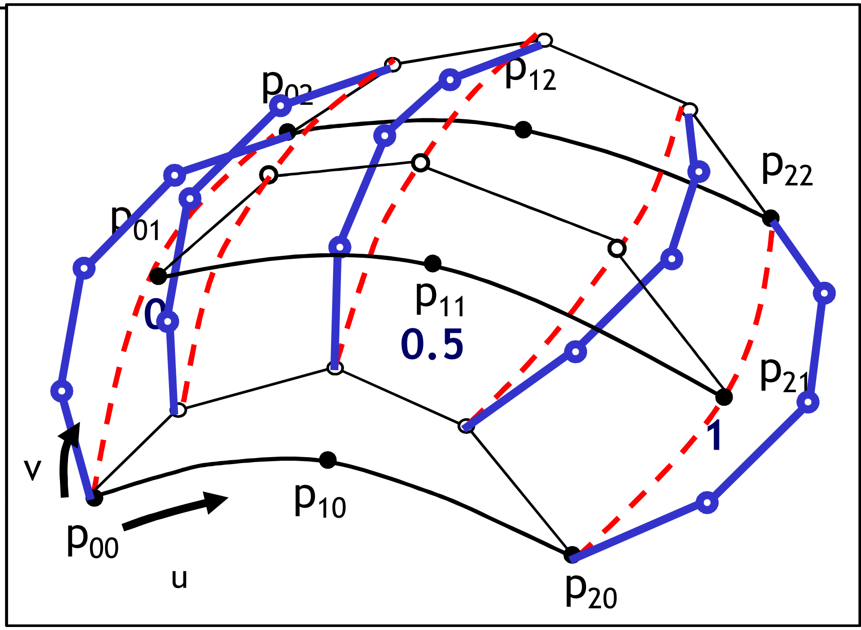
u 방향 B-spline curve 생성

3.3.2.5 Sample code of bicubic B-spline Surface Interpolation (2)

```
// Interpolate u-directional B-spline curve
CubicBsplineCurve* u_curve = new CubicBsplineCurve[nV];
for(int j = 0; j < nV; j++){
    u_curve[j].SetKnot( m_UKnot );
    u_curve[j].Interpolate(  pFittingPoint[j], nU );
}
```

```
// Generate v-directional Fitting Point
int nvFittingPoint = u_curve[0].m_nControlPoint;
Vector** vFittingPoint = new Vector [ nvFittingPoint ];
for(int j=0; j < nvFittingPoint; j++){
    vFittingPoint[j] = new Vector[ nV ];
    for( int i = 0; i < nV; i++){
        vFittingPoint[j][i] = u_curve[i].m_ControlPoint[j];
    }
}
```

....



u 방향 B-spline curve 의 조정점을
Fitting Point로 하여
v 방향 B-spline Curve 생성

3.4. Generation of Bezier surfaces by tensor-product approach

3.4.1 Tensor-product approach

3.4.2 Tensor-product biquadratic Bezier surface

3.4.3 Tensor-product bicubic Bezier surface

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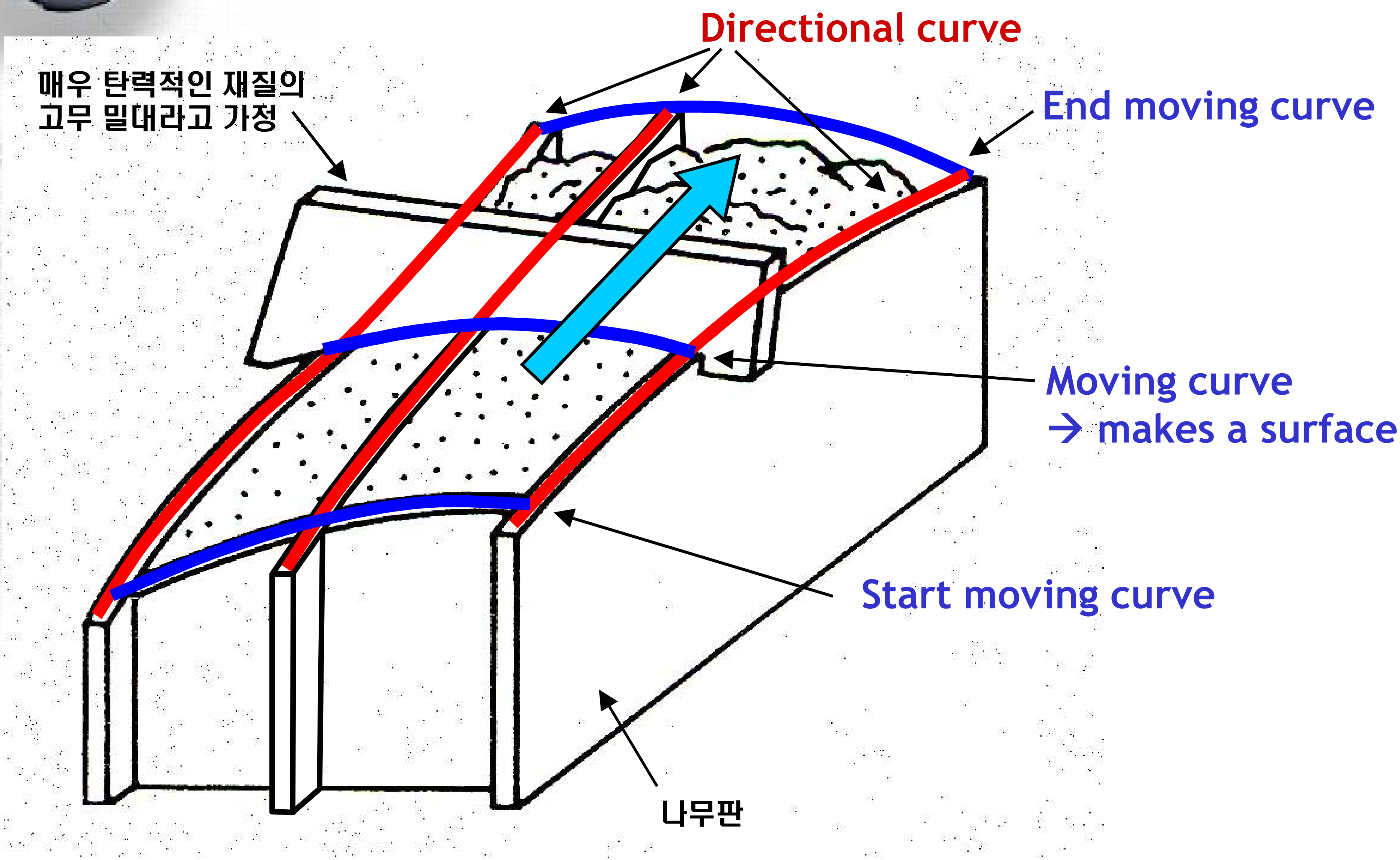
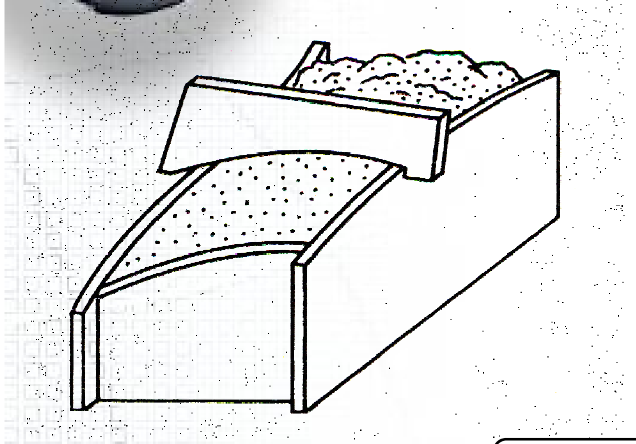
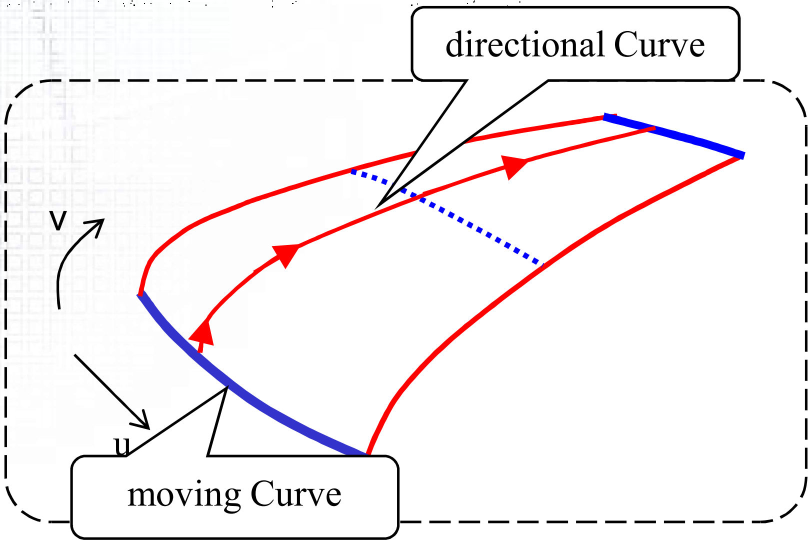
3.4.1 Tensor product approach (1)



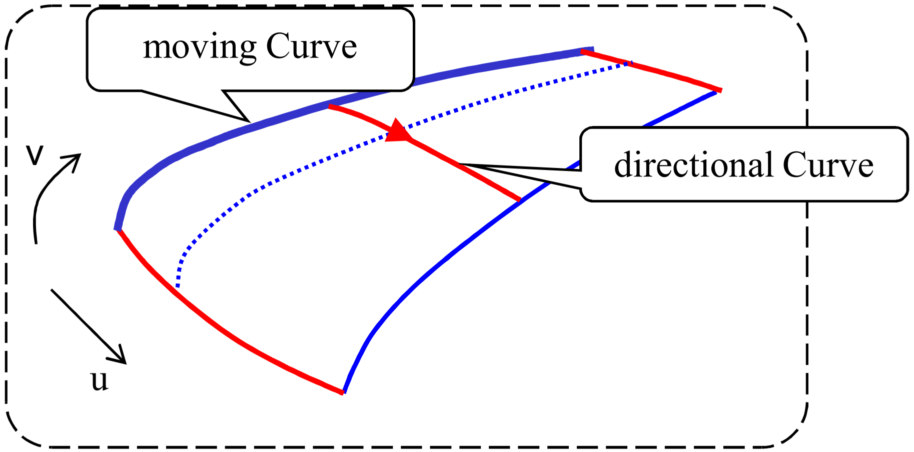
3.4.1 Tensor product approach (2)



- moving curve가 일정한 차수의 Bezier curve 이고, moving curve의Bezier control points의 궤적을 나타내는 directional curve도 Bezier curve일 때, 이러한 방법으로 생성되는 곡면을 "Tensor product Bezier surface" 라고 한다.



