

# **Electrical Systems I**

# **Development of Integrated Vehicle Control System of “Fine-X” Which Realized Freeer Movement.**



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**Toyota Motor Corporation.**



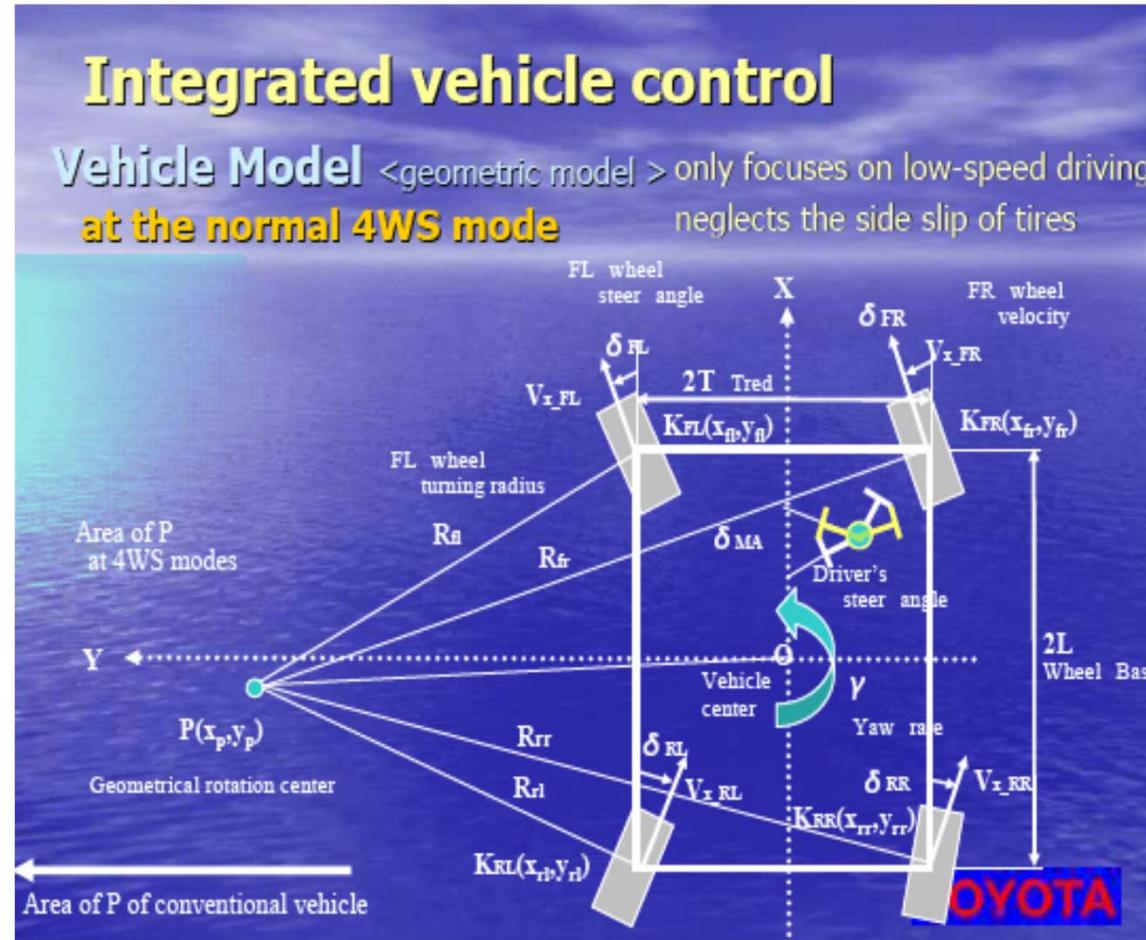
# TOYOTA Freer Movement Control System



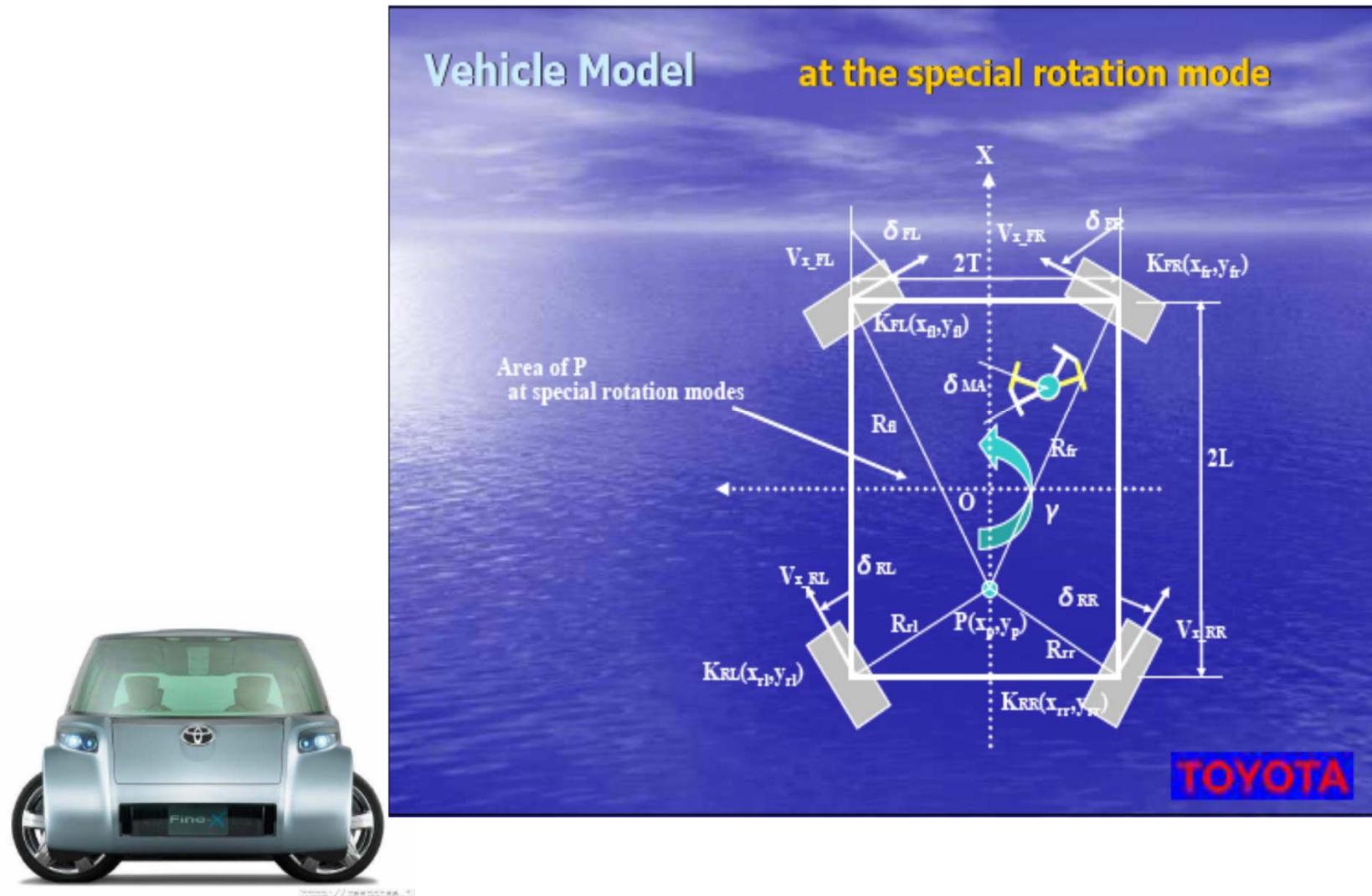
4Wheel independent drive  
4wheel independent steering  
4wheel independent braking  
By 'wheel-in-motor'



