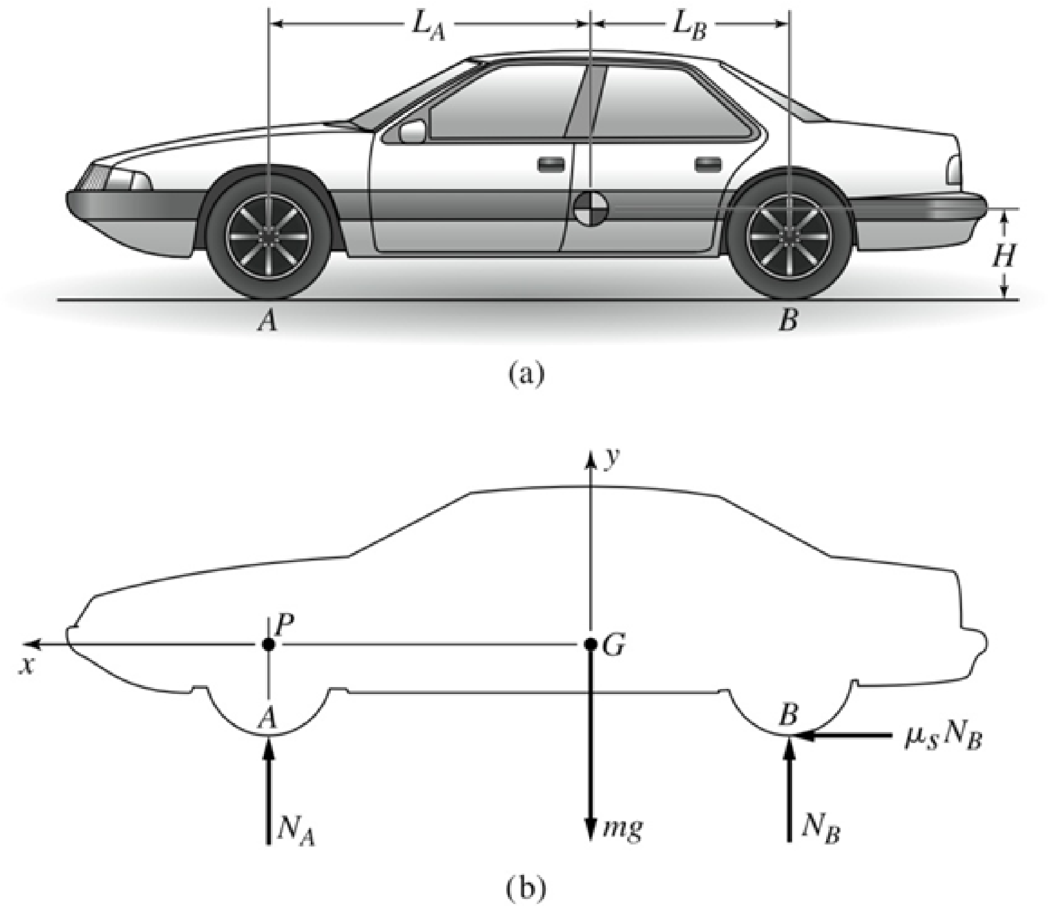
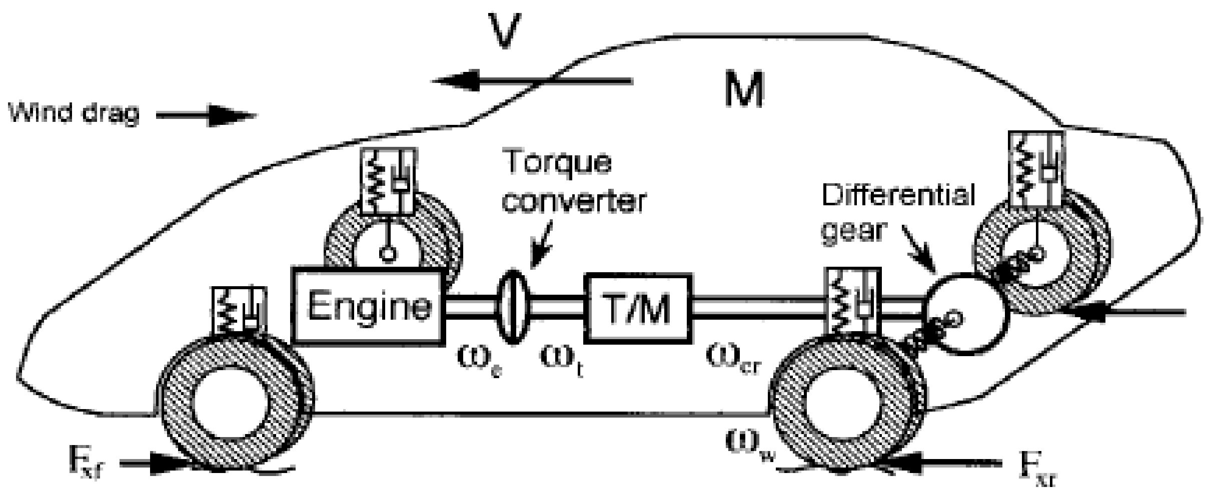
Mechanical Systems II

