Optimal Design of Energy Systems

Chapter 3

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Chapter 3. Economics

3.1 Introduction

Basis of engineering decision

- Economics (optimization)
  - Minimum investment cost
  - Minimum total lifetime cost
- Noneconomic factors
  - Legal concerns
  - Social concerns
  - Environmental concerns
  - Aesthetic concerns

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3.2 Interest

└ Rental charge for the use of money

The worth of money – 2 dimensions (dollar amount + time)

3.3 Lump sum, annual compounding

└─ Interest available at the end of each year

	At start of year	Value (i=0.1)	At end of year
Start	P	100	P+Pi
1 year	P(1+i)	110	P(1+i)+P(1+i)i
2	$P(1+i)^2$	121	
3	$P(1+i)^3$	133	
4		146	⋮
20	⋮	673	
30		1745	
40	$P(1+i)^{40}$	4526	$P(1+i)^n$

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3.4 Lump sum, compound more often than annually

Nominal annual interest rate : $\frac{days}{365} \times i$

$$S = P\left(1 + \frac{i}{m}\right)^{m \times n}$$

Number of years

Number of compounding periods per year

3.5 Compound-Amount Factor (f/p) and Present-Worth Factor (p/f)

Future worth = (present worth) (f/p)

Present worth = (future worth) (p/f)

$$f/p = \left(1 + \frac{i}{m}\right)^{mn} \quad \text{and} \quad p/f = \frac{1}{(1 + i/m)^{mn}}$$

Where i = nominal annual interest rate

n = number of years

m = number of compounding periods per year

3.5 Compound-Amount Factor (f/p) and Present-Worth Factor (p/f)

<Example> invest $5000, compound 5% interest quarterly, after 5yr ?

$$\text{Future amount} = (\text{present amount})\left(f\ /\ p,\ \frac{0.05}{4},\ 20\,periods \right)$$

where the meaning of the convention is (factor, rate, period).

$$\text{Future amount} = (\$5000)\left(1+\frac{0.05}{4}\right)^{20} = (\$5000)(1.2820) = \$6410$$

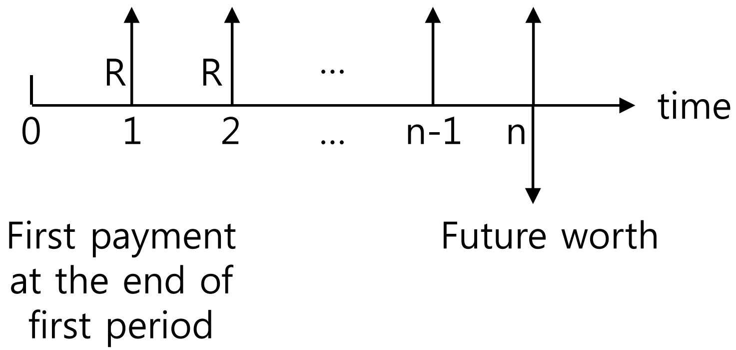
3.6 Future worth (f/a) of a uniform series of amounts

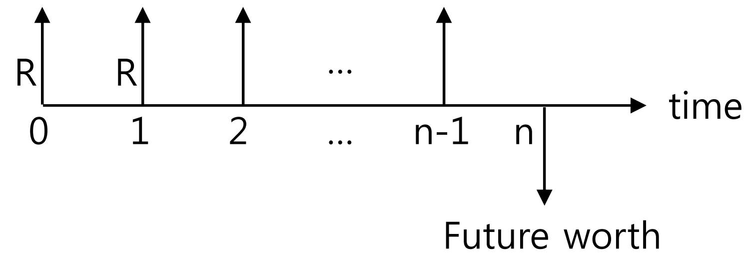
R : uniform amount at each time period

S : future worth

$$S = R(1+i)^{n-1} + R(i+i)^{n-2} + \cdots + R(1+i) + R$$

$$S = R(1+i)^{n} + R(i+i)^{n-1} + \cdots + R(1+i)$$





3.6 Future worth (f/a) of a uniform series of amounts

$$f\ /\ a = S\ /\ R = \frac{(1+i)^n - 1}{(1+i) - 1}$$

Series compound
amount factor(SCAF)

$$a\ /\ f = \frac{i}{(1+i)^n - 1}$$

sinking fund factor (SFF)

Regular amount R=(future worth) * (a/f)

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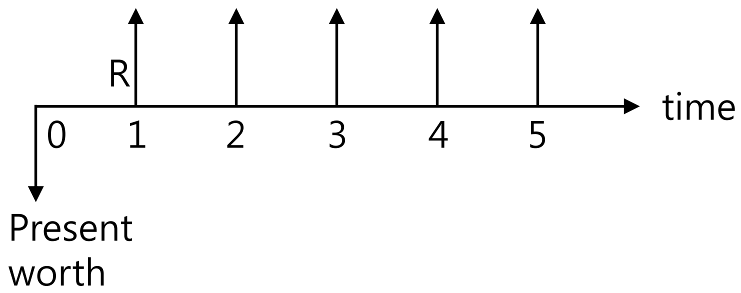
3.7 Present worth (p/a) of a uniform series of amounts



Present worth = $R\left[\dfrac{1}{1+i}+\dfrac{1}{(1+i)^2}+\cdots+\dfrac{1}{(1+i)^n}\right]$

Series present worth factor : $p\,/\,a=\dfrac{\dfrac{1}{1+i}\left(1-\dfrac{1}{(1+i)^n}\right)}{1-\dfrac{1}{1+i}}=\dfrac{(1+i)^n-1}{i(1+i)^n}$

