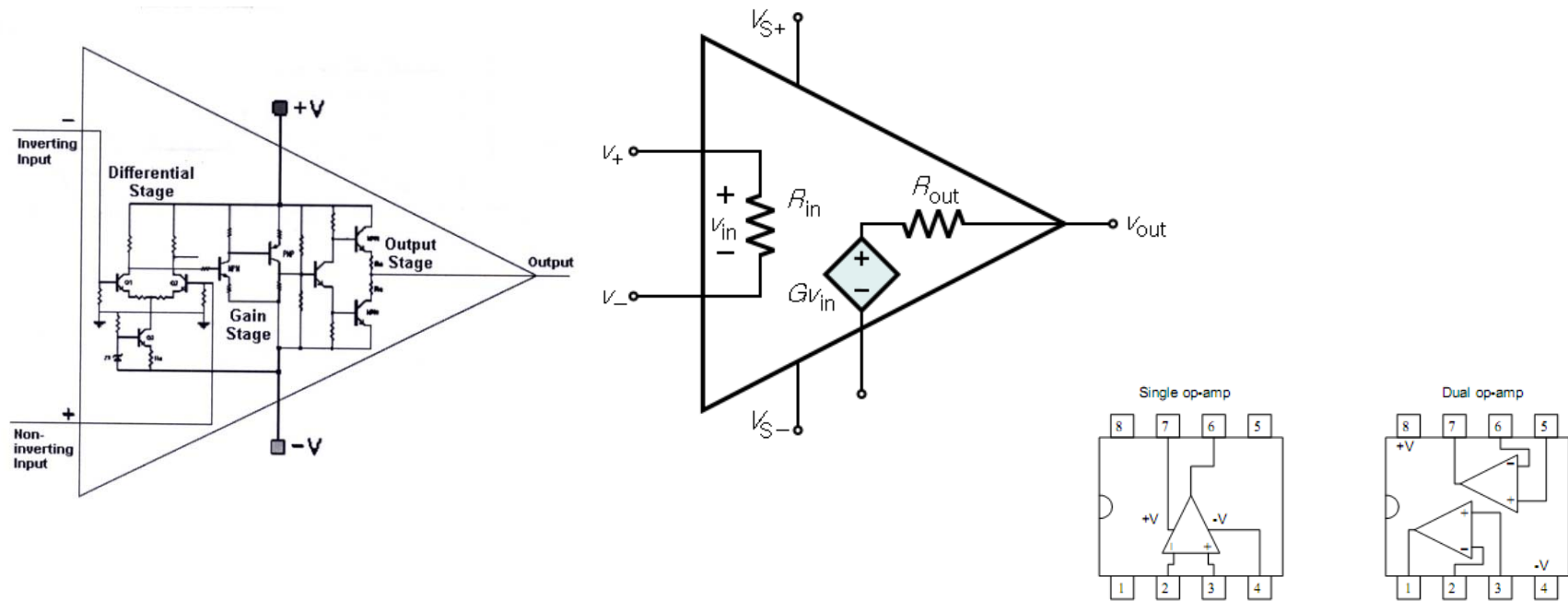


Electrical Systems II

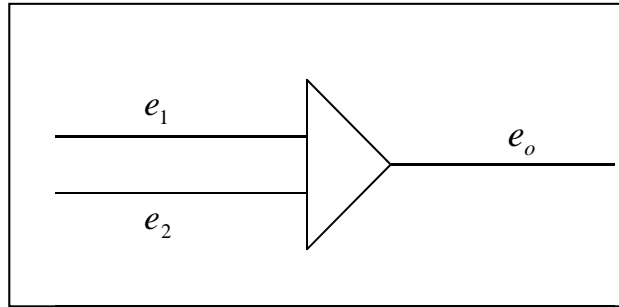
Operational Amplifiers



Golden Rules of OP Amp

1. The voltage difference between V_- and V_+ is Zero.
2. The inputs draw no current.

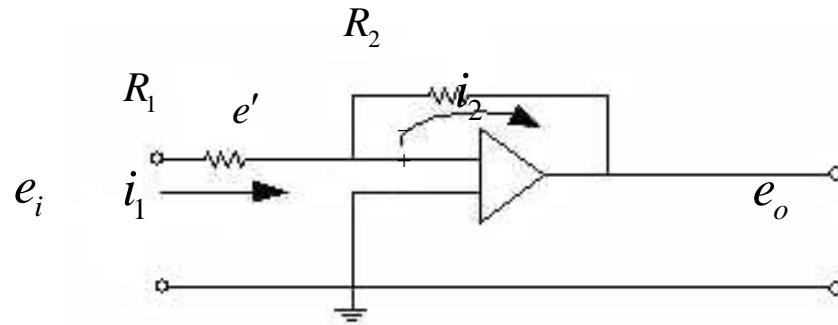
Operational Amplifiers



$$e_0 = K (e_2 - e_1)$$

- i) $K = 10^5 \sim 10^6$ for dc signals and ac of less than 10 Hz frequency.
- ii) $K = 1$ for ac 1 MHz \sim 50MHz
- iii) Ideal CP amps
 - $K = \infty$
 - no current flow into the input terminals
 - the output voltage is not affected by the load connected to the output terminal

Inverting Amplifier



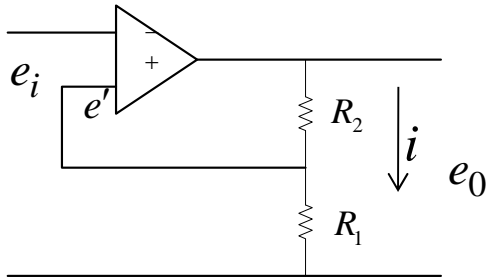
$$\text{i) } i_1 = \frac{e_i - e'}{R_1} \quad i_2 = \frac{e' - e_o}{R_2}$$

$$\text{ii) } i_{op} = 0, \quad i_1 = i_2, \quad \frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