r(u) 곡선을 v 방향으로 sweeping

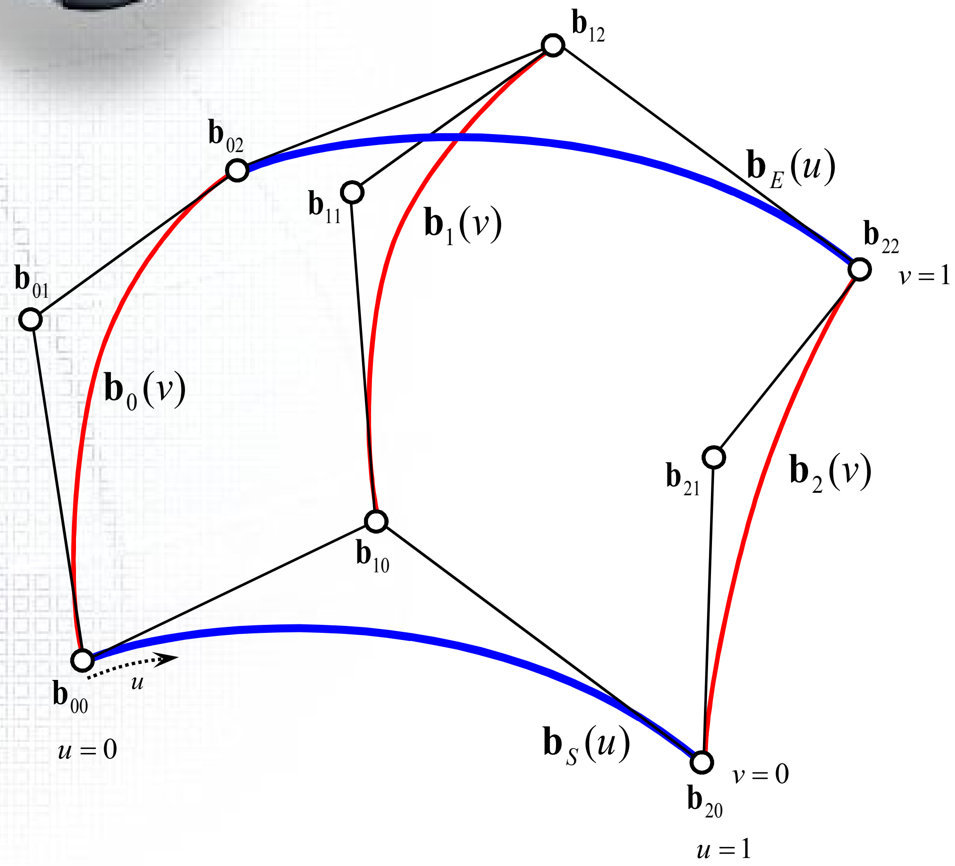


r(v) 곡선을 u 방향으로 sweeping

* CAGD, Ch14.3, Tensor Product Approach

3.4.2 Tensor-product bi-quadratic Bezier surface (1)

- Given: Control Points of bi-quadratic Bezier Surface
- Find: Points on bi-quadratic Bezier Surface



- Given 3x3 Points $\mathbf{b}_{ij}$,
- Generate start/end moving curves and directional curves in quadratic Bezier form

$$\mathbf{b}_E(u) = \mathbf{b}_{02}B_0^2(u) + \mathbf{b}_{12}B_1^2(u) + \mathbf{b}_{22}B_2^2(u)$$

$$\mathbf{b}_S(u) = \mathbf{b}_{00}B_0^2(u) + \mathbf{b}_{10}B_1^2(u) + \mathbf{b}_{20}B_2^2(u)$$

$$\mathbf{b}_0(v) = \mathbf{b}_{00}B_0^2(v) + \mathbf{b}_{01}B_1^2(v) + \mathbf{b}_{02}B_2^2(v)$$

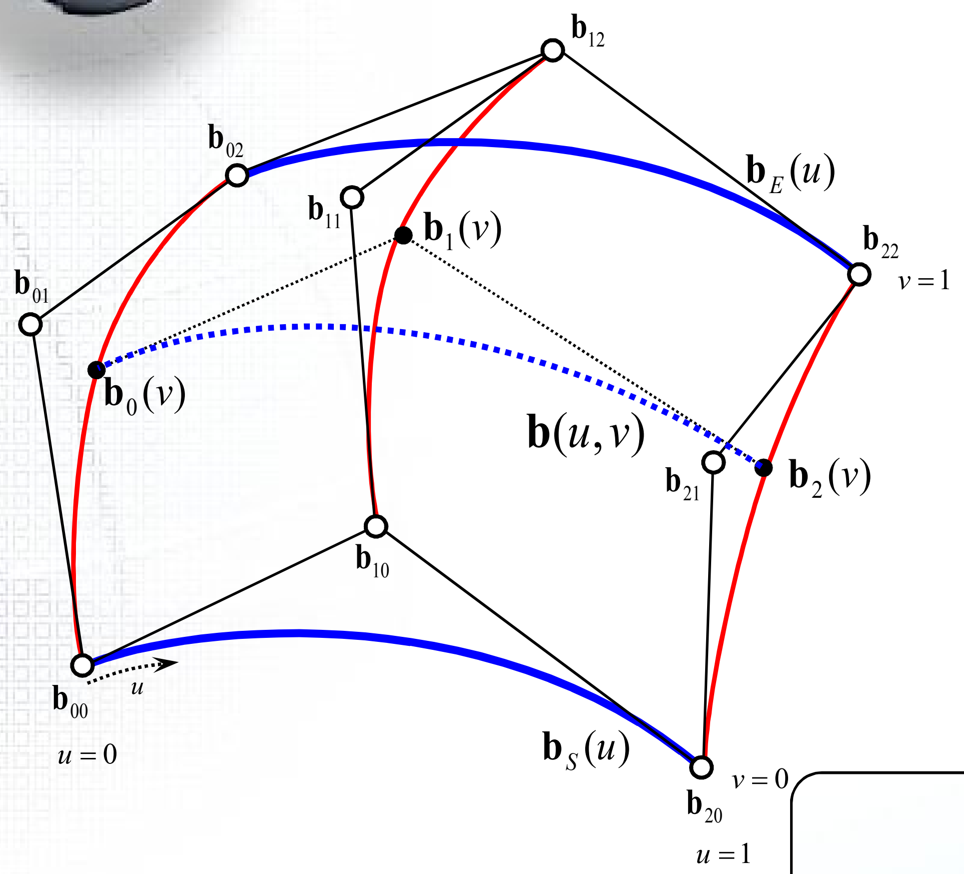
$$\mathbf{b}_1(v) = \mathbf{b}_{10}B_0^2(v) + \mathbf{b}_{11}B_1^2(v) + \mathbf{b}_{12}B_2^2(v)$$

$$\mathbf{b}_2(v) = \mathbf{b}_{20}B_0^2(v) + \mathbf{b}_{21}B_1^2(v) + \mathbf{b}_{22}B_2^2(v)$$

$$\begin{bmatrix} \mathbf{b}_0(v) \\ \mathbf{b}_1(v) \\ \mathbf{b}_2(v) \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} \\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix} \begin{bmatrix} B_0^2(v) \\ B_1^2(v) \\ B_2^2(v) \end{bmatrix}$$

3.4.2 Tensor-product bi-quadratic Bezier surface (2)

- Given: Control Points of bi-quadratic Bezier Surface
- Find: Points on bi-quadratic Bezier Surface



- Given 3x3 Points $\mathbf{b}_{ij}$,
- Moving curve can be represented in the following form:

$$\mathbf{b}(u,v) = \mathbf{b}_0(v)B_0^2(u) + \mathbf{b}_1(v)B_1^2(u) + \mathbf{b}_2(v)B_2^2(u)$$

$$= \begin{bmatrix} B_0^2(u) & B_1^2(u) & B_2^2(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_0(v) \\ \mathbf{b}_1(v) \\ \mathbf{b}_2(v) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{b}_0(v) \\ \mathbf{b}_1(v) \\ \mathbf{b}_2(v) \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} \\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix} \begin{bmatrix} B_0^2(v) \\ B_1^2(v) \\ B_2^2(v) \end{bmatrix}$$

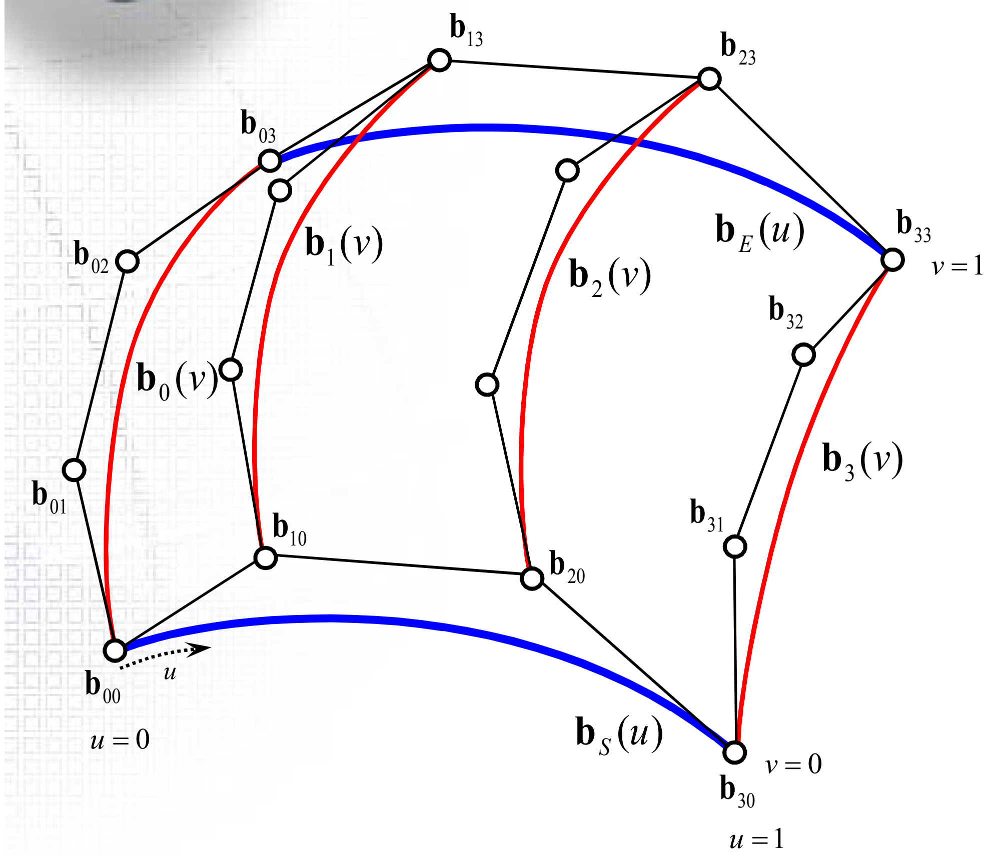
$$\mathbf{b}(u,v) = \begin{bmatrix} B_0^2(u) & B_1^2(u) & B_2^2(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} \\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix} \begin{bmatrix} B_0^2(v) \\ B_1^2(v) \\ B_2^2(v) \end{bmatrix}$$

$$= \sum_{j=0}^{2} \sum_{i=0}^{2} \mathbf{b}_{ij} B_i^2(u) B_j^2(v)$$

Bezier surface control points

3.4.2 Tensor-product bi-cubic Bezier surface (1)

- Given: Control Points of bi-cubic Bezier Surface
- Find: Points on bi-cubic Bezier Surface



