Introduction to fluid flow in rock (Week 7, 12 Oct)

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Last week's lecture

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- Some useful steady state and transient solutions
  - Terminology
  - Flow to a well in confined aquifer
    - Steady state solution
    - Transient Theis solution
  - Method of measuring hydraulic conductivity and specific storage
    - Curve matching method, Time drawdown method & Distance drawdown method

Modification of transient equation
Time-drawdown Method (example)

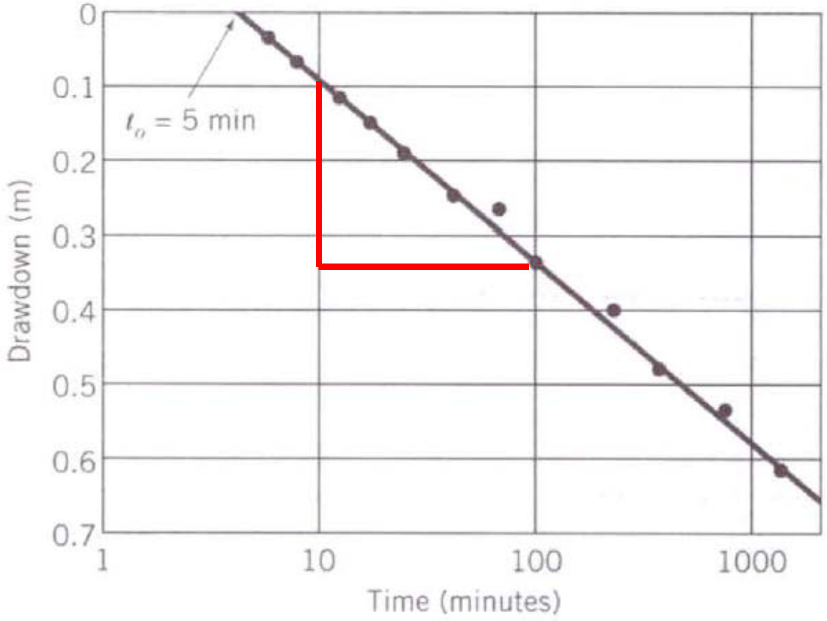


Figure 6.7 Semilogarithmic plot of drawdown versus time in an observation well.

- 305 m from a well pumping at a rate of 5.43 x 10^3 m^3/day
- Drawdown from 10 to 100 sec is 0.24 m.

$$T = \frac{2.3Q}{4\pi s}\log\frac{t_2}{t_1} = \frac{2.3\times 5.43\times 10^3 m^3/day}{4\times 3.14\times 0.24m}$$
$$= 4.1\times 10^3 m^2/day$$

- We can select any point in the graph. When s=0 is selected for convenience, $s = \frac{2.3Q}{4\pi T}\log\underbrace{\frac{2.25Tt}{r^2S}}_{1}$

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25\times 4.1\times 10^3 m^2/day\times 5\min}{93025m^2\times 1440\min/day}$$
$$= 4.1\times 10^3 m^2/day$$

Domenico & Schwartz, 1998

Q & A

다름이 아니라 지열에너지를 공부하고 있는데 궁금한 점이 있어서 여쭈어보려 메일을 보내게 되었습니다.

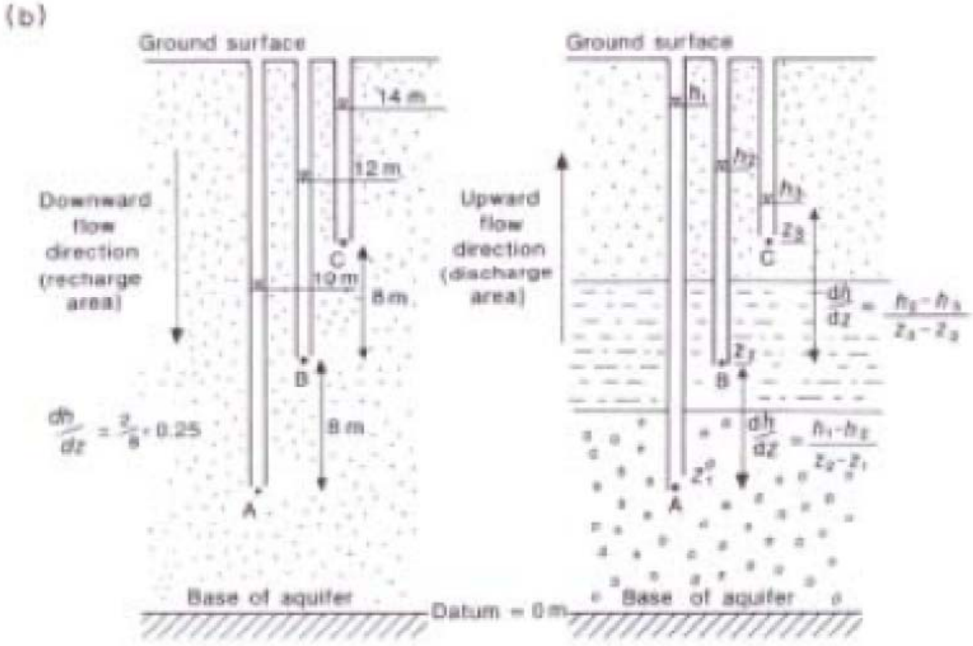
첫째로, 1D steady solutions 의 radial conduction in cylindrical wall 에서

$T(x) = C_1 \ln r + C_2$ 이렇게 잡던데 왜 $\ln r$ 로 하는지 잘 모르겠습니다.