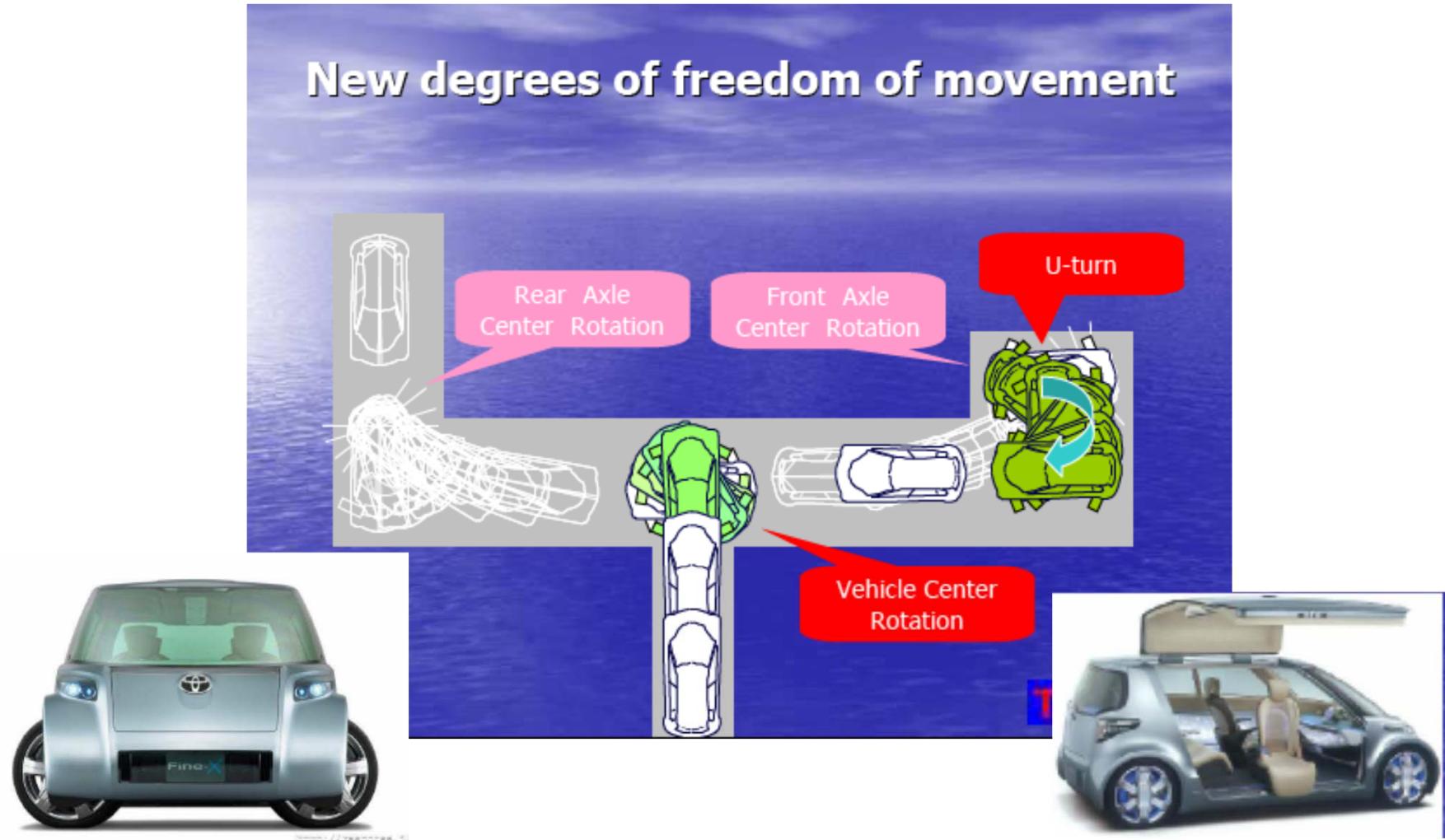
# TOYOTA Freer Movement Control System



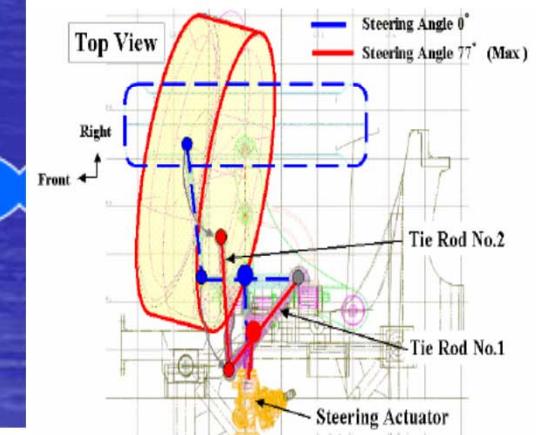
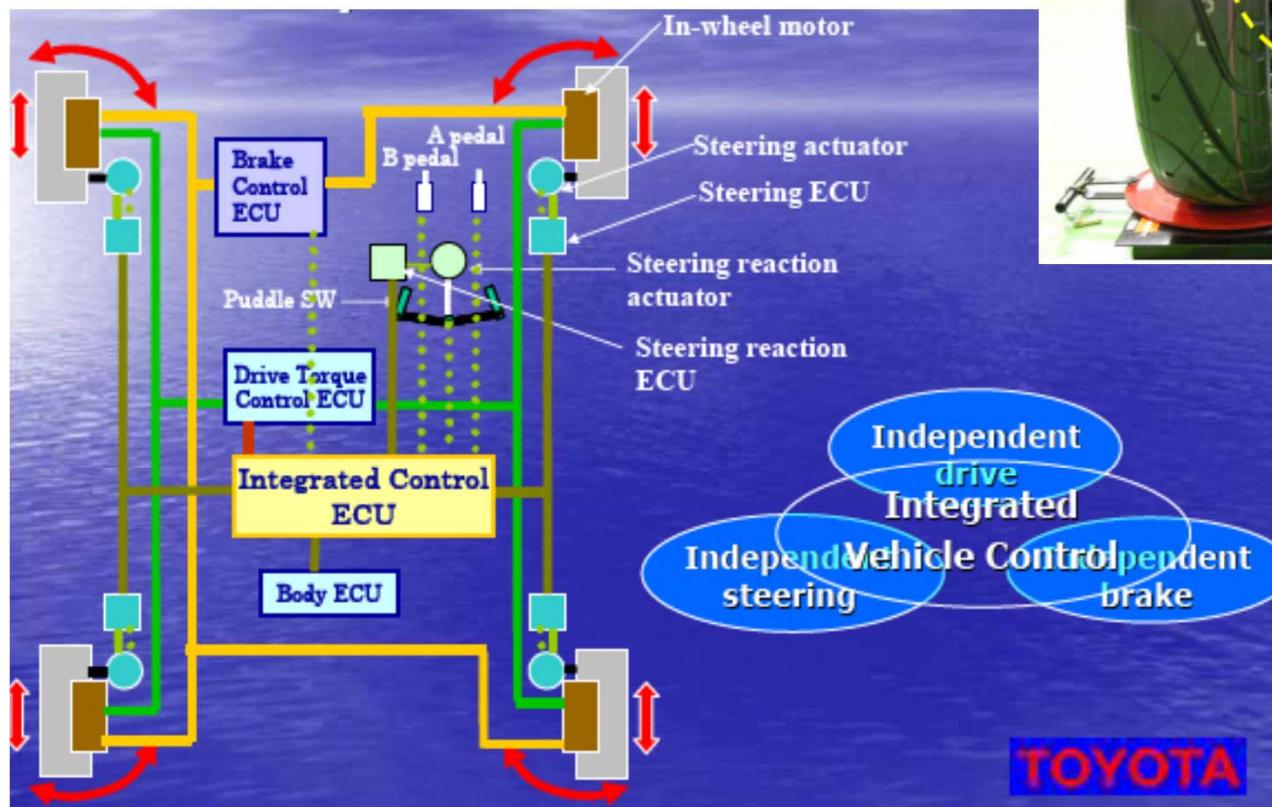
# TOYOTA Freer Movement Control System