- Given 4x4 Points $\mathbf{b}_{ij}$,
- Generate start/end moving curves and directional curves in cubic Bezier form

$$\mathbf{b}_E(u)=\mathbf{b}_{03}B_0^3(u)+\mathbf{b}_{13}B_1^3(u)+\mathbf{b}_{23}B_2^3(u)+\mathbf{b}_{33}B_3^3(u)$$

$$\mathbf{b}_S(u)=\mathbf{b}_{00}B_0^3(u)+\mathbf{b}_{10}B_1^3(u)+\mathbf{b}_{20}B_2^3(u)+\mathbf{b}_{30}B_3^3(u)$$

$$\mathbf{b}_0(v)=\mathbf{b}_{00}B_0^3(v)+\mathbf{b}_{01}B_1^3(v)+\mathbf{b}_{02}B_2^3(v)+\mathbf{b}_{03}B_3^3(v)$$

$$\mathbf{b}_1(v)=\mathbf{b}_{10}B_0^3(v)+\mathbf{b}_{11}B_1^3(v)+\mathbf{b}_{12}B_2^3(v)+\mathbf{b}_{13}B_3^3(v)$$

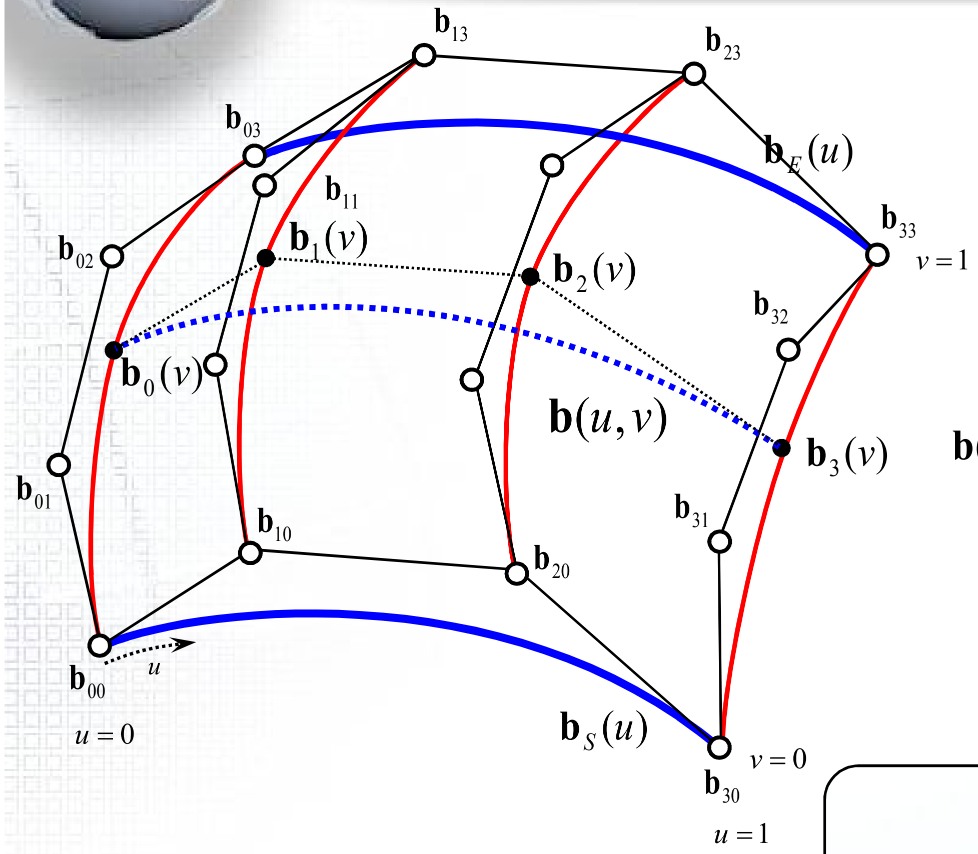
$$\mathbf{b}_2(v)=\mathbf{b}_{20}B_0^3(v)+\mathbf{b}_{21}B_1^3(v)+\mathbf{b}_{22}B_2^3(v)+\mathbf{b}_{23}B_3^3(v)$$

$$\mathbf{b}_3(v)=\mathbf{b}_{30}B_0^3(v)+\mathbf{b}_{31}B_1^3(v)+\mathbf{b}_{32}B_2^3(v)+\mathbf{b}_{33}B_3^3(v)$$

$$\begin{bmatrix}\mathbf{b}_0(v)\\ \mathbf{b}_1(v)\\ \mathbf{b}_2(v)\\ \mathbf{b}_3(v)\end{bmatrix}=\begin{bmatrix}\mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} & \mathbf{b}_{03}\\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} & \mathbf{b}_{13}\\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} & \mathbf{b}_{23}\\ \mathbf{b}_{30} & \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33}\end{bmatrix}\begin{bmatrix}B_0^3(v)\\ B_1^3(v)\\ B_2^3(v)\\ B_3^3(v)\end{bmatrix}$$

3.4.2 Tensor-product bi-cubic Bezier surface (2)

- Given: Control Points of bi-cubic Bezier Surface
- Find: Points on bi-cubic Bezier Surface



- Given 4x4 Points $\mathbf{b}_{ij}$,
- Moving curve can be represented in the following form:

$$\mathbf{b}(u,v) = \mathbf{b}_0(v)B_0^3(u) + \mathbf{b}_1(v)B_1^3(u) + \mathbf{b}_2(v)B_2^3(u) + \mathbf{b}_3(v)B_3^3(u)$$

$$= \begin{bmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_0(v) \\ \mathbf{b}_1(v) \\ \mathbf{b}_2(v) \\ \mathbf{b}_3(v) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{b}_0(v) \\ \mathbf{b}_1(v) \\ \mathbf{b}_2(v) \\ \mathbf{b}_3(v) \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} & \mathbf{b}_{03} \\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} & \mathbf{b}_{13} \\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} & \mathbf{b}_{23} \\ \mathbf{b}_{30} & \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33} \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

$$\mathbf{b}(u,v) = \begin{bmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{b}_{00} & \mathbf{b}_{01} & \mathbf{b}_{02} & \mathbf{b}_{03} \\ \mathbf{b}_{10} & \mathbf{b}_{11} & \mathbf{b}_{12} & \mathbf{b}_{13} \\ \mathbf{b}_{20} & \mathbf{b}_{21} & \mathbf{b}_{22} & \mathbf{b}_{23} \\ \mathbf{b}_{30} & \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33} \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

