

10-4

**Linear Systems Analysis
in the Time Domain IV
- Transient Response -**

Matrix Exponential

$$\mathbf{x}(t) = e^{At} \mathbf{x}(0) \quad \dots(\alpha), \quad \dot{\mathbf{x}} = A \mathbf{x} \quad \dots(\beta)$$

Matrix exponential e^{At}

$$e^{At} = I + At + \frac{1}{2!} A^2 t^2 + \frac{1}{3!} A^3 t^3 + \dots \quad (*1)$$

$$\left(\because e^{at} = 1 + at + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \dots \right)$$

$$\begin{aligned} \frac{d}{dt}(e^{At}) &= A + A^2 t + \frac{1}{2} A^3 t^2 + \dots = A \left\{ I + At + \frac{1}{2} A^2 t^2 + \dots \right\} \\ &= A e^{At} \end{aligned}$$

$$\dot{\mathbf{x}}(t) = \frac{d}{dt}(e^{At}) \mathbf{x}(0) = A e^{At} \mathbf{x}(0) = A \mathbf{x}(t)$$

$\Rightarrow (\alpha)$ is the solution of the matrix differential equation (β)

Matrix Exponential

- $e^{At} = \exp(At) = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!}$

- $\frac{d(e^{At})}{dt} = A e^{At}$

- $e^{(A+B)t} = e^{At} e^{Bt}$ if $AB = BA$

- $e^{(A+B)t} \neq e^{At} e^{Bt}$ if $AB \neq BA$

$$\Rightarrow e^{(A+B)t} = I + (A+B)t + \frac{(A+B)^2}{2!} t^2 + \frac{(A+B)^3}{3!} t^3 + \dots$$

$$e^{At} e^{Bt} = \left(I + At + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \dots \right) \left(I + Bt + \frac{B^2 t^2}{2!} + \frac{B^3 t^3}{3!} + \dots \right)$$

$$= I + (A+B)t + \frac{A^2 t^2}{2!} + ABt^2 + \frac{B^2 t^2}{2!} + \frac{B^3 t^3}{3!} + \frac{A^2 B t^3}{2!} + \frac{AB^2 t^3}{2!} + \frac{B^3 t^3}{3!} \dots$$

How to Evaluate e^{At}

Hence,

$$e^{(A+B)t} - e^{At}e^{Bt} = \frac{BA - AB}{2!}t^2 + \frac{BA^2 + ABA + B^2A + BAB - 2A^2B - 2AB^2}{3!}t^3 + \dots$$

The difference between $e^{(A+B)t}$ and $e^{At}e^{Bt}$ vanishes, if A and B commute.

How to Evaluate e^{At}

(*1) \Rightarrow a) Diagonalized Form $A = \begin{bmatrix} \lambda_1 & & \underline{0} \\ & \lambda_2 & \\ \underline{0} & & \lambda_3 \end{bmatrix}$

$$e^{At} = I + At + \frac{1}{2} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}^2 t^2 + \frac{1}{3!} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}^3 t^3 + \dots$$

$$= \begin{bmatrix} 1 + \lambda_1 t + \frac{1}{2} \lambda_1^2 t^2 + \frac{1}{3!} \lambda_1^3 t^3 + \dots & & \\ & \ddots & \\ & & \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_3 t} \end{bmatrix}$$

How to Evaluate e^{At}

b) Jordan Form

$$A = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}, \quad \rightarrow \quad e^{At} = \begin{bmatrix} e^{\lambda_1 t} & te^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_1 t} & 0 \\ 0 & 0 & e^{\lambda_2 t} \end{bmatrix}$$

$$A = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 1 \\ 0 & 0 & \lambda_1 \end{bmatrix}, \quad \rightarrow \quad e^{At} = \begin{bmatrix} e^{\lambda_1 t} & te^{\lambda_1 t} & \frac{1}{2}t^2 e^{\lambda_1 t} \\ 0 & e^{\lambda_1 t} & te^{\lambda_1 t} \\ 0 & 0 & e^{\lambda_1 t} \end{bmatrix}$$

How to Evaluate e^{At}

c) General A : diagonalize !!