그리고 함께 나오는 $T_{s,1}$, $T_{s,2}$, r_1, r_2 에 관한 그래프는 지수함수인가요 로그함수 인가요?

둘째로, Hydraulic head 를 할 때 나오는 이 그림의 의미를 잘 모르겠습니다.

- Determination of flow direction from piezometric measurement



Q & A

첨부한 설명을 참조하렴. krdt/dr 이 상수가 되어야 하므로 r 을 우항으로 보내면 lnr 로 표시가 되어야 함을 알 수 있단다. 그리고, T 는 최종 표현이 로그함수로 표시가 되어있으니 당연히 로그함수와 유사하게 변화한다.

두 번째는 유체유동의 방향이 전체수두(=피압수두에 의해 측정된 값)의 값에 따라 정해짐을 보여주기 위한 그림으로써 왼쪽에는 C 점이 가장 전체수두의 값이 크므로 유체는 아래로 흐른다고 할 수 있고, 오른쪽 그림에는 비록 A 층이 아래에 있지만 전체 수두의 값이 크므로 유체유동은 위로 향한다는 것을 보여준다. 특히 오른쪽 그림은 A 층을 사암층, B 층을 세일 층 등으로 가정하여 초기 석유 등의 생산 과정에 비유할 수 도 있겠다.

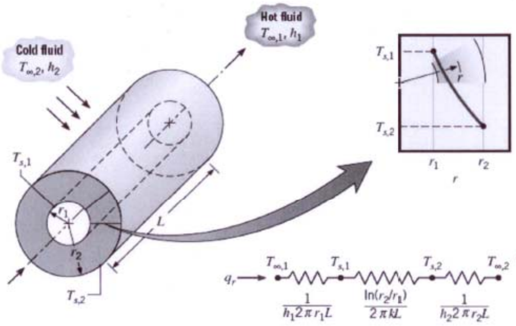


FIGURE 3.6 Hollow cylinder with convective surface conditions.

- 1D, Steady state, no heat generation,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)=0$$

$$q_r = -kA\frac{\partial T}{\partial r} = -k(2\pi rL)\frac{\partial T}{\partial r}$$

- Heat transfer rate is a constant in the radial direction.

$kr\frac{\partial T}{\partial r} = \text{constant}.$

- Assuming constant k,

$$T(r) = C_1 \ln r + C_2$$

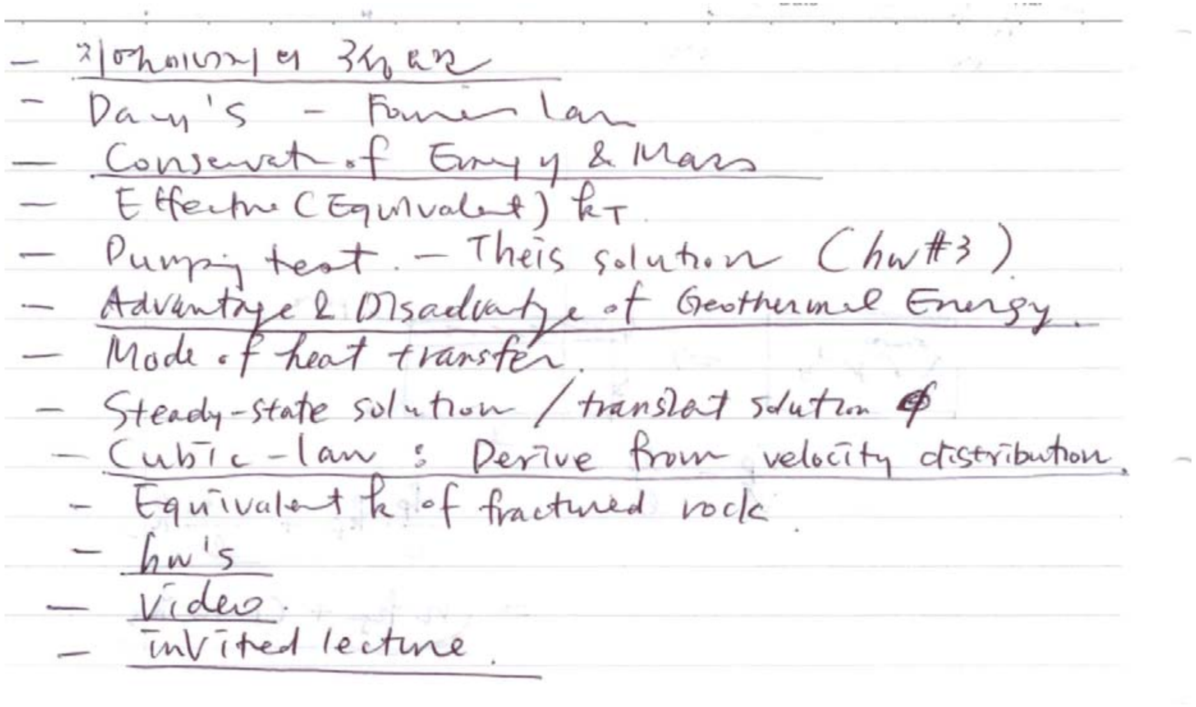
$\frac{\partial T}{\partial r} = \boxed{\frac{1}{kr}} \times \text{constant}$