$$\text{iii) } \text{since } e_o = K(0 - e'), \quad K \geq 1, \quad K \approx \infty \quad \Rightarrow \quad e' \approx 0$$

$$\frac{e_i}{R_1} = \frac{-e_o}{R_2} \quad \Rightarrow \quad e_o = -\frac{R_2}{R_1} e_i \quad G = -\frac{R_2}{R_1}$$

Non-inverting Amplifier



$$e' = e_i \quad (\because K \approx \infty)$$

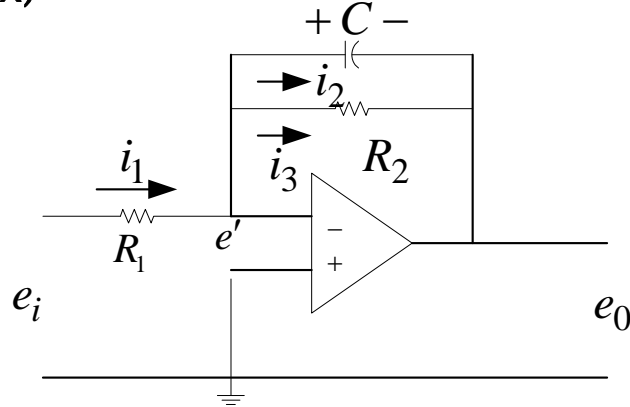
$$i = \frac{e_o - e_i}{R_2} = \frac{e_i}{R_1}$$

$$\Rightarrow e_o = \left(1 + \frac{R_2}{R_1}\right) e_i$$

$$G = \left(1 + \frac{R_2}{R_1}\right)$$

Example of Operational Amplifier Circuit

ex)



$$e' = 0, \quad i_1 = i_2 + i_3$$

$$i_1 = \frac{e_i}{R_1}, \quad \frac{dv_c}{dt} = \frac{1}{C} i_3 = \left(-\frac{de_o}{dt} \right)$$

$$(v_c = e' - e_o = -e_o)$$

$$i_3 = \frac{e' - e_o}{R_2} = -\frac{e_o}{R_2}$$

$$\Rightarrow \frac{e_i}{R_1} = C \frac{dv_c}{dt} - \frac{1}{R_2} e_o = C \frac{de_o}{dt} - \frac{1}{R_2} e_o$$