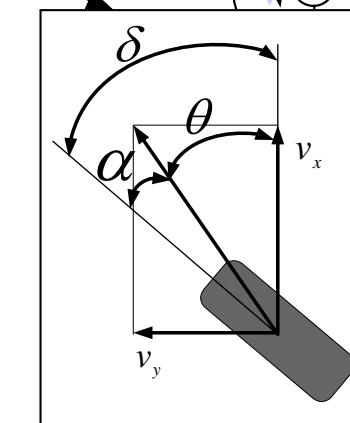
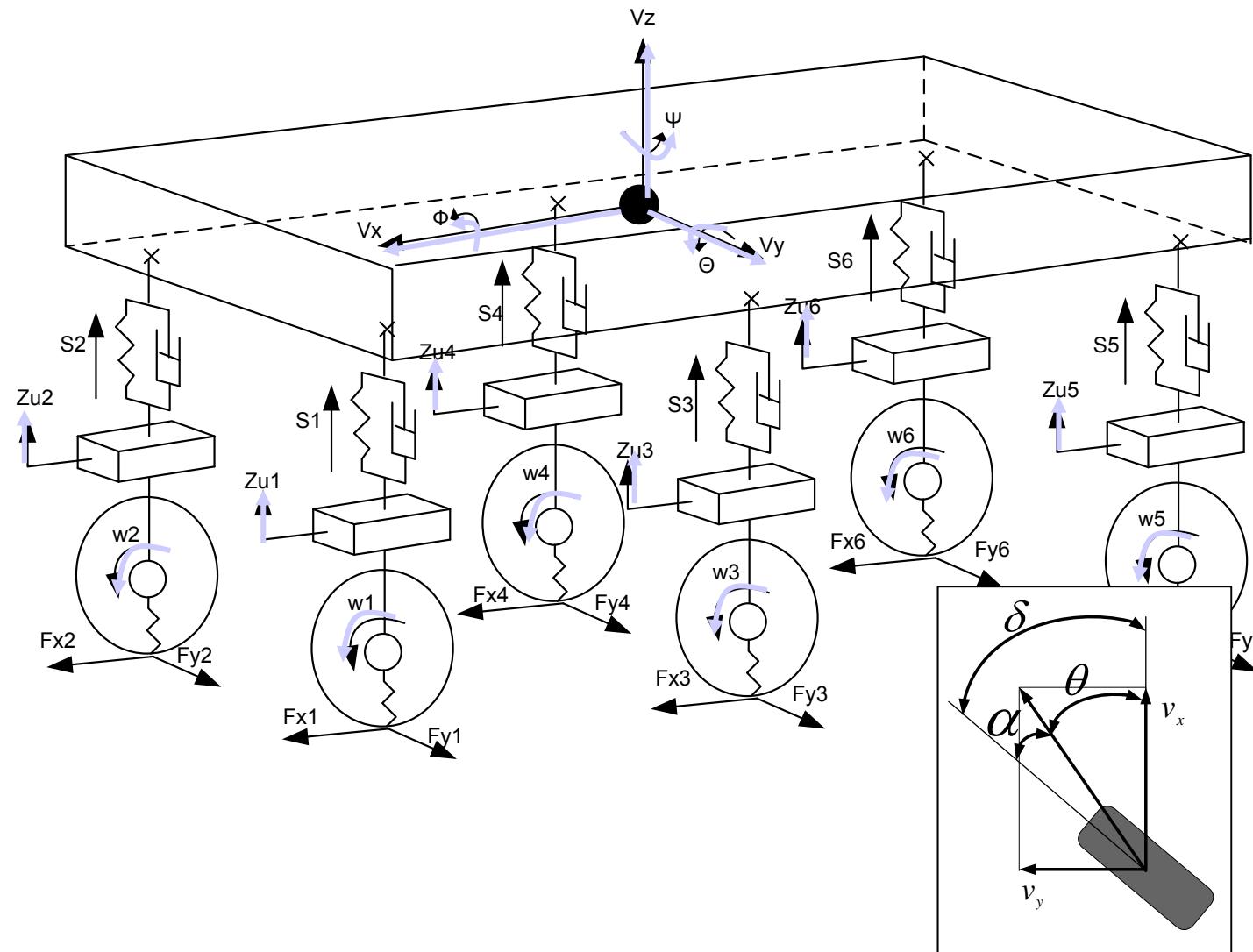
# TOYOTA Freer Movement Control System for Auto-Parking



# TOYOTA Freer Movement Control System for Auto-Parking



# 6WD6WS Vehicle



$$u = \begin{bmatrix} \Delta\delta_f \\ \delta_r \\ \delta_m \end{bmatrix}$$

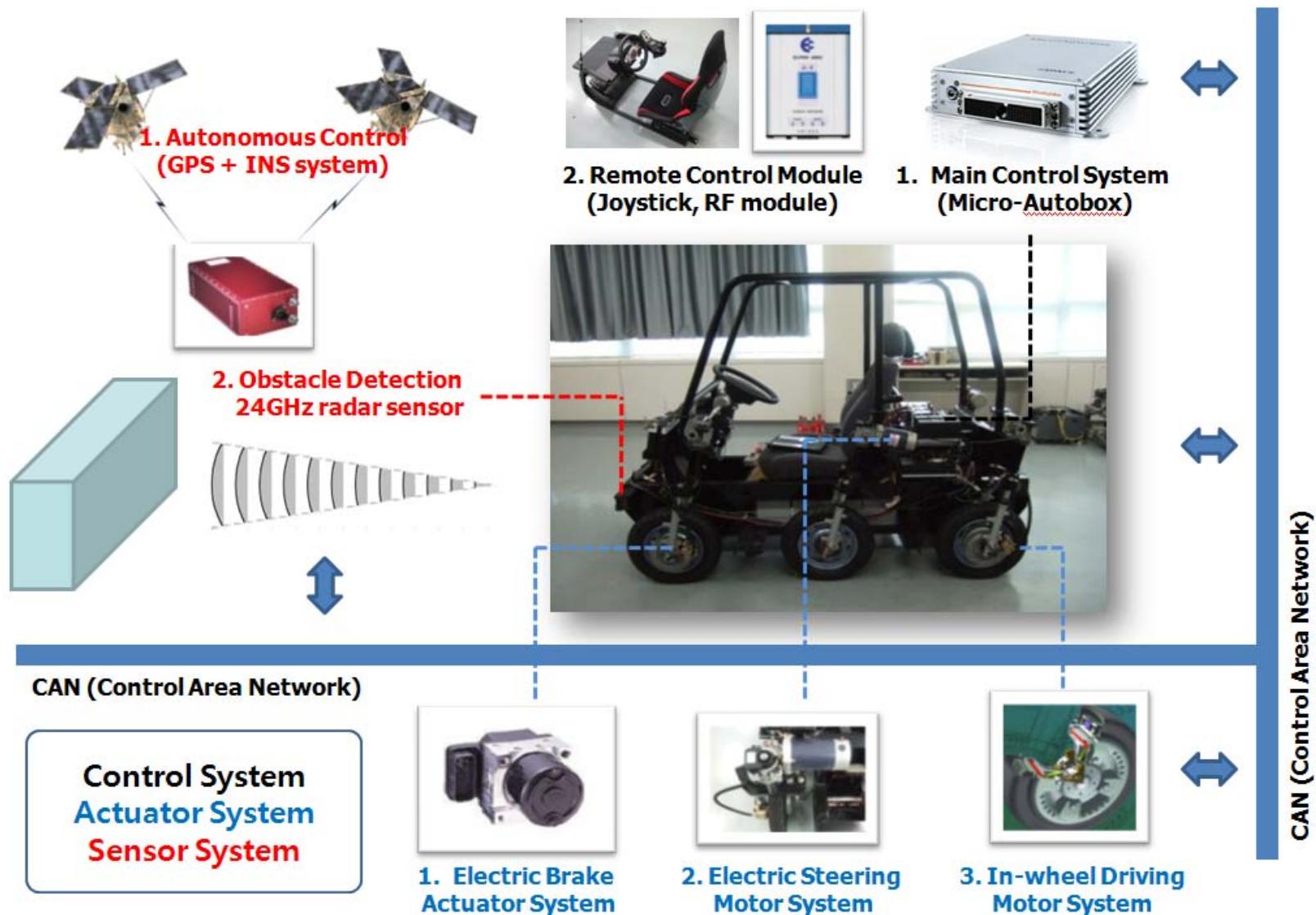
$$\delta_m = X_1\beta + X_2r$$

The right-hand side of the diagram contains two parts. On the left, a vector  $u$  is defined as a column matrix containing  $\Delta\delta_f$ ,  $\delta_r$ , and  $\delta_m$ . On the right, the formula  $\delta_m = X_1\beta + X_2r$  is given. Below these equations is a small diagram of a steering actuator, which is a cylinder with a rod extending from its side, representing the physical mechanism for applying the steering moment.

