Week 12, 17 &19 May

# Mechanics in Energy Resources Engineering - Chapter 8 Applications of Plane Stress

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# outline

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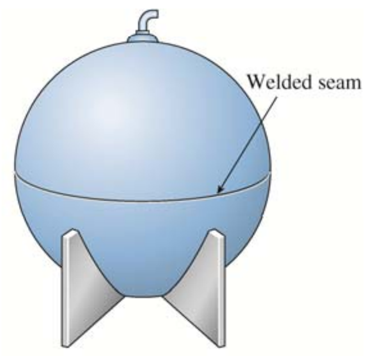
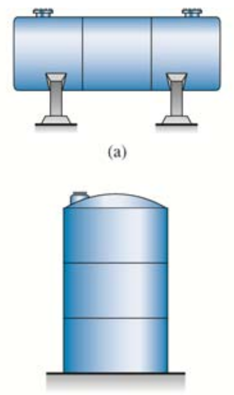
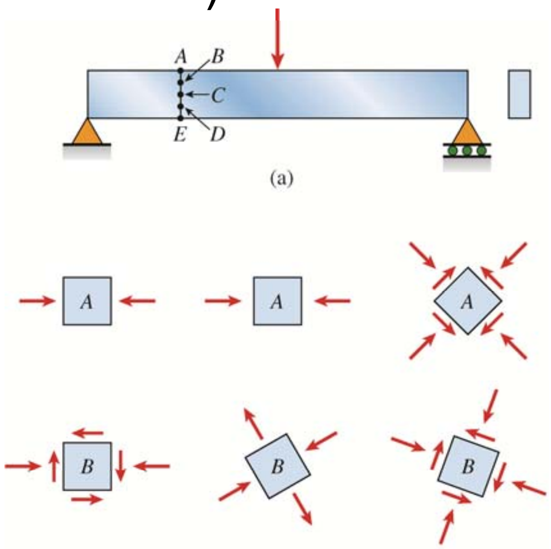
- <u>Chapter 8: Practical Examples of Plane Stress or Strain</u>
  - Introduction
  - Spherical Pressure Vessels (구형압력용기)
  - Cylindrical Pressure Vessels (원통형압력용기)
  - Maximum Stresses in Beams (보에서의 최대응력)
  - Combined Loadings (조합하중)



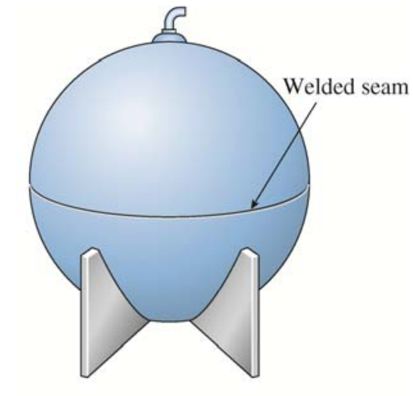



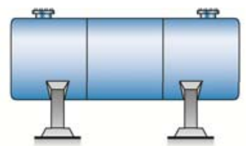



# Pressure Vessel (압력용기)



- Pressure Vessels:
  - closed structures containing liquids or gases under pressure
  - Tanks, pipes, and pressurized cabins in aircrafts
  - t (thickness) <<< r (radius or other dimension) → shell structures
- Thin-walled pressure vessels of spherical shape
  - r > 10 x t
- Cylindrical Pressure Vessels





(a)

# Example
# Compressed Air Energy Storage

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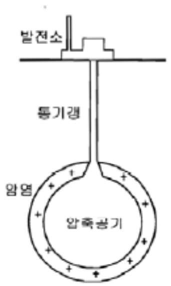
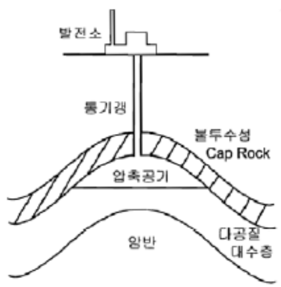
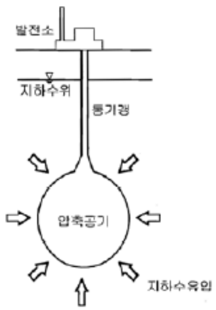
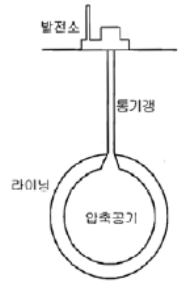
## CAES 지하 공간

### 지하 압축공기 저장 방식

국내 적용 가능 방식

| 구 분 | 암염 공동형 | 대수층형 | 암반 공동형 | |
|---|---|---|---|---|
| | | | 무복공식 | 복공식 |
| 모식도 |  |  |  |  |
| 장 점 | 건설비 저렴<br>기밀성 양호 | 건설비 저렴 | LPG저장기지와 유사 | 천심도 가능 |
| 단 점 | 입지 제한적<br>국내 가능성 없음 | 입지 제한적<br>국내 가능성 없음 | 대심도 굴착<br>입지 다소 제한적<br>(신선한 암반) | 공사비 유동적 |
| 사례 | 상업운전 중:<br>독일 Huntrof<br>미국 Alabama | (검토 및 계획)<br>미국 Iowa(ISEP project) 등 | (pilot 규모 실증)<br>일본 Kamioka 광산 | (pilot plant 운영)<br>일본Sunakagawa 광산<br>(pilot 규모 실증 예정)<br>한국 신예미철광산 |

자료제공: 류동우 (2010)

# Spherical Pressure Vessel (구형압력용기)

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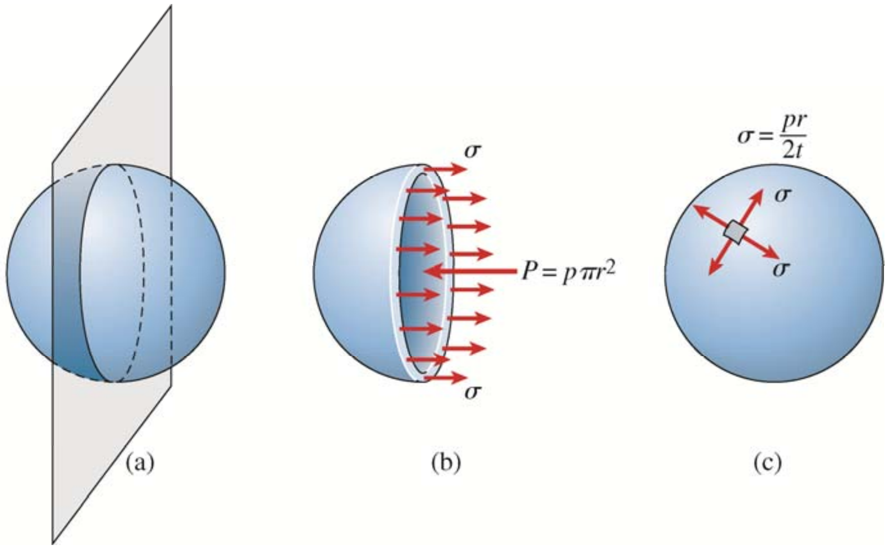
- Stresses in sphere vessel
  - Free Body Diagram : isolate half of the shell & fluid
  - Tensile stress (σ) in the wall & fluid pressure p



# Spherical Pressure Vessel
# (구형압력용기)

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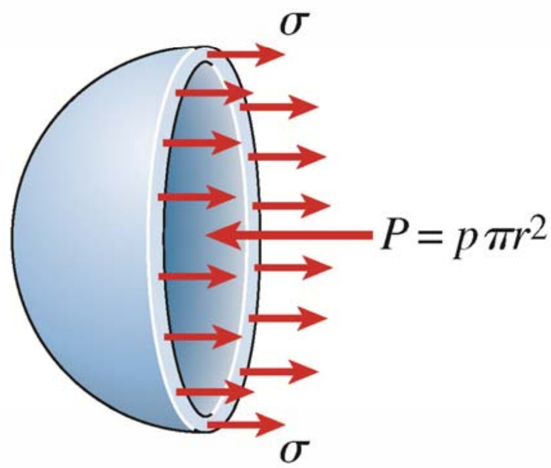
- Fluid pressure
  - Uniform
  - Resultant pressure force P $\qquad P = p(\pi r^2)$
- Tensile stress in the wall
  - Uniform around the circumference
  - Uniform across thickness (thin wall)
  - Resultant tensile stress $\qquad \sigma(2\pi r_m t)$




$$mean\ radius, r_m = r + \frac{t}{2}$$

$$\sigma = \frac{pr^2}{2r_m t} \xrightarrow[\text{Use inner radius}]{} \boxed{\sigma = \frac{pr}{2t}}$$