Capital recovery factor : $a\,/\,p$

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3.10 Bonds

- Face value will be returned at maturity
- Semiannual interest
- May be sold

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3.10 Bonds

P_b : price to be paid for bond now

i_c : current interest rate

i_b : interest rate on bond

n : years to maturity

$$P_b\left(1+\frac{i_c}{2}\right)^{2n} = FV + FV\,\frac{i_b}{2}\,\frac{\left(1+\frac{i_c}{2}\right)^{2n}-1}{\frac{i_c}{2}}$$

Face value

$(\frac{f}{p}, \frac{i_c}{2}, 2n)$

Future worth of investment

$(\frac{f}{a}, \frac{i_c}{2}, 2n)$

Future worth of uniform series of the semiannual interest payment on the bond

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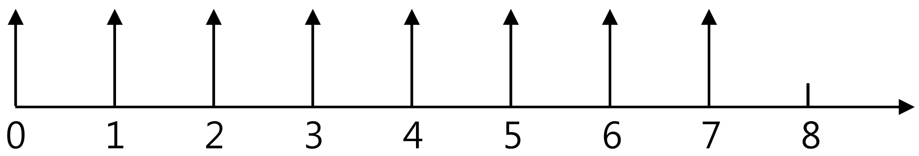
3.11 Shift in time of a series

$$\left(\frac{f}{a}\right)_{shift} = (1+i)\,\frac{(1+i)^n - 1}{i}$$



Actual situation : first amount appears at time 0

no amount appears at the end

3.14 Evaluating Potential Investments

<Example> The rate of return on each building?

Economic data	Building A	Building B
First cost	$800,000	$600,000
Annual income from rent	160,000	155,000
Annual operating and maintenance cost	73,000	50,300
Anticipated selling price	960,000	540,000

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3.14 Evaluating Potential Investments

<Example> The rate of return on each building?

Building A :

$$800{,}000 = (160{,}000 - 73{,}000)(p/a,\ i\%,\ 5) + (960{,}000)(p/f,\ i\%,\ 5)$$

Building B :

$$600{,}000 = (155{,}000 - 50{,}300)(p/a,\ i\%,\ 5) + (540{,}000)(p/f,\ i\%,\ 5)$$

$$i = \begin{cases} 13.9\% & \text{building A} \\ 16.0\% & \text{building B} \end{cases}$$

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3.15 Taxes

└ Money for operating the government

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3.16 Depreciation

- Annual expense
- For replacement of the facility at the end of its life

- Straight line depreciation = $\dfrac{\text{First cost} - \text{salvage cost}}{\text{\# of years of tax life}}$

- Sum of the year's digits (SYD) method = $\dfrac{N + 1 - t}{N(N+1)/2} (P - S)$

  t : year in question

  N : tax life

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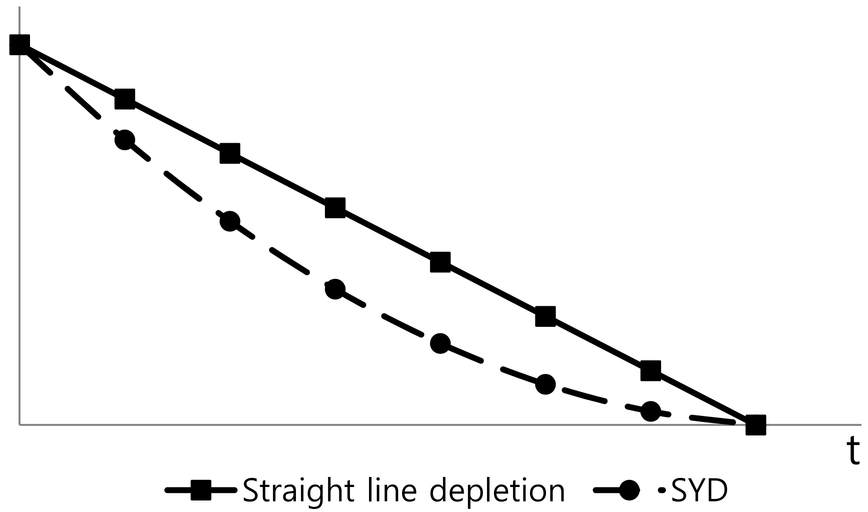
3.16 Depreciation



SYD : greater depreciation in the early portion of the life

: more of tax is paid in later years

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3.18 Continuous compounding

└ daily compounding

- compound amount factor for continuous compounding

$$\frac{f}{p} = \left(1 + \frac{i}{m}\right)^{mn}$$

$$\left(\frac{f}{p}\right)_{cont} = \lim_{m \to \infty} \left(1 + \frac{i}{m}\right)^{mn}$$

$$\ln\left(\frac{f}{p}\right)_{cont} = \lim_{m \to \infty} m\,n \ln\left(1 + \frac{i}{m}\right) = i \times n$$

$$\left(\frac{f}{p}\right)_{cont} = e^{i \times n}$$

3.18 Continuous compounding

- Series compound amount factor with continuous compounding

$$\left(\frac{f}{a}\right)_{cont} = e^{i(n-1)} + e^{i(n-2)} + \cdots + e^{i} + 1 = \frac{e^{in} - 1}{e^{i} - 1}$$

- Continuous flow future worth factor

$1 per year → m equal amount spread uniformly over the entire year

$$\left(\frac{f}{a}\right)_{flow} = \frac{1}{m}(1 + \frac{i}{m})^{mn-1} + \cdots + \frac{1}{m}(1 + \frac{i}{m})^{1} + \frac{1}{m} = \frac{\frac{1}{m}\left\{(1 + \frac{i}{m})^{mn} - 1\right\}}{(1 + \frac{i}{m}) - 1}$$

$$\lim_{m \to \infty}\left(\frac{f}{a}\right)_{flow} = \frac{e^{in} - 1}{i}$$