$$= \sum_{j=0}^{3} \sum_{i=0}^{3} \mathbf{b}_{ij} B_i^3(u) B_j^3(v)$$

Ch 4. Term Project

Generation Ship hull surfaces
by interpolating given points

- Given: $\mathbf{P}_{i,j}$
- Find : $\mathbf{d}_{i,j}$

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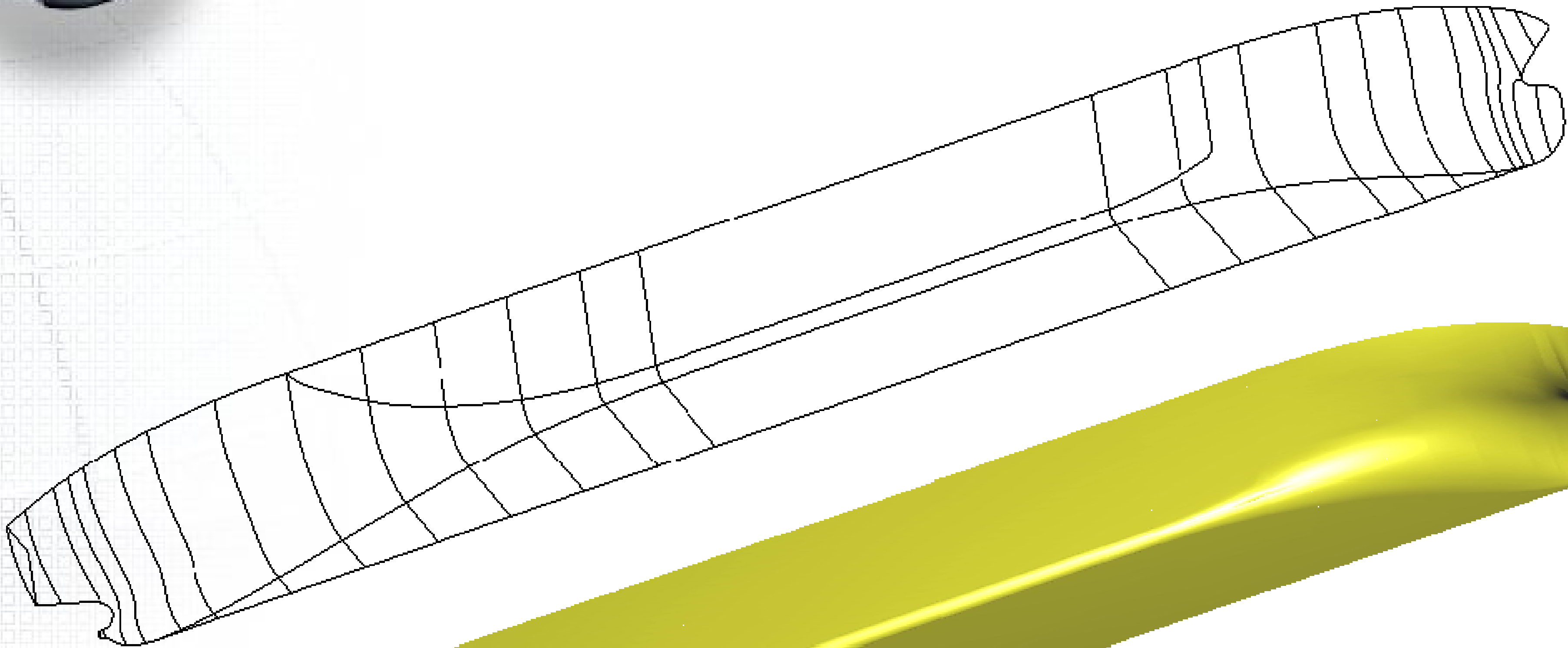
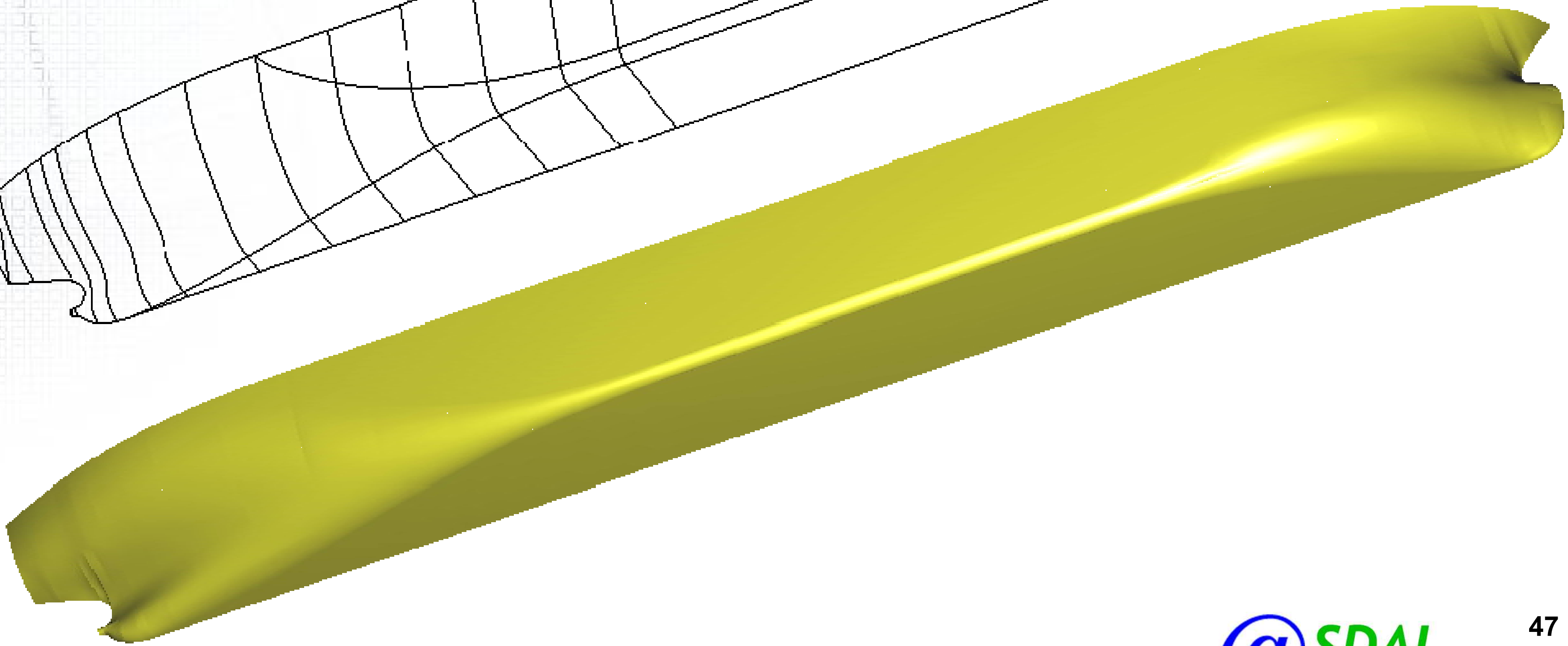
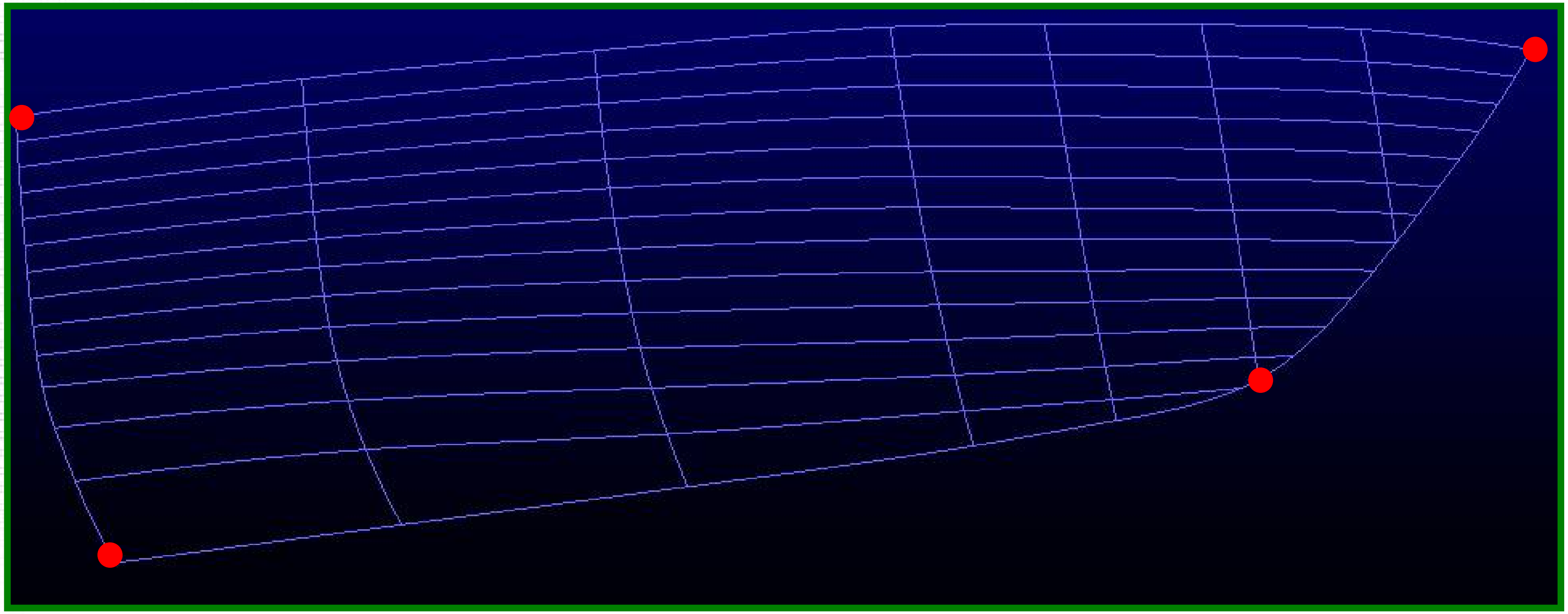
4.1 단일 B-spline 곡면 patch를 이용한 선형곡면 생성 프로그램 구현





4.2 간단한 요트 형상의 선수부 곡선그물망 형상

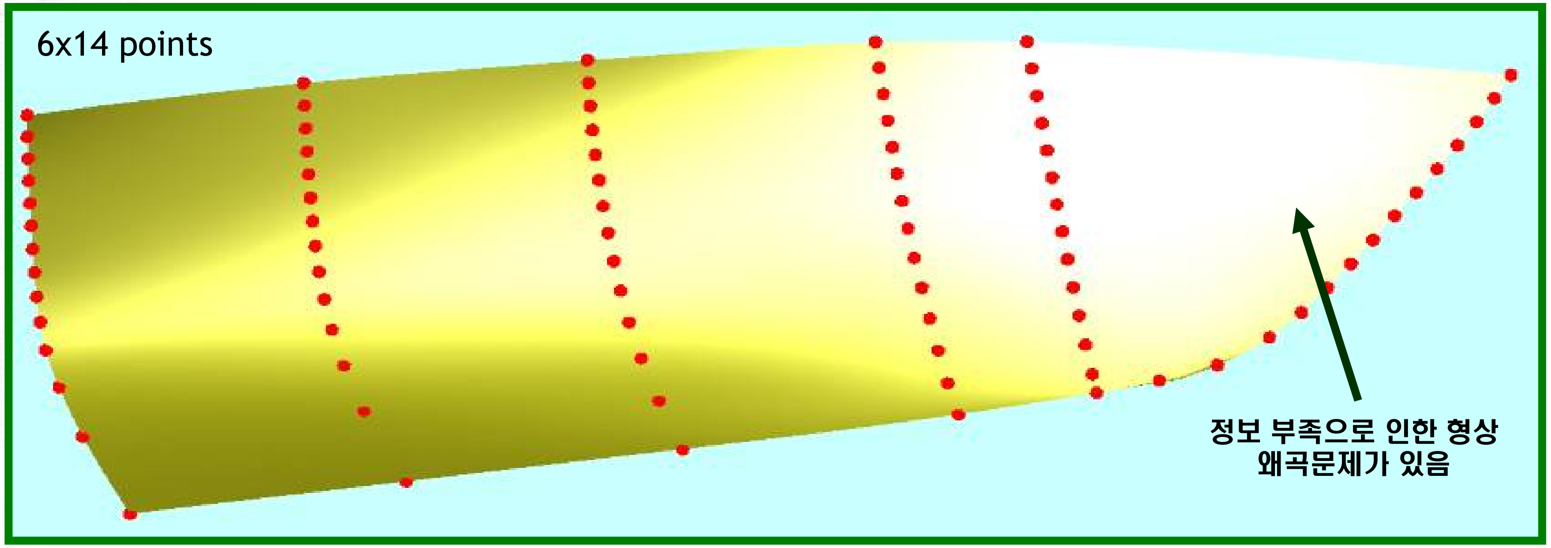
* 출처 : 서울대 조선해양공학과 2005년 2학년 교과목 『 조선해양공학계획』 강좌 중에 학생들이 설계한 선형



사각형 패치의 꼭지점 결정

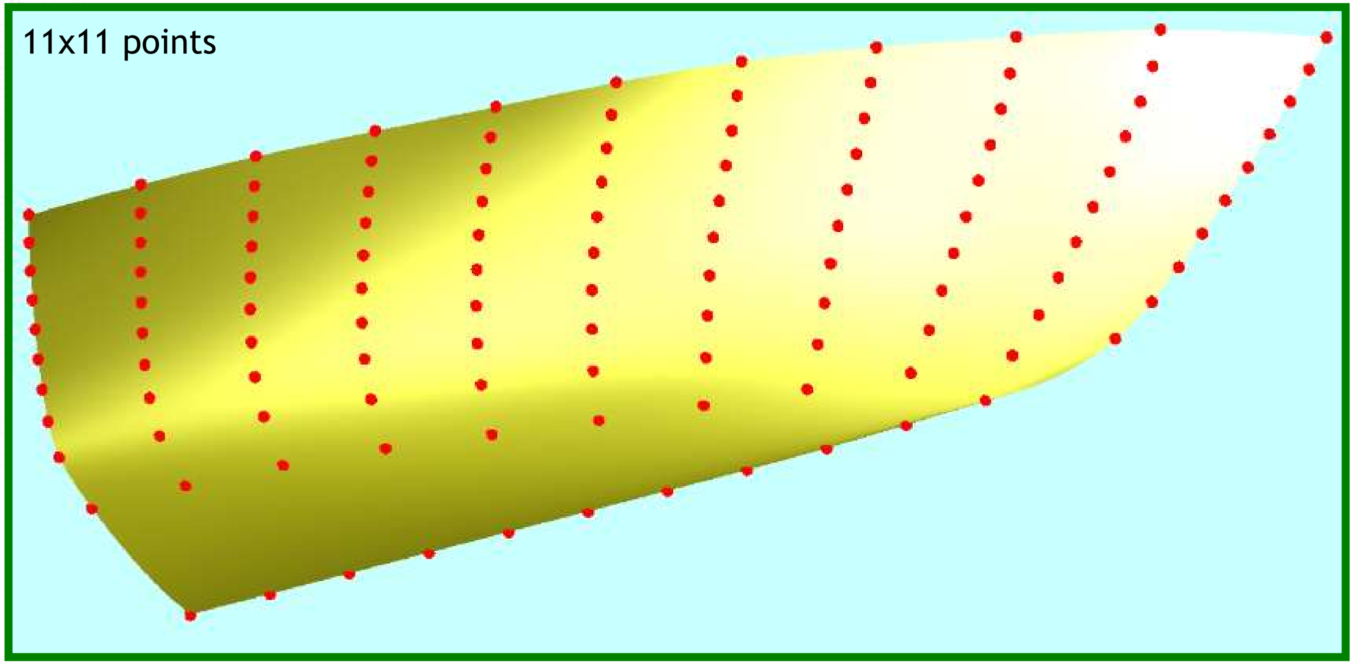
4.3 요트 형상의 곡선그물망으로부터 선형곡면 생성결과 (1)