<u>Tensile stress in the wall of spherical shell</u>

# Spherical Pressure Vessel (구형압력용기)
## Stresses at the outer surface

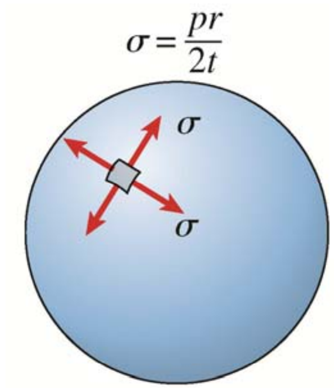


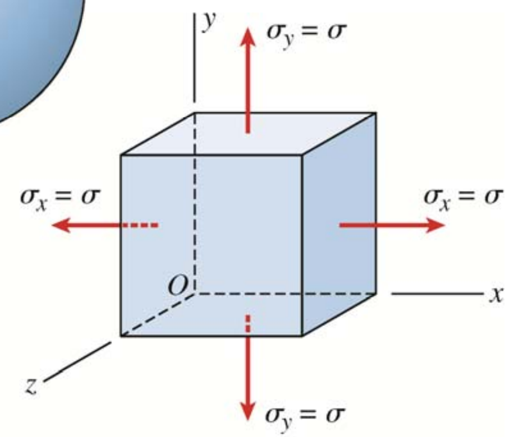
- Stress states at the outer & inner surface are different
  - Outer surface: free of loads → biaxial stress
  - Inner surface: internal pressure act in z direction → triaxial stress
- Stresses at the outer surface
  - Principal stresses

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}, \ \sigma_3 = 0$$

  - Maximum shear stress (out-of-plane rotation)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma}{2} = \frac{pr}{4t}$$



# Spherical Pressure Vessel (구형압력용기)
## Stresses at the inner surface



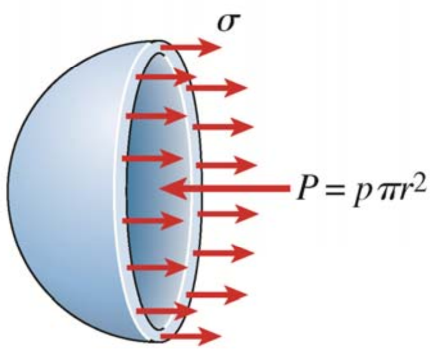
- Stresses at the outer surface
  - Principal stresses

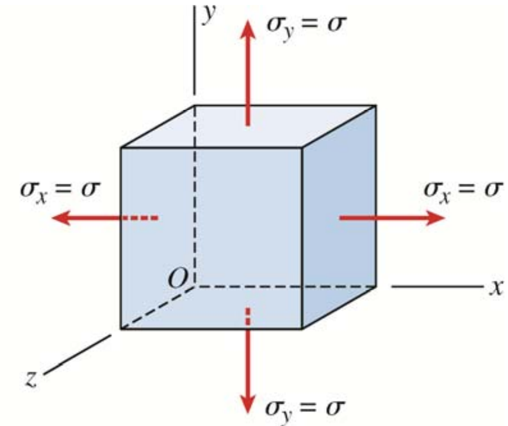
$$\sigma_1 = \sigma_2 = \frac{pr}{2t}, \ \sigma_3 = -p$$

  - Maximum shear stress (out-of-plane rotation)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma + p}{2} = \frac{pr}{4t} + \frac{p}{2}$$

$$\xrightarrow[\text{approximation}]{} \qquad \tau_{\max} = \frac{pr}{4t} + \cancel{\frac{p}{2}}$$



# Spherical Pressure Vessel (구형압력용기)
## General Comments

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- Limitation of thin-shell theory as applied to pressure vessels
  - Very thin, r/t > 10
  - Internal pressure exceed external pressure
  - Analysis based on internal pressure (not weight…)
  - Formula applicable except near the stress concentration

# Cylindrical Pressure Vessels (원통형압력용기)

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- Cylindrical pressure vessel with circular cross section
  - Pressurized pipe (water, LNG, …)