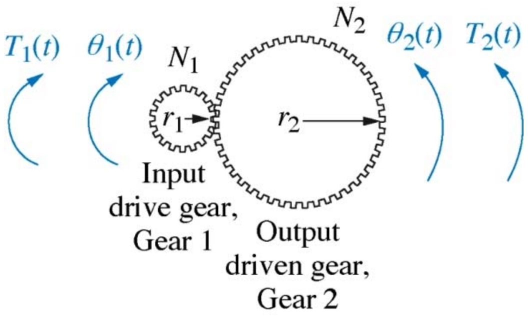
Maximum vehicle acceleration

Rear wheel drive




V
M
Wind drag
Torque
converter
Differential
gear
Engine
T/M
ω_e
ω_t
ω_{cr}
ω_w
F_{xf}
F_{xr}

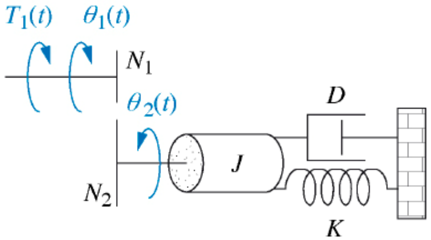
systems with Gears



$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

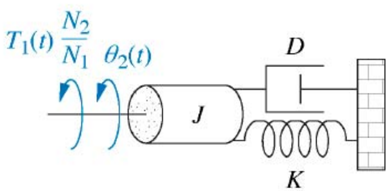
Torque relationship? $$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2}$$

(*a*) (*b*)



(*a*)

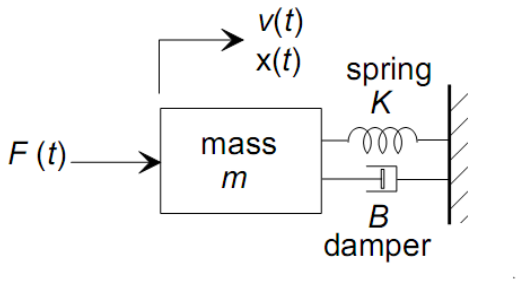
Represent the system with gears without gears.