# BLDC Wheel-in-Motor of 6WD6WS Vehicle



# Configuration of 6WD6WS Vehicle



# Sectional View of BLDC Motor



# Video of 6WD/6WS Vehicle equipped with Wheel-in Motor

Parallel & Circular Turning



평행\_제자리선회.wmv

Remote Control



Remote\_general\_recent.avi

Autonomous Driving



Autonomous\_path\_tracking.avi

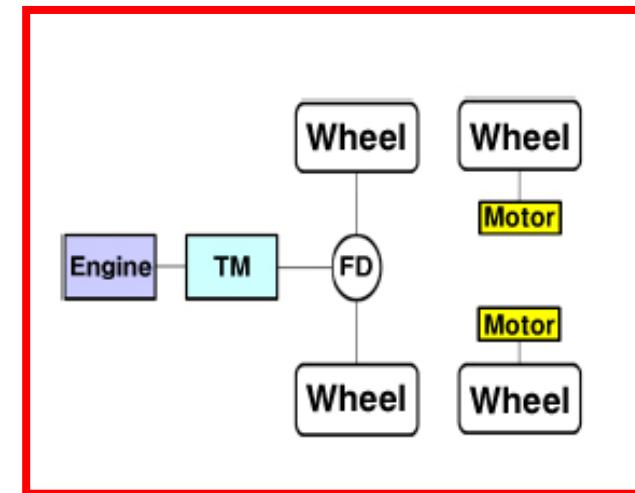
# Hybrid and Electrical Vehicles

## ● Advantages

- Minimized both costs and technical issues
- Even weight distribution
- Simplest packaging
- Improved vehicle chassis control

## ● Targets

- To maintain or improve on Euro 4 emissions
- To achieve a 30% overall reduction in CO<sub>2</sub> tail pipe emissions of the baseline vehicle operating with the same fuel, over the NEDC cycle as per EC/98/69 and EC/70/220 for vehicles less than 3500kg.
- Equivalent fuel economy ≥ 60mpg=25.4kpl (50% improvement in mpg on baseline vehicle)

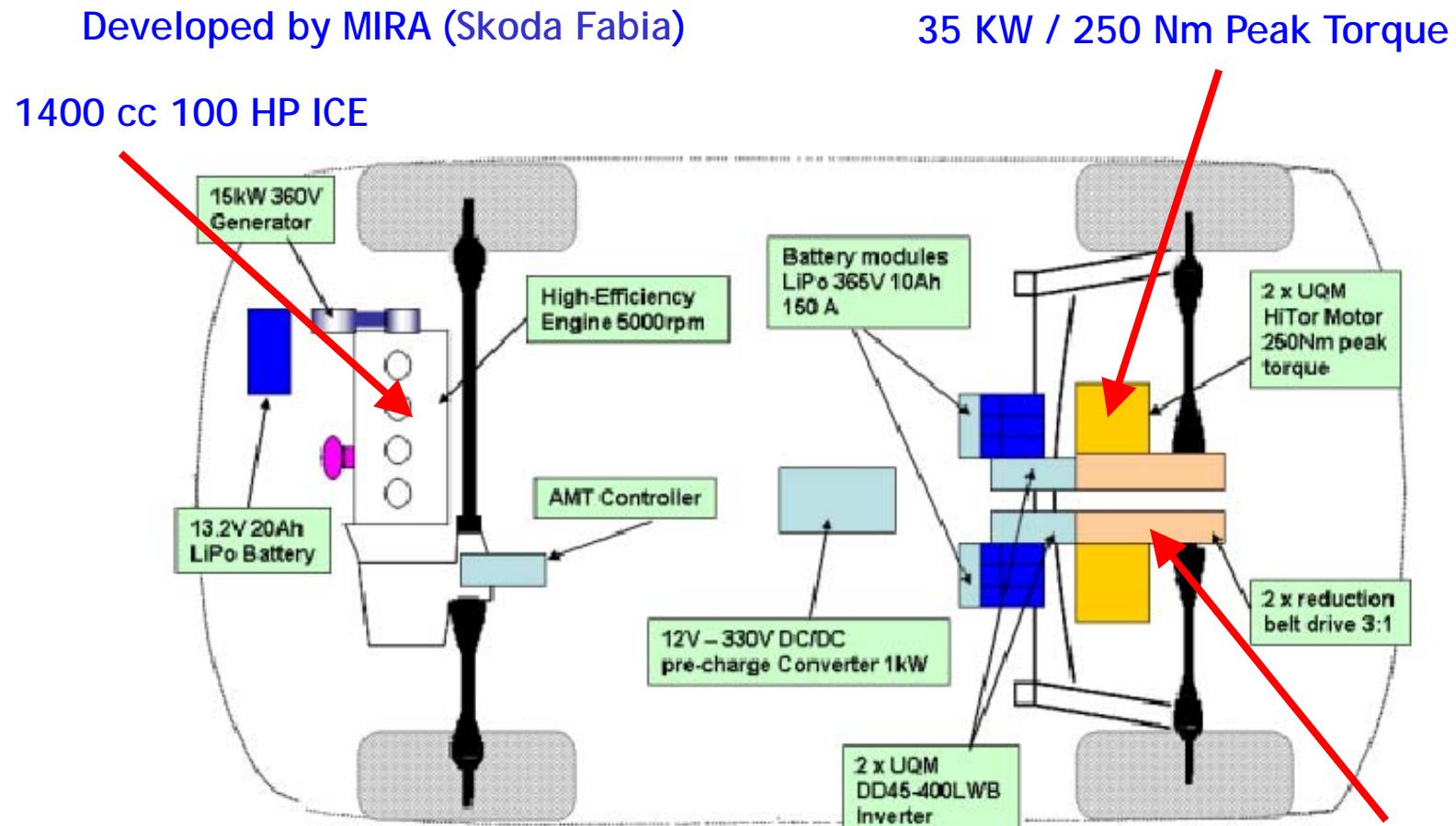


# Hybrid and Electrical Vehicles

Skoda Fabia (Compact SUV)



# Hybrid 4WD Vehicle Configuration



A total weight of around 150kg:  
motors = 2x45kg, inverters=2x15kg, structure=30kg

3:1 reduction belt drive  
→ Peak torque = 750 Nm

# 4 WD In-wheel Electric Vehicle

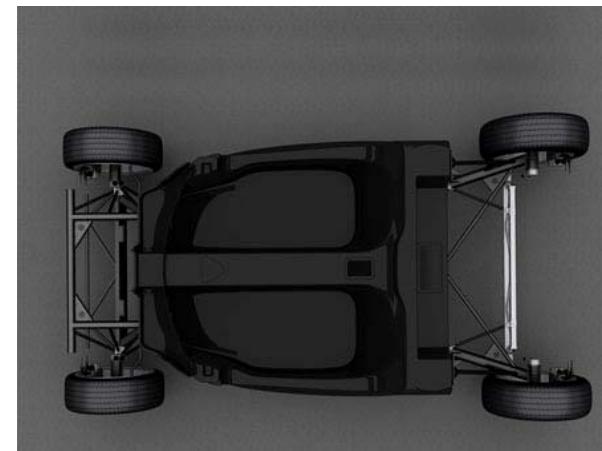
Michelin Active Wheel with in-wheel motor, suspension and brake system