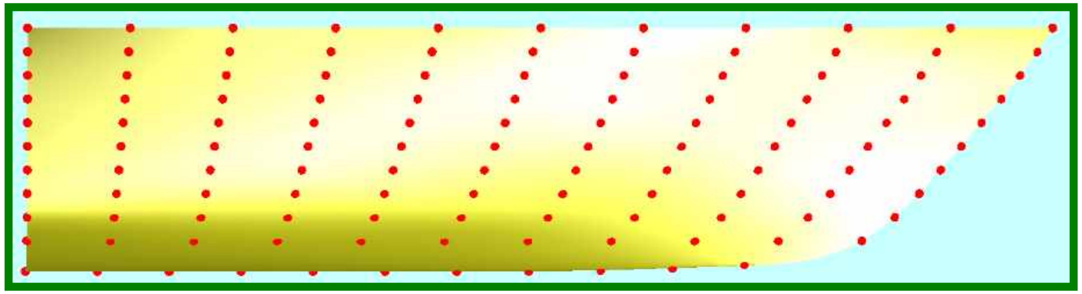
- Offset table 형식으로 점을 추출한 후,
  이 점들로 부터 bicubic B-spline 선형곡면을 생성한 결과



4.3 요트 형상의 곡선그물망으로부터 선형곡면 생성결과 (2)

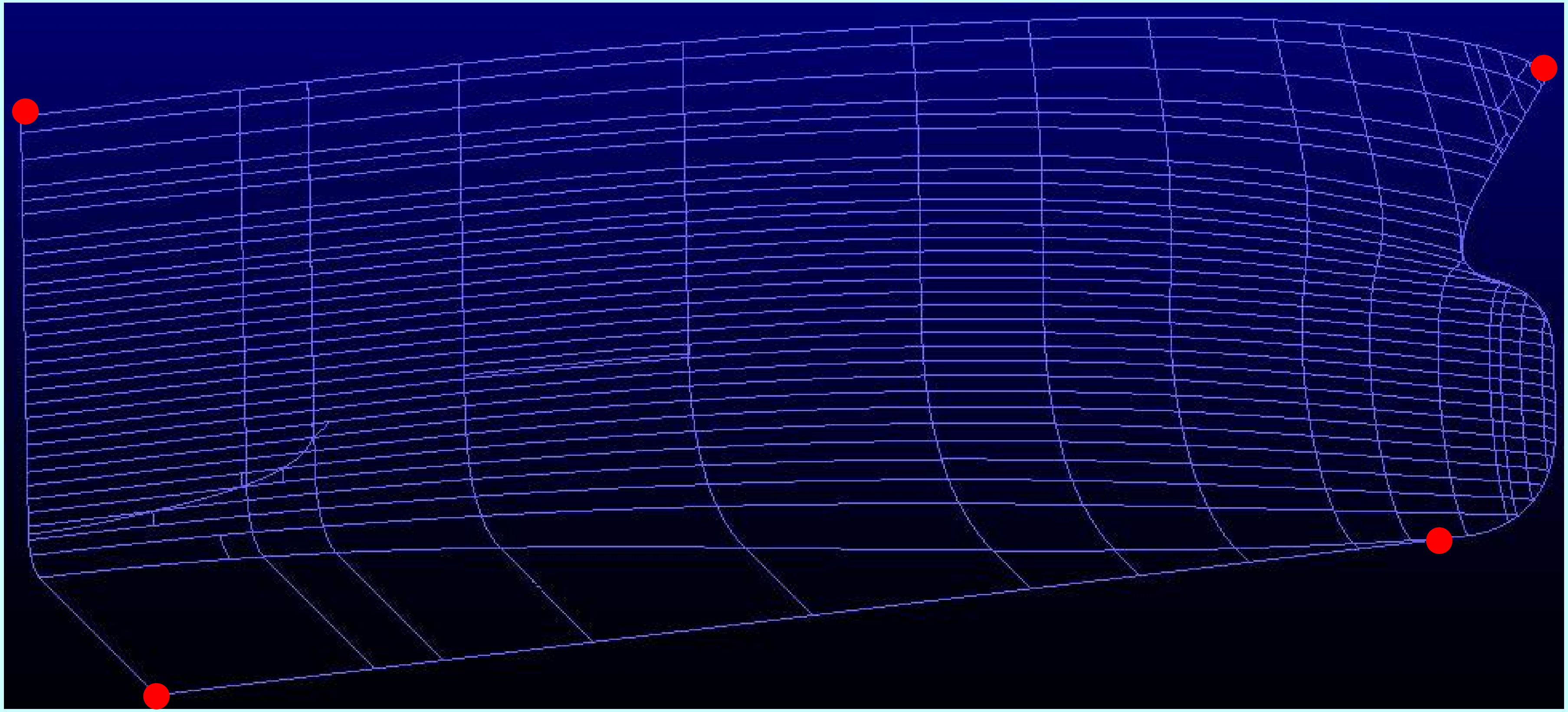
- ❑ 점들의 x좌표 사이의 거리가 일정하도록 점을 추출한 후, 이 점들로부터 bicubic B-spline 선형곡면을 생성한 결과





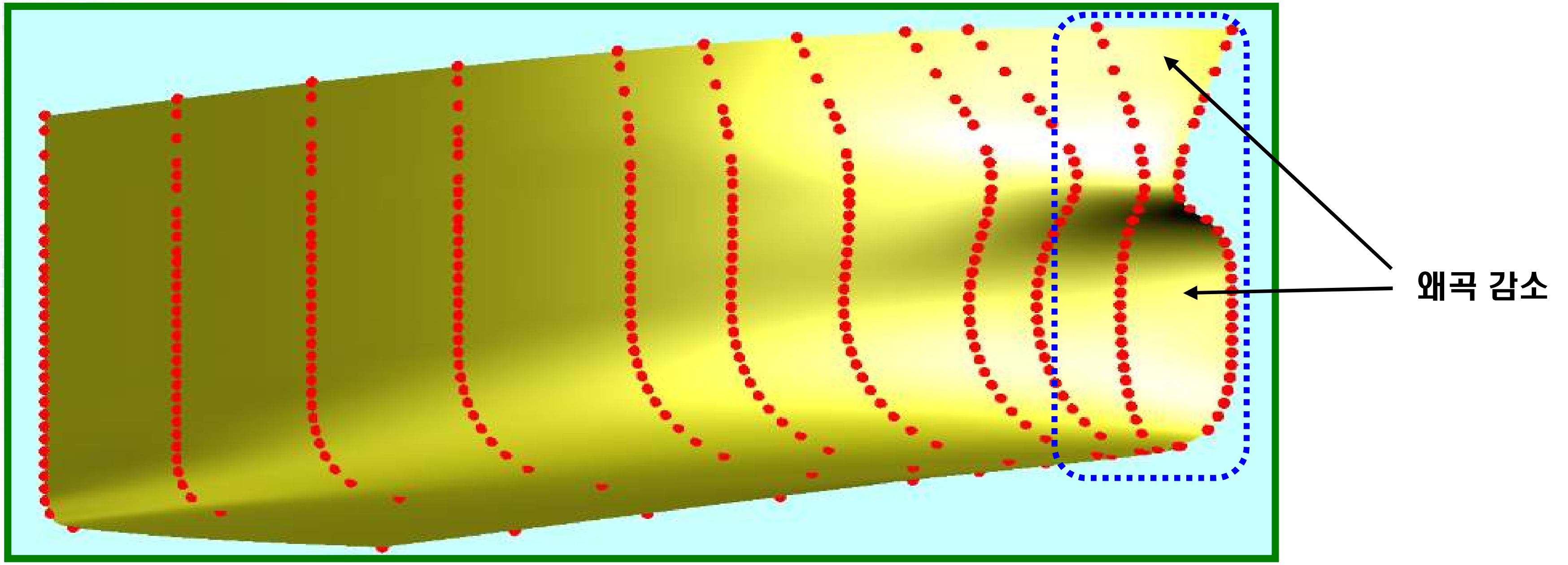
4.4 구상선수를 갖는 단축선의 선수부 곡선그물망 형상

선수부



4.5 구상선수부 곡선그물망으로부터 선수부 선형곡면 생성결과

❑ 주어진 곡선그물망 이외의 보조선을 생성하여 곡선 보간에 적합한 점 data를 생성한 후, 점 data로부터 선수부 선형곡면을 생성한 결과



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- G. Farin, Curves and Surfaces for CAGD, 5^{th} Edi., Academic Press, 2002
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참고 슬라이드

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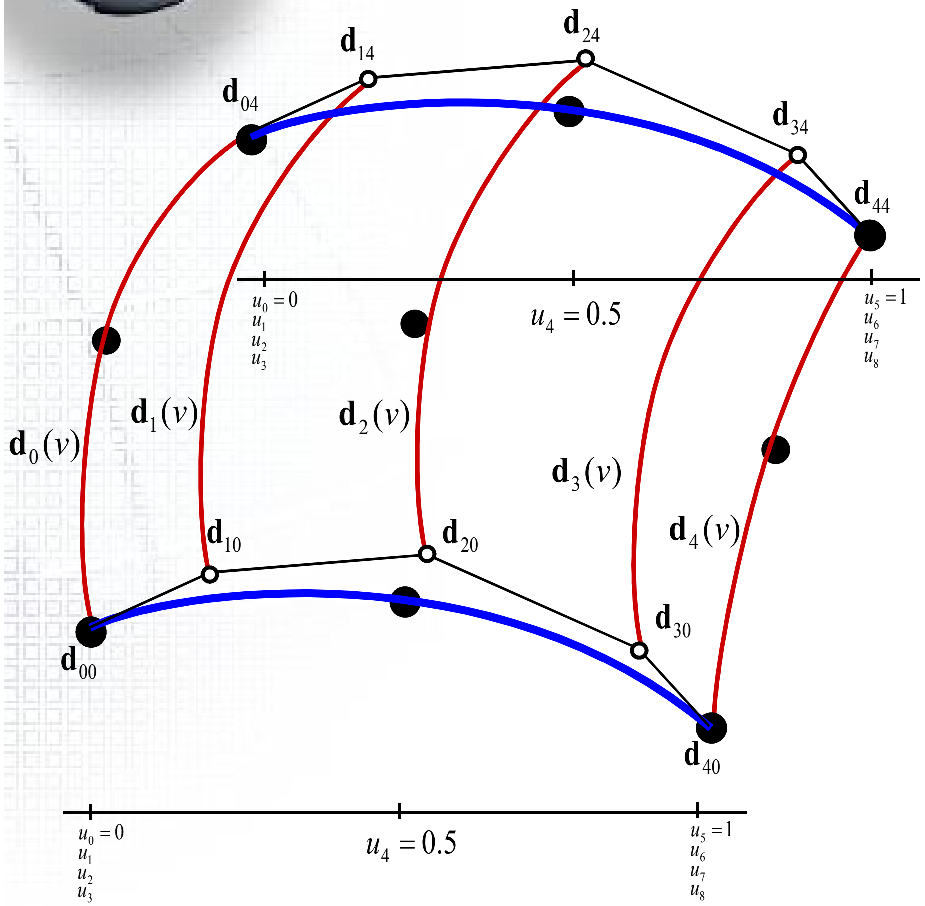
Bezier Curve and B-spline Curve