$$T_{s,1} = C_1 \ln r_1 + C_2 \;\; and \;\; T_{s,2} = C_1 \ln r_2 + C_2$$

$T = C_1 \ln r + C_2.$

Mid-term exam

- From Dr Min's notebook…



- 지하에너지의 3가지 요소
- Darcy's - Fourier law
- Conservation of Energy & Mass
- Effective (Equivalent) k_T.
- Pumping test. - Theis solution (hw#3)
- Advantage & Disadvantage of Geothermal Energy.
- Mode of heat transfer.
- Steady-state solution / transient solution ϕ
- Cubic-law : Derive from velocity distribution.
- Equivalent k of fractured rock.
- hw's
- video.
- invited lecture.

- Closed-book, 75 minutes, English/Korean

Today

- Mixing laws - Effective thermal conductivity, heat capacity
- Forced convection and free convection
- Conductive-convection equation
  - Derivation
  - Peclet Number
  - 1D equation
- Dimensional analysis
- Free convection

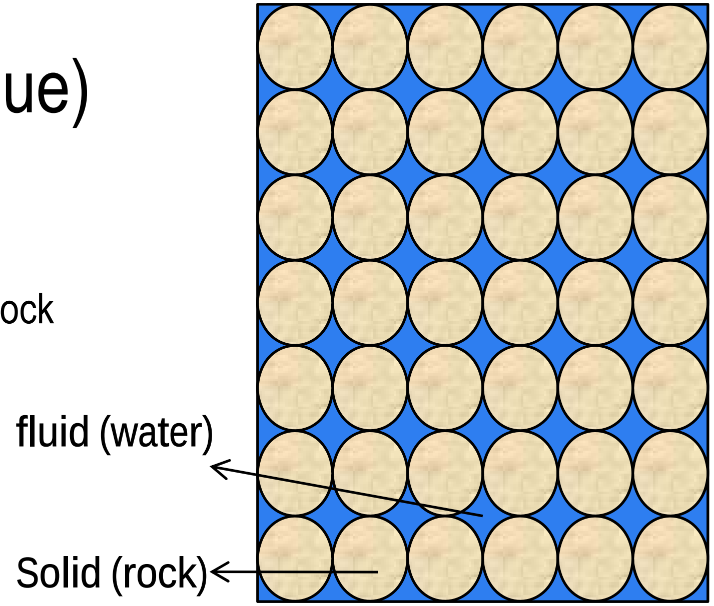
Effective thermal conductivity

Mixing Laws

- Model can predict an effective (equivalent) thermal conductivity of fluid-saturated rock with known properties (porosity, k_f, k_s).
- Laboratory measurement of thermal conductivity on all rock types of possible porosity can be time consuming.
- Weighted arithmetic mean (largest value)

$$k_e = k_f n + k_s (1 - n)$$

  - k_e: effective (equivalent) thermal conductivity of saturated rock
  - k_f: thermal conductivity of fluid (e.g., water)
  - k_s: thermal conductivity of solid (e.g., rock)
  - n: porosity



Effective thermal conductivity

Mixing Laws

- Weighted arithmetic mean (largest value) – by considering volume fraction

$$k_e = k_f n + k_s (1-n)$$

  - k_e: effective (equivalent) thermal conductivity of saturated rock
  - k_f: thermal conductivity of fluid (e.g., water)
  - k_s: thermal conductivity of solid (e.g., rock)
  - n: porosity

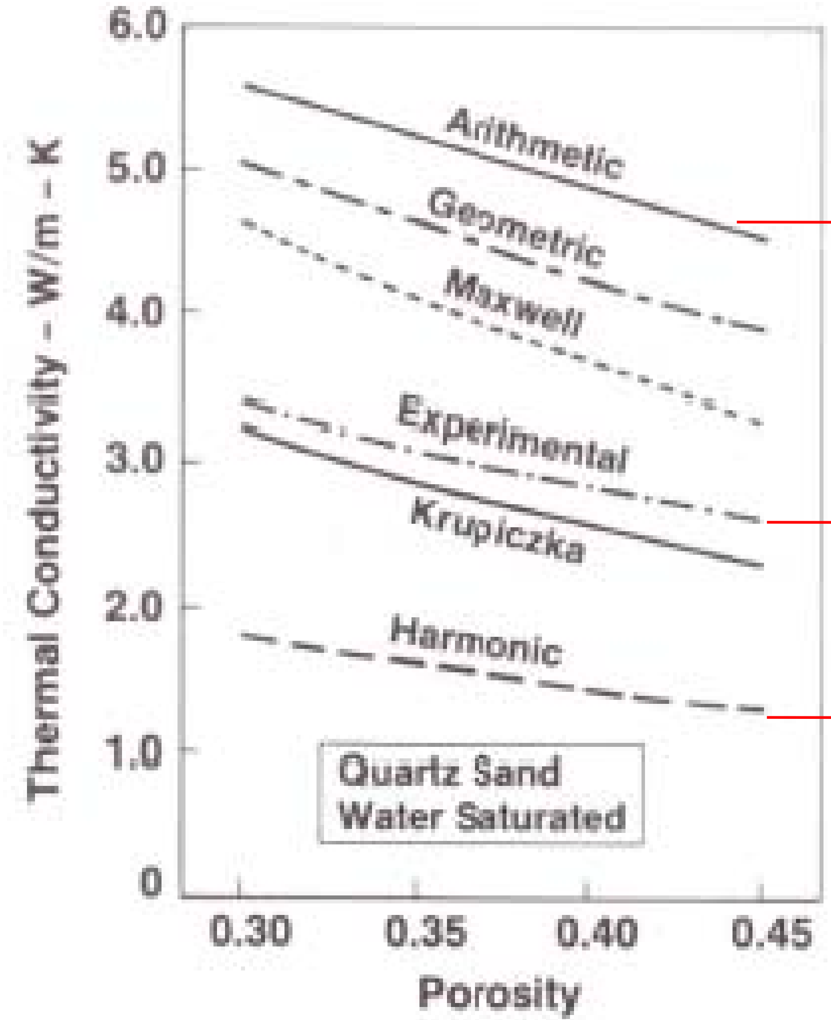
- Weighted harmonic mean (lowest value)