Electric vehicle equipped with Front two in-wheel motors

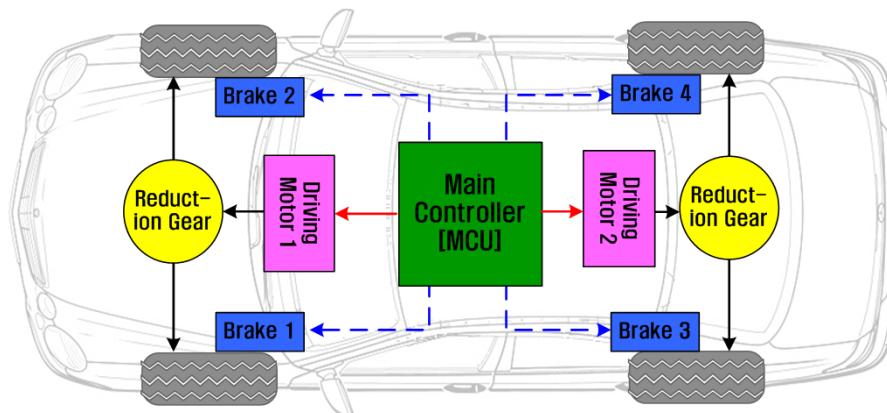


Electric vehicle equipped with four in-wheel motors

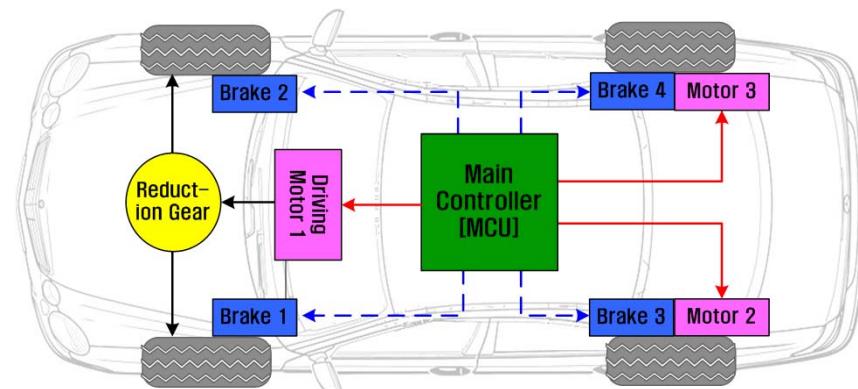


# 4 WD Electric Vehicle Combinations

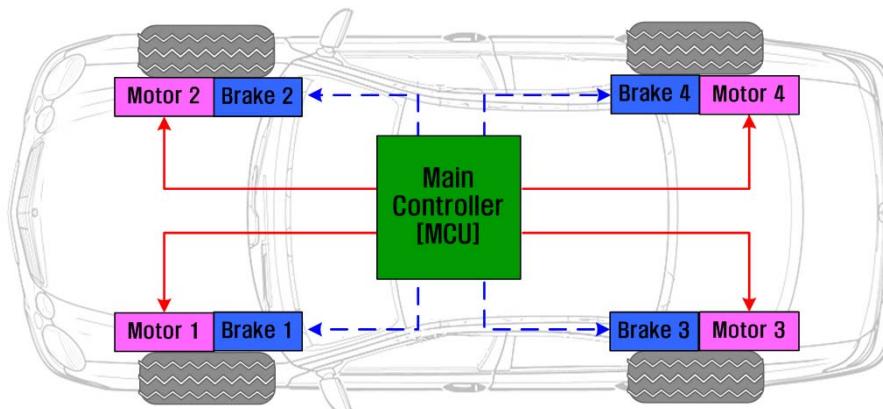
- Front/rear Two In-line Motors



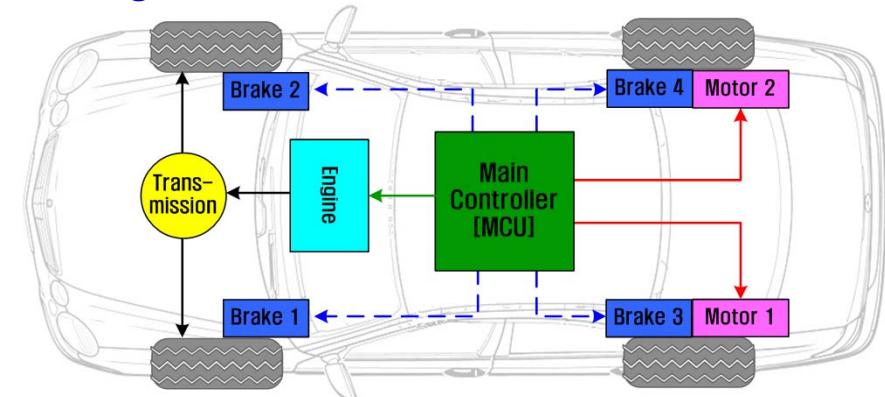
- Front In-line Motor/Rear In-wheel motors



- Four In-wheel Motors



- Front Engine Drive/Rear Two In-wheel Motors



# 6WD Skid Steering Vehicle

**Crusher**



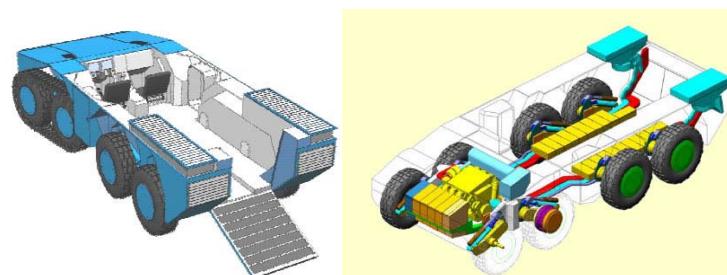
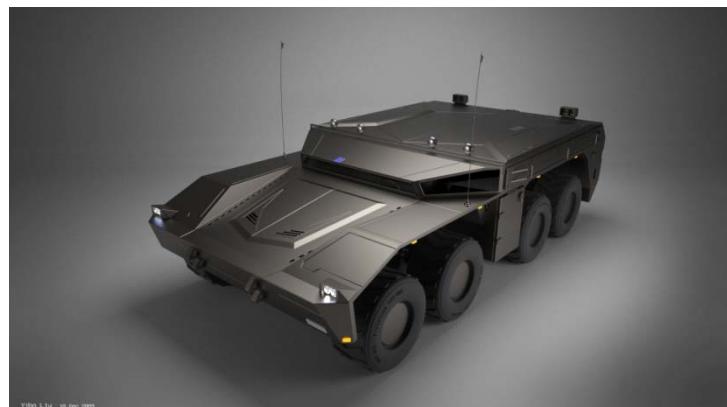
[\*\*Crusher highlights.wmv\*\*](#)

**APD**



[\*\*Autonomous Platform Demonstrator  
\(APD\) Overview.wmv\*\*](#)

# 8WD/4WS Vehicle Equipped with 8 In-wheel Motor AHED



AHED\_8x8\_vehicle\_vedio.wmv

# Basic Elements

- **Active Elements:** OP Amp etc.  
(Has transistors/amplifiers that require active source of power to work)

- **Passive Elements:** Inductor, Resistor, Capacitor etc.  
(Simply respond to an applied voltage or current.)

- **Current :** the rate of flow of charge
- **Charge :** (electric charge) the integral of current with respect to time [C]

$$i = \frac{dq}{dt} \quad [\text{ampere}] = \frac{[\text{coulomb}]}{[\text{sec}]}$$

- **Voltage :** electromotive force needed to produce a flow of current in a wire [V]

Change in energy as the charge is passed through a component.

$$[V] = [J/C]$$

- **Power:** product of voltage and current

$$[W] = [J/\text{Sec}]$$

## Basic Elements - Resistance

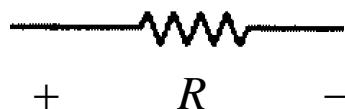
- Resistance : the change in voltage required to make a unit change in current.

Analogous to → Damping Element

$$R = \frac{\text{Change in voltage}}{\text{Change in current}} = \frac{[V]}{[A]} = [\text{Ohm}(\Omega)]$$

- Resistor

$$V_R = R \cdot i_R \quad R = \frac{V_R}{i_R}$$



## Basic Elements - Capacitance

- Capacitance: the change in the quantity of electric charge required to make a unit change in voltage.

Analogous to → Spring Element (Stores Potential Energy)

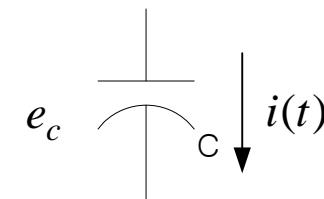
$$C = \frac{[\text{Coulomb}]}{[V]} = [\text{Farad } (F)]$$

- Capacitor: two conductor separated by non-conducting medium.

$$i = dq/dt, \quad e_c = q/C \quad \rightarrow \quad i = C \frac{de_c}{dt}, \quad de_c = \frac{1}{C} i dt$$

$$\therefore e_c(t) = \frac{1}{C} \int_0^t i dt + e_c(0)$$

$$I(s) = CsV(s), \quad V(s) = \frac{1}{Cs} I(s)$$



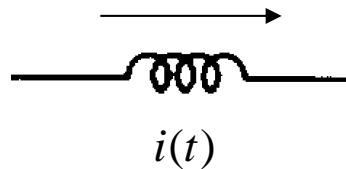
## Basic Elements - Inductance

- Inductance: An electromotive force induced in a circuit, if the circuit lies in a time-varying magnetic field.

