# Introduction to fluid flow in rock (Week5, 28, 30 Sept)

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- Fluid flow in porous media
  - Darcy's law
  - Permeability
  - Diffusion equation of fluid flow in porous media
- Fluid flow in fractured media
  - Cubic law
  - Permeability defined in fractured rock
  - Discrete Fracture Network
- Convective heat transfer

# **Content of today's lecture**



- Fluid flow in porous media
- Darcy's law
- Permeability vs. Hydraulic Conductivity
- Diffusion Equation for fluid flow in porous media

# Fluid flow in porous media



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- Problem of great importance to geological/energy resources engineering;
  - Groundwater hydrology

agroundwater migration, tunnel inflow, Contaminant transport,

Oil/gas extraction

 $\mathfrak{A}$ Reservoir engineering

- Rock/soil mechanics

ର୍କ୍ଷ Stability (pore pressure) of underground structure ର୍କ୍ଷ Fault mechanics

Geothermal Energy

# Fluid flow in porous media



- Application of fluid mechanics to porous media
  - Fluid (water, oil, gas) flows through the pores of the rock
  - Porosity of rock: volumetric fraction of (interconnected) pore
  - Permeability: the ease with which fluid can move through a porous rock

## **Representative Elementary Volume** (REV)



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 minimum volume (or a range) beyond which the characteristics of the domain remain basically constant



*Figure 3.14* Diagram illustrating the representative elementary volume (from Hubbert, 1956). Reprinted with permission of the Amer. Inst. Mining, Met. and Petrol. Engrs.





#### • n: porosity,Vv: void volume, V: total volume

$$n = \frac{V_v}{V}$$

Table 2.1 Range in Values of Porosity

Material	Porosity (%)
SEDIMENTARY	
Gravel, coarse	24-36
Gravel, fine	25-38
Sand, coarse	31-46
Sand, fine	26-53
Silt	34-61
Clay	34-60
SEDIMENTARY ROCKS	
Sandstone	5-30
Siltstone	21-41
Limestone, dolomite	0-40
Karst limestone	0-40
Shale	0-10
CRYSTALLINE ROCKS	
Fractured crystalline rocks	0-10
Dense crystalline rocks	0-5
Basalt	5-35
Weathered granite	34-57
Weathered gabbro	42-45

In part from Davis (1969) and Johnson and Morris (1962).



Figure 2.18 Porosity versus depth curves. Curve A from Athy (1930) for shales; curve B from Blatt (1979) for sandstones. Data for Blatt's curve represent 1000-ft averages of 17,367 porosity measurements (from an unpublished manuscript by Atwater and Miller).

Domenico & Schwartz (1998)

# **Darcy's law**



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Fig. 6.1. Darcy's experiment (Hubbert's version).

- Q: volumetric flow rate (m<sup>3</sup>/sec)
- q: volumetric flow rate per unit area (fluid flux or specific discharge) (m/sec)
- K: hydraulic conductivity (m/sec), the ease with which fluid can move through a porous rock
- h: hydraulic head

$$Q = -KA \frac{\delta h}{\delta l} = -KiA$$
  $q = -K \frac{\delta h}{\delta l} = -Ki$ 

$$q_x = -K_x \frac{\partial h}{\partial x}$$
  $q_y = -K_y \frac{\partial h}{\partial y}$   $q_z = -K_z \frac{\partial h}{\partial z}$   $\mathbf{q} = -K\nabla h$ 

Middleton and Wilcock, 1994

# **Hydraulic Head**



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*Figure 3.2* Diagram showing elevation, pressure, and totapressure head head for a point in the flow field.

