Optimal Design of Energy Systems

Chapter 5 Modeling Thermal Equipment

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Chapter 5. Modeling Thermal Equipment

5.1 Using Physical Insight

- Heat exchanger (HX)
- Distillation separator
- Turbomachinery

- Design condition
- Off-design condition

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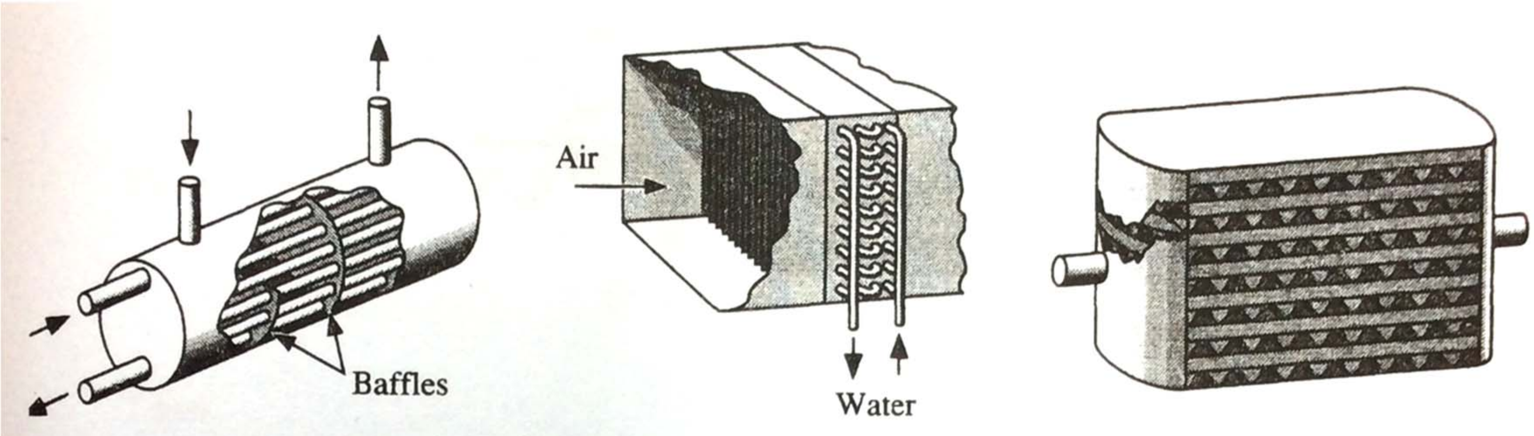
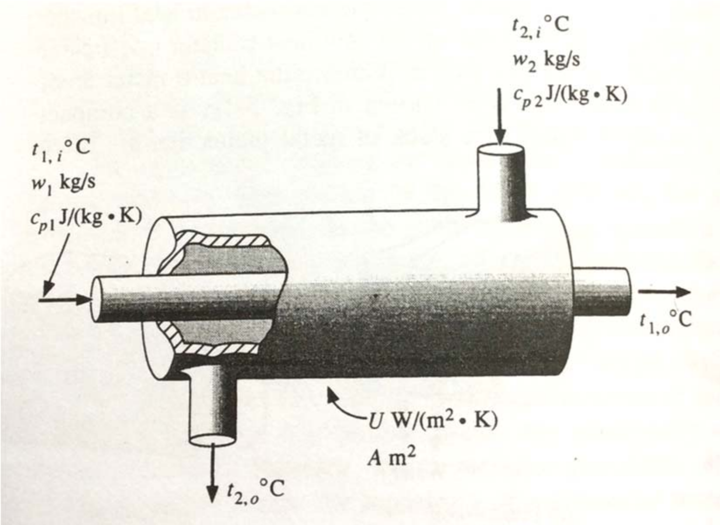
5.2 Selecting and simulating a Heat-Exchanger