$\dot{x} = Ax + Bu$ i) $Q(\lambda) = \det[\lambda I - A] = 0$: Characteristic equation.

$$A : n \times n$$

$$n \text{ sol} = \lambda_i \quad i = 1, \dots, n$$

ii) λ_i : eigenvalue of A

n distinct eigenvalues.

iii) $(\lambda_i I - A)p_i = 0$ p_i : eigen vector

$$Ap_i = \lambda_i p_i$$

$$A \begin{bmatrix} p_1 & \cdots & p_n \end{bmatrix} = \begin{bmatrix} p_1 & \cdots & p_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}, \quad AP = P\Lambda$$

$$\Rightarrow P^{-1}AP = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} = \Lambda \quad \text{Diagonalizable if } \lambda_i : n - \text{distinct eigenvalues}$$

How to Evaluate e^{At}

$$A p_i = \lambda_i p_i$$

if $p_i^T p_i = 1$, p_i is a normalized eigenvector.

Orthogonality of eigenvector

if $A = A^T$

$$\begin{cases} A p_i = \lambda_i p_i \\ A p_j = \lambda_j p_j \end{cases} \rightarrow \begin{cases} p_j^T A p_i = \lambda_i p_j^T p_i = 0 \\ p_i^T A p_j = \lambda_j p_i^T p_j = 0 \end{cases}$$

$$A \begin{bmatrix} p_1 & \cdots & p_n \end{bmatrix} = \begin{bmatrix} p_1 & \cdots & p_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

$$P^T A P = \Lambda$$

How to Evaluate e^{At}

$$\text{let } P\hat{x} = x, \quad \dot{x} = P\dot{\hat{x}}$$

$$\dot{x} = Ax + Bu$$

$$P\dot{\hat{x}} = AP\hat{x} + Bu$$

$$\dot{\hat{x}} = P^{-1}AP\hat{x} + P^{-1}Bu = \Lambda\hat{x} + P^{-1}Bu$$

$$\hat{x}(t) = e^{\Lambda t} \hat{x}(0) + \int_0^t e^{\Lambda(t-\tau)} P^{-1} Bu(\tau) d\tau$$

$$P\hat{x}(t) = P e^{\Lambda t} P^{-1} x(0) + \int_0^t P e^{\Lambda(t-\tau)} P^{-1} Bu(\tau) d\tau$$

$$\therefore x(t) = P e^{\Lambda t} P^{-1} x(0) + \int_0^t P e^{\Lambda(t-\tau)} P^{-1} Bu(\tau) d\tau$$

How to Evaluate e^{At}

d) General A (\rightarrow Jordan form)

repeated λ_i : multiple eigenvalue

$$\det[\lambda_i I - A] = (\lambda - \lambda_1)(\lambda - \lambda_2)^2 \quad A: 3 \times 3$$

$$(\lambda_1 I - A)p_1 = 0 \quad \Rightarrow \quad p_1$$

$$(\lambda_2 I - A)p_2 = 0 \quad \Rightarrow \quad p_2$$

if $\text{rank}(\lambda_2 I - A) = 2$ then p_3 ?

Find p such that

$$AP = P \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & 1 \\ & & \lambda_2 \end{bmatrix} \Rightarrow A p_1 = \lambda_1 p_1,$$

$$A p_2 = \lambda_2 p_2$$

$$A p_3 = p_2 + \lambda_2 p_3, \quad (A - \lambda_2 I)p_3 = p_2$$

Multiple Eigenvalue - Diagonal Form

$$\text{ex1) } A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad Q(\lambda) = \det|\lambda I - A| = \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 0 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 1)^2 (\lambda - 2)$$

eigenvalue : $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 2$

eigenvector : $Ap_i = \lambda_i p_i \quad (\lambda_i I - A) p_i = 0$

$$\lambda_{1,2} = 1, \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} p_{1,2} = 0 \Rightarrow \text{choose } p_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 2, \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} p_3 = 0 \Rightarrow p_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Multiple Eigenvalue - Diagonal Form

$$\mathbf{P} = [p_1 \quad p_2 \quad p_3] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{P}^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \Lambda$$