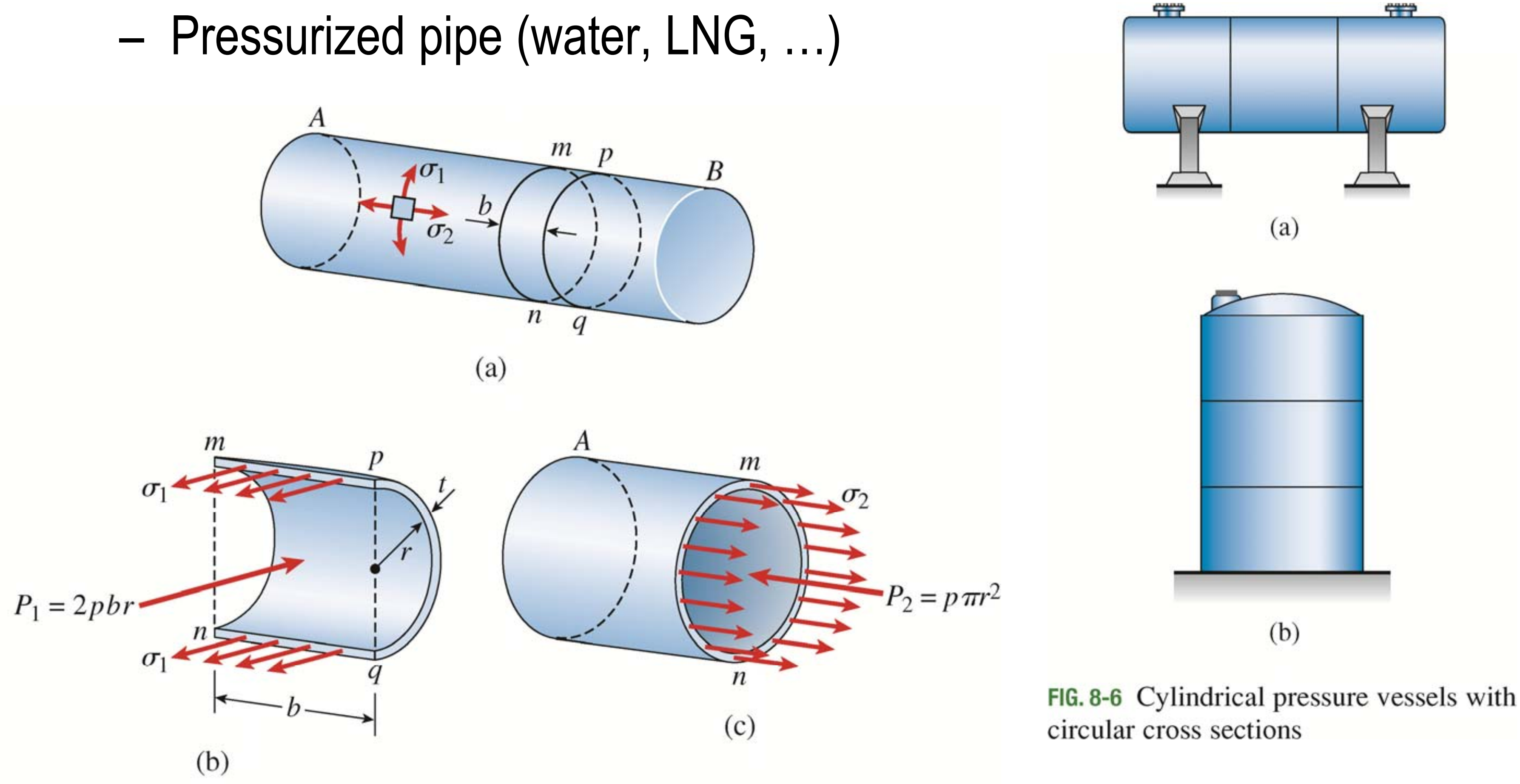



FIG. 8-6 Cylindrical pressure vessels with circular cross sections

# Cylindrical Pressure Vessels (원통형압력용기)
# Circumferential Stress (원주응력)

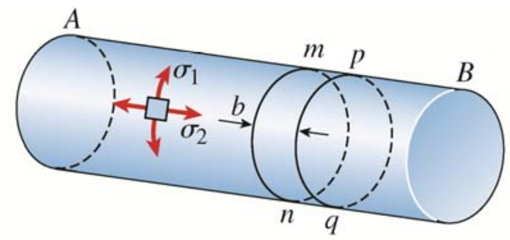


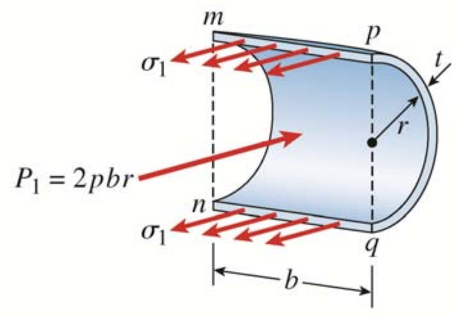
- Circumferential Stress (or hoop stress)
  - Stress acting perpendicular to the axis of the tank
  - Free body diagram
  - From force equilibrium,

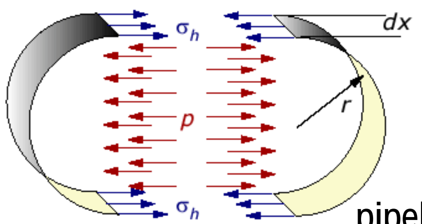
$$\sigma_1(2bt) - 2pbr = 0$$

  - Circumferential stress in a pressurized cylinder

$$\boxed{\sigma_1 = \frac{pr}{t}}$$



# Cylindrical Pressure Vessels (원통형압력용기)
## Longitudinal Stress (길이방향응력)

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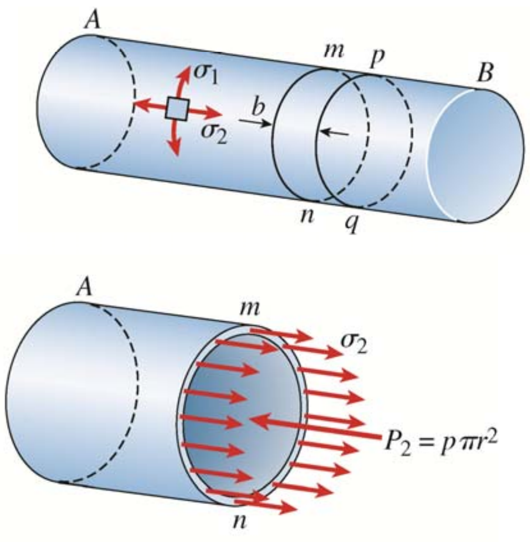
- Longitudinal Stress
  - Stress acting parallel to the axis of the tank
  - Free body diagram
  - From force equilibrium,

$$\sigma_2(2\pi rt) - p\pi r^2 = 0$$

  - Longitudinal stress in a pressurized cylinder

$$\sigma_2 = \frac{pr}{2t}$$



(c)

# Cylindrical Pressure Vessels (원통형압력용기)
# Stresses at the Outer Surface

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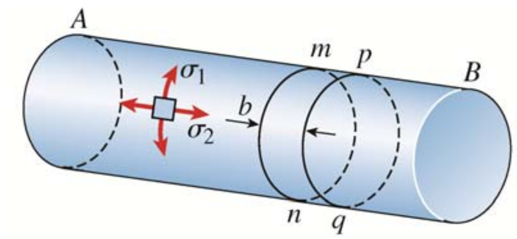
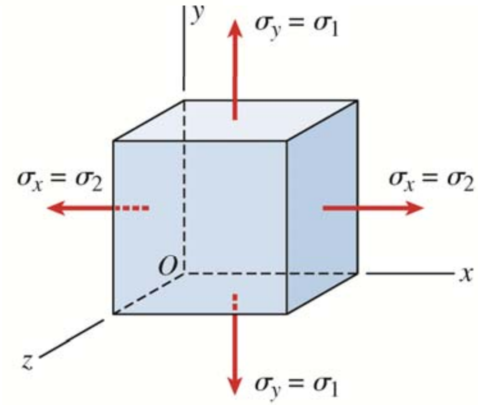



- Principal stresses

$$\sigma_1 = \frac{pr}{t}, \sigma_2 = \frac{pr}{2t}, \sigma_3 = 0$$

- Maximum shear stress
  - Maximum in-plane shear stress

$$(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1}{4} = \frac{pr}{4t}$$




  - Maximum out-of-plane shear stress

$$(\tau_{\max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1}{2} = \frac{pr}{2t} \qquad (\tau_{\max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{\sigma_2}{2} = \frac{pr}{4t}$$

  - Absolute maximum shear stress (largest of above three)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

Occurs on a plane rotated 45 ° about x-axis

# Cylindrical Pressure Vessels (원통형압력용기)
## Stresses at the Inner Surface

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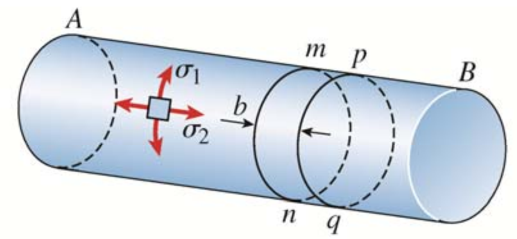
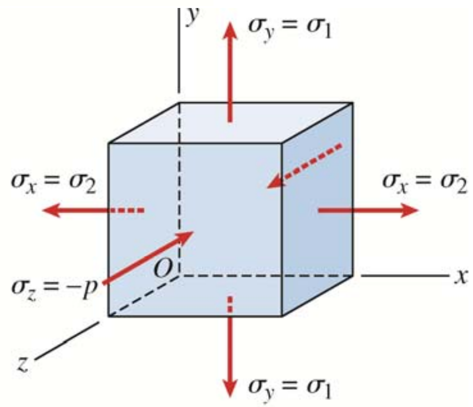
- Principal stresses

$$\sigma_1 = \frac{pr}{t}, \sigma_2 = \frac{pr}{2t}, \sigma_3 = -p$$




- Maximum shear stress
  - Maximum in-plane shear stress

$$(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1}{4} = \frac{pr}{4t}$$




  - Maximum out-of-plane shear stress

$$(\tau_{\max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{2t} + \frac{p}{2} \qquad (\tau_{\max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{pr}{4t} + \frac{p}{2}$$

  - Absolute maximum shear stress (largest of above three)

$$(\tau_{\max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{2t} + \frac{p}{2}$$

<u>Occurs on a plane rotated 45 ° about x-axis</u>

# Example 8-2

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- r = 1.8 m, t = 20 mm, E = 200GPa, v = 0.3, p = 800 kPa
- (a) circumferential and longitudinal stress,
- (b) maximum in-plane and out-of-plane shear stresses,
- (c) circumferential and longitudinal strains,
- (d) normal stress and shear stress acting perpendicular and parallel to the welded seam