항목		Bezier Curve	B-spline Curve
Make Curve	Given	Bezier Control Point $\mathbf{b}_i$ Parameter t Bernstein Polynomial Func. $B_i^n(t)$	B-spline Control Point $\mathbf{d}_i$ Parameter u B-spline Basis Func. $N_i^n(u)$
	Find	Bezier Curve $\mathbf{r}(t)$ $\mathbf{r}(t)=\mathbf{b}_0B_0^n(t)+\mathbf{b}_1B_1^n(t)+.......+\mathbf{b}_nB_n^n(t).$	B-spline Curve $\mathbf{r}(u)$ $\mathbf{r}(u)=\mathbf{d}_0N_0^3(u)+\mathbf{d}_1N_1^3(u)+\mathbf{d}_2N_2^3(u)+\cdots+\mathbf{d}_{D-1}N_{D-1}^3(u)$
Basis Function (*Blending*)		Bernstein Polynomial Function $B_i^n(t)=\binom{n}{i}t^i(1-t)^{n-i},$ $\binom{n}{i}={}_nC_i=\begin{cases}\dfrac{n!}{i!(n-i)!} & \textbf{if } 0\le i\le n\\ \mathbf{0} & \textbf{else}\end{cases}$	B-spline Basis Function (Cox-de boor Recursive Formula) $N_i^n(u)=\dfrac{u-u_{i-1}}{u_{i+n-1}-u_{i-1}}N_i^{n-1}(u)+\dfrac{u_{i+n}-u}{u_{i+n}-u_i}N_{i+1}^{n-1}(u)$ $N_i^0(u)=\begin{cases}1 & \textbf{if } u_{i-1}\le u<u_i\\ 0 & \textbf{else}\end{cases},\ \sum_{i=0}^{D-1}N_i^n(u)=1$
Constructive Approach		de Casteljau Algorithm $\mathbf{b}_i^k(t)=(1-t)\mathbf{b}_i^{k-1}+t\mathbf{b}_{i+1}^{k-1}$	de Boor Algorithm $\mathbf{d}_i^k(u)=\dfrac{u_{i+n-k}-u}{u_{i+n-k}-u_{i-1}}\mathbf{d}_{i-1}^{k-1}(u)+\dfrac{u-u_{i-1}}{u_{i+n-k}-u_{i-1}}\mathbf{d}_i^{k-1}(u)$
Interpolation	Given	Points on Curve: $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$	Points on Curve: $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$
	Find	Bezier Control Point $\mathbf{b}_i$	B-spline Control Point $\mathbf{d}_i$

3.3.2.1 B-spline surface interpolation 과정 (1)

- Given: 곡면 상의 9개 점
- Find: Control Points of bicubic B-spline Surface



- Given 3x3 fitting points
- (1) interpolate start/end moving B-spline curves with same u-knots, which are swept along directional curves

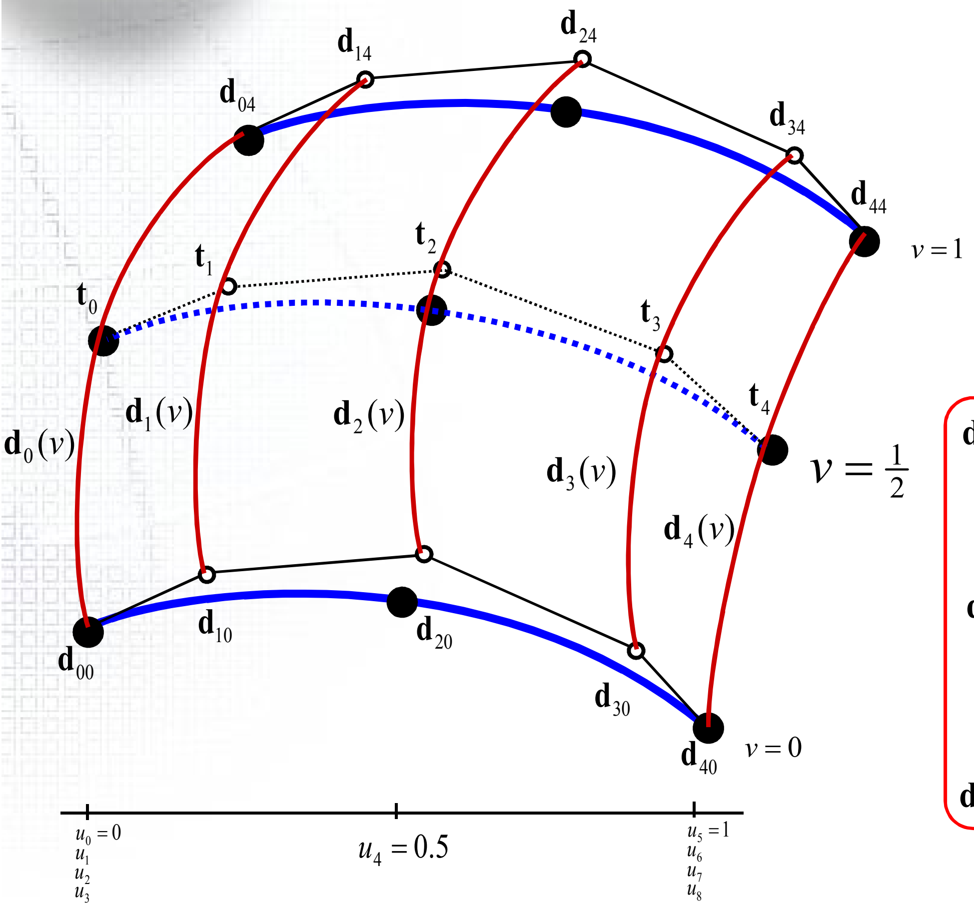
$\mathbf{d}_0(1) = \mathbf{d}_{04} \quad \mathbf{d}_1(1) = \mathbf{d}_{14} \quad \mathbf{d}_2(1) = \mathbf{d}_{24} \quad \mathbf{d}_3(1) = \mathbf{d}_{34} \quad \mathbf{d}_4(1) = \mathbf{d}_{44}$

$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_0(v) \\ \mathbf{d}_1(v) \\ \mathbf{d}_2(v) \\ \mathbf{d}_3(v) \\ \mathbf{d}_4(v) \end{bmatrix}$$

$\mathbf{d}_0(0) = \mathbf{d}_{00} \quad \mathbf{d}_1(0) = \mathbf{d}_{10} \quad \mathbf{d}_2(0) = \mathbf{d}_{20} \quad \mathbf{d}_3(0) = \mathbf{d}_{30} \quad \mathbf{d}_4(0) = \mathbf{d}_{40}$

3.3.2.1 B-spline surface interpolation 과정 (2)

- Given: 곡면 상의 9개 점
- Find: Control Points of bicubic B-spline Surface

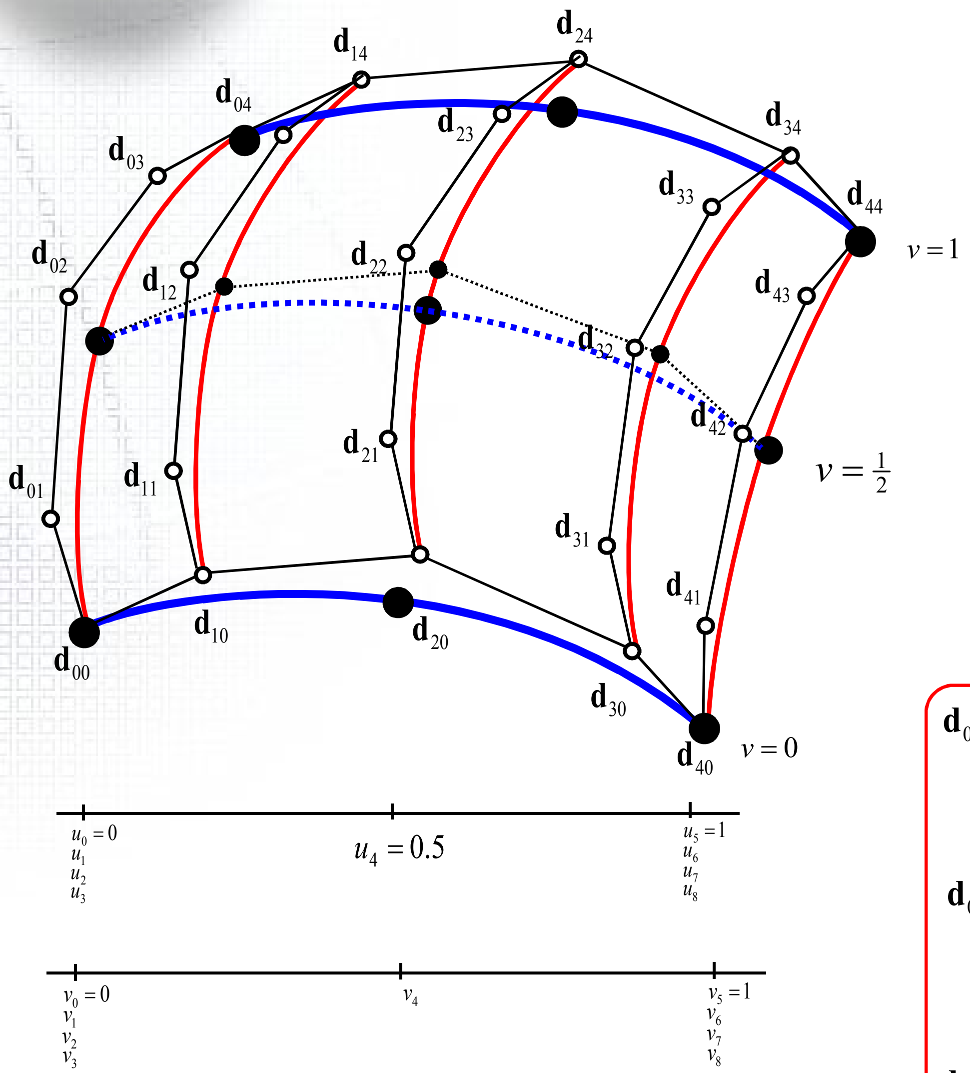


- Given 3x3 fitting points
- (1) interpolate start/end moving B-spline curves with same u-knots, which are swept along directional curves
- (2) interpolate moving curves at v=v* with same u-knots

