ζ : variation of fluid content (dimensionless)

$$\zeta = \frac{\Delta V_f^{(2)}}{V}$$

$$\Delta V_f = \Delta V_f^{(1)} + \Delta V_f^{(2)}$$

variation of fluid volume due to compression or dilation of fluid

variation of fluid volume due to fluid exchange.

in drained test, there is no change of pore pressure and $\Delta V_f^{(1)}$ is zero.

$$\begin{pmatrix} \varepsilon = \frac{1}{K}\sigma + \frac{\alpha}{K}p \\ \zeta = \frac{\alpha}{K}\sigma + \frac{\alpha}{KB}p \end{pmatrix}$$

$$\frac{1}{M} \equiv \left.\frac{\delta\zeta}{\delta p}\right|_{\epsilon=0}. \qquad \alpha = \left.\frac{\zeta}{\varepsilon}\right|_{p=0}, \quad \alpha = \left.-\frac{\sigma}{p}\right|_{\varepsilon=0}.$$

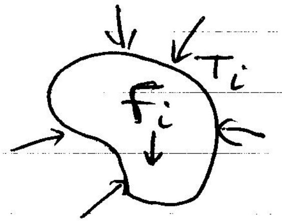
$$B = \left.-\frac{p}{\sigma}\right|_{\zeta=0}, = \left.-\frac{\varepsilon}{\zeta}\right|_{\sigma=0}.$$

1872, MSc thesis.

* Maxwell-Betti reciprocal theorem

Date . . No.

"The work W_{12} that would be done by the first set of forces if they acted through the displacement that are due to the second set of forces is equal to the work W_{21} that would be done by the second set of forces if they acted through the displacement that are due to the first set of forces".



ex) $\{\underline{F}^1, \underline{T}^1\}$, $\{\underline{F}^2, \underline{T}^2\}$

$\{\tau^1, \varepsilon^1, u^1\}$ $\{\tau^2, \varepsilon^2, u^2\}$.

$$W_{12} = \frac{1}{2}\iint_{\partial B} T^1 \cdot u^2 \, dA + \frac{1}{2}\iiint F^1 \cdot u^2 \, dV$$

$$= \frac{1}{2}\iiint_{B} \mathrm{trace}(\tau^1 \varepsilon^2)\, dV$$

$W^{12} = W^{21}$

$$\tau = \lambda \, \mathrm{trace}(\varepsilon) I + 2G\varepsilon .$$

$$W_{12} = \frac{1}{2}\iiint \mathrm{trace}\left[\{\lambda \mathrm{trace}(\underline{\underline{\varepsilon}}^1) I + 2G \underline{\underline{\varepsilon}}^1\} \underline{\underline{\varepsilon}}^2\right] dV$$

$$= \frac{1}{2}\iiint \mathrm{trace}\left[\{\lambda \mathrm{trace}(\underline{\underline{\varepsilon}}^1)\underline{\underline{\varepsilon}}^2 + 2G \underline{\underline{\varepsilon}}^1 \underline{\underline{\varepsilon}}^2\}\right] dV$$

$$= \frac{1}{2}\iiint \{\lambda \mathrm{trace}(\varepsilon^1)\mathrm{trace}(\varepsilon^2) + 2G \, \mathrm{trace}(\underline{\underline{\varepsilon}}^1 \underline{\underline{\varepsilon}}^2)\} dV.$$

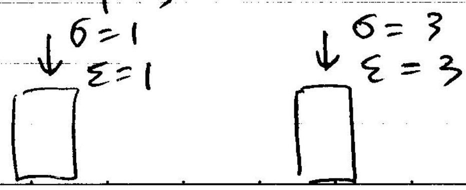
↑ Symmetric with respect to superscripts 1 & 2.

$\therefore W_{12} = W_{21}$

application : - ~~rep~~ relationships between pore & bulk compressibility (Geertsma (1957))

- Selvadurai (1982) : displacement caused by a point load to a rigid foundation on an elastic half-space
- BEM (Brady, 1979)

simple example)



$$\frac{1}{2}\times(1\times 3) = \frac{1}{2}(3\times 1)$$

11 May 2019
3 March

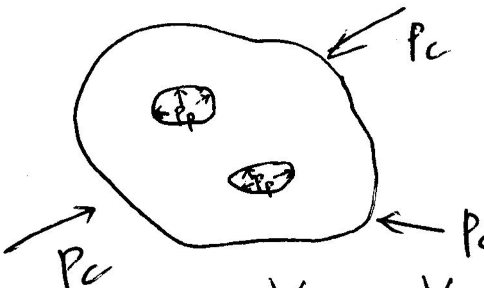
Poroelasticity.

poromechanics, hydroelasticity.

Biot (1941)
Frenkel (1939)

Biot did (derived) in a difficult way. not clear interpretation.

he didn't really talk about pore or fluid.
~~he that~~ thought water is incompressible.
water is more compressible than rock.



V_b = bulk volume
V_p = pore volume
V_m = mineral volume.
matrix? → this also has different meaning!

① Hydrostatic condition.

$t_i = -P n_i$

$$V_b = V_p + V_m$$

$$\phi = \frac{V_p}{V_B} = \text{porosity}.$$

$$e = \frac{V_p}{V_m} = \frac{\phi}{1-\phi} = \text{void ratio}.$$

$$C_{Bc} = -\frac{1}{V_b}\left(\frac{\partial V_b}{\partial P_c}\right)_{P_p} = \frac{1}{K_{bc}}$$

P_p → constant

$$C_{BP} = +\frac{1}{V_b}\left(\frac{\partial V_b}{\partial P_p}\right)_{P_c}$$

$$C_{pc} = -\frac{1}{V_p}\left(\frac{\partial V_p}{\partial P_c}\right)_{P_p}$$

$$C_{PP} = +\frac{1}{V_p}\left(\frac{\partial V_p}{\partial P_p}\right)_{P_c}$$

under (+)

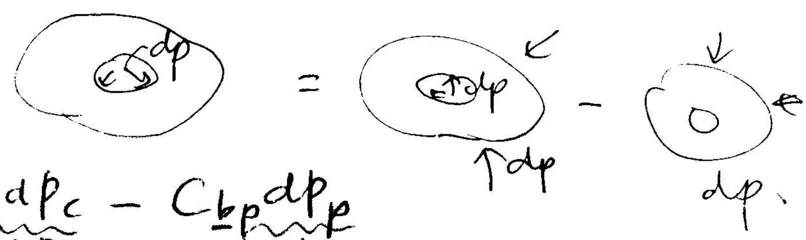
$$d\varepsilon_b = -\frac{dV_b}{V_b} = -\frac{1}{V_b}\left(\frac{\partial V_b}{\partial P_c}dP_c + \frac{\partial V_b}{\partial P_p}\partial P_p\right)$$

bulk strain increment

$$d\varepsilon_b = -\frac{1}{V_b}\left(-V_b C_{bc}\partial P_c + V_b C_{Bp} dP_p\right)$$