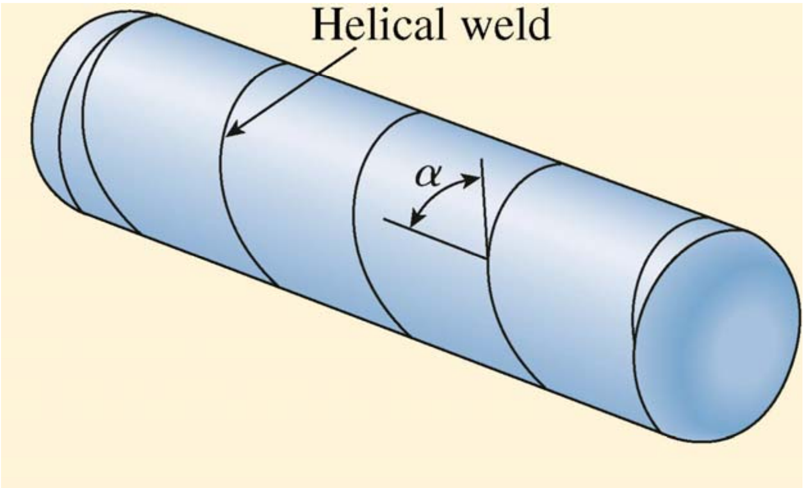


FIG. 8-9 Example 8-2. Cylindrical pressure vessel with a helical weld

# Schedule

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- Week 12 17 May 19 May : (ch.7) + ch. 8
- Week 13 24 May ~~26 May~~ (?) : ch.9 (deflection)
- Week 14 31 May : ch.9 (deflection)
- Week 15 7 June 9 June : ch.9 + ch.10 (indeterminate)

- 14 June : Final Exam + beer party

# Maximum Stresses in Beams

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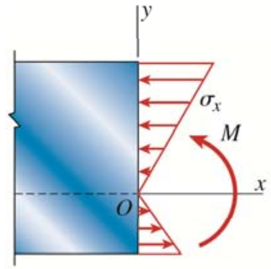
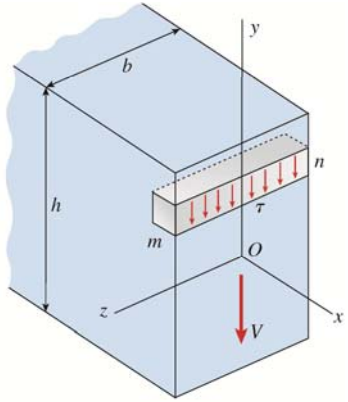
- Flexure formula $\sigma = -\dfrac{My}{I}$
  - Maximum at the farthest distance from the neutral axis

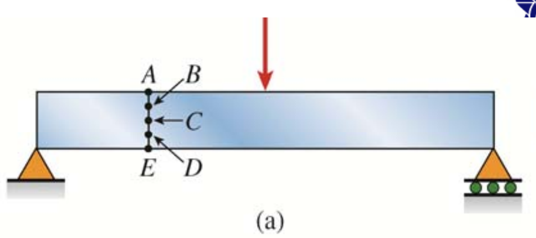


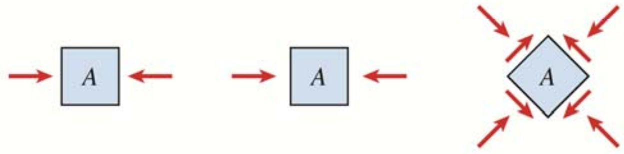

- Shear formula $\tau = \dfrac{VQ}{Ib}$
  - Highest at the neutral axis



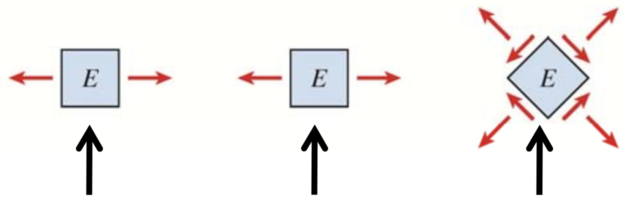

- Other places???
- Using above formulas, a more complete picture of stresses in the beam can be obtained.

# Maximum Stresses in Beam




- Direction & magnitudes of $\sigma_1$, $\sigma_2$, $\tau_{max}$ vary continuously.
- Investigations in many cross section →
  - Stress trajectories
  - Stress contours




Normal stress & shear stress

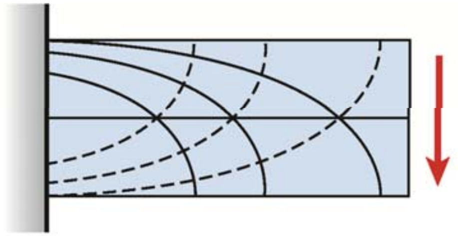
Principal Stresses

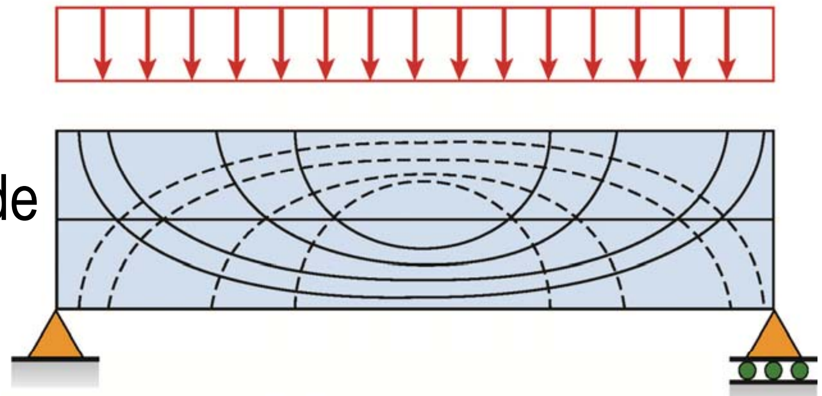
Maximum shear stress & normal stress

# Maximum Stresses in Beams



- Stress Trajectories (응력 궤적)
  - Two systems of orthogonal curves
  - Shows the directions of principal stresses
  - Tensile and compressive stresses always intersect at right angles
  - No information about the magnitude





: Compressive principal stress
: Tensile compressive stress

# Maximum Stresses in Beams

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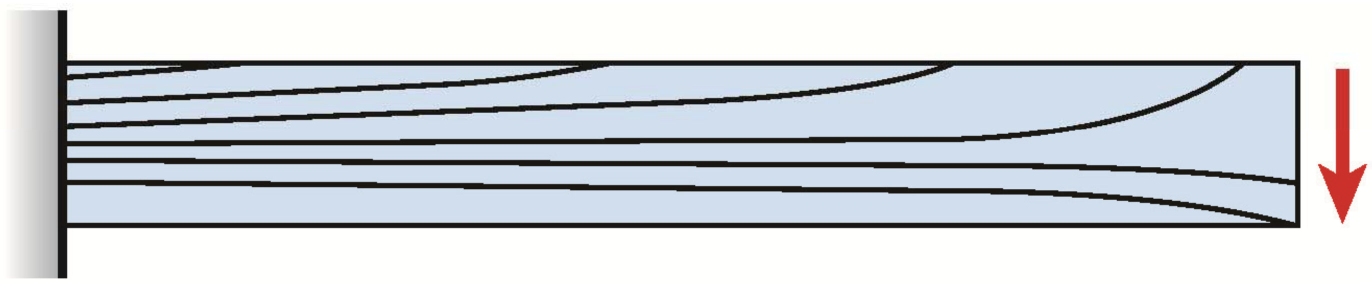
- Stress Contour (응력등고선)
  - Curve connecting points of equal principal stress*
  - Magnitude is constant as we move along a stress contour
  - Give no information about the direction of principal stress



*actually a single component of stress can be also plotted.