$\mathbf{d}_0(1)=\mathbf{d}_{04}$	$\mathbf{d}_1(1)=\mathbf{d}_{14}$	$\mathbf{d}_2(1)=\mathbf{d}_{24}$	$\mathbf{d}_3(1)=\mathbf{d}_{34}$	$\mathbf{d}_4(1)=\mathbf{d}_{44}$
$\mathbf{d}_0(\frac{1}{2})=\mathbf{t}_0$	$\mathbf{d}_1(\frac{1}{2})=\mathbf{t}_1$	$\mathbf{d}_2(\frac{1}{2})=\mathbf{t}_2$	$\mathbf{d}_3(\frac{1}{2})=\mathbf{t}_3$	$\mathbf{d}_4(\frac{1}{2})=\mathbf{t}_4$
$\mathbf{d}_0(0)=\mathbf{d}_{00}$	$\mathbf{d}_1(0)=\mathbf{d}_{10}$	$\mathbf{d}_2(0)=\mathbf{d}_{20}$	$\mathbf{d}_3(0)=\mathbf{d}_{30}$	$\mathbf{d}_4(0)=\mathbf{d}_{40}$
$\mathbf{d}_0(v)$	$\mathbf{d}_1(v)$	$\mathbf{d}_2(v)$	$\mathbf{d}_3(v)$	$\mathbf{d}_4(v)$

3.3.2.1 B-spline surface interpolation 과정 (3)

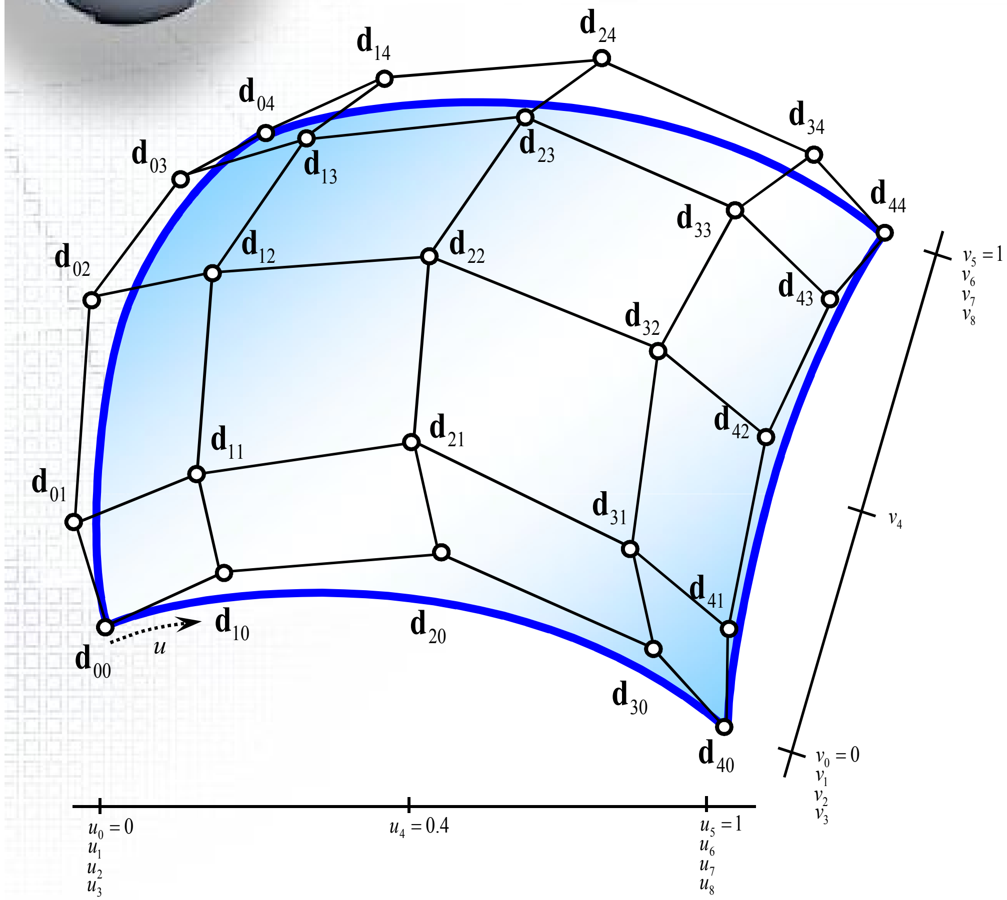
- Given: 곡면 상의 9개 점
- Find: Control Points of bicubic B-spline Surface



- Given 3x3 fitting points
- (1) interpolate start/end moving B-spline curves with same u-knots, which are swept along directional curves
- (2) interpolate moving curves at v=v* with same u-knots
- (3) generate directional B-spline curves with same v-knots by interpolating the control points of the moving B-spline curves
- The control points of the directional curves are the control points of final bicubic B-spline surface

$\mathbf{d}_0(1)=\mathbf{d}_{04}$	$\mathbf{d}_1(1)=\mathbf{d}_{14}$	$\mathbf{d}_2(1)=\mathbf{d}_{24}$	$\mathbf{d}_3(1)=\mathbf{d}_{34}$	$\mathbf{d}_4(1)=\mathbf{d}_{44}$
$\mathbf{d}_0(\frac{1}{2})=\mathbf{t}_0$	$\mathbf{d}_1(\frac{1}{2})=\mathbf{t}_1$	$\mathbf{d}_2(\frac{1}{2})=\mathbf{t}_2$	$\mathbf{d}_3(\frac{1}{2})=\mathbf{t}_3$	$\mathbf{d}_4(\frac{1}{2})=\mathbf{t}_4$
$\mathbf{d}_0(0)=\mathbf{d}_{00}$	$\mathbf{d}_1(0)=\mathbf{d}_{10}$	$\mathbf{d}_2(0)=\mathbf{d}_{20}$	$\mathbf{d}_3(0)=\mathbf{d}_{30}$	$\mathbf{d}_4(0)=\mathbf{d}_{40}$

B-spline surface 정리



$$\mathbf{r}(u,v) = \sum_{j=0}^{5}\sum_{i=0}^{5}\mathbf{d}_{ij}N_i^3(u)N_j^3(v)$$

$$= \sum_{i=0}^{5}\left(\sum_{j=0}^{5}\mathbf{d}_{ij}N_j^3(v)\right)\cdot N_i^3(u)$$

$$= \sum_{j=0}^{5}\left(\sum_{i=0}^{5}\mathbf{d}_{ij}N_i^3(u)\right)\cdot N_j^3(v)$$

$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

Programming Guide (1)

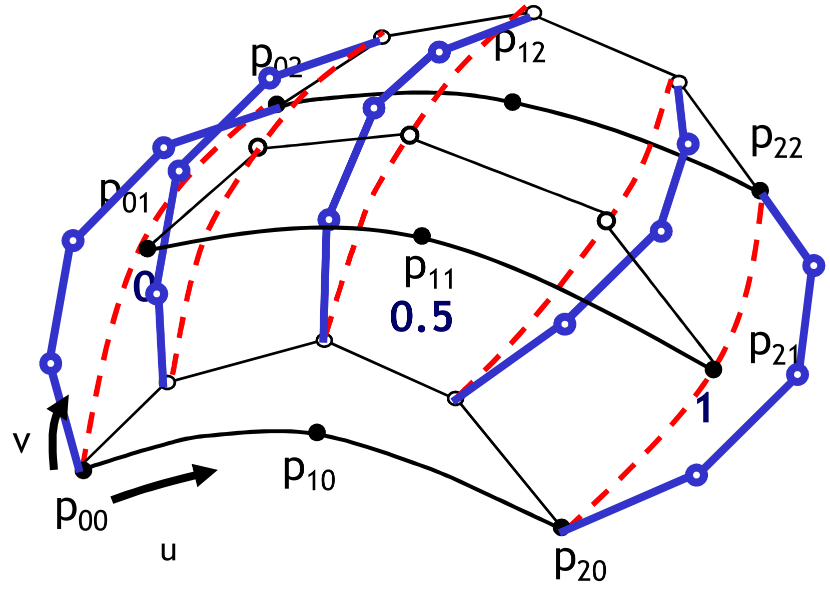
(1) B-spline Surface 생성

(2) u,v 방향 knot 구함

① 점 사이의 거리비를 통해 knot 계산
② 마지막 knot로 나누어 Normalize 함
③ 평균을 구함

(3) u 방향 B-spline 곡선의 Control Point 구함

(4) u방향의 B-spline 곡선의 Control point를 사용하여 v 방향의 B-spline 곡선을 생성하는 Control Point 생성



Programming Guide (2)

Given : 곡면을 지나는 공간상의 점 (MxN 개)

Find : Cubic B-spline Surface의 Control Points

(Q) 4x3의 점이 주어졌을 때, u,v 방향 knot 개수와 Control point 개수는?

(A) u 방향 : 4 + 3 x 2 = 10 개

v 방향 : 3 + 3 x 2 = 9 개

Control Point 개수 : 6 x 5 = 30 개

(Q) MxN의 점이 주어졌을 때, u,v 방향 knot 개수와 Control point 개수는?

(A) u 방향 : M + 3 x 2 = M + 6 개

v 방향 : N + 3 x 2 = N + 9 개

Control Point 개수 : (M+2) x (N+2) 개

Term Project #2
B-Spline Surface Interpolation

Advanced
Ship
Design
Automation
Laboratory

Term Project 2. B-Spline Surface Interpolation
- 과제 개요

과제 목표: 입력 받은 점을 지나는 B-Spline Surface를 Interpolation 한다.