we just want to have more general relationship that include non-linear stage

$\dfrac{-dV_b}{V_b}$



$$d\varepsilon_b = C_{bc}\, dP_c - C_{bp}\, dP_p$$

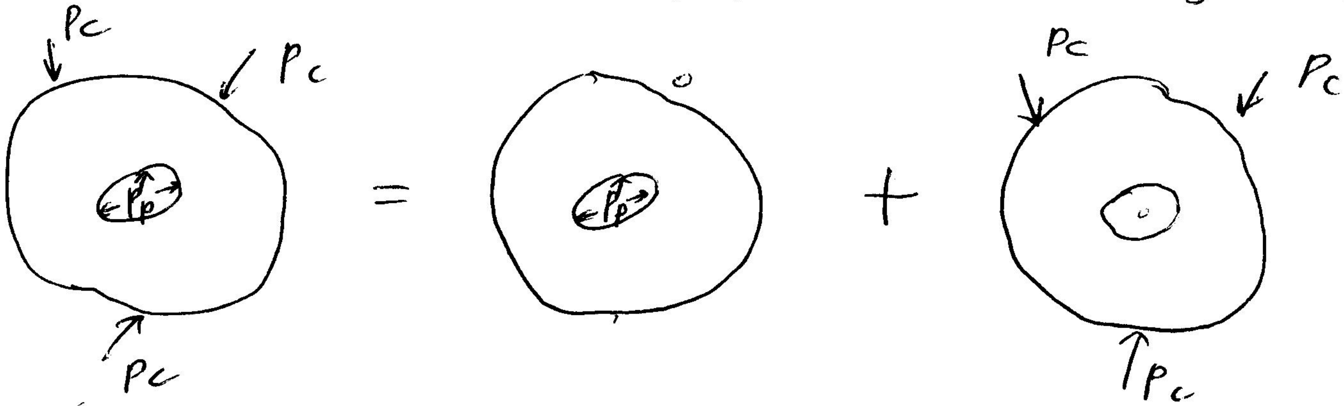
$$d\varepsilon_p = C_{pc}\, dP_c - C_{pp}\, dP_p$$

$$= \frac{-dV_p}{V_p} = ?$$

So far they are just definition.

Main assumption,

mineral phase is isotropic, elastic and homogeneous.



(I) $\qquad \dfrac{1}{V_p}\cdot\left(\dfrac{\partial V_p}{\partial p_c}\right)$ $\qquad$ (II) $\qquad$ (III)

Let all pressuric increment be equal to "dp"

$$dV_b\ (I) = dV_b\ (II) + dV_b\ (III).$$

$$dV_b = -V_b\,(C_{bc}\, dP_c - C_{bp}\, dP_p) \quad (I)$$

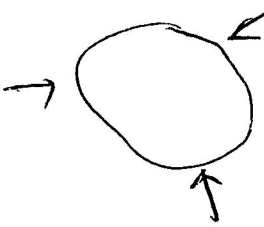
$$dV_b = -V_b\,(0 - C_{bp}\, dP_p) = V_b C_{bp}\, dp \quad (II)$$

$$dV_{pb} = -V_b\,(C_{bc}\, dp - 0) = -V_b C_{bc}\, dp \quad (III) \longrightarrow$$

$$V_b\,(C_{bp} - C_{pc})\, dp = -V_b\,(C_{bc} - C_{bp})\, dp.$$

now left hand side.

First imagine that the rock has no pores. and under uniform pressures.



dp. $\quad T_i = dp\, n_i = dp\, \delta_{ij}\, n_j$

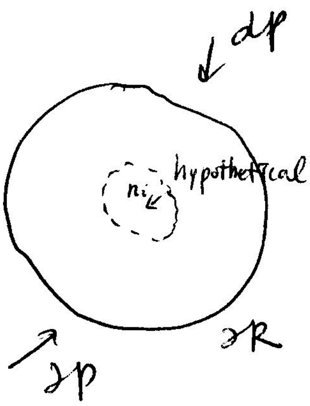
$$\tau_{ij} = (dp)\,\delta_{ij} = \begin{pmatrix} dp & 0 & 0 \\ 0 & dp & 0 \\ 0 & 0 & dp \end{pmatrix}.$$

claim $\tau_{ij} = (dp)\,\delta_{ij}$ inside R

satisfy equilbrm eqn. $\quad \tau_{ij,j} + f_i = 0.$

it is uniform regardless of its shape.

$(dp)\,\delta_{ij,j}$



what is the traction on that surface.

$$T_i = \tau_{ij} n_j = dp\, \delta_{ij} n_j = dp\, n_i$$

i.e. no shear, uniform pressure of magnitude dp.

✓ If I took out the hypothetical part, we should replace the stress.

$d\varepsilon_m = C_m dp$, $\frac{dV_m}{V_m} \cdot \frac{1}{dp} = C_m$

$\rightarrow d\varepsilon_b = C_m dp$.

$\therefore \quad dV_b \; (I) = -V_b C_m dp$

$$C_m = -\frac{1}{V_b}\left(\frac{\partial V_b}{\partial p_c}\right), \qquad dV_b = -V_b C_m dp_c = -V_b C_m dp.$$

$$-V_b C_m dp = -V_b (C_{BC} - C_{BP}) dp.$$

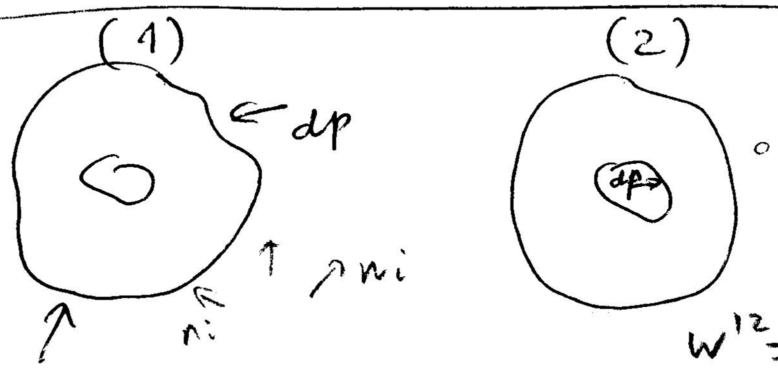
$$C_{BC} - C_{BP} = C_m \quad \cdots\cdots (1)$$

HW exactly the same argument for pore change.

$$C_{PC} - C_{PP} = C_m \quad \cdots\cdots (2)$$

C_m, C_{BC}, C_{BP}, C_{PC}, C_{PP}.

4 unknowns, 2 eqns.



$W^{12} = -dP \times [\, dV_b(0, dp)]$

Betti's reciprocal theorem. $= W^{21} = dP \times [dV_p(dP, 0)]$

W^{12} work done by the first set of applied loads acting through the displacement due to the 2nd set of loads.

$$= -dp \cdot \underbrace{V_b C_{BP} dp}_{\Delta V_b \text{ due to loading state(2)}} = -V_b C_{BP} (dp)^2$$

deform -ni direction

$$W^{21} = dP(-V_p \cdot C_{pc} dP) = -V_p C_{pc} (dp)^2.$$

$$W_{12} = W_{21}, \quad V_b C_{Bp} = V_p C_{pc}.$$

$$\underline{C_{Bp} = \frac{V_p}{V_b} C_{pc} = \phi C_{pc}}$$

We don't need to assume isotropy so far.
doesn't require rock to be isotropic or homogeneous.
now we have three eqns, for four unknowns.

$$\boxed{\begin{aligned} C_{BC} - C_{Bp} &= C_m \\ C_{pc} - C_{pp} &= C_m \\ C_{Bp} &= \phi C_{pc} \end{aligned}}$$

(if we had four eqns every rock has same property! why!)

Once we kn one, we can calculate the others.

easy to measure, $C_{pp} \to$ very difficult
but petroleum engineer wants to know that!

