Lecture 9
Thermal Systems

Thermal System

- $q = K\Delta\theta$

$\Delta\theta$: temperature difference $[°C]$

q : heat flow rate $[kcal/sec]$

K : coefficient $[kcal/(sec \cdot °C)]$

specific heat : α $[kcal/(kg \cdot °C)]$

Heat capacitance : $C = m \cdot \alpha$ $[kcal/°C]$

Heat Flow Rate

Coefficient K :

$$K = \frac{kA}{\Delta x} \quad (conduction)$$

$$= HA \quad (convection)$$

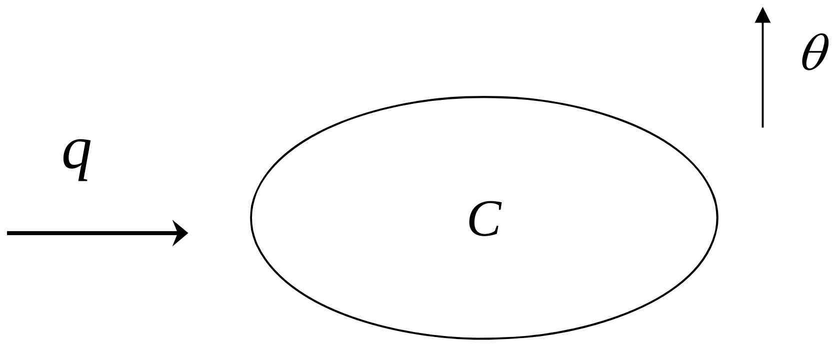
$$k = thermal\ conductivity,\ [kcal / ms°C]$$

$$A = area\ normal\ to\ heat\ flow,\ [m^2]$$

$$\Delta x = thickness\ of\ conductor,\ [m]$$

$$H = convection\ coefficient,\ [kcal / m^2 s°C]$$

Heat Balance Equation



$$q \cdot dt = C \cdot d\theta$$

$$\frac{d\theta}{dt} = \frac{q}{C}$$

Thermal Resistance / Capacitance

- Thermal resistance

$$R = \frac{\text{change in temperature difference } [^\circ C]}{\text{change in heat flow rate } [kal/sec]}$$

$$q = \frac{\Delta\theta}{R}, \qquad R = \frac{1}{K}$$

- Thermal capacitance

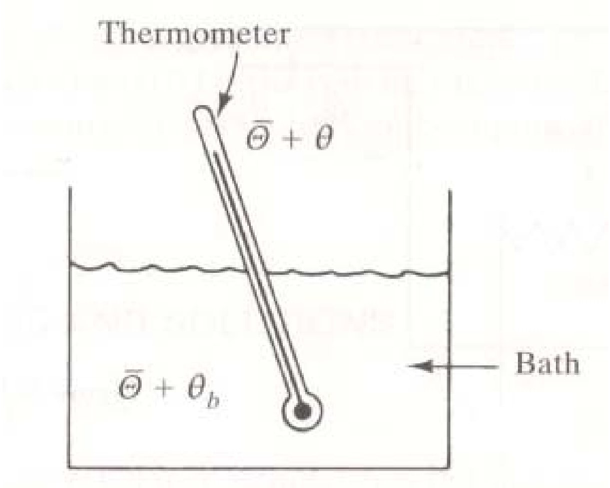
$$C = \frac{q}{d\theta} = \frac{\text{change in heat stored } [kcal]}{\text{change in temperature } [^\circ C]}$$