# Example of Stress Contour
## Saint-Venant Principle

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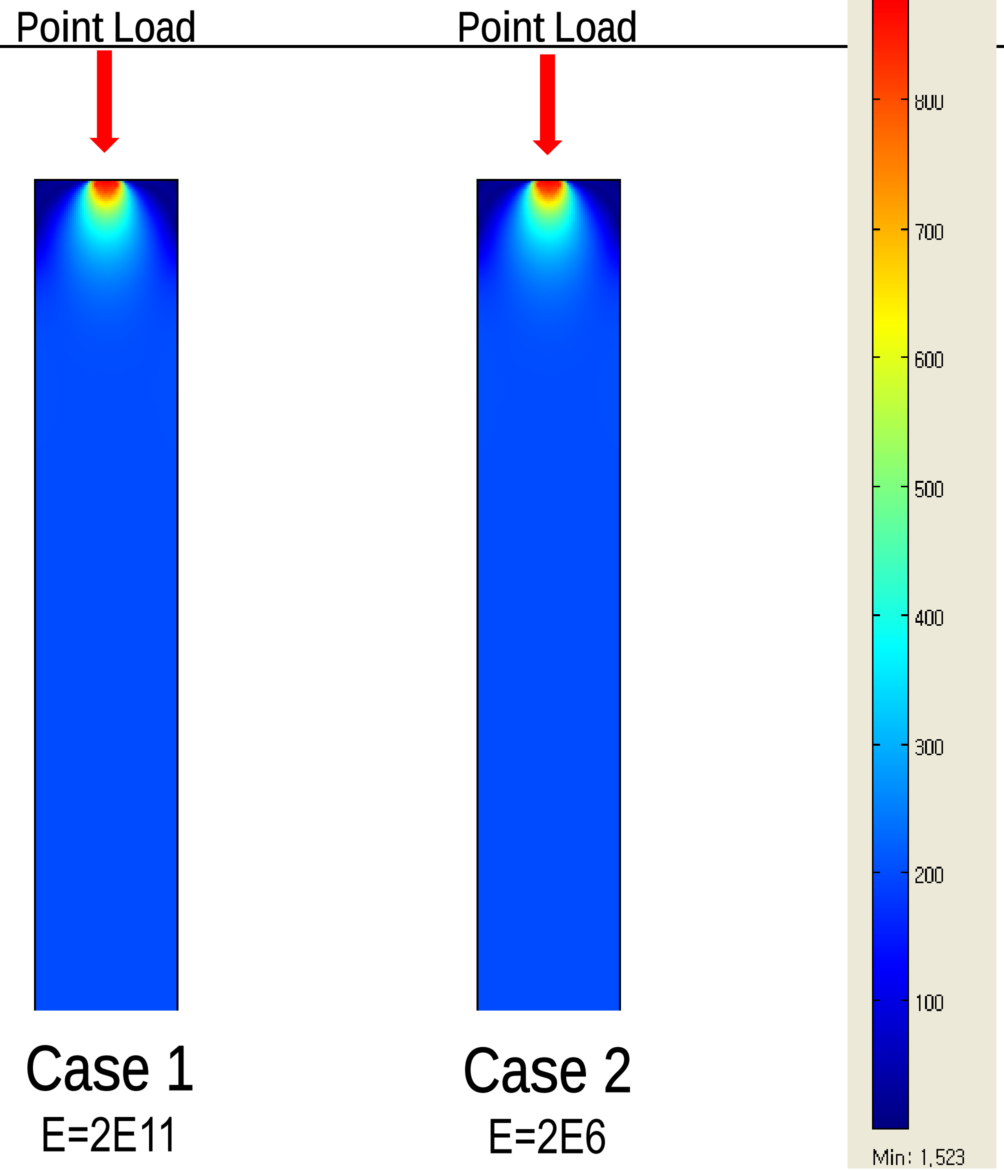


Case 1
E=2E11

Case 2
E=2E6

# Example 8-3

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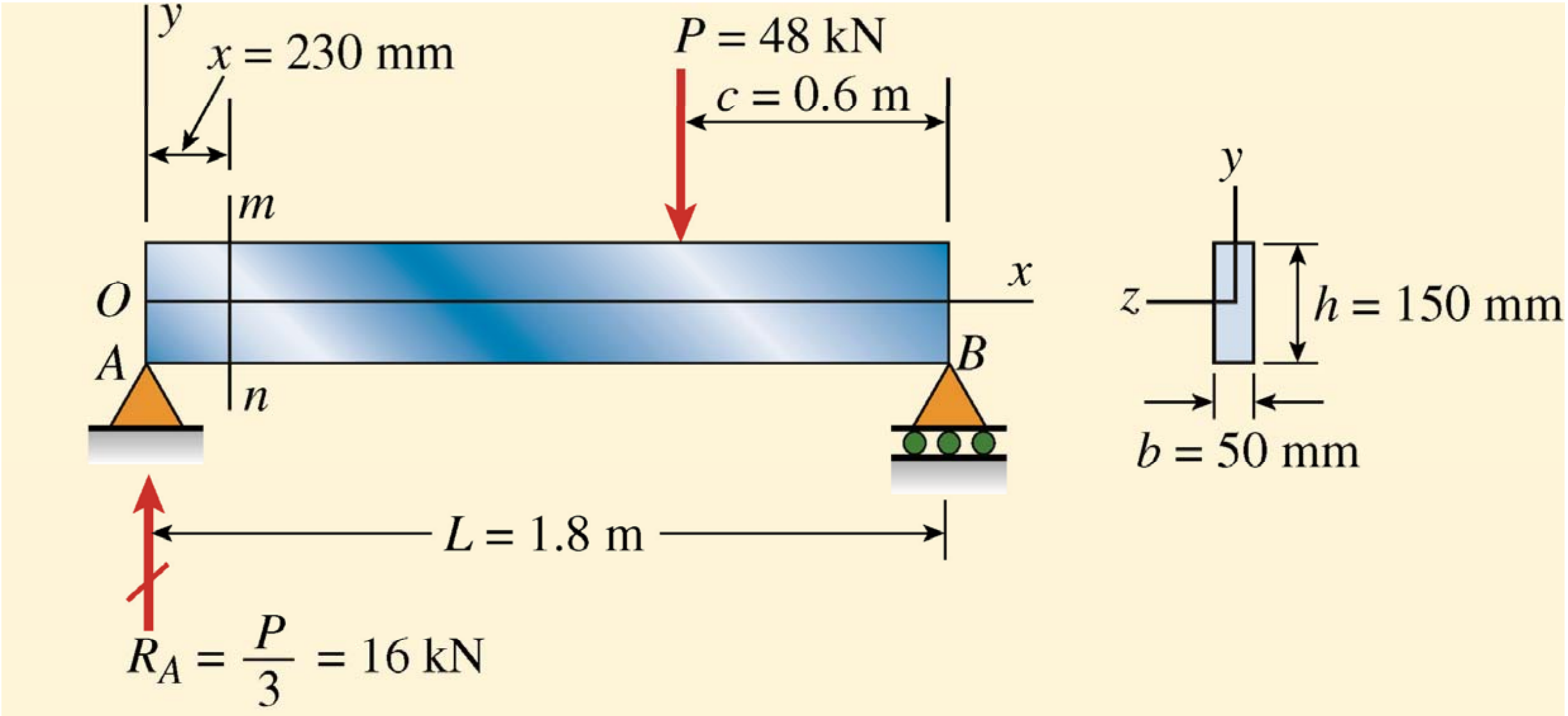
- Principal stresses and maximum shear stresses at cross section mn?