- Piezometer: a tube(pipe) used to measure water-level elevations in field situation
- Elevation head: elevation at the base of piezometer
- Pressure head: length of water column.
- Total head: potential energy of
   the fluid = elevation head +

$$h = z + \frac{P}{\rho_w g}$$

# Hydraulic head



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- Determination of flow direction from piezometric measurement



# Hydraulic head



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#### - Determination of flow direction from piezometric measurement



# Permeability



- K (hydraulic conductivity, 수리전도도) is related to medium & fluid  $K = \frac{\rho g k}{\mu}$ 
  - $\mu$ : viscosity (점성도) of fluid, unit: Pa·s, water: ~10<sup>-3</sup> Pa·s = 1 cp
  - $\rho$ : density of fluid, unit: kg/m<sup>3</sup>, water: 10<sup>3</sup> kg/m<sup>3</sup>
  - g: acceleration due to gravity
  - k: permeability, unit: m<sup>2</sup>

# Permeability



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Rock Type	k (m2)	k (Darcies)	K (m/s)
Coarse gravels	10 <sup>-9</sup> -10 <sup>-8</sup>	10 <sup>3</sup> -10 <sup>4</sup>	10 <sup>-2</sup> -10 <sup>-1</sup>
Sands, gravels	10 <sup>-12</sup> -10 <sup>-9</sup>	10 <sup>0</sup> -10 <sup>3</sup>	10 <sup>-5</sup> -10 <sup>-2</sup>
Fine sands, silts	10 <sup>-16</sup> -10 <sup>-12</sup>	10 <sup>-4</sup> -10 <sup>0</sup>	10 <sup>-9</sup> -10 <sup>-5</sup>
Clays, shales	10 <sup>-23</sup> -10 <sup>-16</sup>	10 <sup>-11</sup> -10 <sup>-4</sup>	10 <sup>-16</sup> -10 <sup>-9</sup>
Dolomites	10 <sup>-12</sup> -10 <sup>-10</sup>	10 <sup>0</sup> -10 <sup>2</sup>	10 <sup>-5</sup> -10 <sup>-3</sup>
Limestones	10 <sup>-22</sup> -10 <sup>-12</sup>	10 <sup>-10</sup> -10 <sup>0</sup>	10 <sup>-15</sup> -10 <sup>-5</sup>
Sandstones	10 <sup>-17</sup> -10 <sup>-11</sup>	10 <sup>-5</sup> -10 <sup>1</sup>	10 <sup>-10</sup> -10 <sup>-4</sup>
Granites, Gneiss	10 <sup>-20</sup> -10 <sup>-16</sup>	10 <sup>-8</sup> -10 <sup>-4</sup>	10 <sup>-13</sup> -10 <sup>-9</sup>
Basalts	10 <sup>-19</sup> -10 <sup>-13</sup>	10 <sup>-7</sup> -10 <sup>-1</sup>	10 <sup>-12</sup> -10 <sup>-6</sup>

k (permeability, 투수율) is a measure of only 'medium'

 Also called, coefficient of permeability, intrinsic permeability

1 darcy =  $0.987 \times 10^{-12} \text{ m}^2 \sim 10^{-12} \text{ m}^2 = 10^{-5} \text{ m/sec}$ 

1 m/sec = 10<sup>-7</sup> m<sup>2</sup>  
Answer to  
Kyungjin's question  

$$K = \frac{\rho g k}{\mu} = \frac{10^3 \times 10 \times k}{10^{-3}} = 10^7 \times k$$

 Permeability has very large variation → very important to characterize/determine its value <sub>Jaeger et al., 2007</sub>

# **Darcy's law**



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• Expressed in terms of pressure and elevation head,

$$q = -K\nabla\left(z + \frac{P}{\rho_w g}\right)$$

• When all piezometers are bottomed at the same elevation

$$q = -\frac{k}{\mu} \nabla p \qquad \qquad \checkmark \qquad \qquad K = \frac{\rho g k}{\mu}$$

• In 1D,

$$q = -\frac{k}{\mu} \frac{dP}{dl}$$





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- Nature of Darcy's velocity
- Fluid flux, q: superficial velocity
- linear or pore velocity (v) of groundwater with porosity, n

$$v = \frac{q}{n}$$

• Pore velocity (v) will be always larger than the superficial velocity.