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x}$$

$$\dot{\hat{\mathbf{x}}} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \hat{\mathbf{x}} = \Lambda \hat{\mathbf{x}}$$

$$\hat{\mathbf{x}}(t) = e^{\Lambda t} \hat{\mathbf{x}}(0), \quad \mathbf{P} \hat{\mathbf{x}} = \mathbf{x} \quad \Rightarrow \quad \mathbf{P}^{-1} \mathbf{x}(t) = e^{\Lambda t} \mathbf{P}^{-1} \mathbf{x}(0)$$

$$\therefore \mathbf{x}(t) = \mathbf{P} e^{\Lambda t} \mathbf{P}^{-1} \mathbf{x}(0) = e^{\mathbf{A}t} \mathbf{x}(0)$$

$$e^{\mathbf{A}t} = \mathbf{P} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \mathbf{P}^{-1} = \begin{bmatrix} e^t & 0 & e^t \\ 0 & e^t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix}$$

Multiple Eigenvalue - Jordan Form

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \quad Q(\lambda) = \det |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & -2 \\ 0 & \lambda - 1 & -3 \\ 0 & 0 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 1)^2 (\lambda - 2)$$

eigenvalue : $\lambda_{1,2} = 1, \quad \lambda_3 = 2$

eigenvector : $A p_i = \lambda_i p_i \quad (\lambda_i I - A) p_i = 0$

$$\lambda_{1,2} = 1, \quad \begin{bmatrix} 0 & -1 & -2 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix} p_{1,2} = 0 \quad \Rightarrow \quad p_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 2, \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} p_3 = 0 \quad \Rightarrow \quad p_3 = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$$

**We can find only
1 eigenvector
for $\lambda = 1$**

Multiple Eigenvalue - Jordan Form

If not able to find 3 independent vectors, find p_2 which transforms A as Jordan form.

$$AP = PJ = P \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$A \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A p_1 & A p_2 & A p_3 \end{bmatrix} = \begin{bmatrix} p_1 \lambda_1 & p_1 + \lambda_1 p_2 & \lambda_3 p_3 \end{bmatrix}$$

So that, $A p_1 = \lambda_1 p_1$ $[\lambda_1 I - A] p_1 = 0 \Rightarrow p_1$

$$A p_2 = p_1 + \lambda_1 p_2 \quad [A - \lambda_1 I] p_2 = p_1 \Rightarrow p_2$$

$$A p_3 = \lambda_3 p_3 \quad [A - \lambda_3 I] p_3 = 0 \Rightarrow p_3$$

Multiple Eigenvalue - Jordan Form

$$[A - \lambda_1 I] p_2 = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} p_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{let, } p_2 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$b + 2c = 1, \quad 3c = 0, \quad c = 0 \quad \rightarrow \quad b = 1, \quad \text{choose } p_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow P^{-1}AP = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{Jordan form}$$

Multiple Eigenvalue - Jordan Form

$$\text{if } \dot{\mathbf{x}} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x} = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}(0) ?$$

$$\text{let } \mathbf{P}\hat{\mathbf{x}} = \mathbf{x}, \quad \dot{\mathbf{x}} = \mathbf{P}\dot{\hat{\mathbf{x}}}$$

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \hat{\mathbf{x}} = \mathbf{J} \hat{\mathbf{x}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \hat{\mathbf{x}}$$

$$e^{\mathbf{J}t} = \begin{bmatrix} e^t & te^t & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix}, \quad \hat{\mathbf{x}}(t) = e^{\mathbf{J}t} \hat{\mathbf{x}}(0)$$

$$\mathbf{x}(t) = \mathbf{P} e^{\mathbf{J}t} \mathbf{P}^{-1} \mathbf{x}(0) = e^{\mathbf{A}t} \mathbf{x}(0)$$

Multiple Eigenvalue - Jordan Form

$$\begin{aligned}\therefore e^{At} = P e^{Jt} P^{-1} &= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^t & te^t & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & te^t & 5e^{2t} \\ 0 & e^t & 3e^{2t} \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & te^t & (-5e^t - 3te^t + 5e^{2t}) \\ 0 & e^t & (-3e^t + 3e^{2t}) \\ 0 & 0 & e^{2t} \end{bmatrix}\end{aligned}$$

Summary

1. Diagonal Form

$$A = \begin{bmatrix} \lambda_1 & & \underline{0} \\ & \lambda_2 & \\ \underline{0} & & \lambda_3 \end{bmatrix} \rightarrow e^{At} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_3 t} \end{bmatrix}$$