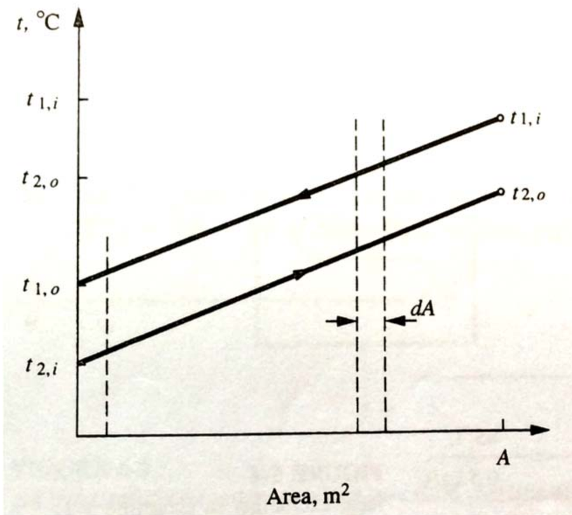
- (a) Shell and tube   liquid : liquid
- (b) Finned coil   liquid : vapor(gas)
- (c) Compact > $700 m^2/m^3$



(a) Shell-and-tube (b) Finned-coil (c) Compact

5.3 Counterflow Heat-Exchanger – most favorable ΔT





$$q = w_1 c_{p1}(t_{1,i} - t_{1,o})$$

$$q = w_2 c_{p2}(t_{2,o} - t_{2,i})$$

$$q = UA\Delta t_{lm} \qquad \Delta t_{lm} = \frac{(t_{1,i} - t_{2,o}) - (t_{1,o} - t_{2,i})}{\ln \dfrac{t_{1,i} - t_{2,o}}{t_{1,o} - t_{2,i}}}$$

5.3 Counterflow Heat-Exchanger – most favorable ΔT

Known : $t_{1,i}$ $t_{2,i}$ $W_1(=w_1c_{p1})$ $W_2(=w_2c_{p2})$ UA

Unknown : $t_{1,o}$ $t_{2,o}$ q

$$W_1(t_{1,i}-t_{1,o})=W_2(t_{2,o}-t_{2,i}) \qquad (1)$$

$$W_1(t_{1,i}-t_{1,o})=UA\frac{(t_{1,i}-t_{2,o})-(t_{1,o}-t_{2,i})}{\ln[(t_{1,i}-t_{2,o})/(t_{1,o}-t_{2,i})]} \qquad (2)$$

$$(1)\rightarrow \quad t_{2,o}=t_{2,i}+\frac{W_1}{W_2}(t_{1,i}-t_{1,o}) \qquad (3)$$

$$(3)\rightarrow(2) \quad W_1(t_{1,i}-t_{1,o})=UA\frac{\left[t_{1,i}-\left\{t_{2,i}+\frac{W_1}{W_2}(t_{1,i}-t_{1,o})\right\}\right]-(t_{1,o}-t_{2,i})}{\ln\dfrac{t_{1,i}-\left\{t_{2,i}+\frac{W_1}{W_2}(t_{1,i}-t_{1,o})\right\}}{t_{1,o}-t_{2,i}}}$$

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5.3 Counterflow Heat-Exchanger – most favorable ΔT

$$\ln \frac{t_{1,i} - \left\{ t_{2,i} + \frac{W_1}{W_2}(t_{1,i} - t_{1,o}) \right\}}{t_{1,o} - t_{2,i}} = UA\left(\frac{1}{W_1} - \frac{1}{W_2}\right) = D$$

$$t_{1,o} = \frac{t_{1,i}\left(\frac{W_1}{W_2} - 1\right) + t_{2,i}(1 - e^D)}{\frac{W_1}{W_2} - e^D} \qquad (4)$$

$(4) \rightarrow (3) \quad t_{2,o} \qquad (5)$

$(4),(5) \quad q$

5.4 Special Case of Counterflow Heat-Exchanger

$$W_1 = W_2 = W$$

$$(4) \rightarrow t_{1,o} = t_{1,i} - (t_{1,i} - t_{2,i}) \frac{1 - e^D}{W_1 / W_2 - e^D}$$