# Example 8-3

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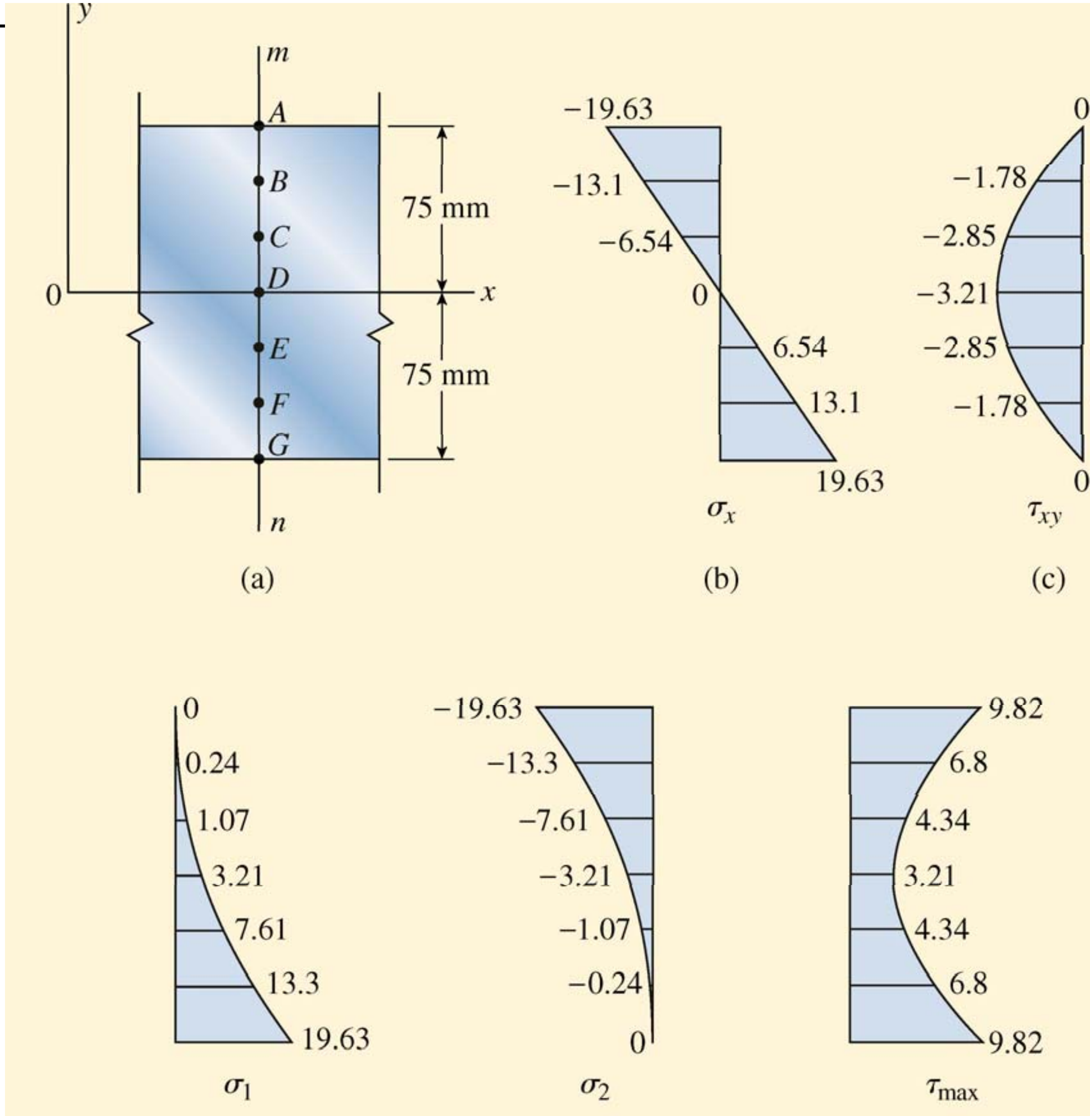

# Combined Loadings

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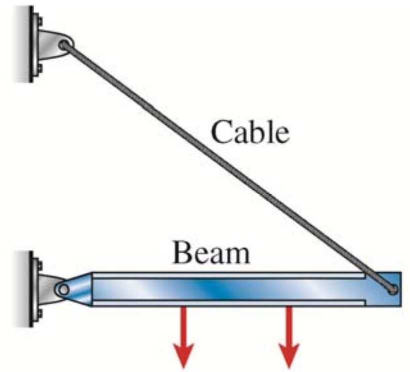
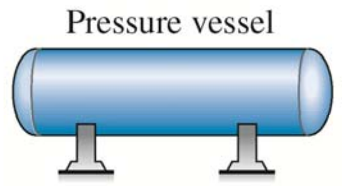
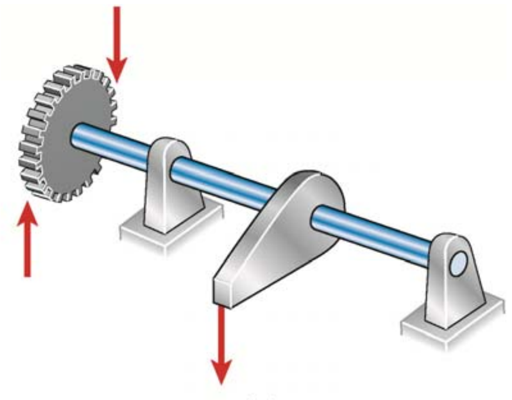
- Ch.1 & 2: axially loaded bars
- Ch. 3: shaft in torsion
- Ch. 4,5,6 beams in bending
- Ch. 8 pressure vessel




- In reality, above loadings are combined.



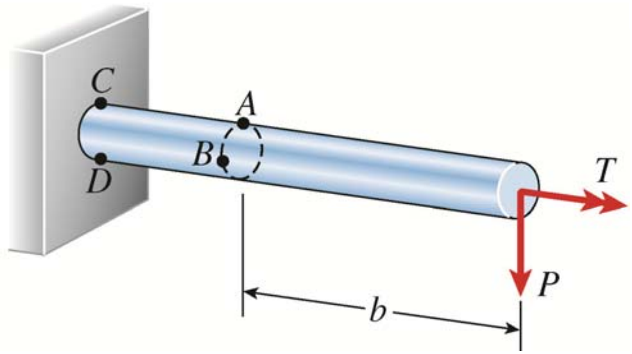
# Combined Loadings



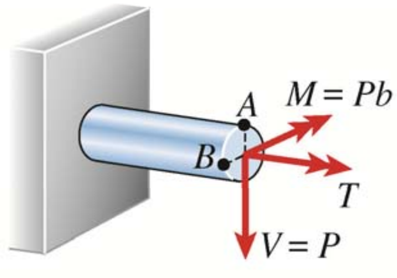
- Combined loading
  - analyzed by superimposing (중첩)the stresses and strains
  - Condition for superposition:
    - Stresses and strains linear functions of the applied loads
    - No interaction between loads
  - Superposition is very common in engineering work
  - No new theories are involved

# Combined Loading
## Illustration of the method






- Two types of load:
  - Torque T & vertical load P
  - Point A & B for investigation

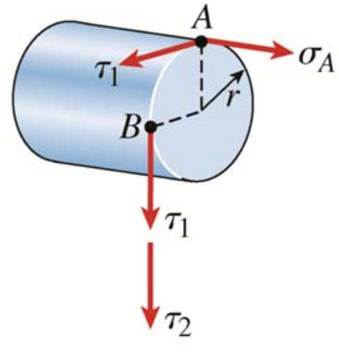



- Twisting moment ($A_{horizontal}$ & $B_{vertical}$)

$$\tau_1 = \frac{Tr}{I_p} = \frac{2T}{\pi r^3} \qquad I_p = \pi r^4 / 2$$

- Bending moment (only at A)