Analogous to → Inertia Element (Stores Kinetic Energy)

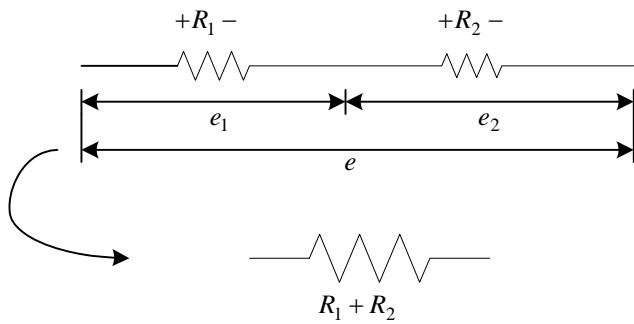
$$L = \frac{[V]}{[A/\text{sec}]} = [\text{Henry (H)}]$$

- Inductor:  $e_L = L \frac{di_L}{dt}$        $V(s) = LsI(s)$   
 $\therefore i_L(t) = \frac{1}{L} \int_0^t e_L dt + i_L(0)$        $I(s) = \frac{1}{Ls} V(s)$



# Series & Parallel Resistance

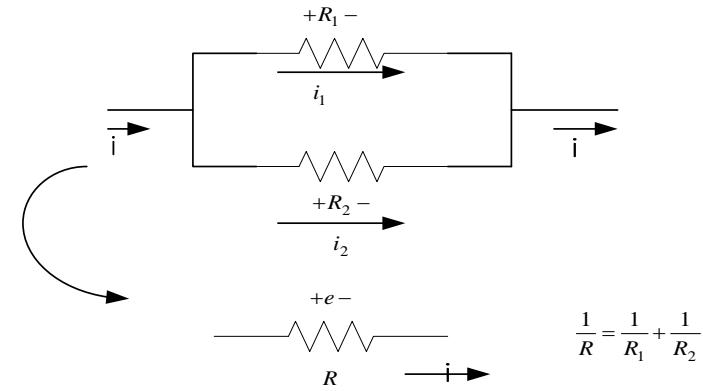
- Series Resistance



$$e_1 = iR_1, \quad e_2 = iR_2$$

$$e = e_1 + e_2 = i(R_1 + R_2)$$

- Parallel Resistance



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

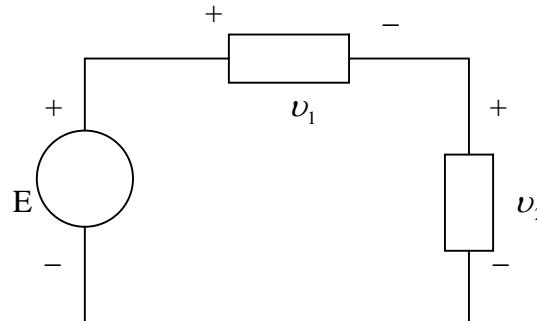
$$-e_1 + e_2 = 0 \Rightarrow e_1 = e_2$$

$$i = i_1 + i_2 = \frac{e_1}{R_1} + \frac{e_2}{R_2} = e\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{e}{R}$$

Series/Parallel Capacitance?

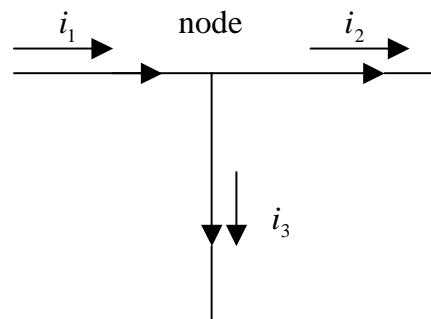
## Kirchhoff's laws

1. The algebraic sum of the potential difference around a closed path equals zero.



$$-v_1 - v_2 + E = 0$$

2. The algebraic sum of the currents entering (or leaving) a node is equal to zero.

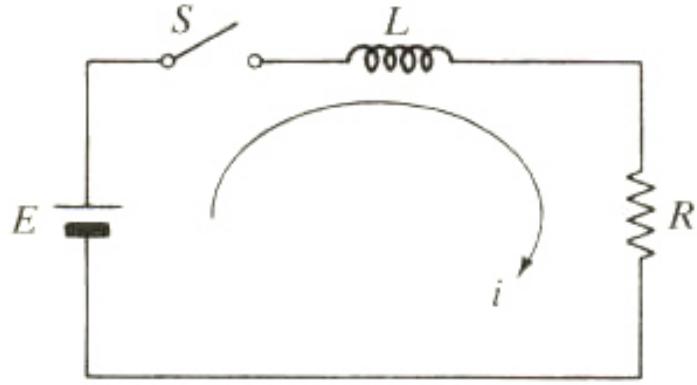


$$i_1 - i_2 - i_3 = 0$$

Current in = Current out

$$\rightarrow i_1 = i_2 + i_3$$

# Mathematical Modeling of Electrical Systems



The switch  $S$  is closed at  $t=0$

$$E - L \frac{di}{dt} - Ri = 0 \quad \text{or} \quad L \frac{di}{dt} + Ri = E$$

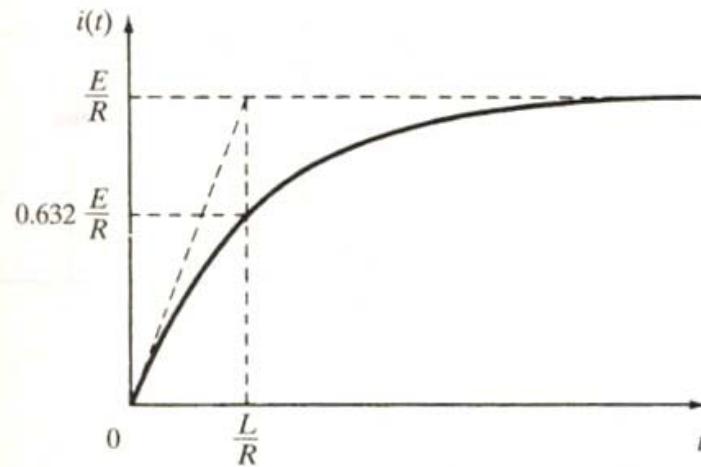
At the instant that switch  $S$  is closed,  
the current  $i(0) = 0$

Laplace Transformation :  $L[sI(s) - i(0)] + RI(s) = \frac{E}{s}$

$$i(0) = 0 \quad \rightarrow \quad (Ls + R)I(s) = \frac{E}{s}$$

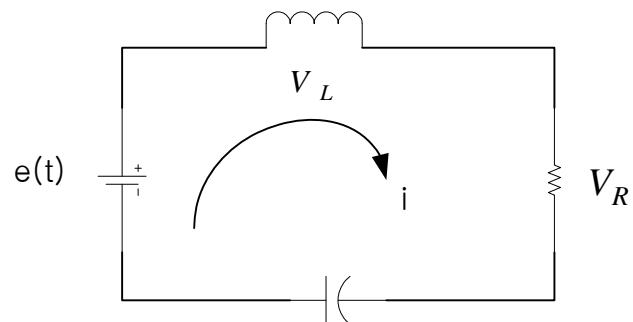
$$I(s) = \frac{E}{s(Ls + R)} = \frac{E}{R} \left[ \frac{1}{s} - \frac{1}{s + (R/L)} \right]$$

$$\therefore i(t) = \frac{E}{R} \left[ 1 - e^{-(R/L)t} \right]$$



# Examples of Circuit Analysis

ex) R-L-C Circuit



$$-V_L - V_R - V_C + e(t) = 0$$

$$V_L = L \frac{di}{dt}, \quad V_R = iR, \quad V_c = \frac{1}{C} \int i dt + V_c(t)$$

$$\frac{dV_c}{dt} = \frac{1}{C} i$$

$$L \frac{di}{dt} + Ri = e(t) - V_c(t)$$

$$\text{Laplace Transform, } (Ls + R + \frac{1}{Cs})I(s) = E(s)$$

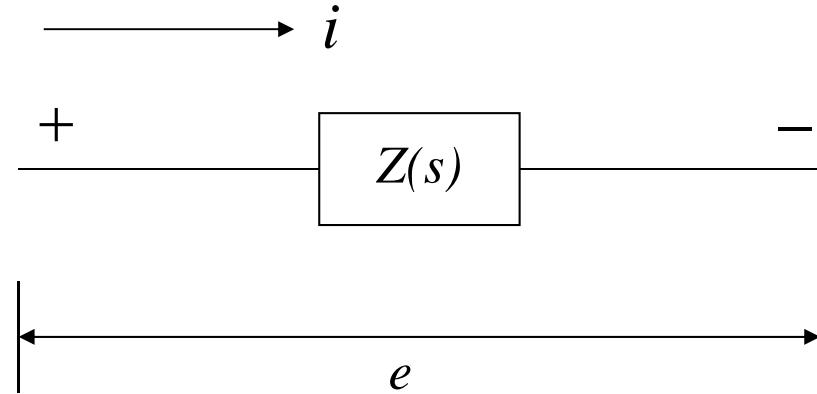
$$\frac{I(s)}{E(s)} = \frac{1}{(Ls + R + \frac{1}{Cs})}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$V_o(s) = \frac{\frac{1}{Cs}}{(Ls + R + \frac{1}{Cs})} E(s) = \frac{1}{(LCs^2 + RCs + 1)} E(s)$$

Step Response?