SKIP

$\underline{V_m.\ \phi}$ $\to$ Let's try other terms. instead of $\underline{V_p, V_b}$

$$V_m = V_B - V_p$$

$$\frac{dV_m}{V_m} = \frac{dV_b}{V_m} - \frac{dV_p}{V_m} = \frac{V_b}{V_m}\left(\frac{dV_b}{V_b}\right) - \frac{V_p}{V_m}\left(\frac{dV_p}{V_p}\right)$$

$$= \frac{V_b}{(1-\phi)V_b}\frac{dV_b}{V_b} - \frac{\phi}{1-\phi}\frac{V_b}{V_b}\frac{dV_p}{V_p}$$

$$d\varepsilon_m = \frac{1}{1-\phi} d\varepsilon_b - \frac{\phi}{1-\phi} d\varepsilon_p$$

$$= \frac{1}{1-\phi}(C_{BC} dP_c - C_{Bp} dP_p) - \frac{\phi}{1-\phi}(C_{pc} dP_c - C_{pp} dP_p)$$

$$= \frac{1}{1-\phi}\Big[\underbrace{(C_{BC} - \underbrace{\phi C_{pc}}_{C_{Bp}})}_{C_m} dP_c - (\underbrace{C_{Bp}}_{\phi C_{pc}} - \phi C_{pp}) dP_p\Big]$$

$$\phi(C_{pc} - C_{pp})$$

$$= \frac{1}{1-\phi}(C_m dP_c - \phi C_m dP_p)$$

$$= \frac{C_m}{1-\phi}(dP_c - \phi\, dP_p) = C_m\, d\left(\frac{P_c - \phi P_p}{1-\phi}\right)$$

mineral

mineral $\quad d\varepsilon_m = C_m \, dp$

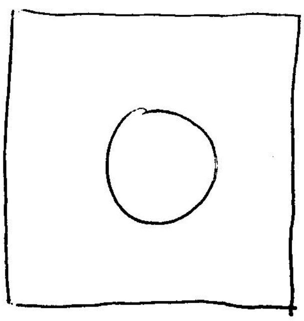
$\langle \varepsilon_m \rangle = C_m \langle p \rangle$

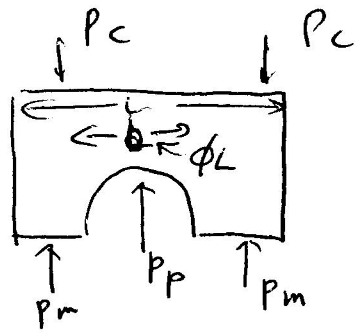
average

$$\therefore \quad \varepsilon_m = C_m \left(\frac{P_c - \phi P_p}{1-\phi} \right) \quad \text{average stress}$$

equation regardless of pore shape.

Technical note.





W into page.

Force acting down $= P_c L W$

" up $= P_p \phi L W + P_m (1-\phi) L W$

$$P_c L W = P_p \phi L W + P_m (1-\phi) L W$$

$$P_c = P_p \phi + P_m (1-\phi)$$

$$P_m = \frac{P_c - \phi P_p}{1-\phi}$$

$$d\varepsilon_m = \frac{C_m}{1-\phi} \left(dP_c - \phi \, dP_p \right)$$

$$\phi = \frac{V_p}{V_B} \Rightarrow \frac{d\phi}{\phi} = d\log\phi = d\log V_p - d\ln V_p$$

$$= \frac{dV_p}{V_p} - \frac{dV_B}{V_B}$$

$$= -(C_{pc}\,dP_c - C_{pp}\,dP_p) + (C_{Bc}\,dP_c - C_{Bp}\,dP_p)$$

$$= (C_{Bc} - C_{pc})\,dP_c - (C_{Bp} - C_{pp})\,dP_p$$

$C_{Bc} - C_m - (C_{pc} - C_m)$

$$d\phi = -\left[(1-\phi)\,C_{Bc} - C_m\right] d(P_c - P_p) \qquad C_{Bc} - C_{pc} = C_m + C_{Bp} - C_{pc}$$

$= C_m$

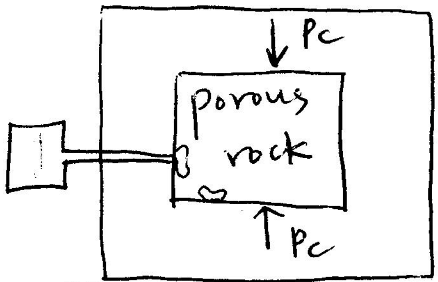
$P_c - nP_p = P_{\text{effective}}$.

diffent paramters have diffent effective pressure ⌋ skip

Terzaghi actully confused many (including himself)!

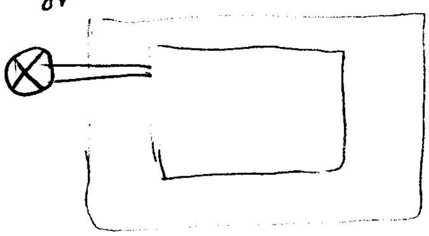
deformation of fluid saturated rock at first takes place in an undrained manner → after some time drained

undrained condition. → ~~confined pressure & pore pressure are not independent~~ !



~~un~~ drained

or



undrained

→ this is common in real case.

$$d\varepsilon_b = C_{Bc}\, dP_c - C_{Bp}\, dP_p.$$
$$d\varepsilon_p = C_{pc}\, dP_c - C_{pp}\, dP_p$$

) → always true ~~even when~~ P_p & P_c are coupled.

$$d\varepsilon_p = \frac{dV_p}{V_p} = \frac{dV_f}{V_f} = C_f \cdot dP_p$$

$$C_{pc} dP_c - C_{pp} dP_p = C_f \cdot dP_p$$
$$C_{pc} dP_c = (C_{pp} + C_f)\, dP_p.$$
$$dP_p = \boxed{\frac{C_{pc}}{(C_{pp} + C_f)}}\, dP_c.$$

B. Skempton's coefficient (1954).

$$B = \frac{C_{pp} + C_m}{C_{pp} + C_f} < 1 \quad (C_f > C_m)$$

$$= \frac{C_{pp}}{C_{pp} + C_f} = \frac{1}{1 + \left(\frac{C_f}{C_{pp}}\right)}$$

(20 times ~~of pore~~)

sandstone

$C_m = 0.286 \times 10^{-4}$ /MPa.

$C_{bc} = 1.31 \times 10^{-4}$ /MPa

$C_{pp} = 11.8 \times 10^{-4}$ /MPa

$C_{f,air} = 9.87$ / MPa → $C_{bu} = 1.31 \times 10^{-4}$/MPa

$C_{f,water} = 5 \times 10^{-4}$ /MPa → $C_{bu} = 0.672 \times 10^{-4}$ /MPa

Air $C_{air} = \infty$. $B \to 0$.

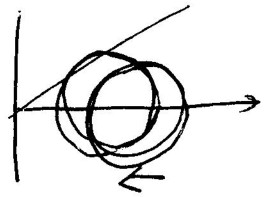
$C_{f,hypothetical} = 0$. → $C_{bu} = 0.261 \times 10^{-4}$/MPa

ζ : variation of fluid content

$= \frac{\Delta V_f^{(2)}}{V}$. $\Delta V_f = \Delta V_{f(1)} + \Delta V_f^{(2)}$

variation of

Implication of Skempton.



if rock is behaving in undrained way more likelihood of failure.

also micromechanics, —— . bone.