#### **Reminder** Heat Diffusion Equation







- *E<sub>g</sub>*: some energy conversion process, e.g., chemical or nuclear reaction
- $E_{st}$ : rate of change of thermal energy stored by the matter

# **Analogy with Heat diffusion equation**



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Heat transfer Energy Conservation

 $\Delta E_{st} = E_{in} - E_{out} + E_g$ 

 $E_{st}$ : Energy stored in control volume  $E_{in}$ : Energy entering the control volume  $E_{out}$ : Energy leaving the control volume  $E_g$ : Thermal Energy Generation Fluid flow in porous rock Mass Conservation

 $\Delta m_{st} = m_{in} - m_{out} + m_g$ 

 $m_{st}$ : mass stored in control volume  $m_{in}$ : mass entering the control volume  $m_{out}$ : mass leaving the control volume  $m_{g}$ : mass Generation

## **Diffusion equation** Derivation (1)



- We follow the exactly same procedure with the heat diffusion equation
- Energy conservation  $\rightarrow$  mass conservation
- Heat  $\rightarrow$  fluid flow
- Temperature  $\rightarrow$  hydraulic head
- Hydraulic conductivity  $\rightarrow$  hydraulic conductivity
- Fourier's law  $\rightarrow$  Darcy's law

### **Diffusion equation** Derivation (2)



- The basis for developing such an equation is a conservation of "mass".
- When there is no mass generated (= there is no sink or source)→ a conservation of fluid mass statement may be given as; mass inflow rate – mass outflow rate = change in mass storage with time.



## **Diffusion equation** Derivation (3)



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- Mass inflow rate through the face ABCD =  $Q_w q_x \Delta y \Delta z$ where  $\rho_w$ : density of fluid,  $q_x$ : specific discharge or flow flux (L/T)
- Mass outflow rate through the face EFGH =  $\left[\rho_w q_x + \frac{\partial(\rho_w q_x)\Delta x}{\partial x}\right]\Delta y\Delta z$

 $\partial v$ 

- Net outflow rate is the difference between the inflow and outflow, Net outflow rate through EFGH =  $\frac{\partial(\rho_w q_x)\Delta x \Delta y \Delta z}{\partial x}$
- Making similar calculations for the remainder of the cube, Net outflow rate through CDHG =  $-\frac{\partial(\rho_w q_y)\Delta x \Delta y \Delta z}{\partial (\rho_w q_y)\Delta x \Delta y \Delta z}$

Net outflow rate through BCGF =  $\frac{\partial(\rho_w q_z) \Delta x \Delta y \Delta z}{\partial x \Delta y \Delta z}$ 

#### **Diffusion equation** Derivation (4)



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- Adding these results, Net outflow rate through <u>ALL</u> the faces =  $-\left[\frac{\partial(\rho_w q_x)}{\partial x} + \frac{\partial(\rho_w q_y)}{\partial y} + \frac{\partial(\rho_w q_z)}{\partial z}\right]\Delta x \Delta y \Delta z$ - Right hand side of the conservation statement is merely a change
  - in mass storage with time (please confirm if units are mass/time),

$$\frac{\partial(\rho_w n)}{\partial t} \Delta x \Delta y \Delta z$$

Collecting above two equations and dividing by unit volume,

$$-\left[\frac{\partial(\rho_w q_x)}{\partial x} + \frac{\partial(\rho_w q_y)}{\partial y} + \frac{\partial(\rho_w q_z)}{\partial z}\right] = \frac{\partial(\rho_w n)}{\partial t}$$

 Above equations states that net outflow rate per unit volume equals the time rate of change of fluid mass per unit volume.

## **Diffusion equation** Derivation (5)



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- By further assuming that density of fluid does not vary spatially,



Time rate of change of fluid volume within the unit volume = gain or loss in fluid volume per time within the unit volume

- By substituting Darcy's law into specific discharge term, q;

 $\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] = \frac{1}{\rho_w} \frac{\partial(\rho_w n)}{\partial t}$ 

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = \frac{1}{\rho_w} \frac{\partial(\rho_w n)}{\partial t}$$

 We need to do something with the term in the right (because it is not expressed in terms of 'h'.