Input Data

5 4

10.0 10.0 0.0

10.0 20.0 5.0

10.0 30.0 5.0

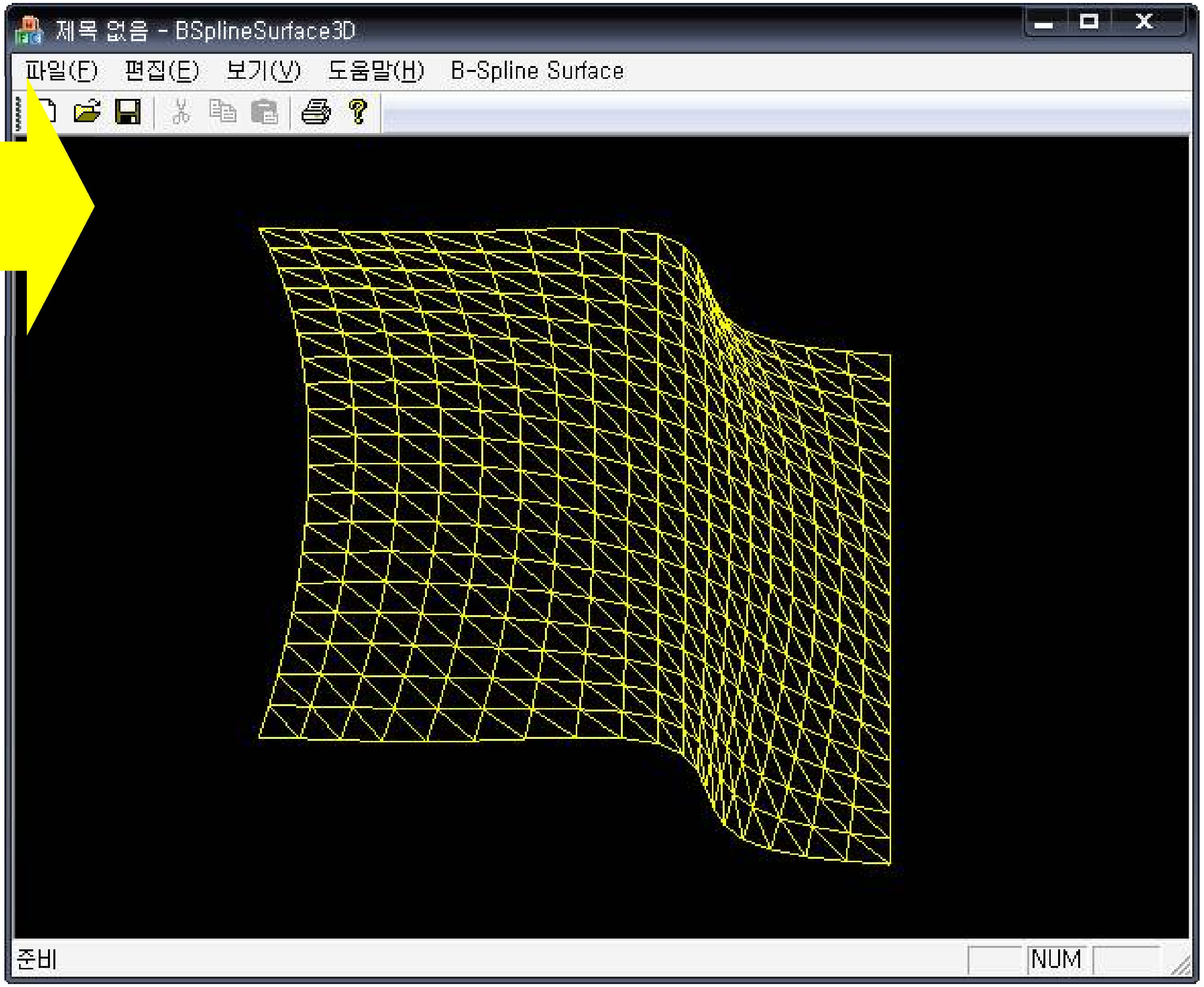
10.0 40.0 0.0

20.0 10.0 5.0

20.0 20.0 10.0

20.0 30.0 10.0

20.0 40.0 5.0



Term Project 2. B-Spline Surface Interpolation
- Input Data Format

Input Data

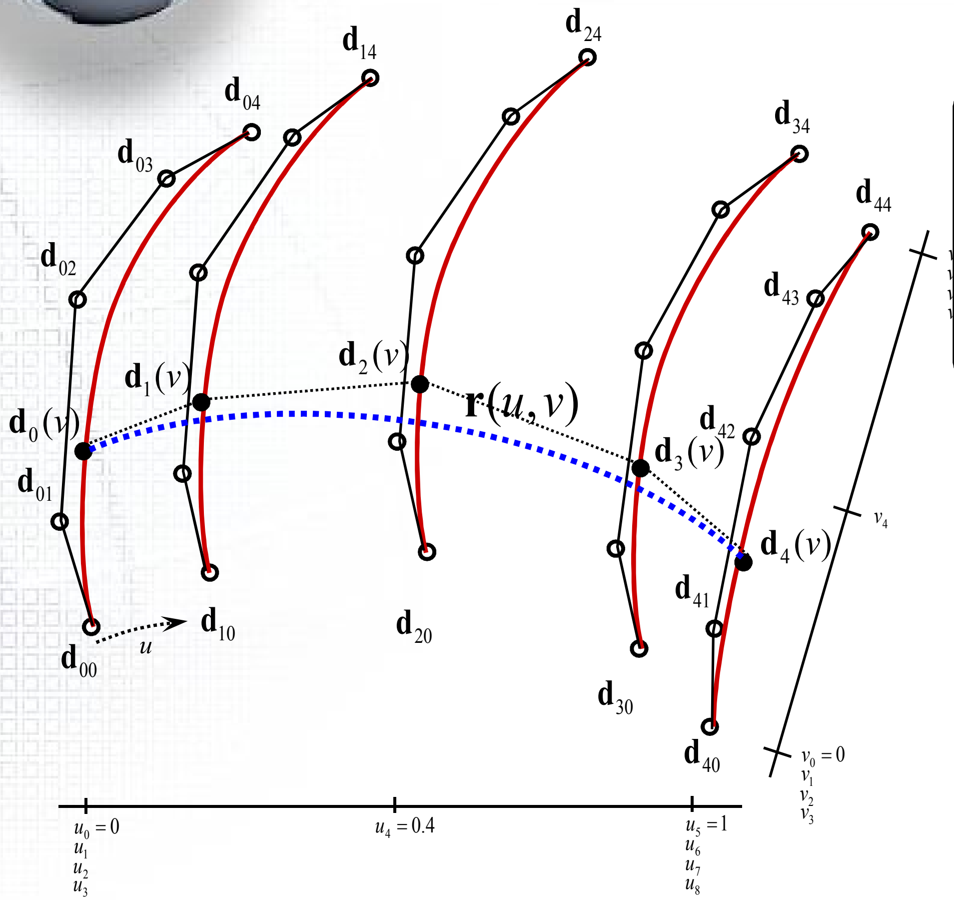
5 4 → 입력 받을 점의 u, v 방향 개수

10.0 10.0 0.0
10.0 20.0 5.0
10.0 30.0 5.0
10.0 40.0 0.0

20.0 10.0 5.0
20.0 20.0 10.0
20.0 30.0 10.0
20.0 40.0 5.0
…

→ 입력 받는 점의 3차원 좌표 (x, y, z)

bicubic B-spline Surface Interpolation



Cox-de Boor Recurrence Formula

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1 & \textbf{if } \; u_{i-1} \le u < u_i \\ 0 & \textbf{else} \end{cases} \quad , \sum_{i=0}^{D-1} N_i^n(u) = 1$$

$$\mathbf{r}(u,v) = \begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix} \begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

Term Project 2. B-Spline Surface Interpolation

- 과제 수행 시 프로그램 작성 부분

```
double CBSplineSurface::N(int n, int i, double u, int uv)
{
    // check invalid condition
    if (m_nNumOfKnot_U < 1) return 0.0;
    if (m_nNumOfKnot_V < 1) return 0.0;
    if (m_nDegree < 2) return 0.0;

    // initialize
    double val = 0.0;

    ///////////////////////////////////////////////////////////////////////////
    // B-Spline basis function 함수를 작성하시오.


    //
    ///////////////////////////////////////////////////////////////////////////

    return val;
}

Vector CBSplineSurface::GetPoint(double u, double v)
{
    // return value
    Vector vec(0.0, 0.0, 0.0);

    // check invalid condition
    if (m_nDegree < 2) return vec;
    if (m_nNumOfCP_U < 1) return vec;
    if (m_nNumOfCP_V < 1) return vec;

    ///////////////////////////////////////////////////////////////////////////
    // parameter u, v가 주어졌을 때 곡면 상의 점을 구하는 함수를 작성하시오.

    //
    ///////////////////////////////////////////////////////////////////////////

    return vec;
}
```

Class	CBSplineSurface
Function	N
내용	u, v parameter**에 따라** B-spline basis function**을 계산**

Cox-de Boor Recurrence Formula

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1 & \textbf{if} \;\; u_{i-1} \le u < u_i \\ 0 & \textbf{else} \end{cases}, \quad \sum_{i=0}^{D-1} N_i^n(u) = 1$$

Term Project 2. B-Spline Surface Interpolation
- 과제 수행 시 프로그램 작성 부분

```
Vector CBSplineSurface::GetPoint(double u, double v)
{
    // return value
    Vector vec(0.0, 0.0, 0.0);

    // check invalid condition
    if (m_nDegree < 2) return vec;
    if (m_nNumOfCP_U < 1) return vec;
    if (m_nNumOfCP_V < 1) return vec;

    ////////////////////////////////////////////////////////////////////////////
    // parameter u, v가 주어졌을 때 곡면 상의 점을 구하는 함수를 작성하시오.

    //
    ////////////////////////////////////////////////////////////////////////////

    return vec;
}
```

Class	CBSplineSurface
Function	GetPoint
내용	**주어진** parameter u, v**에 대한 곡면 상의 점을 구하는 함수**

$$\mathbf{r}(u,v)=\begin{bmatrix} N_0^3(u) & N_1^3(u) & N_2^3(u) & N_3^3(u) & N_4^3(u) \end{bmatrix}\begin{bmatrix} \mathbf{d}_{00} & \mathbf{d}_{01} & \mathbf{d}_{02} & \mathbf{d}_{03} & \mathbf{d}_{04} \\ \mathbf{d}_{10} & \mathbf{d}_{11} & \mathbf{d}_{12} & \mathbf{d}_{13} & \mathbf{d}_{14} \\ \mathbf{d}_{20} & \mathbf{d}_{21} & \mathbf{d}_{22} & \mathbf{d}_{23} & \mathbf{d}_{24} \\ \mathbf{d}_{30} & \mathbf{d}_{31} & \mathbf{d}_{32} & \mathbf{d}_{33} & \mathbf{d}_{34} \\ \mathbf{d}_{40} & \mathbf{d}_{41} & \mathbf{d}_{42} & \mathbf{d}_{43} & \mathbf{d}_{44} \end{bmatrix}\begin{bmatrix} N_0^3(v) \\ N_1^3(v) \\ N_2^3(v) \\ N_3^3(v) \\ N_4^3(v) \end{bmatrix}$$

Term Project 2. B-Spline Surface Interpolation
- 과제 수행 시 프로그램 작성 부분

```
bool CBSplineSurface::Interpolation(Vector** pPoint, int nNumOfPointU, int nNumOfPointV)
{
    // check invalid condition
    if (pPoint == NULL) return false;
    if (nNumOfPointU < 1) return false;
    if (nNumOfPointV < 1) return false;

    // initialize
    int i = 0;
    int j = 0;

    double* pAvgKnotU = NULL;
    int nNumOfAvgKnotU = 0;

    double* pAvgKnotV = NULL;
    int nNumOfAvgKnotV = NULL;


    //////////////////////////////////////////////////////////////////////////
    // bicubic B-Spline surface interpolation 함수를 작성하시오.

    // create curve (u direction or v direction)


    // set knot using chord length


    // normalize knot


    // set average knot


    // set average knot to each curve


    // interpolation



    // create curve (v direction of u direction)
```

Class	CBSplineSurface
Function	Interpolation
내용	**곡면 상의 주어진 점을 지나는** bicubic B-spline surface**의** Control Point**를 구하는 함수**