$$k_e = \left(n / k_f + (1-n) / k_s \right)^{-1}$$

- Maxwell model (somewhere between)

$$k_e = k_f \left\{ \left[2nk_f + (3-2n)k_s \right] / \left[(3-n)k_f + nk_s \right] \right\}$$

Effective thermal conductivity
Mixing Laws



→ largest

→ experiment

→ smallest

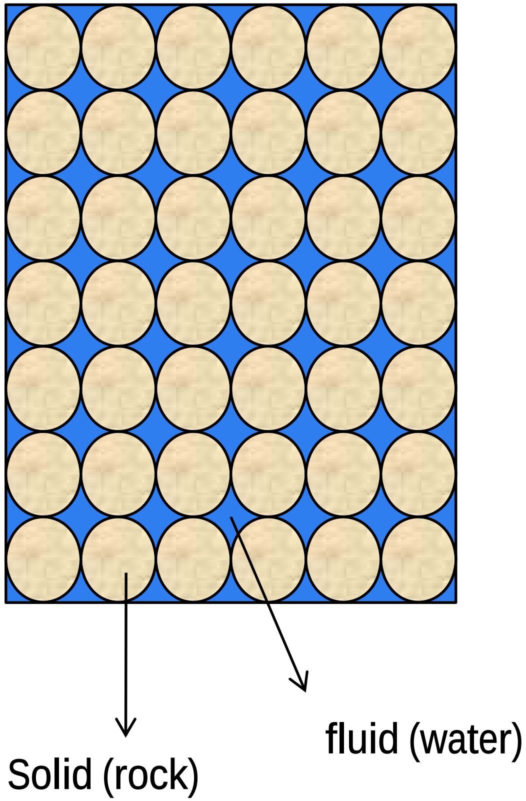
Mixing law models predictions of thermal conductivity as a function of porosity for water-saturated quartz sand compared with experimental measurement (Somerton, 1992)

Effective volumetric heat capacity
Mixing Laws

- Heat capacity of fluid-saturated rock

$$\rho c = n\rho_w c_w + (1-n)\rho_s c_s$$

  - ρ_w: density of fluid
  - ρ_s: density of solid (e.g., rock)
  - c_w: heat capacity of fluid (e.g., water)
  - c_s: heat capacity of solid (e.g., rock)
  - n: porosity



Convection
Forced convection and free convection

- Forced convection
  - Flow is caused by external forces
- Free (natural) convection
  - Flow driven by density variation (← thermal expansion of water)
- Forced and free convections are two limiting cases and they can exist together (mixed convection)

Conduction-convection equation
Derivation(1)

- Fourier's law

$$q_x'' = -k\frac{\partial T}{\partial x},\ q_y'' = -k\frac{\partial T}{\partial y},\ q_z'' = -k\frac{\partial T}{\partial z}$$

- Fourier's law (conduction) + fluid motion (convection)