## **Diffusion equation** Derivation (5)



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$$\frac{1}{\rho_{w}}\frac{\partial(\rho_{w}n)}{\partial t} = S_{s}\frac{\partial h}{\partial t}$$



 $S_s$ : specific storage (L<sup>-1</sup>) – a measure of the volume of water(fluid) withdrawn from or added to the unit volume when h changes a unit amount. A difficult concept to grasp but remember that this has to do with three things: porosity, compressibility of water & compressibility of rock pore.

Finally we obtain the diffusion equation for fluid flow for porous media

Baloon ~ pore Air ~ water

$$\frac{\partial}{\partial x}\left(K_{x}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{y}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{z}\frac{\partial h}{\partial z}\right) = S_{s}\frac{\partial h}{\partial t}$$

Higher head (pressure) → more water due to compressed water & expansion of pore

#### **Diffusion equation** Derivation (6)



- When K is isotropic, the equation becomes;  $\frac{\partial}{\partial x} \left( K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t}$
- When K is homogeneous (does not change spatially);
- $K \left[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] = S_s \frac{\partial h}{\partial t} \qquad \qquad \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K} \frac{\partial h}{\partial t}$   $- \text{ By putting} \quad \frac{K}{S_s} = c \qquad \qquad \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{1}{c} \frac{\partial h}{\partial t}$  - c: hydraulic diffusivity (L<sup>2</sup>/T) $\nabla^2 h = \frac{S_s}{Q} \frac{\partial h}{\partial t}$

# **Diffusion equation** Derivation (7)



- For steady state;  

$$K \left[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] = S_s \frac{\partial h}{\partial t}$$

$$\frac{\partial^2 h}{\partial t} = \frac{\partial^2 h}{\partial t} + \frac{\partial^2 h}{\partial t} = \nabla^2 h = 0$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \nabla^2 h = 0 \quad \longleftarrow \quad \text{Laplace Equation}$$

#### **Diffusion equation** Expression in terms of hydraulic pressure



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- From,  $h = z + \frac{P}{\rho_w g}$
- When z = 0, which is when the elevation is the same (i.e., plate with the same elevation)  $h = 0 + \frac{P}{\rho_w g}$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\mu S}{k} \frac{\partial p}{\partial t} = \frac{1}{c} \frac{\partial p}{\partial t}$$

Diffusion equation expressed in terms of pore hydraulic pressure

#### **Diffusion equations** Heat diffusion and porous media flow



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- Heat diffusion equation

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

- Diffusion equation for fluid flow in porous media

$$\frac{\partial}{\partial x} \left( K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial h}{\partial z} \right) + \dot{q} = S_s \frac{\partial h}{\partial t}$$





Group	Members	Title
1	Han Na Kim, Hun Joo Lee, Hun Hoe Cho	Case studies of Geothermal Power Plant and remaining issues for Korea
2	Eun Ji Oh, Jong oong Joo, Chang jo Choi	Case studies of Soultz site (France) and Cooper Basin (Australia)
3	Min-Su Kim, Jin son, Sung-Min Kim, Sung-Chan Oh	Is EGS applicable in Korea?
4	Ki Yeol Kim, Young Soo Lee, Seong Moon Kim	Environmental Impacts
5	Bona Park, YeongSook Park, Kyungjin Yoo	Geothermal Heat Pump – Comparison between installations of GHP in Korea
6	Taehyun Kim, Jaewon Lee	Thermally induced permeability changes for a Geothermal Reservoir Simulation
7	Sunghoon Ryu, Seungbum Choi	Study on Drilling & Borehole stability for Geothermal Energy Development
8	Jiyoung Shin, DongKeun Lee	Numerical Simulation of conductive heat transfer under the biaxial stress and fracture presence

# Fluid flow in fractured rock



- In many rock types (especially hard rocks), fractures are the main pathways of fluid flow – note that hard rocks are attractive for underground space.
- Understandings on fluid flow in fractures are essential.
  - Enhanced Geothermal System
  - Fractured Oil Reservoir
  - Geological repository of high level nuclear waste
  - Underground structure (mines, tunnels and oil storages)