Lets define undrained compressibility.

$$d\varepsilon_b = C_{Bc}\, dP_c - C_{Bp} \frac{B\, dP_c}{dP_p} = (C_{Bc} - B C_{Bp})\, dP_c .$$

C_B undrained

$$C_{Bu} = \left(\frac{\partial \varepsilon_b}{\partial P_c}\right)_{\zeta=0} = C_{Bc} - B\, C_{Bp} \qquad B = \frac{C_{pc}}{C_{pp} + C_f}$$

$$C_{Bp} = C_{Bc} - C_m .$$

$$= C_{Bc} - B(C_{Bc} - C_m) = (1-B) C_{Bc} + B C_m$$

$$C_{Bu} = \boxed{\frac{\phi C_{bc} (C_f - C_m) + C_m (C_{Bc} - C_m)}{\phi (C_f - C_m) + (C_{Bc} - C_m)}}$$

Gassmann's Eqn. (1951)

→ most common way of expressing

→ very important in seismic wave.

$$V_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \qquad (C_{bc})$$

situation in seismic is undrained condition.

$$V_s = \sqrt{\frac{\mu}{\rho}} .$$

"4D seismic"

Fortunlon sandstone

$C_{Bc} = 1.31 \times 10^{-4}$ /MPa

$C_m = 0.286 \times 10^{-4}$ /MPa .

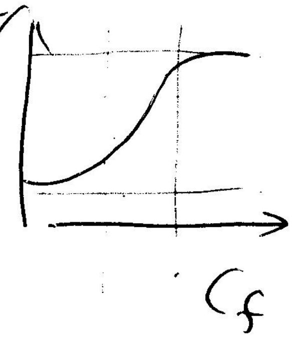
$C_m : C_{Bc} \fallingdotseq 1 : 4$

$C_w = 5 \times 10^{-4}$ /MPa.

$C_{air} = 1.25$ /MPa

$\phi = 0.085$

$C_{Bu} = 0.573 \times 10^{-4}$ /MPa .



Hooke's law for poroelasticity.

non-porous rock

$$\varepsilon_{ij} = \frac{1}{2G}\left(\tau_{ij} - \frac{\nu}{1+\nu}\tau_{kk}\delta_{ij}\right)$$

$$\varepsilon_b = \varepsilon_{ii} = \frac{1}{2\mu}\left(\tau_{ii} - \frac{3\nu}{1+\nu}\tau_{kk}\right) = \frac{3\tau_{ii}}{2\mu\,3}\left(\frac{1-2\nu}{1+\nu}\right)$$

$$\frac{\tau_{11}+\tau_{22}+\tau_{33}}{3} = \tau_m = \text{"}p_c\text{"}$$

$$= \frac{3(1-2\nu)}{E}p_c = \frac{1}{K}p_c$$

Biot coefficient, α,

$$d\varepsilon_b = C_{Bc}\,dP_c - C_{Bp}\,dP_p$$

$$\varepsilon_b = C_{Bc}P_c - C_{Bp}P_p.$$

① if $P_p = 0$, $\varepsilon_b = C_{Bc}P_c$. $\quad C_{Bc} = \frac{1}{K}$.

② if $P_p \neq 0$.

P_p causes a bulk strain of $-C_{Bp}P_p$.

$$\varepsilon_b = \varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33} = -C_{Bp}P_p.$$

For isotropic rock, $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = -\frac{C_{Bp}}{3}P_p$

$\varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 0$ under hydrostatic stress.

$$\begin{pmatrix} -\frac{C_{Bp}P_p}{3} & 0 & 0 \\ 0 & -\frac{C_{Bp}P_p}{3} & 0 \\ 0 & 0 & -\frac{C_{Bp}P_p}{3} \end{pmatrix} \rightarrow -\frac{C_{Bp}}{3}P_p\delta_{ij}$$

simply add this term!

porous rock

$$\varepsilon_{ij} = \frac{1}{2\mu}\left(\tau_{ij} - \frac{\nu}{1+\nu}\tau_{kk}\delta_{ij}\right) - \frac{C_{Bp}}{3}P_p\delta_{ij}$$

$$\varepsilon_b = \varepsilon_{ii} = \frac{1}{2\mu}\left(\tau_{ii} - \frac{3\nu}{1+\nu}\tau_{kk}\right) - C_{Bp}P_p = \frac{3}{2\mu}\left(\frac{\tau_{ii}}{3}\right)\left(\frac{1-2\nu}{1+\nu}\right) - C_{Bp}P_p$$

$$= \frac{1}{K}P_c - C_{Bp}P_p = C_{Bc}P_c - C_{Bp}P_p$$

$$= C_{Bc}\left(P_c - \frac{C_{Bp}}{C_{Bc}}P_p\right) = \frac{1}{K}(P_c - \alpha P_p)$$

Biot's coefficient

$$= 1 - \frac{K}{K_m} < 1.$$

$$\alpha = \frac{C_{BP}}{C_{BC}} = \frac{C_{BC} - C_m}{C_{BC}} = 1 - \frac{C_m}{C_{BC}} < 1$$

in general, $1.5\phi < \alpha < 1$.

(Fatt, 1957) $\alpha = 0.85$.

$$C_{\beta p} = \alpha C_{BC} = \frac{\alpha}{K}.$$

last time?

$$\boxed{\varepsilon_{ij} = \frac{1}{2\mu}\left(\tau_{ij} - \frac{\nu}{1+\nu}\tau_{kk}\delta_{ij}\right) - \frac{\alpha}{3K}P_p\delta_{ij}}$$

invert this to form τ_{ij} in terms of ε_{ij}.

$$\underset{\varepsilon_{kk}}{\varepsilon_{ii}} = \frac{\tau_{kk}}{3K} - \frac{3\alpha P_p}{3K} = \frac{\tau_{kk}}{3K} - \frac{\alpha}{K}P_p,$$

$$3K\varepsilon_{ii} = \tau_{kk} - 3\alpha P_p, \quad \tau_{kk} = \overset{3K\varepsilon_{kk}}{3K\varepsilon_{ii}} + 3\alpha P_p.$$

$$2\mu\varepsilon_{ij} = \tau_{ij} - \frac{\nu}{1+\nu}(3K\underset{kk}{\varepsilon_{ii}} + 3\alpha P_p)\delta_{ij} - 2\mu\frac{\alpha}{3K}P_p\delta_{ij}, \qquad 2\mu = \frac{E}{1+\nu}$$

$$3K = \frac{E}{1-2\nu}$$

$$2\mu\varepsilon_{ij} = \tau_{ij} - \frac{3K\nu}{1+\nu}\underset{kk}{\varepsilon_{ii}}\delta_{ij} - \frac{3\alpha\nu}{1+\nu}P_p\delta_{ij} - \left(\frac{1-2\nu}{1+\nu}\right)\alpha P_p\delta_{ij} \qquad \frac{2\mu}{3K} = \frac{1-2\nu}{1+\nu}.$$

$$= \tau_{ij} - \frac{3\nu}{1+\nu}\frac{2\mu(1+\nu)}{3(1-2\nu)}\underset{kk}{\varepsilon_{ii}}\delta_{ij} - \frac{1+\nu}{1+\nu}\alpha P_p\delta_{ij} \qquad K = \frac{2\mu(1+\nu)}{3(1-2\nu)}.$$

$$= \tau_{ij} - \underbrace{\frac{\nu}{1-2\nu}2\mu}_{\lambda}\underset{kk}{\varepsilon_{ii}}\delta_{ij} - \alpha P_p\delta_{ij}$$

$$= \tau_{ij} - \lambda\varepsilon_{kk}\delta_{ij} - \alpha P_p\delta_{ij}$$

$$\tau_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} + \alpha P_p\delta_{ij}$$

$$\boxed{\underbrace{\tau_{ij} - \alpha P_p\delta_{ij}}_{\tau_{ij}^{effective}} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}}$$

Hooke's law for poreelastic material.

def. $$\begin{cases} A_{ij}^{iso} = \left(\frac{A_{kk}}{3}\right)\delta_{ij} \\ A^{dev} = A_{ij} - A_{ij}^{iso} \end{cases}$$

hw $$\begin{cases} \varepsilon_{ij}^{iso} = \frac{1}{3K}\tau_{ij}^{iso} - \alpha P_p\delta_{ij} \\ \varepsilon_{ij}^{dev} = \frac{1}{2\mu}\tau_{ij}^{dev} \end{cases}$$

in a fully saturated, single phase case.