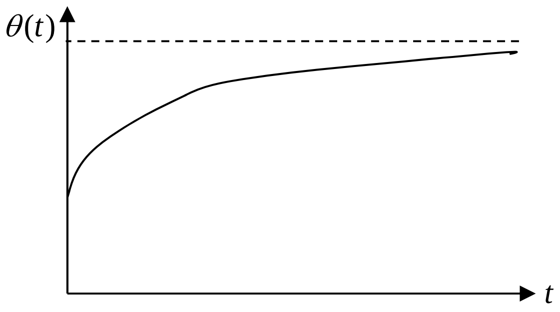
$$= m \cdot c \quad \left(\text{mass } [kg] \cdot \text{specific heat } [kcal/(kg \cdot {}^\circ C)]\right)$$

$$\frac{d\theta}{dt} = \frac{q}{C}$$

Thermal System : Thermometer System

$$q = K\Delta\theta, \quad R = \frac{1}{K}, \quad q = \frac{\Delta\theta}{R}$$





ambient temperature $\bar{\theta}$: *constant*

bath temperature $\bar{\theta} + \theta_b$, $\quad \theta_b$: *constant*

$$q\,dt = C\,d\theta$$

C : *heat capacitance of the thermometer*

C_b : *heat capacitance of the fluid*

R : *thermal resistance*

$$q = \frac{(\bar{\theta} + \theta_b) - (\bar{\theta} + \theta)}{R} = \frac{\theta_b - \theta}{R}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{q}{c} = \frac{1}{RC}(\theta_b - \theta)$$

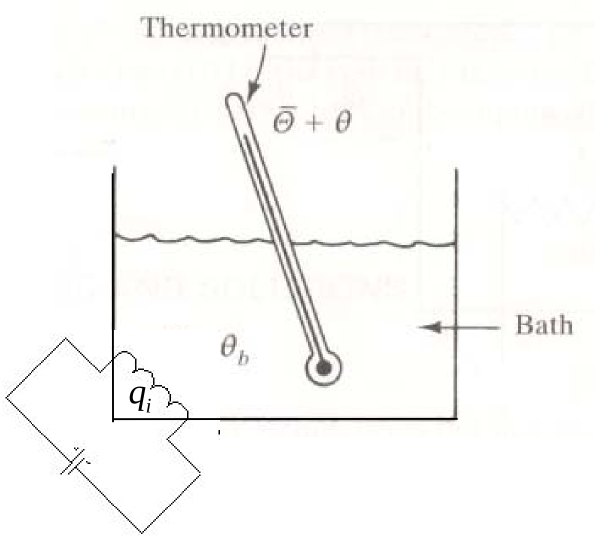
$$\therefore T.F = \frac{\theta(s)}{\theta_b(s)} = \frac{1}{RCs + 1}$$

$$\Rightarrow \theta_b(t) = \theta_b, \qquad \theta(t) = \theta_b\left(1 - e^{-\frac{1}{RC}t}\right)$$

Thermal System : Thermometer System

When q_i applied,

$$\begin{cases} \dfrac{d\theta_b}{dt} = \dfrac{1}{C_b}(q_i - q) \\ q = \dfrac{1}{R}(\theta_b - \theta) \\ \dfrac{d\theta}{dt} = \dfrac{1}{C} q \end{cases}$$