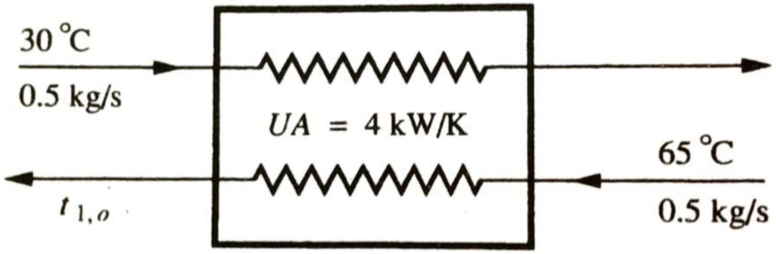
$$\frac{1-e^D}{W_1/W_2 - e^D} = \frac{1 - \left\{ 1 + \frac{UA}{W_1}\left(1 - \frac{W_1}{W_2}\right) + \frac{1}{2}\left[\frac{UA}{W_1}\left(1 - \frac{W_1}{W_2}\right)\right]^2 + \cdots \right\}}{\frac{W_1}{W_2} - \left\{ 1 + \frac{UA}{W_1}\left(1 - \frac{W_1}{W_2}\right) + \frac{1}{2}\left[\frac{UA}{W_1}\left(1 - \frac{W_1}{W_2}\right)\right]^2 + \cdots \right\}} = \frac{-(UA/W_1) - \frac{1}{2}(UA/W_1)^2(1 - W_1/W_2) + \cdots}{-1 - (UA/W_1) - \frac{1}{2}(UA/W_1)^2(1 - W_1/W_2) + \cdots}$$

$$\frac{1-e^D}{W_1/W_2 - e^D} = \frac{UA/W}{1 + UA/W}$$

$$\therefore t_{1,o} = t_{1,i} - (t_{1,i} - t_{2,i}) \frac{UA/W}{1 + UA/W}$$

5.4 Special Case of Counterflow Heat-Exchanger

$$t_{1,o} = \frac{t_{1,i} + t_{2,i} UA / W}{1 + UA / W}$$



$c_p = 4.19 kJ / (kg \cdot K)$

$$t_{1,o} = \frac{65 + 30 \frac{4}{2.1}}{1 + \frac{4}{2.1}} = \frac{122.14}{2.90} = 42.1^oC$$

$$t_{2,o} = 30 + (65 - 42.1) = 52.9^oC$$

$$\Delta t = 12.1^oC$$

5.5 Evaporators and Condensers

Vapor ↑ Liquid   Vapor ↓ Liquid

Refrigerant - working fluid

└ during evaporation and condensation

→ constant temperature (→ constant pressure)

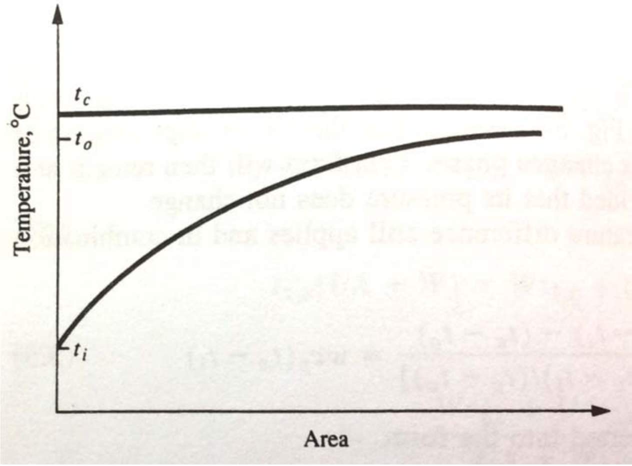
5.5 Evaporators and Condensers

$$\Delta t_{lm} = \frac{(t_c - t_i) - (t_c - t_o)}{\ln \dfrac{t_c - t_i}{t_c - t_o}}$$

$$q = UA\Delta t_{lm} = wc_p(t_o - t_i)$$

Known : $t_c \quad t_i \quad UA \quad W$

Unknown : t_o



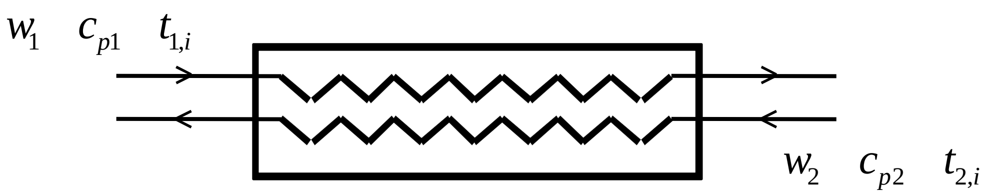
$$\frac{UA}{W} = \ln \frac{t_c - t_i}{t_c - t_o} \Rightarrow \exp\left(-\frac{UA}{W}\right) = \frac{t_c - t_o}{t_c - t_i} = \frac{t_c - t_i - (t_o - t_i)}{t_c - t_i} = 1 - \frac{t_o - t_i}{t_c - t_i}$$

$$\Rightarrow t_o = t_i + (t_c - t_i)\left\{1 - \exp\left(-\frac{UA}{W}\right)\right\}$$