$V_f = V_p$. m_f : mass of fluid.

$$m_f = \rho_f \cdot V_f = \rho_f V_p \Rightarrow V_p = \frac{m_f}{\rho_f}$$

incremental pore volume, dV_p.

$$dV_p = \frac{dm_f}{\rho_f} - \frac{m_f}{\rho_f^2} d\rho_f = \frac{dm_f}{\rho_f} - \underbrace{\frac{m_f}{\rho_f}}_{V_p} \cdot \underbrace{\frac{d\rho_f}{\rho_f}}_{C_f dP_p}$$

(?)

The change in the volumetric content of a certain region of a rock is broken into two parts.

$$= \frac{dm_f}{\rho_f} - V_p \cdot C_f dP_p.$$

$$\frac{dV_p}{V_b} = \frac{1}{V_b} \frac{dm_f}{\rho_f} - \frac{V_p}{V_b} C_f \cdot dP_p$$

$$C_f = -\frac{1}{V_f} \frac{dV_f}{dP_p}$$

additional fluid mj into the region.

$$\frac{dV_f}{V_b} = \underbrace{\frac{1}{V_b} \frac{dm_f}{\rho_f}}_{\text{mass transfer}} - \phi C_f \cdot dP_p.$$

$d\xi$ = excess fluid content (Biot)

due to compressn of pore fluid that is already in the region.

$$d\xi = \frac{dV_p}{V_b} + \phi C_f dP_p$$

$$\frac{V_p}{V_b} \cdot \frac{dV_p}{V_p} + \phi C_f dP_p.$$

$V_p = \phi V_b$

$$= \frac{1}{V_b} V_p(-C_{pc} dP_c + C_{pp} dP_p) + \phi C_f dP.$$

$$d\xi = -\phi C_{pc} dP_c + \phi C_{pp} dP_p + \phi C_f dP_p$$

$$= -\phi \left(C_{pc} dP_c - (C_{pp} + C_f) dP_p \right)$$

undrained compressn, $d\xi = 0$.

$$\left(\frac{dP_p}{dP_c}\right)_{\xi=0} = \frac{C_{pc}}{C_{pp} + C_f} = B$$

$$d\xi = -\phi C_{pc} \left(dP_c - \frac{1}{B} dP_p \right) = -\frac{\alpha}{K}\left(dP_c - \frac{1}{B} dP_p\right)$$

$$\xi = -\frac{\alpha}{K}\left(P_c - \frac{1}{B} P_p\right) \quad \cdots (1)$$

$$\frac{1}{B} > 1$$

but not common terminology) $\leftarrow$ effective stress coeffnt for ξ contrary to people's idea of coeffnt $B < 1$

$$\varepsilon_b = \frac{1}{K}(P_c - \alpha P_p)$$

$P_c = K\varepsilon_b + \alpha P_p$. → into fluid content. ①

$$\zeta = -\frac{\alpha}{K}\left(K\varepsilon_b + \alpha P_p - \frac{1}{B}P_p\right)$$

$\varepsilon = 0$, $\sigma = -\alpha p$.

$\zeta = \frac{\alpha}{K}\cdot(-\alpha p) + \frac{\alpha}{KB}p$

$\zeta = \frac{\alpha(1-\alpha B)}{KB} p$.

$$K\zeta = -\alpha K\varepsilon_b - \alpha\left(\alpha - \frac{1}{B}\right)P_p .$$

$$P_p' = \boxed{\frac{BK}{\alpha(1-\alpha B)}}(\zeta + \alpha\varepsilon_b) .$$

$$\frac{1}{M} \equiv \left.\frac{\delta\zeta}{\delta p}\right|_{\epsilon=0}$$

M = Biot modulus. (Biot → no physical explan)

there is no simple experiment that can measure M directly. So write explicitly.

$\frac{1}{M}$: specific storage coeffint at constant strain.

$= \frac{\alpha}{K_u B}$.

calculation of K_u

$$~~K\varepsilon_b~~ = P_c - \alpha P_p .$$

$$= P_c - \frac{BK}{1-\alpha B}\cdot(\zeta + \alpha\varepsilon_b)$$

$$K\varepsilon_b\left(1 + \frac{\alpha B}{1-\alpha B}\right) = P_c - \frac{BK}{1-\alpha B}\zeta$$

$$\frac{K\varepsilon_b}{1-\alpha B} = P_c - \frac{BK}{1-\alpha B}\zeta .$$

when $\zeta = 0$.

$$d\varepsilon_b = \frac{1-\alpha B}{K} dP_c = C_{Bu} dP_c$$

$$C_{Bu} = \frac{1-\alpha B}{K} = C_{Bc}(1-\alpha B)$$

$$\therefore \begin{cases} C_{bu} = C_{bc}^{\text{drained}}(1-\alpha B) \\ K_u = \dfrac{K}{1-\alpha B} = K + \alpha^2 M \end{cases}$$

show.

$1-\alpha B = \frac{K}{K_u}$ $\quad 1 - \frac{K}{K_u} = \alpha B.$ $\quad B = \frac{K_u - K}{\alpha K_u}$

$$B = \frac{K_u - K}{\alpha K_u} .$$

Summary.

in Wang.

$$\left\langle \begin{aligned} \varepsilon &= \frac{1}{K} P_c - \frac{\alpha}{K} P_p \\ \zeta &= -\frac{\alpha}{K} P_c + \frac{\alpha}{KB} P_p \end{aligned} \right\rangle \qquad \left\langle \begin{aligned} \varepsilon &= \frac{1}{K}\sigma + \frac{\alpha}{K} p \\ \zeta &= \frac{\alpha}{K}\sigma + \frac{\alpha}{KB} p \end{aligned} \right\rangle$$

$$\alpha = \left.\frac{-\zeta}{\varepsilon}\right|_{P_p=0}. \qquad \alpha = \left.\frac{P_c}{P_p}\right|_{\varepsilon=0} \qquad \alpha = 1 - \frac{K}{K_m}.$$

$$B = \left.\frac{P_p}{P_c}\right|_{\zeta=0}, \qquad B = \left.-\frac{\varepsilon}{\zeta}\right|_{P_c=0}. \qquad B = \frac{C_{pc}}{C_{pp}+C_f}.$$

☆ A few simple types of loading / deformation.