$$q_{T,x}'' = -k_T\frac{\partial T}{\partial x} + n\rho_w c_w T v_x$$

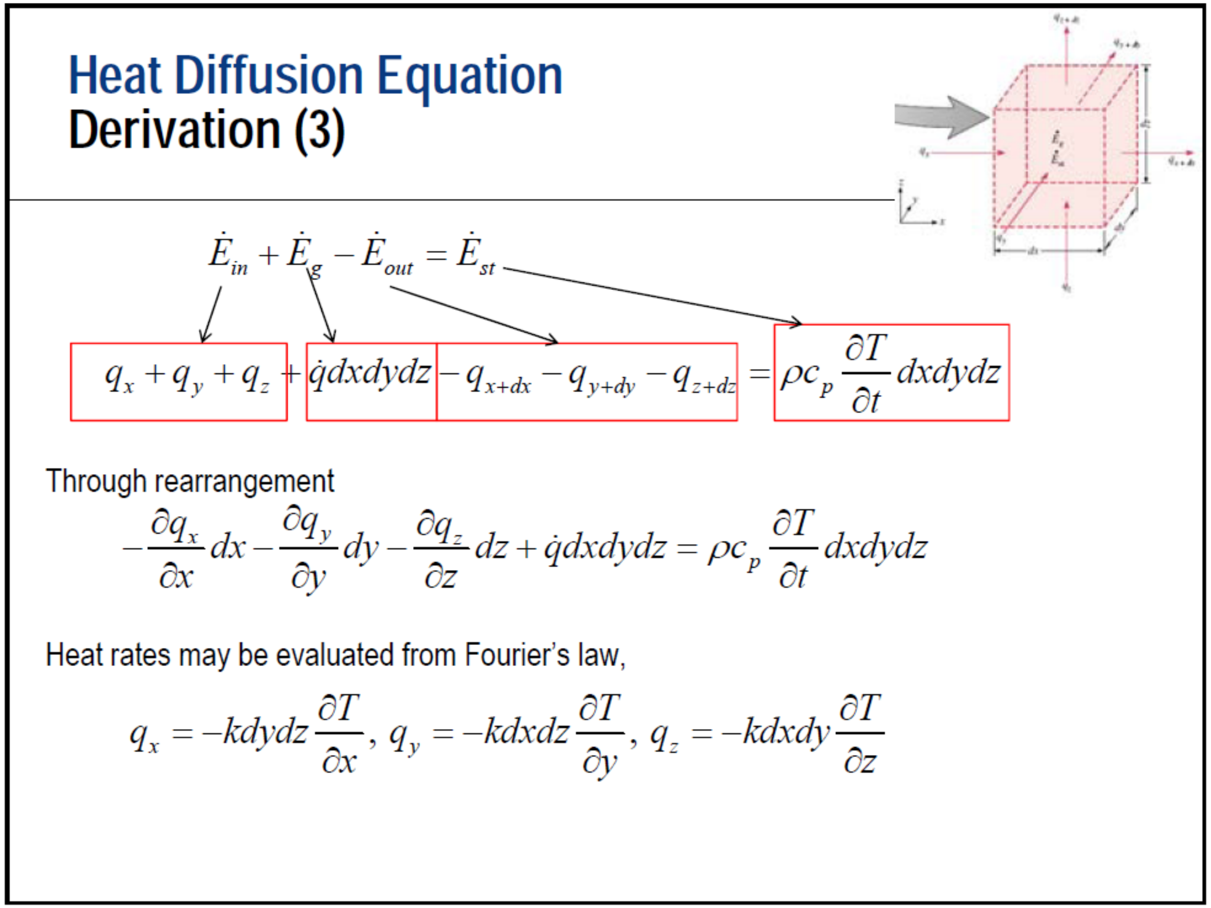
$$q_{T,y}'' = -k_T\frac{\partial T}{\partial y} + n\rho_w c_w T v_y$$

$$q_{T,z}'' = -k\frac{\partial T}{\partial z} + n\rho_w c_w T v_z$$

Contribution from convection!

- $q_{T,x}$'': heat flux in x-direction (W/m2)
- v: velocity (m/sec)
- $\rho_w c_w$: volumetric heat capacity (J/m^{3}K)

Conduction-convection equation
Derivation(2)



Heat Diffusion Equation
Derivation (3)

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\boxed{q_x + q_y + q_z} + \boxed{\dot{q}dxdydz}\boxed{- q_{x+dx} - q_{y+dy} - q_{z+dz}} = \boxed{\rho c_p \frac{\partial T}{\partial t} dxdydz}$$

Through rearrangement

$$-\frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz + \dot{q}dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Heat rates may be evaluated from Fourier's law,

$$q_x = -kdydz\frac{\partial T}{\partial x},\ q_y = -kdxdz\frac{\partial T}{\partial y},\ q_z = -kdxdy\frac{\partial T}{\partial z}$$

Conduction-convection equation
Derivation(3)

$$-div\,\mathbf{q}'' = \rho c \frac{\partial T}{\partial t}$$

$$\mathbf{q}'' = -k_T \nabla T + \boxed{n\rho_w c_w T\mathbf{v}}$$

Contribution from convection!

$$k_T \nabla^2 T - n\rho_w c_w \left[\mathbf{v}\cdot\nabla T + T\nabla\cdot\mathbf{v}\right] = \rho c \frac{\partial T}{\partial t}$$

For Steady state flow

$$\nabla\cdot\mathbf{v} = 0$$

$$\underbrace{k_T \nabla^2 T}_{\text{conduction}} - \underbrace{n\rho_w c_w \mathbf{v}\cdot\nabla T}_{\text{convection}} = \rho c \frac{\partial T}{\partial t}$$

Conservation expression that describe the manner in which energy is moved from one point to another by means of bulk fluid motion and by conduction

diffusion equation
Dimensional Analysis

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- Dimensionless group dictate the nature of diffusion process or demonstrate the competition between two rate process