# Complex Impedance



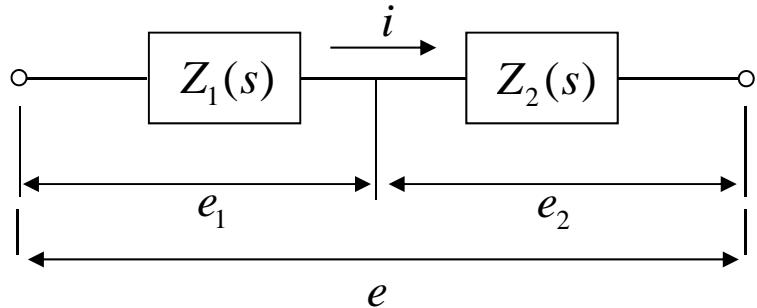
$$I(s) = \frac{E(s)}{Z(s)}$$

$$E(s) = Z(s)I(s)$$

$Z(s)$  : complex impedance

# Complex Impedance

The complex impedance  $Z(s)$  of a two-terminal circuit is : the ratio of  $E(s)$  to  $I(s)$



$$Z(s) = \frac{E(s)}{I(s)}, \quad E(s) = Z(s)I(s)$$

$$E_1(s) = Z_1(s)I(s), \quad E_2(s) = Z_2(s)I(s)$$

$$E(s) = E_1(s) + E_2(s)$$

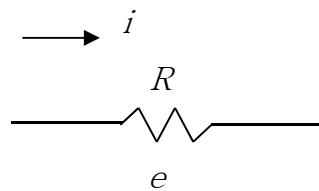
Direct derivation of transfer function,  
without writing differential equations first.

$$= Z_1(s)I(s) + Z_2(s)I(s)$$

$$= (Z_1(s) + Z_2(s))I(s)$$

# Complex Impedance

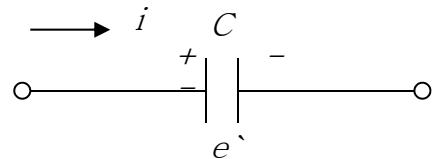
Resistance :



$$e = Ri, \quad E(s) = R I(s)$$

$$Z(s) = R$$

Capacitance :

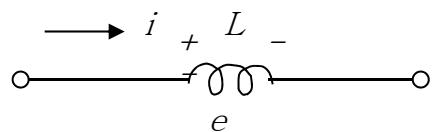


$$\frac{de}{dt} = \frac{1}{C} i$$

$$sE(s) = \frac{1}{C} I(s) \rightarrow E(s) = \frac{1}{Cs} I(s)$$

$$\therefore Z(s) = \frac{1}{Cs}$$

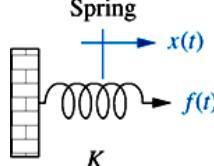
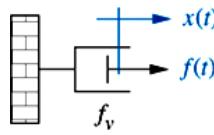
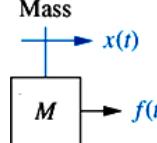
Inductance :



$$e = L \frac{di}{dt}, \quad E(s) = L s I(s)$$

$$\therefore Z(s) = Ls$$

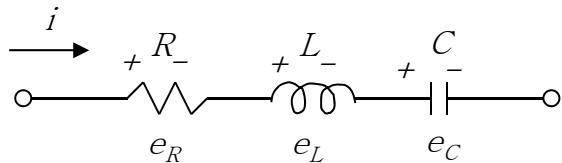
Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau$	$i(t) = C \frac{dv(t)}{dt}$	$v(t) = \frac{1}{C} q(t)$	$\frac{1}{Cs}$	$Cs$
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R} v(t)$	$v(t) = R \frac{dq(t)}{dt}$	$R$	$\frac{1}{R} = G$
 Inductor	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau$	$v(t) = L \frac{d^2q(t)}{dt^2}$	$Ls$	$\frac{1}{Ls}$

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
 Spring	$f(t) = K \int_0^t v(\tau) d\tau$	$f(t) = Kx(t)$	$K$
 Viscous damper	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
 Mass	$f(t) = M \frac{dv(t)}{dt}$	$f(t) = M \frac{d^2x(t)}{dt^2}$	$Ms^2$

# Examples of Complex Impedance

## Series Impedances

ex1)

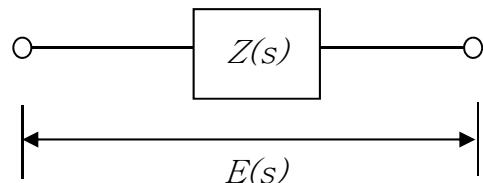


$$e_R = iR, \quad e_L = L \frac{di}{dt}, \quad \frac{de_C}{dt} = \frac{1}{C} i$$

$$e = e_R + e_L + e_C$$

$$\begin{aligned} E(s) &= E_R(s) + E_L(s) + E_C(s) \\ &= RI(s) + LsI(s) + \frac{1}{Cs} I(s) \end{aligned}$$

$$= \left( R + Ls + \frac{1}{Cs} \right) I(s)$$

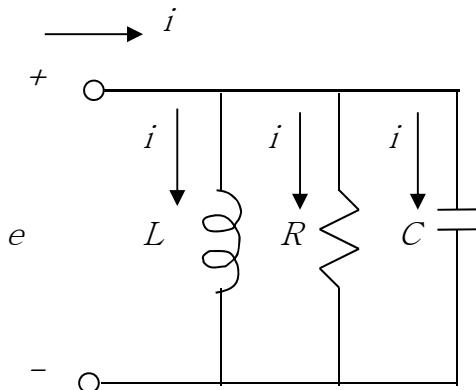


$$\therefore Z(s) = R + Ls + \frac{1}{Cs} = Z_R(s) + Z_L(s) + Z_C(s)$$

# Examples of Complex Impedance

## Parallel Impedances

ex2)



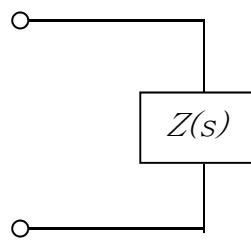
$$i = i_L + i_R + i_C \quad E(s) = Z(s)I(s)$$

$$I(s) = I_L(s) + I_R(s) + I_C(s)$$

$$= \frac{E(s)}{Z_L(s)} + \frac{E(s)}{Z_R(s)} + \frac{E(s)}{Z_C(s)}$$

$$= \left( \frac{1}{Z_L(s)} + \frac{1}{Z_R(s)} + \frac{1}{Z_C(s)} \right) E(s)$$

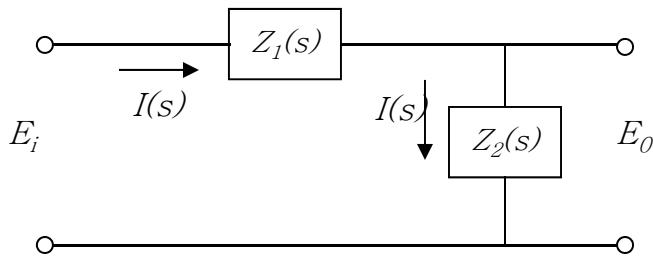
$$= \frac{1}{Z(s)} E(s)$$



$$\therefore Z(s) = \frac{1}{\frac{1}{Z_R(s)} + \frac{1}{Z_L(s)} + \frac{1}{Z_C(s)}} = \frac{1}{\frac{1}{Ls} + \frac{1}{R} + Cs}$$

# Examples of Complex Impedance

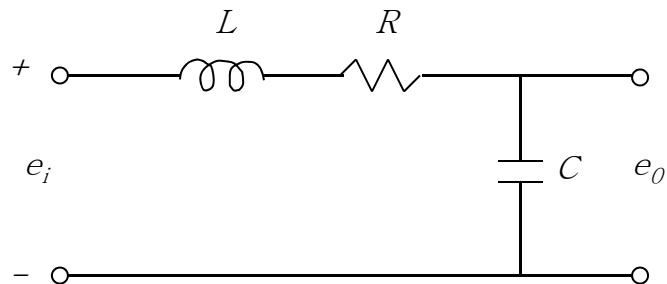
Deriving transfer functions of Electrical circuits by the use of complex impedances.



$$E_i(s) = Z_1(s)I(s) + Z_2(s)I(s), \quad E_o(s) = Z_2(s)I(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

ex)



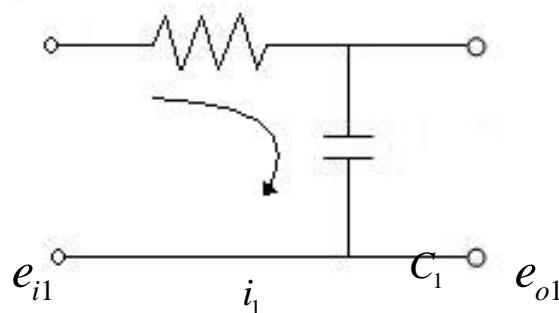
$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$Z_1(s) = Ls + R, \quad Z_2(s) = \frac{1}{Cs}$$