#### Fractured rock fluid flow How do we tackle it?



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Aperture: size of opening measured normal to the fracture wall



**Real rock fracture** 

- Hard to estimate Q due mainly to complex geometry

-Analytical solution exist to calculate Q and velocity profile

Idealized rock fracture

#### Fractured rock fluid flow Cubic Law



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Velocity (v) distribution between parallel plates

$$v = -\frac{1}{8\mu} \left( e^2 - 4y^2 \right) \frac{d}{dx} \left( p + \rho_w gz \right) = -\frac{1}{8\mu} \left( e^2 - 4y^2 \right) \frac{d(\rho_w gh)}{dx}$$

- Navier-Stokes' equation for laminar flow.
- most of geological application involves laminar flow (low Reynolds number, <2000, de Marsily, 1986)</li>

$$\operatorname{Re} = \frac{\rho V d}{\mu}$$

ρ: density of fluidV: mean velocity of fluidD: diameter of the pipeμ: viscosity

#### Fractured rock fluid flow Cubic Law



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 Cubic law: for a given gradient in head and unit width (w), flow rate through a fracture is proportional to the <u>cube</u> of the fracture aperture.

#### **Fractured rock fluid flow Measurement of aperture**



- Direct measurement
  - Insertion of feeler gauge (steel with different thickness)
  - Impression packer
  - Borehole camera
- Indirect measurement
  - Back calculation from the flow rates

$$Q = \frac{\rho_w ge^3}{12\mu} \frac{\partial h}{\partial x} \qquad e = \left(\frac{12\mu Q}{\rho_w g} / \frac{\partial h}{\partial x}\right)^{1/3}$$



#### Fractured rock fluid flow Equivalent permeability





#### Fractured rock fluid flow Equivalent permeability



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A sandstone with K of  $10^{-5}$  cm/s (which is  $10^{-7}$  m/s ~ $10^{-14}$  m<sup>2</sup>~ $10^{-2}$ Darcy~10mD) correspond to aperture 50µm in 1 m interval.

Influence of fracture aperture e and spacing b on hydraulic conductivity K in the direction of a set of smooth parallel fractures in a rock mass (Hoek et al., 2004)

#### Fractured rock fluid flow Characterisation



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- Geometrical properties of fractures
  - Orientation (dip and dip direction), size (trace length in 2D), density (spacing in parallel infinite fracture), location, aperture, roughness
- Characterization method
  - Exposed rock faces

ষ্কscanline sampling: line-based sample, use measuring tape (줄자) of 2-30 m.

ন্থWindow sampling: area-based sample, rectangle of measuring tapes

- Borehole sampling
- We then need to construct a geometric model of fractured rock deterministic or stochastic generation of fractures

#### **Fractured rock fluid flow** Discrete Fracture Network



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• Geometrical models based on the characterisation



Idealized regular fracture model



Discrete Fracture Network (암반균열망)

# **Content of this week's lecture**



- Fluid flow in porous media
- Darcy's law
- Permeability vs. Hydraulic Conductivity
- Diffusion Equation for fluid flow in porous media
- Fluid flow in fractured media
  - Cubic law
  - Permeability defined in fractured rock
  - Characterisation and Discrete Fracture Network (DFN)

# **Content of next week's lecture**



- Monday
  - Some useful solutions for steady and transient state fluid flow
  - Convective heat transfer

- Wednesday
  - Exploration techniques
     By Tae Jong Lee from KIGAM (Korea Institute of Geosciences and Mineral Resources)
  - One question gives 2 points. (attendance of one lecture = 2 points)





- Domenico PA, Schwartz FW, 1998, Physical and Chemical Hydrogeology, 2<sup>nd</sup> Ed., John Wiley & Sons, Inc.
- de Marsily G, 1986, Quantitative Hydrogeology, Academic press, Inc.
- Duncan CW and M CW, 2004, Rock Slope Engineering, 4<sup>th</sup> Ed. (based on Hoek and Bray's 3<sup>rd</sup> edition), Spon Press
- Hiscock KM, 2005, Hydrogeology Principles and Practice, Blackwell publishing