L: length of the model (1D) or some length associated with fluid movement (2D)

$$x^{+} = x / L$$

$$y^{+} = y / L$$

$$z^{+} = z / L$$

$$t^{+} = t / t_e$$

$$h^{+} = h / h_e$$

$$\nabla^{+} = L\nabla$$

$$\nabla^{2+} = L^2\nabla^2$$

+ indicates a dimensionless quantity

L : some characteristic length

t_e : some characteristic time

h_e : some characteristic head

$$\nabla^2 h = \frac{S_s}{K}\frac{\partial h}{\partial t} \longrightarrow \nabla^{2+} h^{+} = \left(\frac{S_s L^2}{K t_e}\right)\frac{\partial h^{+}}{\partial t^{+}}$$

diffusion equation
Dimensional Analysis

- Fourier Number, N_{FO};

$$N_{FO} = \frac{S_s L^2 / K}{t_e} = \frac{L^2 / c}{t_e}$$

$t_e >> L^2 / c$ ⟶ Transient behavior will NOT be observable

$t_e < L^2 / c$ ⟶ Transient behavior is observable

- For heat diffusion,

$$N_{FO} = \frac{L^2 / \alpha}{t_e}$$

Conductive-convection equation
Dimensional Analysis

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- Dimensionless group dictate the nature of diffusion process or demonstrate the competition between two rate process

L: length of the model (1D) or some length associated with fluid movement (2D)

$$x^+ = x/L$$

$$y^+ = y/L$$

$$z^+ = z/L$$

$$t^+ = t/t_e$$

$$T^+ = T/T_e$$

+ indicates a dimensionless quantity
L : some characteristic length
t_e : some characteristic time
T_e : some characteristic temperature

$$\nabla^{2+}T^+ - \left[\frac{n\rho_w c_w v L}{k_T}\right]\nabla^+ T^+ = \left[\frac{L^2(\rho c/k_T)}{t_e}\right]\frac{\partial T^+}{\partial t^+}$$

Conductive-convection equation
Peclet Number (N_{PE})

$$\nabla^{2+}T^{+} - \left[\frac{n\rho_w c_w vL}{k_T}\right]\nabla^{+}T^{+} = \left[\frac{L^2(\rho c/k_e)}{t_e}\right]\frac{\partial T^{+}}{\partial t^{+}}$$

Transport by bulk fluid motion

$$N_{PE} = \frac{n\rho_w c_w vL}{k_T} = \frac{\rho_w c_w qL}{k_T}$$

Transport by conduction

q : specific discharge
L : some characteristic length

- Peclet number (N_{PE}): expresses the transport of energy by bulk fluid motion to the energy transport by conduction.
- Reflects a competition between forced convection and conduction.
- Large N_{PE} → convective transport dominates

Conductive-convection equation
1D form

$$\frac{k_T}{\rho c}\frac{\partial^2 T}{\partial x^2} - \frac{n\rho_w c_w}{\rho c} v_x \frac{\partial T}{\partial x} = \frac{\partial T}{\partial t}$$

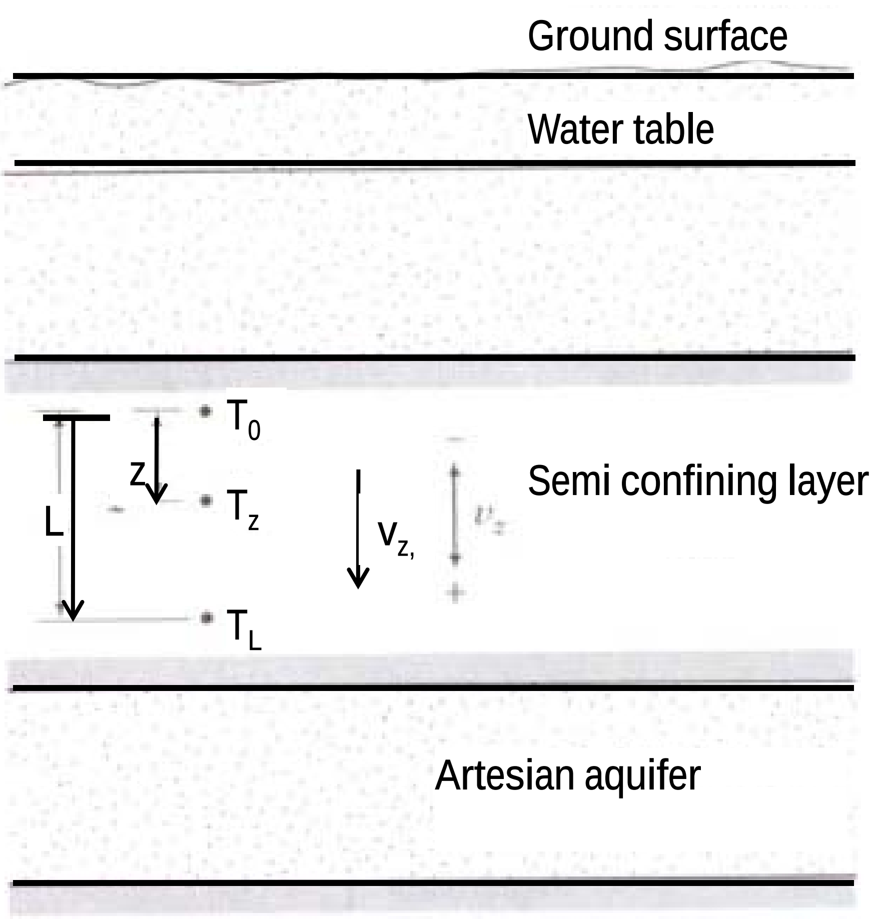
- v_x: mean groundwater velocity in the x-direction
- It is assumed that the temperature of the fluid and the solids are equal.