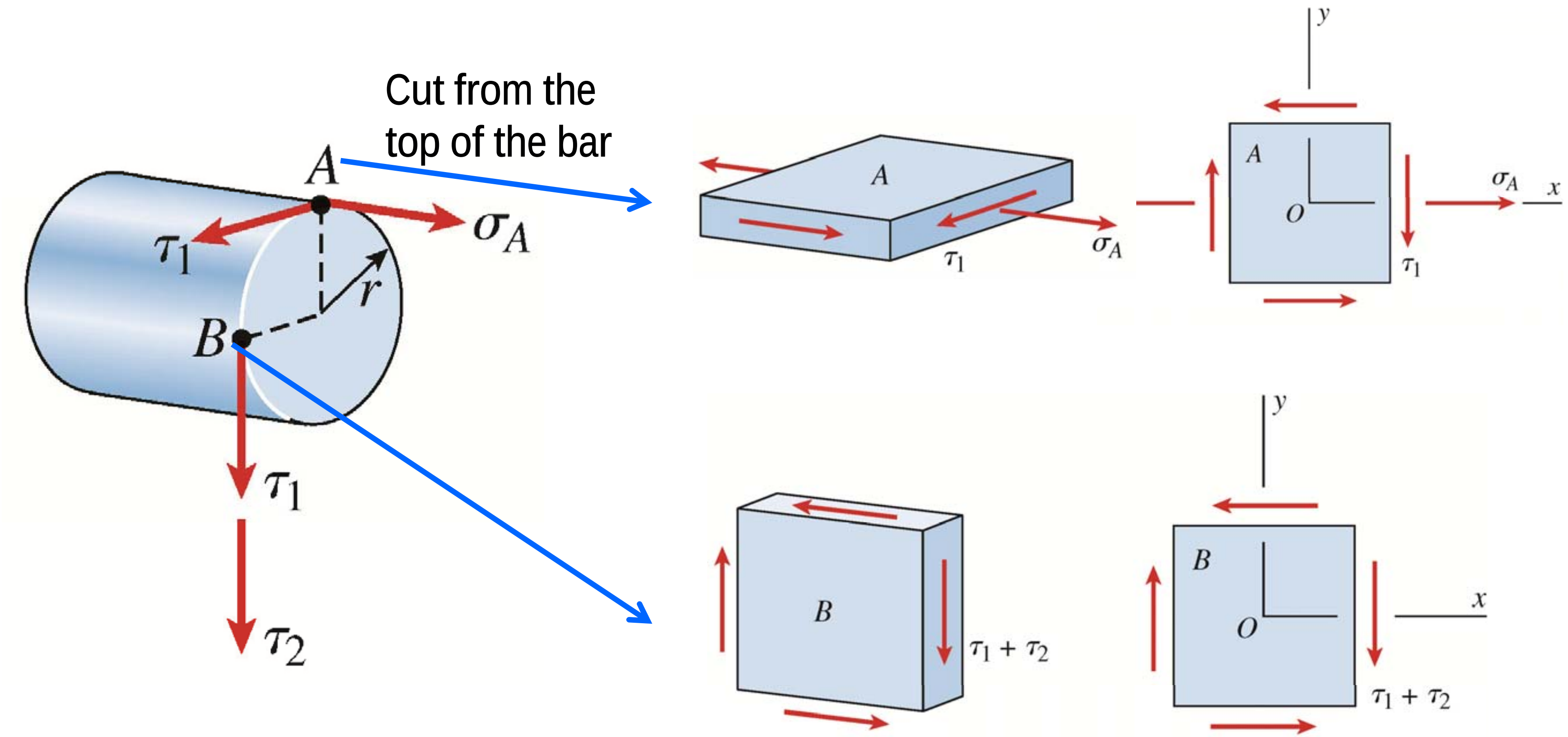
$$\sigma_A = \frac{Mr}{I} = \frac{4M}{\pi r^3}$$




- Shear Stress (only at B)

$$\tau_2 = \frac{4V}{3A} = \frac{4V}{3\pi r^2}$$

# Combined Loading
## Illustration of the method

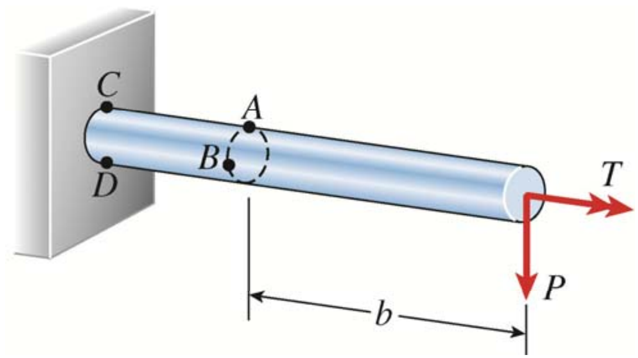



– Stresses at point A & B → $\sigma_1$, $\sigma_2$, $\tau_{max}$, strains (from Hooke's law).

– Also at other points……C, D, E, F….

# Combined Loading
## Illustration of the method

- Critical points: points with maximum or minimum stress
  - Ex) Maximum bending moment? C, D
    Maximum shear : B



# Example 8-4
## The shaft of a helicopter

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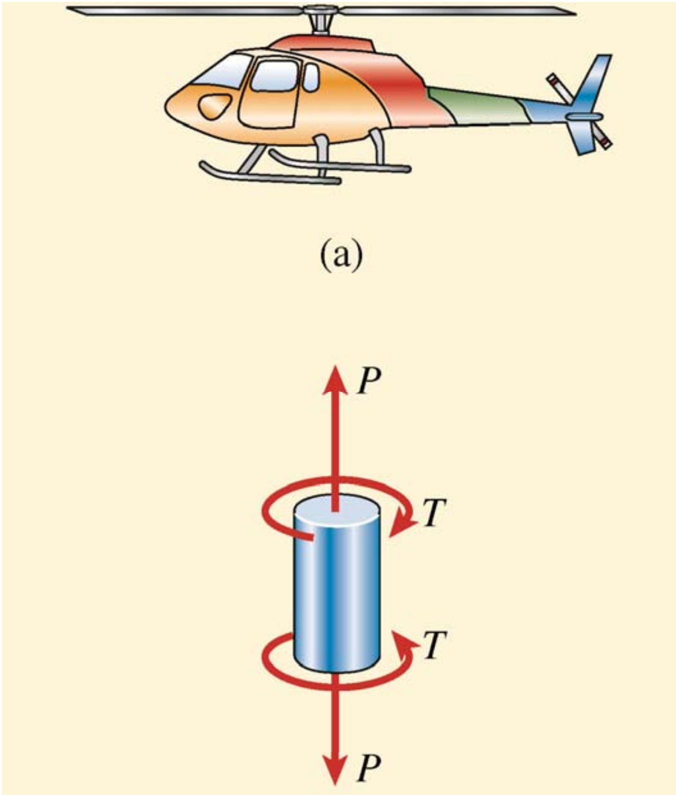
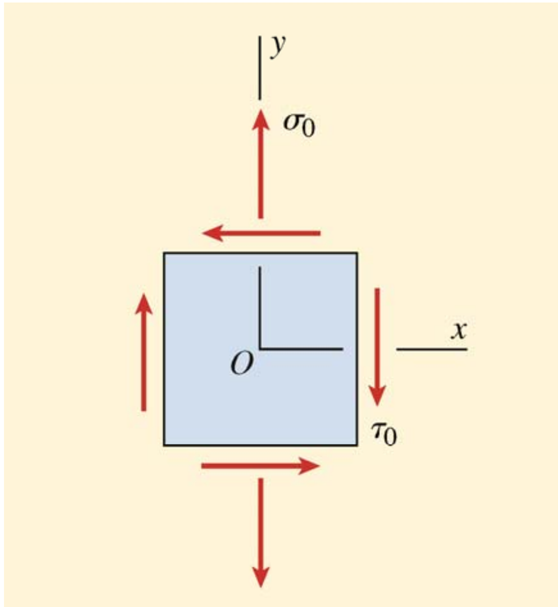
- The shaft of a helicopter: Torsion + axial loading



# Example 8-5
## a thin-walled cylindrical pressure vessel

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- Pressure vessel under internal pressure (p) + axial force (P = 55 kN)
- r = 50 mm, t = 4 mm
- Allowable shear stress: 45 Mpa
- Maximum allowable internal pressure p?