(*b*)

$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

Solution of Differential Equation by Laplace Transformation



$$m\ddot{x} + B\dot{x} + Kx = F.$$

$$y'' + 2y' + 4y = 1 \qquad y(0) = 0\ ,\ y'(0) = 2$$

L.T: $$s^2Y(s) - sy(0) - y'(0) + 2\{sY(s) - y(0)\} + 4Y(s) = \frac{1}{s}$$

$$(s^2 + 2s + 4)Y(s) = \frac{1}{s} + 2 = \frac{2s+1}{s}$$

$$Y(s) = \frac{2s+1}{s(s^2+2s+4)} = \frac{1}{4s} - \frac{1}{4}\frac{s+1-1}{(s+1)^2 + \left(\sqrt{3}\right)^2}$$

$$\therefore\ y(t) = \frac{1}{4} - \frac{1}{4}e^{-t}\cos\sqrt{3}t + \frac{1}{4\sqrt{3}}e^{-t}\sin\sqrt{3}t$$

Work , Energy, and Power

- Mechanical work : $W = F \cdot x$ [N·m] = [Joule]

  = Force × displacement

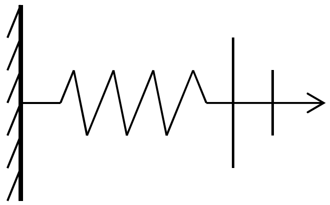
- Energy : capacity or ability to do work. Electrical, Chemical, Mechanical, etc.

- Mechanical energy : Potential energy – position

  Kinetic energy – velocity

Potential Energy

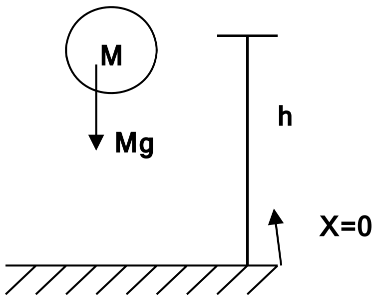
ex1)



$$F = kx$$

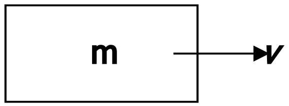
$$dw = F \cdot dx = kx \cdot dx \qquad \int_0^{x1} dw = \int_0^{x1} kx\, dx = \frac{1}{2} kx^2$$

ex2)

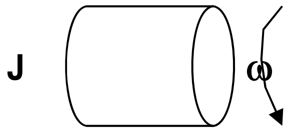


$E_1 = mgx$ **(Potential Energy)**

Kinetic Energy



: $\frac{1}{2}mv^2$



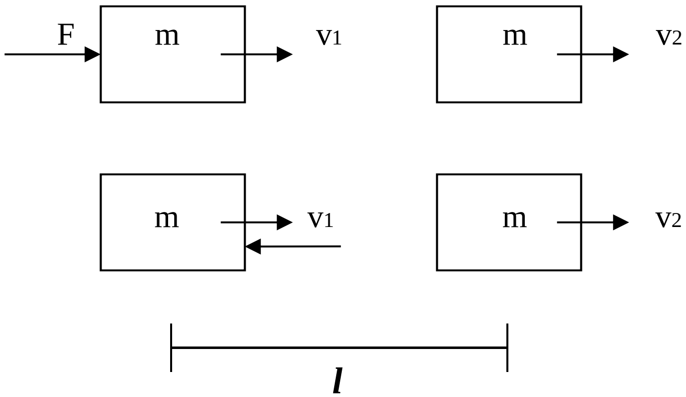
: $\frac{1}{2}J\omega^2$

Work and Energy

System + External Work = E_2

Energy E_1 W

$$E_1 + W = E_2$$



$$E_1 + W = E_2$$

$$\frac{1}{2}mv_1^2 + Fl = \frac{1}{2}mv_2^2$$

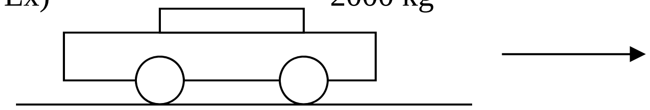
$$\frac{1}{2}mv_1^2 - Fl = \frac{1}{2}mv_2^2$$

Power

Power : time rate & doing work

$$P = \frac{dw}{dt} \left[\frac{Nm}{\sec} \right] = [Watt]$$

Ex) 2000 kg



$V = 72\text{km/h} = 20\text{m/s}$ (in 10sec)