$$\frac{\partial^2 T}{\partial z^2} - \frac{n\rho_w c_w}{k_T} v_z \frac{\partial T}{\partial z} = 0$$

solution

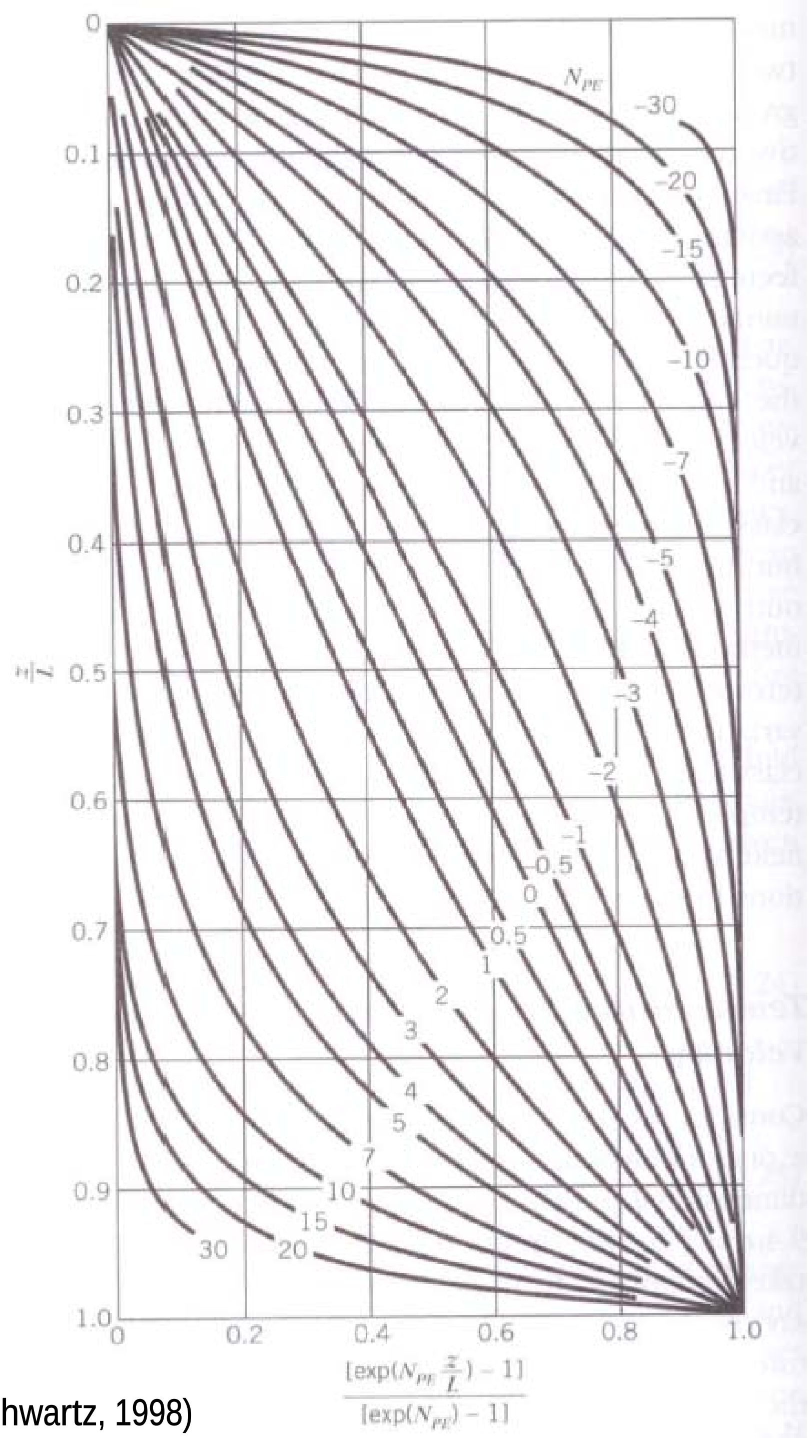
$$T = T_0 + \left(T_L - T_0\right)\frac{\left\{\exp\left(N_{PE}\frac{z}{L}\right)-1\right\}}{\left\{\exp\left(N_{PE}\right)-1\right\}}$$

Conductive-convection equation
1D form



T_0,$T_{z,}$ T_L : measured temeprature

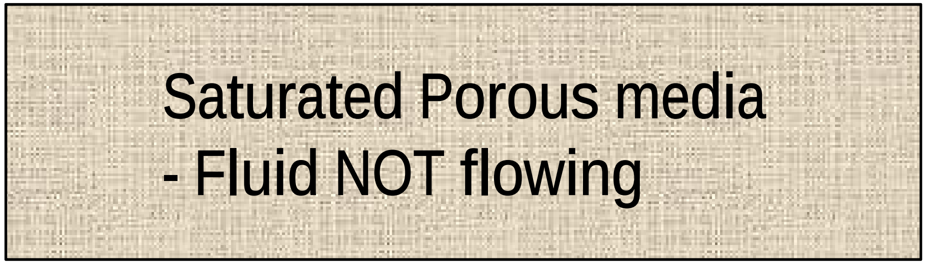
$v_{z,}$: leakage rate (pore linear velocity)



(Domenico and Schwartz, 1998)

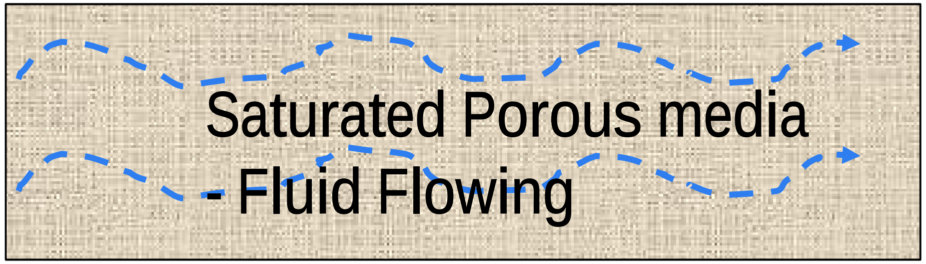
Convective heat transfer

- What will be the factors that make these two cases different?