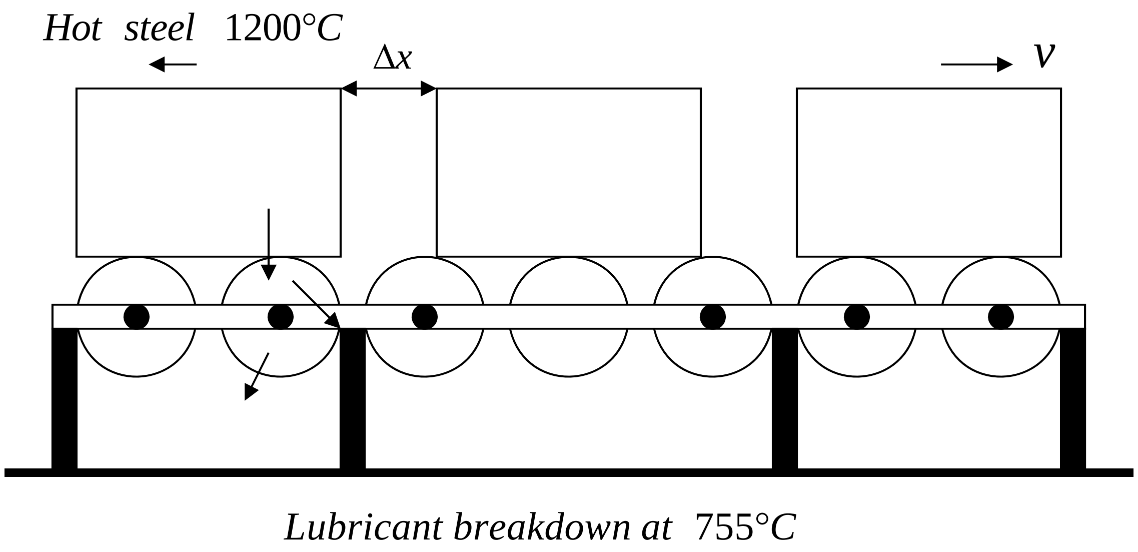
$$\Rightarrow \frac{d\theta_b}{dt} = -\frac{1}{RC_b}\theta_b + \frac{1}{RC_b}\theta + \frac{1}{C_b}q_i, \qquad \frac{d\theta}{dt} = -\frac{1}{RC}\theta + \frac{1}{RC}\theta_b$$

$$(RC_b s + 1)\Theta_b(s) = \Theta(s) + RQ_i(s), \qquad (RCs + 1)\Theta(s) = \Theta_b(s)$$

$$\Rightarrow (R^2 C_b C s^2 + RCs + RC_b s)\Theta(s) = RQ_i(s)$$

$$\therefore \frac{\Theta(s)}{Q_i(s)} = \frac{1}{s\left(RC_b Cs + C_b + C\right)} \approx \frac{1}{sC_b} \qquad (C << C_b, \quad RC \text{ is small})$$

Thermal System : A Steel Processing Plant



- Large slabs of red-hot steel
- $T_{steel} = 1600°F$: *almost constant*
- $T_A = 100°F$ (*ambient temperature*)

Thermal System : A Steel Processing Plant

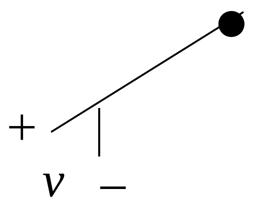
T_A : ambient temperature

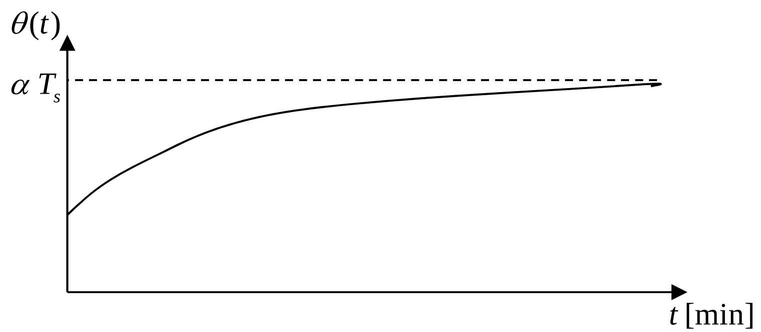
R_1 : thermal resistance between the slab and the rollers (conduction)

R_2 : thermal resistance between the rollers and the ambient air (convection)

and the bearing support (conduction)