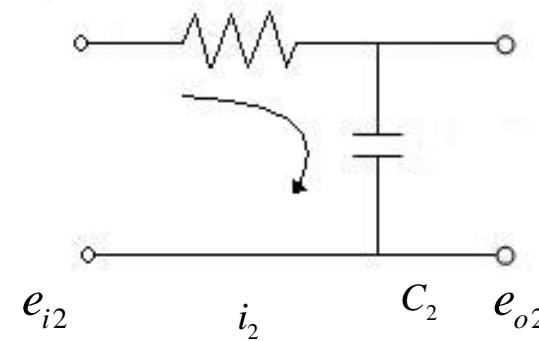
$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$

# Transfer Functions of Cascaded Elements

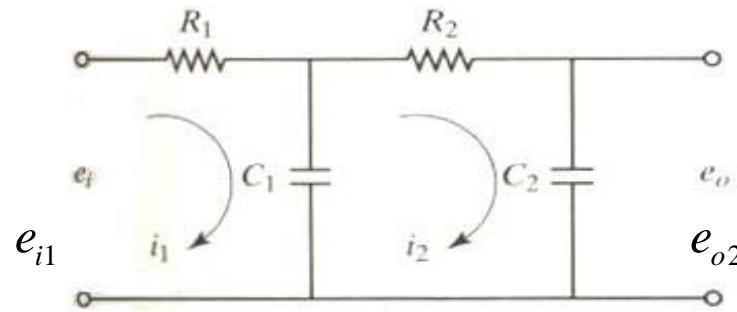
Consider two RC circuits



$$\frac{E_{o1}(s)}{E_{i1}(s)} = \frac{1}{R_1 C_1 s + 1} = G_1(s)$$



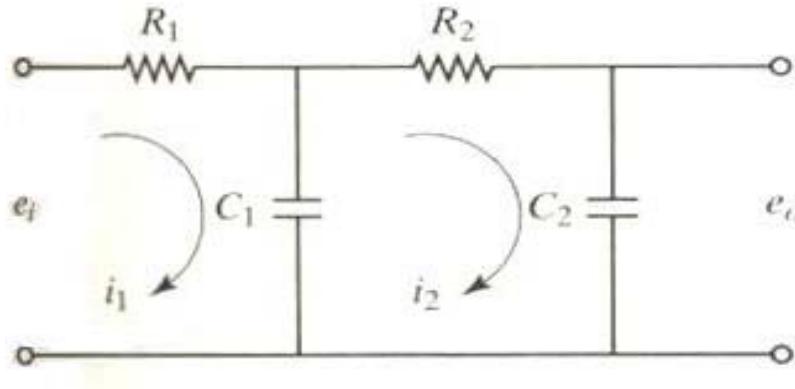
$$\frac{E_{o2}(s)}{E_{i2}(s)} = \frac{1}{R_2 C_2 s + 1} = G_2(s)$$



$$\frac{E_{o2}(s)}{E_{i1}(s)} = ?$$

# Transfer Functions of Cascaded Elements

## Loading Effect



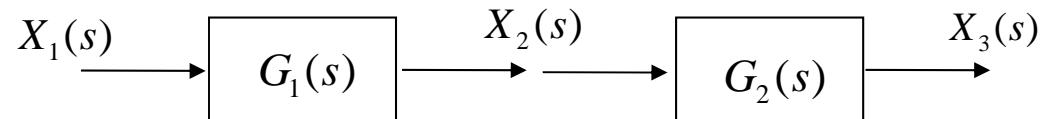
$$E_i(s) = \frac{1}{C_1 s} [I_1(s) - I_2(s)] + R_1 I_1(s), \quad \frac{1}{C_1 s} [I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0, \quad E_o(s) = \frac{1}{C_2 s} I_2(s)$$

$$T.F : \frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} \neq \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$\therefore \frac{E_o(s)}{E_i(s)} \neq \frac{E_{o1}(s)}{E_{i1}(s)} \cdot \frac{E_{o2}(s)}{E_{i2}(s)} \quad \longrightarrow \quad \text{Loading effect}$$

# Transfer Functions of Cascade Elements

## Input Impedance, Output Impedance

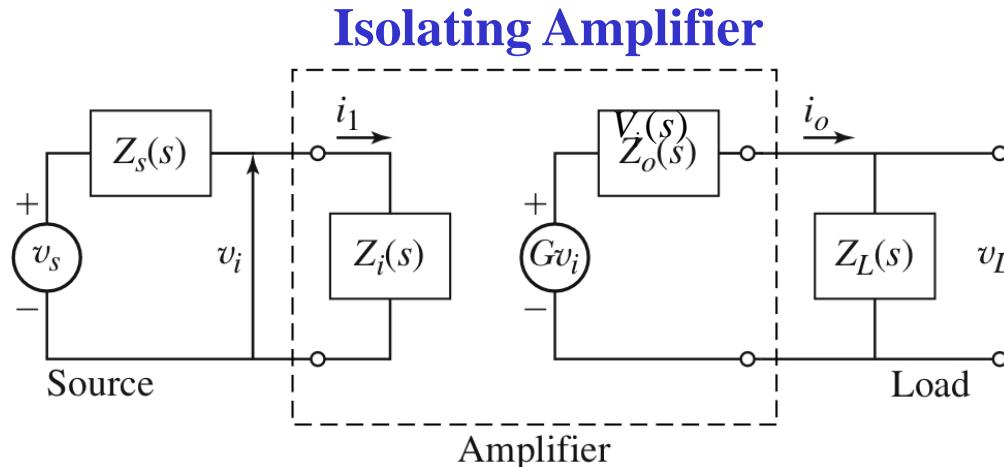


$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_3(s)}{X_2(s)} = G_1(s)G_2(s)$$

If the "input Impedance" of the second element is infinite, the output of the first element is not affected by connecting it to the second element.

Then,  $G(s) = G_1(s)G_2(s)$

# Transfer Functions of Cascade Elements



This amplifier circuit has to

1. Not affect the behavior of the source circuit.
2. Not be affected by the loading circuit.

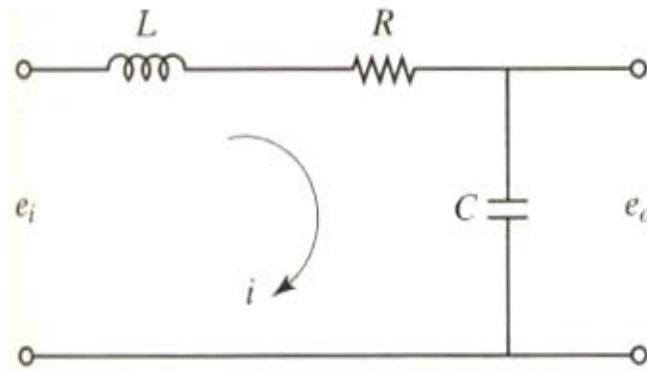
$$V_i(s) = \frac{Z_i(s)}{Z_i(s) + Z_s(s)} V_s(s) \approx V_s(s)$$

This isolating amplifier circuit has to have

1. a very high input impedance,
2. very low output impedance

$$V_L(s) = Z_L(s) I_o(s) = \frac{Z_L(s)}{Z_o(s) + Z_L(s)} G V_o(s) \approx G V_o(s)$$

# State-Space Mathematical Modeling of Electrical Systems



By Kirchhoff's voltage law

$$L \frac{di}{dt} + Ri + v_c = e_i, \quad \frac{dv_c}{dt} = \frac{1}{C} i, \quad e_o = v_c$$

Assume, initial condition is 0,

$$LsI(s) + RI(s) + V_c(s) = E_i(s), \quad V_c(s) = \frac{1}{Cs} I(s)$$

$$T.F : \frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

# State-Space Mathematical Modeling of Electrical Systems

Differential equation :

$$\ddot{e}_o + \frac{R}{L} \dot{e}_o + \frac{1}{LC} e_o = \frac{1}{LC} e_i$$

State variable :

$$x_1 = e_o, \quad x_2 = \dot{e}_o$$

Input and output :

$$u = e_i, \quad y = e_o = x_1$$

State-space equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u, \quad y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

# End of lecture 6-1