$V_0 = 0$

$$\frac{1}{2} m v_0^2 + W = \frac{1}{2} m v^2$$

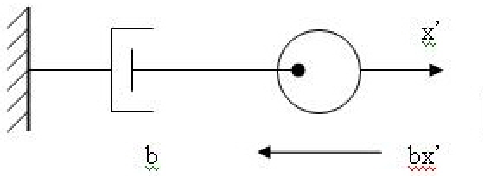
$$W = \frac{1}{2}(2000)(20)^2 = 400 \times 1000 \ Nm = 400 \ kNm[kJ]$$

$$P = \frac{W}{t} = \frac{400 \times 1000}{10} \frac{Nm}{\sec}$$

$$= 40 \times 1000 \ Nm/s = 40 \times 1000 \ W = 40W$$

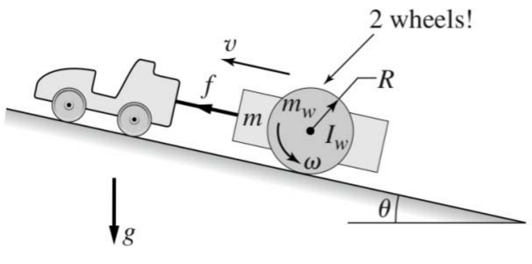
$1hp = 745.7 \ W$ $\qquad \therefore P = 54 \ hp$

Power dissipated in a damper



$$P = Fv = b\dot{x} \cdot \dot{x} = b\dot{x}^2$$

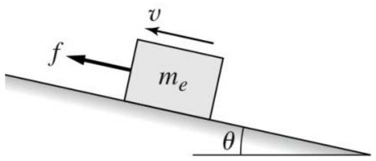
Energy Method for Deriving Equivalent Mass and Inertia



(a)

- **Kinetic Energy of the total System**

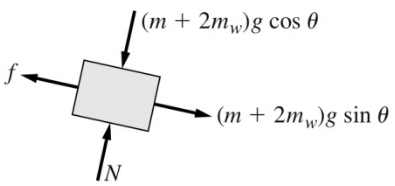
$$KE = \frac{1}{2}mv^2 + \frac{1}{2}(2m_w v^2) + \frac{1}{2}I_w\omega^2$$



(b)

- **Kinetic Energy represented with a single variable**

$$KE = \frac{1}{2}\left(m + 2m_w + 2\frac{I_w}{R^2}\right)v^2$$



(c)

- **Equation of motion using equivalent mass**

$$m_e = \left(m + 2m_w + 2\frac{I_w}{R^2}\right)$$

Energy Method for Deriving Equations of Motion

- Conservative system : No energy dissipation

$$E_1 + W = E_2$$
$$E_2 - E_1 = W$$

- Kinetic Energy T

- Potential Energy U

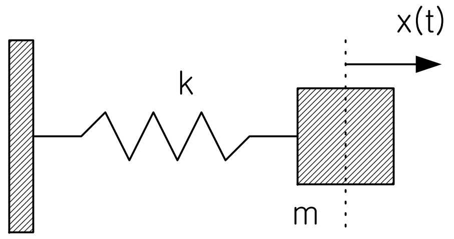
$\Delta(T+U) = \Delta W$

(the change in the total energy)
= (the net work done on the system by the external force)

no external force ; $\Delta W = 0$
$\Delta(T+U) = 0$ T+U = constant

Examples of Energy Method

ex1)