① Unjacketed hydrostatic compression ; $\tau_{xx} = \tau_{yy} = \tau_{zz} = P_p = p$.

$$\varepsilon_b = \frac{(1-\alpha)}{K} p = \frac{p}{K_m}.$$

② Drained uniaxial compression with no lateral strain; $\tau_{zz} > 0$.

$\varepsilon_{xx} = \varepsilon_{yy} = P_p = 0$.

common in soil mechanics tests.

$$\tau_{ij} - \alpha P_p \delta_{ij} = 2\mu \varepsilon_{ij} + \lambda \varepsilon_{kk} \delta_{ij}$$

$$\varepsilon_{zz} = \frac{1}{\lambda + 2G} \tau_{zz}.$$

③ Undrained uniaxial compression with no lateral strain,

$\tau_{zz} \geq 0$, $\varepsilon_{xx} = \varepsilon_{yy} = \zeta = 0$. $(\leftarrow \varepsilon_b = \varepsilon_{zz})$

$P_p = M(\zeta + \alpha\varepsilon_b)$

$M = \frac{BK}{\alpha(1-\alpha B)}$

$$P_p = \alpha M \varepsilon_{zz} = \frac{BK}{1-\alpha B} \varepsilon_{zz}.$$

$$\tau_{zz} = (\lambda + 2G + \alpha^2 M)\, \varepsilon_{zz}.$$

pore pressure induced by axial stress is

$$P_p = \frac{\alpha M}{\lambda + 2G + \alpha^2 M} \tau_{zz}$$

Where do we stand? 25 May 2009

10 March

Equatn of Eqilibrium and fluid flow.

$$\tau_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} + \alpha P_p\delta_{ij}$$
$$= 2\mu\left(\tfrac{1}{2}(u_{i,j}+u_{j,i})\right) + \lambda\varepsilon_{kk}\delta_{ij} + \alpha P_p\delta_{ij}$$
$$\tau_{ij,j} + f_i = \mu(u_{i,jj} + u_{j,ij}) + \lambda u_{k,kj}\delta_{ij} + \alpha P_{p,j}\delta_{ij} = 0$$
$$= \mu u_{i,jj} + \mu u_{j,ji} + \lambda u_{k,ki} + \alpha P_{p,i} + f_i = 0.$$
$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ji} + (f_i + \alpha P_{p,i}) = 0.$$

Biot coefficient $\left(\alpha\frac{\partial P_p}{\partial x_1},\ \alpha\frac{\partial P_p}{\partial x_2},\ \alpha\frac{\partial P_p}{\partial x_3}\right)$

looks like body force.

(mathematically) play the role of additional body force.

P_p P_p if you forget about blood it makes sense.

hw

in terms of excess fluid content, ζ

$$\mu u_{i,jj} + (\lambda + \mu + \alpha^2 M)u_{j,ji} + f_i + \alpha M\zeta_{,i} = 0.$$

$\frac{BK}{\alpha(1-\alpha B)}$

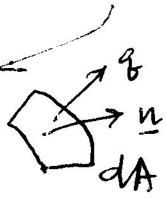
3 eqn & 4 unknowns.

We need one more eqn.

Start with fluid flow vector, q_i (m/s)

we are not talking about individual parts.

REV



$q \cdot n\, dA$ = volumetric flux through dA.

tangential compont q_t is just circulating in the surface of dA.

$$q' = RqR' = \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix}$$

Total Flow out of this region R ∂R

$$= \iint_{\partial R} q\cdot n\, dA = \iint_{\partial R} q_i n_i\, dA = -\frac{\partial}{\partial t}\iiint_R \zeta\, dV = -\iiint_R \frac{\partial\zeta}{\partial t}\, dV.$$

(we already subtracted the compressibility term).

$\frac{\partial}{\partial t}\iiint \zeta\, dV$: mass transfer across the outer boundary

$\iint q\cdot n\, dA$: total volumetric flux of fluid leaving this region per unit time!

div theorem

$$\iiint_R q_{i,i}\, dV = -\iiint \frac{\partial \xi}{\partial t}\, dV$$

$$\iiint_R \left(q_{i,i} + \frac{\partial \xi}{\partial t}\right) dV = 0$$

$$q_{i,i} + \frac{\partial \xi}{\partial t} = 0 .$$

How about $q_{i,i}$?

$\rightarrow$ use Darcy's law.

$$q = -\frac{1}{\mu} \underline{\underline{k}}\ \mathrm{grad}\, P_p \Rightarrow \underline{q_i = -\frac{1}{\mu} k_{ij} P_{p,j}}$$

$$q_1 = -\frac{1}{\mu}\left(k_{11}\frac{\partial P}{\partial x_1} + k_{12}\frac{\partial P}{\partial x_2} + k_{13}\frac{\partial P}{\partial x_3}\right)$$

similarly for q_2 & q_3.

$$\underline{\underline{k}} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}$$

theorem. k is symmetric, ie, $k_{ij} = k_{ji}$ ex) $K_{12} = K_{21}$.

Robert think there is no proof about this
empiricism

For isotropic

$$\underline{k} = k\,\delta_{ij} = \begin{pmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{pmatrix} .$$

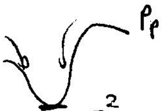
1 Darcy $\doteqdot 10^{-12}\ m^2$

don't use 'K' hydraulic conductivity.

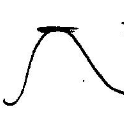
$$q_i = -\frac{k}{\mu}\delta_{ij} P_{p,j} = -\frac{k}{\mu} P_{p,i} = -\frac{k}{\mu}\,\mathrm{grad}\, P$$

$$-\frac{k}{\mu} \cdot P_{p,ii} + \frac{\partial \xi}{\partial t} = 0 \qquad \frac{\partial \xi}{\partial t} = \frac{k}{\mu} P_{p,ii} = \frac{k}{\mu}\nabla^2 P_p$$

$$\frac{\partial \xi}{\partial t} = \frac{k}{\mu} \cdot \nabla^2 P_p$$



$\nabla^2 P_p = \oplus \quad \frac{\partial \xi}{\partial t} > 0 .$



$\nabla^2 P_p = \ominus \quad \frac{\partial \xi}{\partial t} < 0$

k : a definition at a steady state
ability to transmit fluid

a measure of how quickly a material can carry flow away from a source.

$$d\zeta = -\phi\left[C_{pc}\, dP_c - (C_{pp} + C_f)\, dP_p\right] \longrightarrow \text{uncoupled case.}$$

$$\frac{k}{\rho C} = \alpha$$

specific heat capacity, thermal diffusivity.

special case $P_p^c = 0$.

$$d\zeta = \phi\,(C_{pp} + C_f)\, dP_p .$$

$$\phi\,(C_{pp} + C_f)\frac{\partial P_p}{\partial t} = \frac{k}{\mu}\left(\frac{\partial^2 P_p}{\partial x^2} + \frac{\partial^2 P_p}{\partial y^2} + \frac{\partial^2 P_p}{\partial z^2}\right)$$

$$\frac{\partial P_p}{\partial t} = \frac{k}{\phi\,\mu\,(C_{pp} + C_f)}\nabla^2 P_p . \rightarrow \text{important for petroleum}$$

rigid $\rightarrow$ $C_{pp} = 0$, $(C_{pc} = 0)$ $\rightarrow$ $\dfrac{\partial P_p}{\partial t} = \dfrac{k}{\phi\mu C_f}\nabla^2 P_p$. $\dfrac{k}{\mu S}$.