2. Jordan Form

$$A = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix} \rightarrow e^{At} = \begin{bmatrix} e^{\lambda_1 t} & te^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_1 t} & 0 \\ 0 & 0 & e^{\lambda_2 t} \end{bmatrix},$$

$$A = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 1 \\ 0 & 0 & \lambda_1 \end{bmatrix} \rightarrow e^{At} = \begin{bmatrix} e^{\lambda_1 t} & te^{\lambda_1 t} & \frac{1}{2}t^2 e^{\lambda_1 t} \\ 0 & e^{\lambda_1 t} & te^{\lambda_1 t} \\ 0 & 0 & e^{\lambda_1 t} \end{bmatrix}$$

Summary

3. General A

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} u \quad \mathbf{A} \mathbf{p}_i = \lambda_i \mathbf{p}_i, \quad \mathbf{P}^{-1} \mathbf{A} \mathbf{P} = \Lambda$$

let $\mathbf{x} = \mathbf{P} \hat{\mathbf{x}}$

$$\dot{\hat{\mathbf{x}}} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \hat{\mathbf{x}} + \mathbf{P}^{-1} \mathbf{B} u = \Lambda \hat{\mathbf{x}} + \mathbf{P}^{-1} \mathbf{B} u$$

$$\hat{\mathbf{x}}(t) = e^{\Lambda t} \hat{\mathbf{x}}(0) + \int e^{\Lambda(t-\tau)} \mathbf{P}^{-1} \mathbf{B} u(\tau) d\tau$$

$$\mathbf{P} \hat{\mathbf{x}}(t) = \mathbf{x}(t) = \mathbf{P} e^{\Lambda t} \mathbf{P}^{-1} \mathbf{x}(0) + \int \mathbf{P} e^{\Lambda(t-\tau)} \mathbf{P}^{-1} \mathbf{B} u(\tau) d\tau$$

$$\mathbf{P} e^{\Lambda t} \mathbf{P}^{-1} = e^{\mathbf{A} t}$$

Laplace Transformation Method

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad \mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}_0 + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau$$

Laplace Transformation

$$s\mathbf{X}(s) - \mathbf{x}_0 = \mathbf{A}\mathbf{X}(s) + \mathbf{B}u(s), \quad \mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}_0 + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}u(s)$$

$$\therefore e^{\mathbf{A}t} = \mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]$$

$$\begin{aligned}(s\mathbf{I} - \mathbf{A})^{-1} &= \frac{1}{s}\left(\mathbf{I} - \frac{1}{s}\mathbf{A}\right)^{-1} \\ &= \frac{1}{s}\left(\mathbf{I} + \frac{1}{s}\mathbf{A} + \frac{1}{s^2}\mathbf{A}^2 + \frac{1}{s^3}\mathbf{A}^3 \dots\right) \\ &= \frac{1}{s}\mathbf{I} + \frac{1}{s^2}\mathbf{A} + \frac{1}{s^3}\mathbf{A}^2 + \frac{1}{s^4}\mathbf{A}^3 \dots\end{aligned}$$

Laplace Transformation Method

Laplace transformation table

$$\mathcal{L}[t] = \frac{1}{s^2}, \quad \mathcal{L}\left[\frac{1}{(n-1)!}t^{n-1}\right] = \frac{1}{s^n}, \quad \mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\begin{aligned} \Rightarrow \mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right] &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathbf{I} + \frac{1}{s^2}\mathbf{A} + \frac{1}{s^3}\mathbf{A}^2 + \frac{1}{s^4}\mathbf{A}^3 \dots\right] \\ &= \mathbf{I} + \mathbf{A}t + \frac{1}{2!}\mathbf{A}^2 t^2 + \frac{1}{3!}\mathbf{A}^3 t^3 \dots \end{aligned}$$

$$\therefore e^{\mathbf{A}t} = \mathbf{I} + \mathbf{A}t + \frac{1}{2!}\mathbf{A}^2 t^2 + \frac{1}{3!}\mathbf{A}^3 t^3 + \dots$$

Solution of Linear (Time Invariant) State Equation

Method 1. Diagonalization

Method 2. Laplace Transformation

Method 3. Sylvester's Interpolation Formula