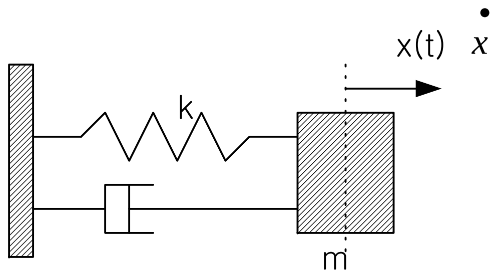
$$T = \frac{1}{2} m\dot{x}^2, \quad U = \frac{1}{2} kx^2, \quad T + U = C$$

$$\rightarrow \qquad \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = 0$$

$$\frac{d}{dt}(T + U) = 0, \quad m\dot{x}\ddot{x} + kx\dot{x} = (m\ddot{x} + kx)\dot{x} = 0$$

$$\dot{x} \neq 0, \quad \therefore (m\ddot{x} + kx) = 0$$

ex2)



$$T = \frac{1}{2} m\dot{x}^2, \quad U = \frac{1}{2} kx^2$$

$$\frac{d}{dt}(T + U) = -b\dot{x}^2, \quad m\dot{x}\ddot{x} + kx\dot{x} = -b\dot{x}\dot{x} = 0$$

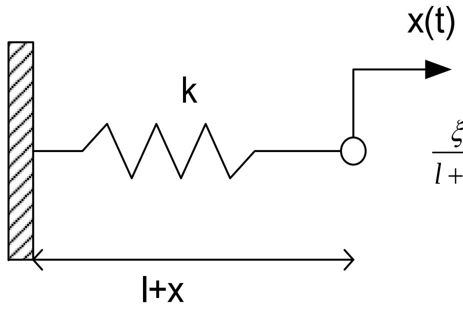
$$(m\ddot{x} + kx + b\dot{x})\dot{x} = 0$$

$$\dot{x} \neq 0, \quad \Rightarrow (m\ddot{x} + kx + b\dot{x}) = 0$$

Examples of Energy Method

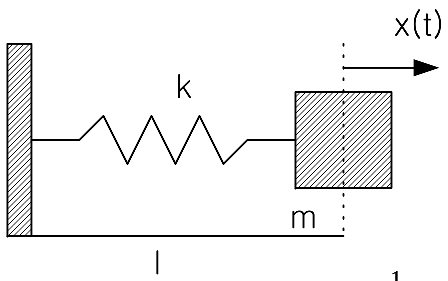
Spring with mass

ex3)



$\frac{\xi}{l+x}\dot{x}$ $\quad \frac{d\xi}{l+x} \bullet m_s$

$$dT = \frac{1}{2} m_s \cdot \frac{d\xi}{l+x} \cdot \left(\frac{\xi}{l+x}\dot{x}\right)^2$$



$m\ddot{x} + kx = 0$

$\omega_n = \sqrt{\frac{k}{m}}$

Potential E $\quad U = \frac{1}{2}kx^2$

Kinetic E $\quad T = \frac{1}{2}m\dot{x}^2$

$$\int_0^{l+x} dT = \int_0^{l+x} \frac{1}{2} m_s \frac{1}{(l+x)^3}\dot{x}^2\xi^2 d\xi$$

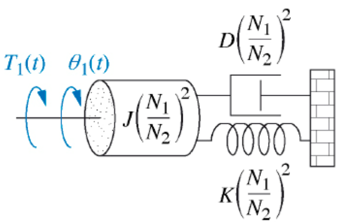
$$= \frac{1}{2} m_s \dot{x}^2 \frac{1}{(l+x)^3} \frac{1}{3}(l+x)^3$$

$$= \frac{1}{2}\left(\frac{1}{3}m_s\right)\dot{x}^2$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{3}m_s\right)\dot{x}^2 = \frac{1}{2}\left(m + \frac{1}{3}m_s\right)\dot{x}^2 \qquad \left(m + \frac{1}{3}m_s\right)\ddot{x} + kx = 0 \qquad \omega_n = \sqrt{\frac{k}{m + \frac{1}{3}m_s}}$$

End of Lecture 3

To be revisited: Transfer function for systems with Gears



(c)

Impedances are reflected from the output to the input, therby eliminating the gears.

$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$(J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2)\theta_1(s) = T_1(s)$$

Rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio,

$$\left(\frac{\theta_2}{\theta_1}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 = \left(\frac{\text{Number of teeth of gear on } \textbf{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textbf{source} \text{ shaft}}\right)^2$$