$$\Rightarrow \frac{de_o}{dt} = -\frac{1}{CR_2} e_o - \frac{1}{CR_1} e_i$$

Examples of Operational Amplifiers

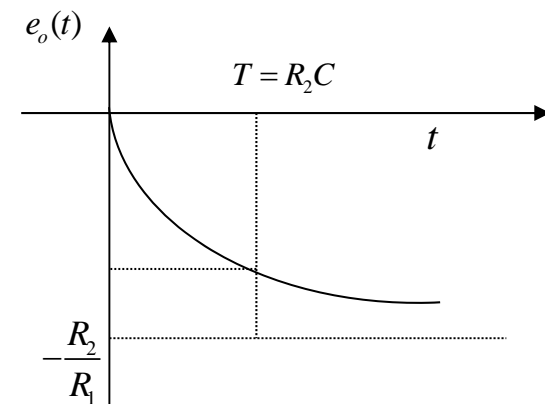
$$\text{Laplace Transform : } E_o(s) \left(s + \frac{1}{CR_2} \right) = -\frac{1}{CR_1} E_i(s)$$

$$T.F = \frac{E_o(s)}{E_i(s)} = -\frac{1}{s + \frac{1}{CR_2}} \cdot \frac{1}{CR_1} = -\frac{R_2}{R_1} \cdot \frac{1}{R_2Cs + 1}$$

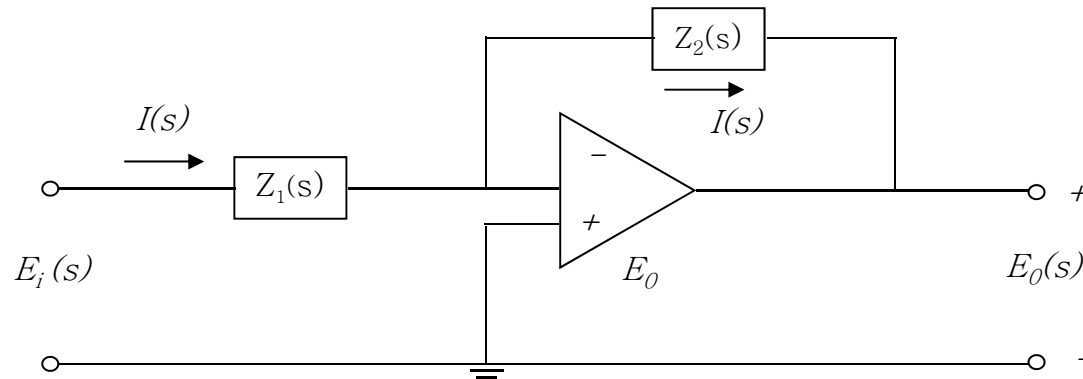
$$\text{Step input response : } E_i(s) = \frac{1}{s}, \quad e_i(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\begin{aligned} E_o(s) &= -\frac{R_2}{R_1} \frac{1}{R_2Cs + 1} \cdot E_i(s) \\ &= -\frac{R_2}{R_1} \frac{1}{R_2Cs + 1} \cdot \frac{1}{s} = -\frac{R_2}{R_1} \left(\frac{1}{s} - \frac{1}{s + (1/R_2C)} \right) \end{aligned}$$

$$e_o(t) = -\frac{R_2}{R_1} \left(1 - e^{(-1/R_2C)t} \right)$$



Examples of Complex Impedance

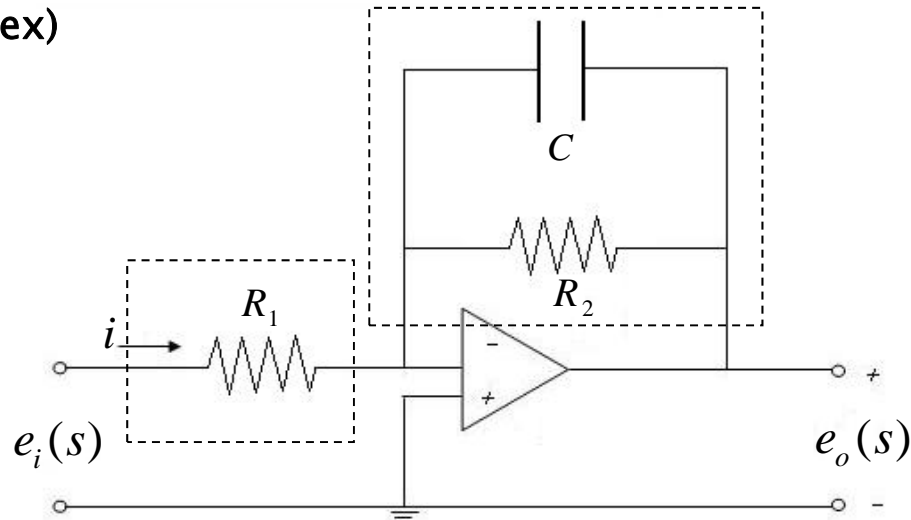


$$E_i(s) = Z_1(s)I(s), \quad E_o(s) = Z_2(s)I(s)$$

$$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Examples of Complex Impedance