Thermocouple : Produce voltage proportional to its temperature

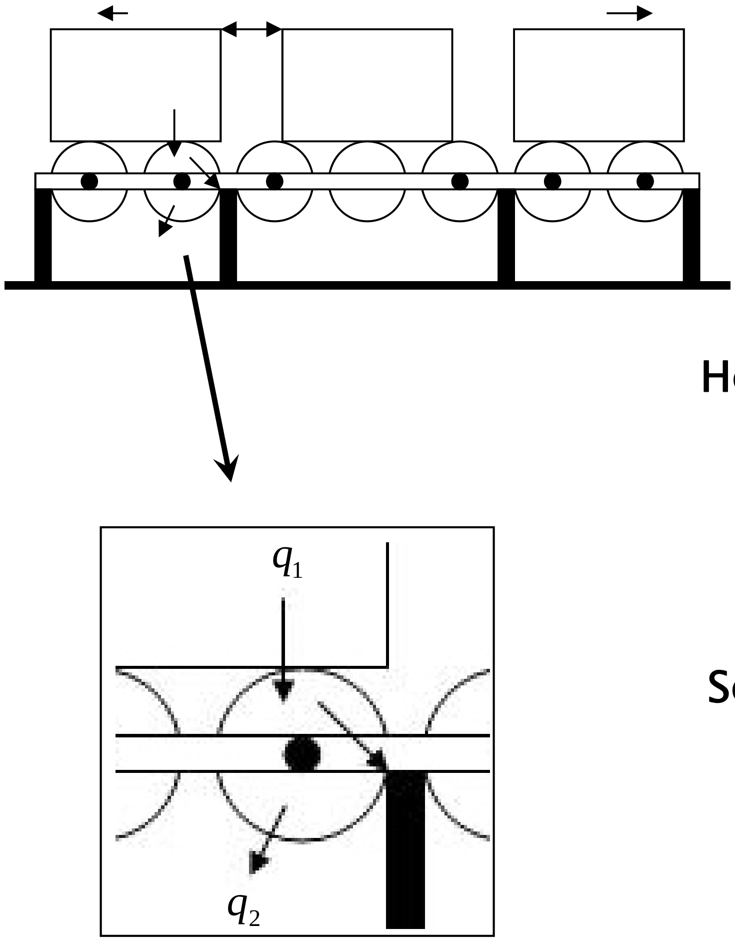




$$roller\ :\quad mass = m,\quad specific\ heat\ capacitance = C_p\left[kcal/kg^\circ C\right]$$

$$heat\ capacitance = C = m \cdot C_p$$

Thermal System : A Steel Processing Plant



$$q_1 = \frac{1}{R_1}(T_s - T_r)$$

$$q_2 = \frac{1}{R_2}(T_r - T_A)$$

Heat balance : $m \cdot C_p \cdot \frac{dT_r}{dt} = q_1 - q_2$

$$= -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)T_r + \frac{1}{R_1}T_s + \frac{1}{R_2}T_A$$

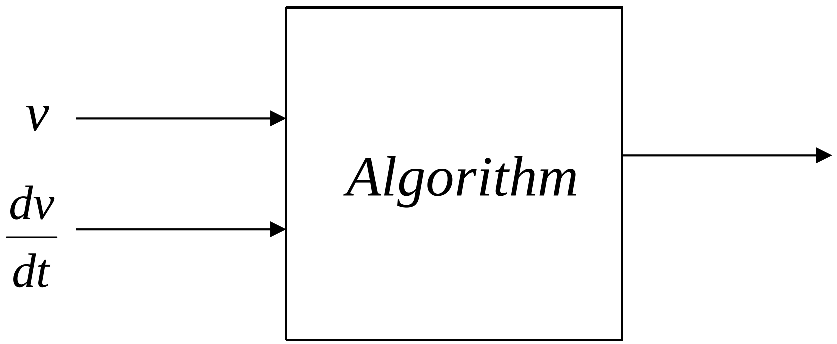
Sensor : $\frac{v(s)}{T_r(s)} = \frac{\alpha}{\tau s + 1}, \quad \tau \dot{v} = -v + \alpha T_r$

$x = v + \beta \dot{v} \qquad (\tau : time\ constant)$

$if,\ x > x_{cr} \Rightarrow stop$

$x < x_{cr} \Rightarrow restart$

Thermal System : A Steel Processing Plant



Stop the process or increase the distance between the slabs Δx

$$x = v + \beta \dot{v}$$

$$if, \quad x > x_{cr} \quad stop$$

End of lecture 9
thermal systems