$\rightarrow$ Hydraulic diffusivity.

For non rigid case,

recall $P_p = M\zeta + \alpha M \varepsilon_b$.

$$\zeta = \frac{1}{M}P_p - \alpha\varepsilon_b$$

$\varepsilon_b = \frac{1}{K}(P_c - \alpha P_p)$

$\zeta = -\frac{\alpha}{K}P_c + \frac{\alpha}{KB}P_p$.

$$\frac{1}{M}\frac{\partial P_p}{\partial t} - \alpha\frac{\partial \varepsilon_b}{\partial t} = \frac{k}{\mu}\nabla^2 P_p$$

diffusivity for constant strain

$$\frac{\partial P_p}{\partial t} = \alpha M\frac{\partial \varepsilon_b}{\partial t} + \frac{M k}{\mu}\nabla^2 P_p . \longrightarrow \text{fourth Equation.}$$

express in terms of pressure/stress

$$\varepsilon_b = \frac{1}{K}(P_c - \alpha P_p)$$

$$\frac{\partial P_p}{\partial t} = \frac{\alpha M}{K}\frac{\partial P_c}{\partial t} - \frac{\alpha^2 M}{K}\frac{\partial P_p}{\partial t} + \frac{M k}{\mu}\nabla^2 P_p .$$

$$\frac{\partial P_p}{\partial t}\left(1 + \frac{\alpha^2 M}{K}\right) = \frac{\alpha M}{K}\frac{\partial P_c}{\partial t} + \frac{M k}{\mu}\nabla^2 P_p . \quad M = \frac{BK}{\alpha(1-\alpha B)} .$$

$$\frac{\partial P_p}{\partial t}\underbrace{\left(1 + \frac{\alpha B}{1-\alpha B}\right)}_{\frac{1}{1-\alpha B}} = \frac{B}{1-\alpha B}\frac{\partial P_c}{\partial t} + \frac{BKk}{\alpha\mu(1-\alpha B)}\nabla^2 P_p .$$

$$\frac{\partial P_p}{\partial t} = B\cdot\frac{\partial \sigma_m}{\partial t} + \frac{BKk}{\mu\alpha}\nabla^2 P_p \rightarrow \text{diffusivity for constant stress.}$$

if we don't consider fluid, we see Skempton

Special case1, τ_m = constant.

$$\frac{\partial P_p}{\partial t} = \frac{BKk}{\mu\alpha}\nabla^2 P_p .$$

↳ diffusivity coeff.

Special case 2, ε_b = constant

$$\frac{\partial P_p}{\partial t} = \frac{BK}{\alpha(1-\alpha B)}\frac{k}{\mu}\cdot\nabla^2 P_p .$$

↳ replace K with $\frac{K}{1-\alpha B}$.

See, p. ~~437-439~~ 188

$$\Rightarrow \frac{\partial \zeta}{\partial t} = \frac{k}{\mu S}\nabla^2 \zeta , \quad S = \frac{1}{M} + \frac{\alpha^2}{\lambda+2\mu} = \frac{1}{M} + \frac{\alpha^2}{K+\frac{4}{3}\mu}$$

no coupling terms?. But coupled in the Boundary Condition

$$= (C_f - C_m)\phi + \left[1 + \frac{2(1+\nu)\alpha}{3(1-\nu)}\right]\alpha C_{BC}$$

(Zimmerman)

---

undrained moduli $\quad (\mu_u = \mu)$

$$K_u = \frac{K}{1-\alpha B} . \qquad G_u = G$$

recall, $\varepsilon^{dev} = \frac{1}{2\mu}\tau_m$

$$\nu = \frac{3K-2\mu}{6K+2\mu} . \qquad \nu_u = \frac{3K_u - 2\mu}{6K_u + 2\mu} .$$

$$\nu_u = \frac{3\cdot\frac{K}{1-\alpha B} - 2\mu}{6\cdot\frac{K}{1-\alpha B} + 2\mu} = \frac{3K - 2\mu(1-\alpha B)}{6K + 2\mu(1-\alpha B)} .$$

$\nu_u > \nu$.

$0 < \nu < \nu_u < 0.5$.

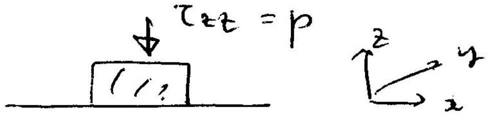
0.15 – 0.25   0.25 – 0.35

soil → 1/2 . not so different

But rock is different.

Terzaghi's problem. (consolidation)

$\tau_{zz} = p$



$T_m > 0$.

time to equilibriate $P_f \uparrow$.

$$t = \frac{L^2}{D} = \frac{L^2}{k}$$

function of both size (L) & k .

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General expression

$$\frac{\partial \zeta}{\partial t} = \frac{k}{\mu} \nabla^2 p_p$$

① uncoupled case

$$\frac{\partial p_p}{\partial t} = \frac{k}{\phi \mu (C_{pp} + C_f)} \nabla^2 p_p ,$$

② uncoupled case — rigid rock

$$\frac{\partial p_p}{\partial t} = \frac{k}{\phi \mu C_f} \nabla^2 p_p .$$

③ $$\frac{\partial p_p}{\partial t} = \alpha M \frac{\partial \varepsilon_b}{\partial t} + \frac{M}{\mu} k \nabla^2 p_p$$

coupled case — expressed in terms of strain.

diffusivity under constant strain.

④ coupled case — expressed in terms of stress.

$$\frac{\partial p_p}{\partial t} = B \frac{\partial \tau_m}{\partial t} + \frac{BKk}{\mu \alpha} \nabla^2 p_p$$

diffusivity for constant stress.

★ Specific storage coefficient, S, $1/\mathrm{Pa}$

ratio of the change in the volume of water added to the storage per unit aquifer volume divided by the change in pore pressure.

$$\frac{\zeta}{P_p} \qquad \frac{\frac{\Delta V_f}{V}}{P_p}.$$

① rigid rock

$$S = \phi C_f \rightarrow \frac{\partial P_p}{\partial t} = \frac{k}{\phi \mu C_f}\nabla^2 P_p, \quad c = \frac{k}{\phi \mu C_f}.$$

② under constant stress, S_σ $\left|\frac{\zeta}{P_p}\right|_{\sigma=0}$.

$$\zeta = -\frac{\alpha}{K}P_c + \frac{\alpha}{KB}P_p$$

$$S_\sigma = \frac{\alpha}{KB} \qquad B = \frac{C_{pc}}{C_{pp}+C_f}, \quad K - C_{Bc}$$

$$\cancel{\frac{\partial P_p}{\partial t} = \frac{k}{\mu S_\sigma}\nabla^2 P_p - \frac{\partial P_c}{\partial t}} \qquad C_{Bc} - C_{Bp} = C_m. \quad C_{Bp} = \phi C_{pc}.$$

$$C_{Bc} - C_{pp} = C_m$$

③ under constant strain, S_ε $\left|\frac{\zeta}{P_p}\right|_{\varepsilon=0}$.