5.6 Heat-Exchanger effectiveness

$$\varepsilon \equiv \frac{q_{actual}}{q_{\max}}$$

$q_{\max}$ ⋯ Maximum possible heat transfer rate (infinite heat transfer area)

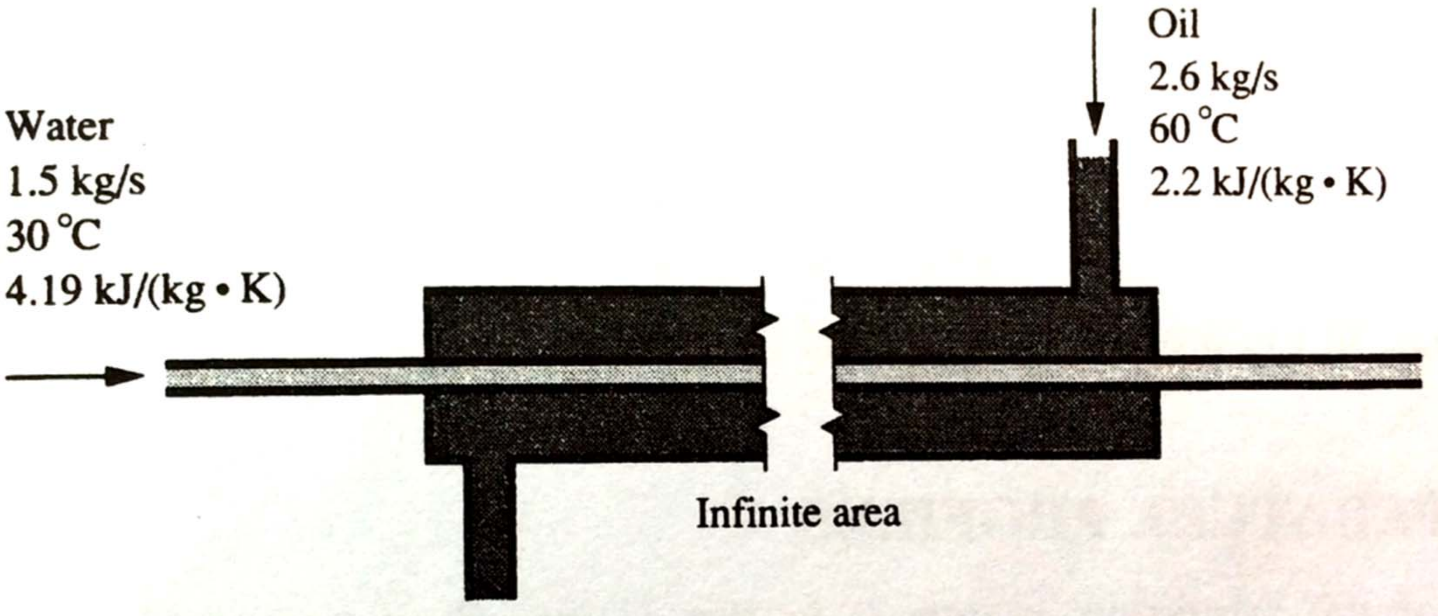


$$q_{1,\max} = w_1 c_{p1}(t_{1,i} - t_{2,i}) \qquad q_{2,\max} = w_2 c_{p2}(t_{1,i} - t_{2,i})$$

$$q_{\max} = \min(q_{1,\max}, q_{2,\max}) = (wc_p)_{\min}(t_{hot,i} - t_{cold,i})$$

5.6 Heat-Exchanger effectiveness

<Example> What is q_{max}?