Saturated Porous media
- Fluid NOT flowing

T_0=100°C T_L=15°C

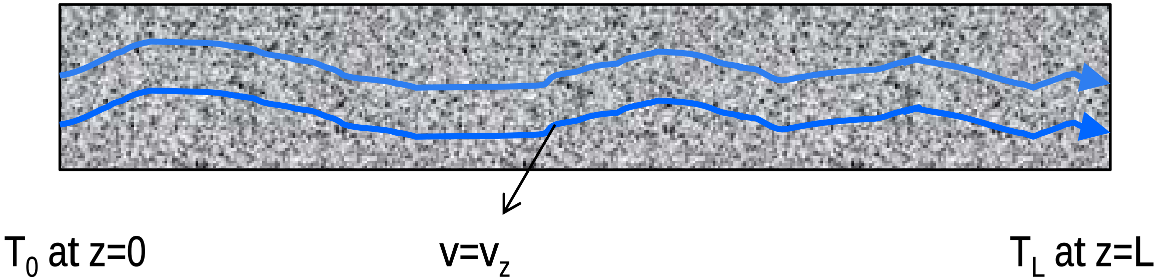


Saturated Porous media
- Fluid Flowing

T_0=100°C T_L=15°C

- In reality, the velocity cannot be taken as constant. → diffusion equation of porous media flow and conduction-convection equation have to be solved simultaneously.

Conductive-convection equation
1D form



- In reality, the velocity cannot be taken as constant. → diffusion equation of porous media flow and conduction-convection equation have to be solved simultaneously.

Thermal expansion

- Linear thermal expansion

$$\frac{\Delta l}{l} = \alpha\left(T - T_0\right)$$

α: coefficient of linear thermal expansion (unit: /K)

T_o: reference temperature

T: new temperature

- Volumetric thermal expansion

$$\frac{\Delta V}{V} = 3\alpha\left(T - T_0\right)$$

- Thermal expansion coefficient of Rock
  - Berea Sandstone: 1.5×10^{-5},
  - Boom clay: 3.3×10^{-6}
  - Water: 6.6×10^{-5}

Convective heat transfer
Free convection

- Linear pore velocity for free convection (under hydrostatic pressure)

$$v = \frac{kg\rho_0 3\alpha(T - T_0)}{n\mu}\frac{\partial z}{\partial x} \qquad \longleftarrow \qquad v = \frac{q}{n}$$

Convective heat transfer
Free convection

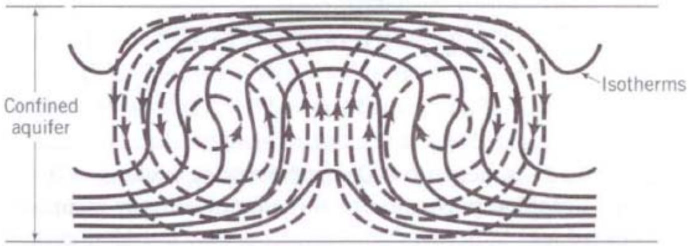


Figure 9.19 Free convection in a confined aquifer (from Donaldson, J., Geophys. Res., v. 67, p. 3449–3459, 1962. Copyright by Amer. Geophys. Union).

- In a buoyancy-driven fluid system (steady-state),

$$\nabla^{2+}T^{+} - \left[\frac{g\rho_0 c_w \rho_w Lk\alpha(T-T_0)}{\mu k_T}\right]\nabla^{+}T^{+} = 0$$

q : specific discharge
L : some characteristic length

$$N_{RA} = \frac{g\rho_0 c_w \rho_w Lk\alpha(T-T_0)}{\mu k_T}$$

- Rayleigh number (N_{RA}): expresses the transport of energy by free convection to the energy transport by conduction.
- Used to establish the conditions for the onset of free convection.
- Onset of free convection: ~ 40 based on horizontal layers with impermeable boundaries (from Domenico and Schwartz, 1998)

(Domenico and Schwartz, 1998)

Today

- Mixing laws - Effective thermal conductivity, heat capacity
- Forced convection and free convection
- Conductive-convection equation
  - Derivation
  - Peclet Number
  - 1D equation
- Dimensional analysis
- Free convection

Next week

- Reservoir Geomechanics
  - Hydraulic fracturing
  - Borehole stability
  - Coupled process
  - …

References

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- Gueguen Y and Palciauskas V, 1994, Introduction to the physics of rocks, Princeton University Press
- Somerton WH, 1992, Thermal properties and temperature-related behavior of rock/fluid systems, Elsevier