$$\varepsilon = \frac{1}{K}P_c - \frac{\alpha}{K}P_p \rightarrow P_c = \alpha P_p.$$

$$\zeta = -\frac{\alpha}{K}P_c + \frac{\alpha}{KB}P_p. \qquad = -\frac{\alpha^2}{K}P_p + \frac{\alpha}{KB}P_p = \frac{\alpha - \alpha^2 B}{KB}P_p$$

$$\frac{\zeta}{P_p} = \frac{\alpha(1-\alpha B)}{KB} = \frac{\alpha}{K_u B}. \qquad \frac{KB}{\alpha(1-\alpha B)} \quad \frac{\alpha(1-\alpha B)}{KB}$$

$$\frac{\partial P_p}{\partial t} = \frac{k}{\mu S_\varepsilon}\nabla^2 P - \alpha M\frac{\partial \varepsilon}{\partial t}.$$

④ 'hydrologic' definition = "the volume of water released ~~from~~ per unit decline of head per unit bulk volume while maintaining the REV in a state of zero lateral strain and constant vertical stress"

$$S_s = (\rho_f g)\cdot\left.\frac{\zeta}{p}\right|_{\varepsilon_{xx}=\varepsilon_{yy}=0,\ \tau_{zz}=0} \qquad \frac{\partial P_p}{\partial t} = \frac{k}{\mu S_{hydro}}\nabla^2 P_p.$$

Specific storage coefficient for hydrogeologist, S_{hydro}

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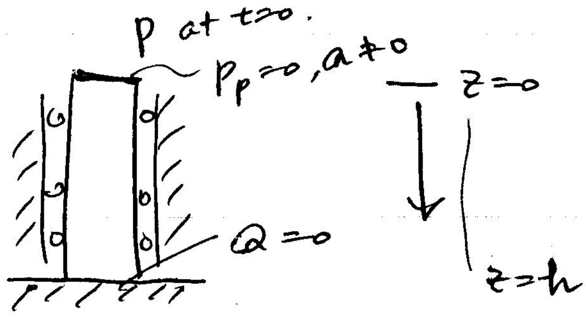
$$S_{hydro} = \frac{S_s}{\rho_f g} = S_\sigma \left(1 - \frac{4\eta B}{3}\right)$$

$$\eta = \frac{1-2\nu}{2(1-\nu)}\alpha$$

1D consolidation.



at t=0, $\sigma = p$, porepressure = $B \cdot p$.
displacement = undrained E.
w

$$\tau_{zz}^{0} = p, \quad \tau_{xx}^{0} = \tau_{yy}^{0} = \nu P(1-\nu).$$

$$\varepsilon_{zz}^{0} = \frac{p}{(\lambda+2G+\alpha^2 M)}. \quad \varepsilon_{xx}^{0} = \varepsilon_{yy}^{0} = 0.$$

shear stress & strains are zero.

$$P_p^{0} = \frac{\alpha M}{(\lambda+2G+\alpha^2 M)} P.$$

$$w^{0} = \frac{p}{\lambda+2G+\alpha^2 M}(z-h)$$

satisfy Navier (P) & diffusion equation. with ε_b or σ.

$$G\nabla^2 \underline{u} + (\lambda+G)\nabla(\nabla\cdot\underline{u}) = -f - \alpha\nabla P_p.$$

$$(\lambda+2G)\frac{\partial^2 w(z,t)}{\partial z^2} = -\alpha\frac{\partial P_p(z,t)}{\partial z}$$

by integrating,

$$(\lambda+2G)\frac{\partial w(z,t)}{\partial z} + \alpha P_p(z,t) = g(t)$$

$\tau_{zz}(z,t) = g(t)$. $z=0 \to \tau_{zz} = p$.

$$\tau_{zz}(z,t) = p.$$

$\varepsilon_{zz} = \frac{\partial w}{\partial z}$ $\quad \varepsilon_{zz}(z,t) = \frac{1}{\lambda+2G}[p - \alpha P_p(z,t)]$ integrati in uniaxial strain $\varepsilon_b = \varepsilon_{zz}$.

$$\frac{\partial \varepsilon_b}{\partial t} = \frac{-\alpha}{\lambda+2G}\frac{\partial P_p}{\partial t}$$

$$+\ \frac{\partial P_p}{\partial t} = \frac{kM}{\mu}\nabla^2 P_p + \alpha M \frac{\partial \varepsilon_b}{\partial t}$$

$$\frac{k}{\mu}\nabla^2 P_p = \left(\frac{1}{M}+\frac{\alpha^2}{\lambda+2G}\right)\frac{\partial P_p}{\partial t} \equiv S\frac{\partial P_p}{\partial t}.$$

diffusivity $D = \frac{k}{\mu S}$: coefficient of consolidation.

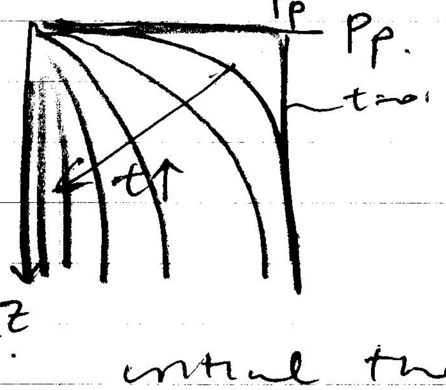
Solve this solution.

$t=0$, $P_p(z,0) = P_p^{\circ} = \frac{\alpha M}{\lambda+2G+\alpha^2 M}\,p$.

$P_p(z=0,t) = 0$, $\frac{\partial P_p}{\partial z}(z=h,t) = 0$.

The solution of this can be obtained from heat conduction Equation. (Carslaw & Jaeger (1959))

$$P_p(z,t) = \frac{\alpha M p}{\lambda+2G+\alpha^2 M}\sum_{n=1,3\ldots}^{\infty}\frac{4}{n\pi}\sin\left(\frac{n\pi z}{2h}\right)\exp\left(\frac{-n^2\pi^2 kt}{4\mu S h^2}\right)$$



$D = 1\times10^{-12}\ m^2$

critical time, 5. $\exp(-5) < 0$.

$$t_{eq} = \frac{20\mu S h^2}{\pi^2 k} \doteq 2\mu S h^2/k.$$

Berea sandstone

t a few second ~ several years.

$C_f = 5\times10^{-4}$/MPa.

$k = 190$ mD $= 190\times10^{-15}\ m^2$. Table 7.2.

$\mu = 10^{-3}$ Pa·s. < Berea Sandstone / Westerly granite.

$$S = \phi C_f = 0.2 \times 5\times 10^{-4} \times 10^{-6}$$

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$$\frac{2\times 10^{-3} \times 0.2\times 5\times 10^{-10} \times 1}{\pi^2 \times 180\times 10^{-15}} = \frac{100}{\pi^2 \times 180} =$$

$$w(z,t) = \frac{P}{\lambda+2G}\left[(z-h) + \frac{\alpha^2 M h}{\lambda+2G+\alpha^2 M}\sum_{n=1,3}^{\infty} \frac{8}{n^2\pi^2}\cos\left(\frac{n\pi z}{2h}\right) \times \exp\left(\frac{-n^2\pi^2 k t}{4\mu S h^2}\right)\right]$$

w at $z=0$,

$$w(0,t) = \frac{-Ph}{\lambda+2G}\left[1 - \frac{\alpha^2 M}{\lambda+2G+\alpha^2 M}\sum_{n=1,3}^{\infty}\frac{8}{n^2\pi^2}\exp\left(\frac{-n^2\pi^2 k t}{4\mu S h^2}\right)\right]$$

$t=0$ $\quad \frac{1}{1} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. $\quad w = \frac{-Ph}{(\lambda+2G+\alpha^2 M)}$.

undrained uniaxial strain.

$t=\infty$. $\quad w = \frac{-Ph}{\lambda+2G}$.