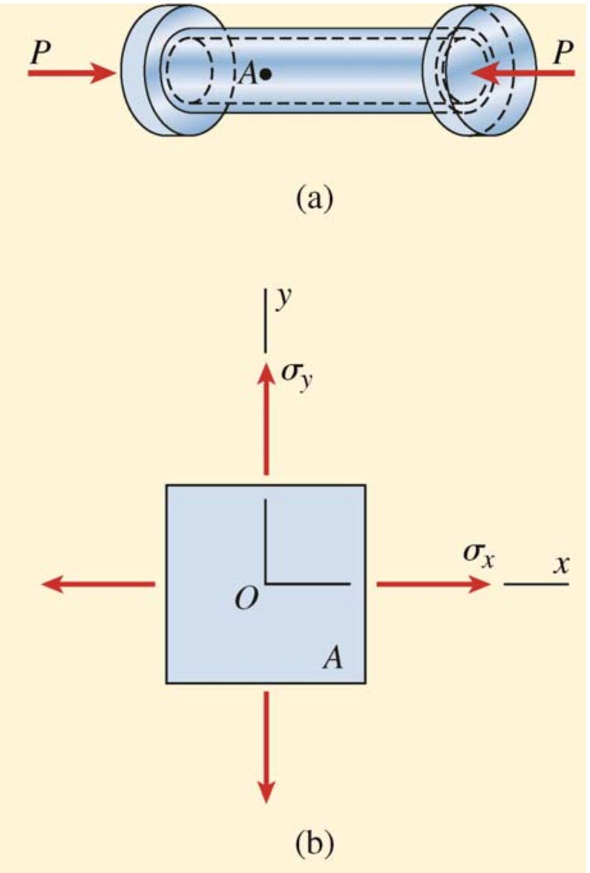


FIG. 8-25 Example 8-5. Pressure vessel subjected to combined internal pressure and axial force

# Example 8-6
## Wind pressure against a sign

- Wind pressure : 2.0 kPa
- Principal stresses and maximum shear stresses at points A and B?

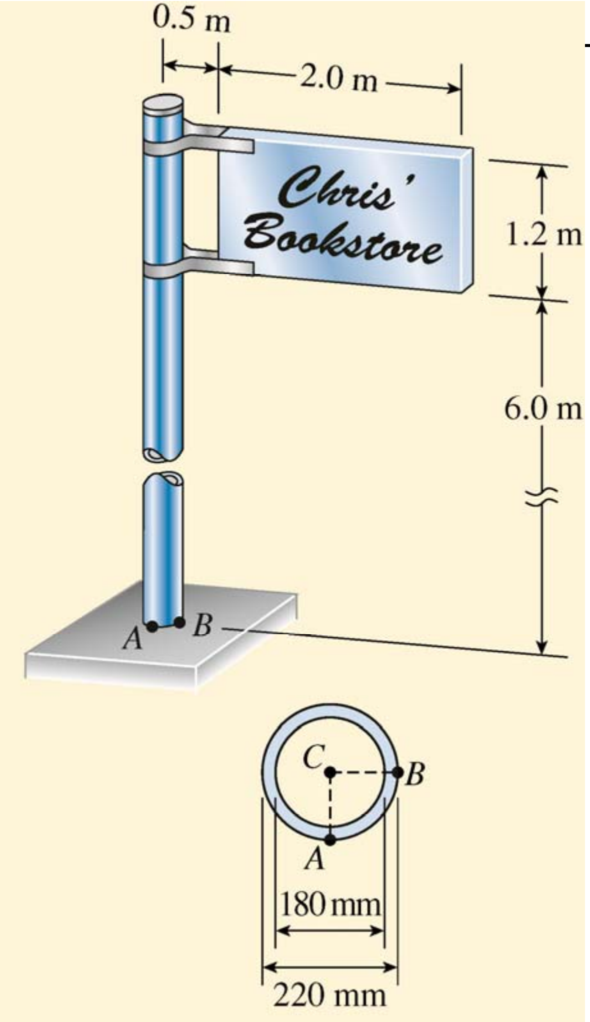


FIG. 8-26 Example 8-6. Wind pressure against a sign (combined bending, torsion, and shear of the pole)

# Example 8-6
# Wind pressure against a sign

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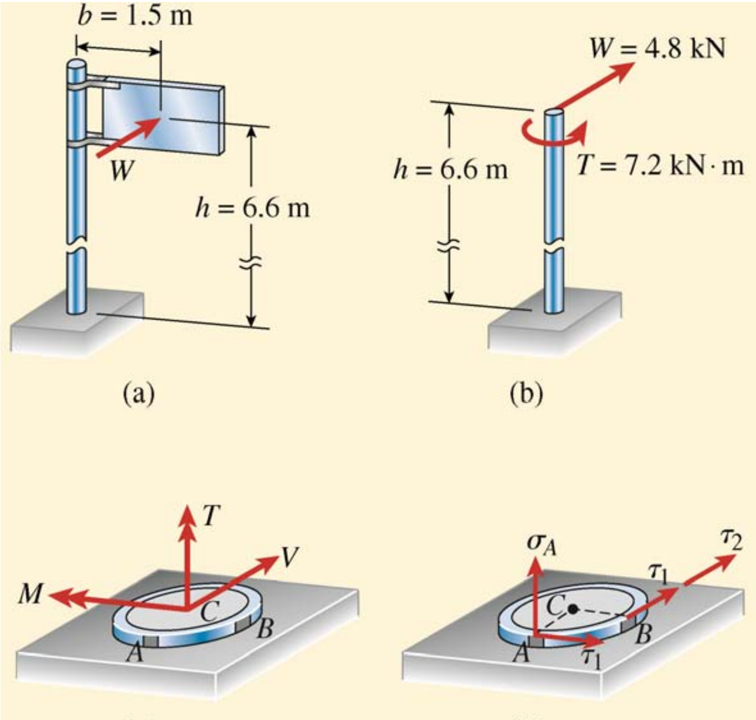
- Wind pressure : 2.0 kPa
- Principal stresses and maximum shear stresses at points A and B?



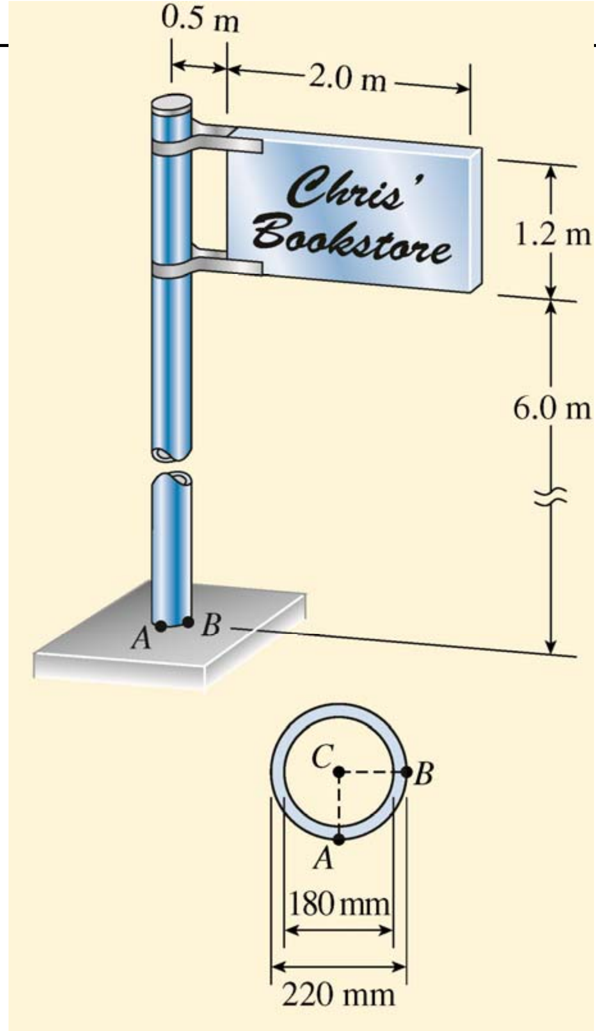




**FIG. 8-26** Example 8-6. Wind pressure against a sign (combined bending, torsion, and shear of the pole)

# summary

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- Chapter 8: Practical Examples of plane stress or strain
  - Introduction
  - Spherical Pressure Vessels
  - Cylindrical Pressure Vessels
  - Maximum Stresses in Beams
  - Combined Loadings