ex)



$$Z_1(s) = R_1$$

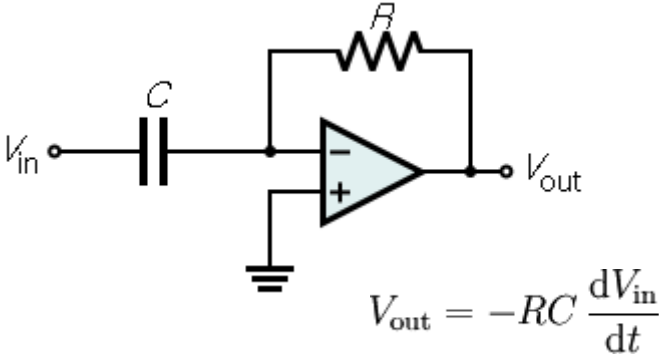
$$Z_2(s) = \frac{1}{Cs + \frac{1}{R_2}} = \frac{R_2}{R_2Cs + 1}$$

$$I(s) = Cs \cdot E(s) + \frac{1}{R_2} E(s)$$

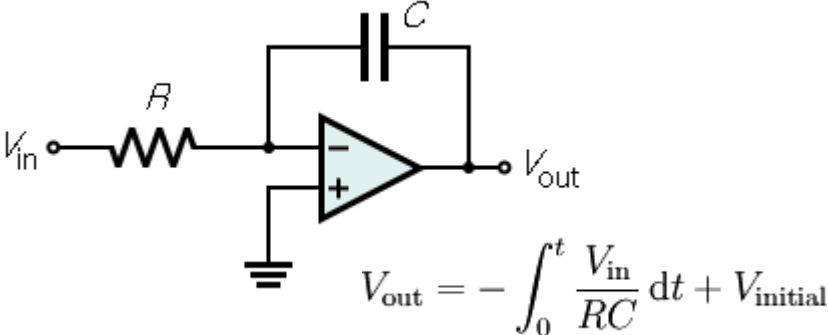
$$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_2}{R_1} \frac{1}{R_2Cs + 1}$$

Other Examples

Differentiator/High Pass Filter

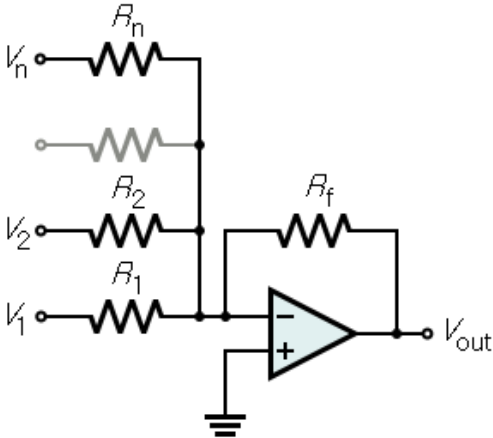


Integrator/Low Pass Filter



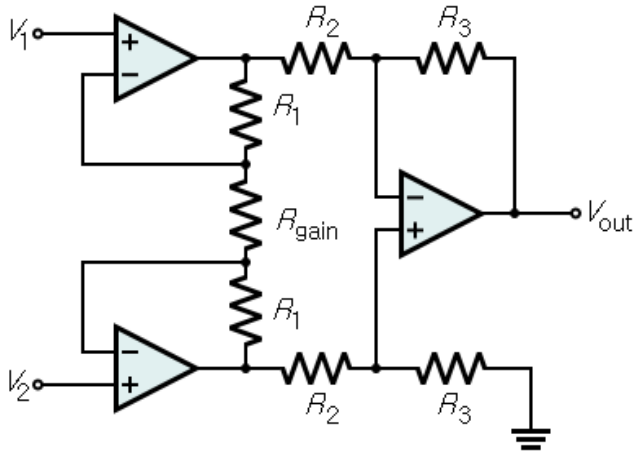
Other Examples

Summing Amplifier



$$V_{out} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} \right)$$

Instrumentation Amplifier



$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}} \right) \frac{R_3}{R_2}$$