5.6 Heat-Exchanger effectiveness

<Solution>

Case 1) $q_{oil,\max} = (2.6\ kg/s)[2.2\ kJ(kg \cdot K)](60 - 30\,^oC) = 171.6\ kW$

Water leaves at $30\,^oC + \dfrac{171.6\ kW}{(1.5\ kg/s)[4.19\ kJ(kg \cdot K)]} = 57.3\,^oC$

Case 2) $q_{water,\max} = (1.5\ kg/s)[4.19\ kJ(kg \cdot K)](60 - 30\,^oC) = 188.6\ kW$

Oil leaves at $60\,^oC - \dfrac{188.6\ kW}{(2.6\ kg/s)[2.2\ kJ(kg \cdot K)]} = 27\,^oC$

Case 2 is impossible ! $\therefore q_{\max} = \min(q_{oil,\max}, q_{water,\max}) = q_{oil,\max}$

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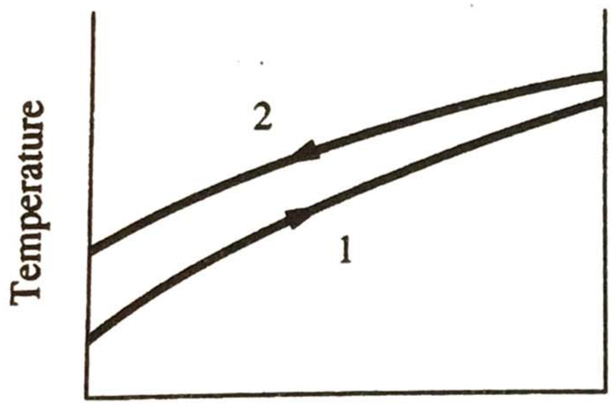
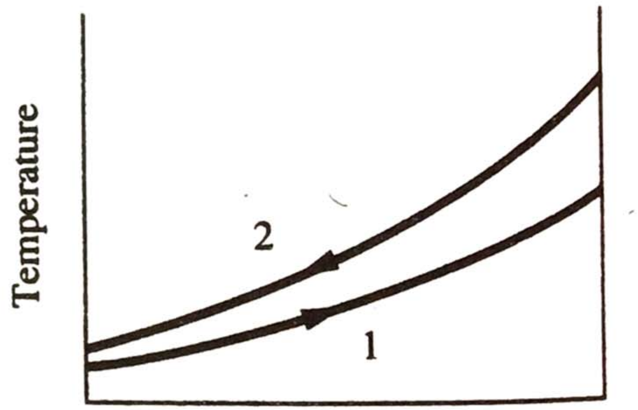
5.7 Temperature profiles



$(wc_p)_1 \rightarrow \min$



$(wc_p)_2 \rightarrow \min$

5.8 Effectiveness of a Counterflow heat exchanger

If $(wc_p)_1 = (wc_p)_{\min}$

$$\varepsilon = \frac{q_{actual}}{q_{\max}} = \frac{(wc_p)_1 \; (t^{(1)}_{cold,out} - t^{(1)}_{cold,in})}{(wc_p)_{\min} \; (t^{(2)}_{hot,in} - t^{(1)}_{cold,in})}$$

$$t_{1,o} = \frac{t_{1,i}\left(\frac{W_1}{W_2} - 1\right) + t_{2,i}(1 - e^D)}{\frac{W_1}{W_2} - e^D}$$

$$t_{1,o} - t_{1,i} = \frac{-t_{1,i}\left(\frac{W_1}{W_2} - e^D\right) + t_{1,i}(\frac{W_1}{W_2} - 1) + t_{2,i}(1 - e^D)}{\frac{W_1}{W_2} - e^D} = (t_{2,i} - t_{1,i})\,\frac{1 - e^D}{\frac{W_1}{W_2} - e^D}$$

5.8 Effectiveness of a Counterflow heat exchanger

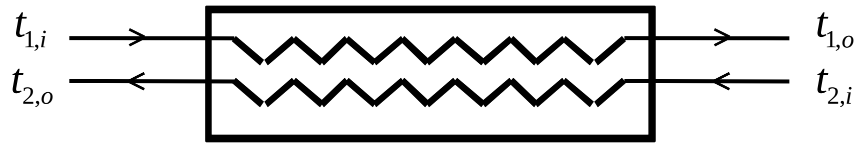
$$\therefore \varepsilon = \frac{1 - e^{D}}{\dfrac{W_1}{W_2} - e^{D}}$$

$$D = UA\left(\frac{1}{W_1} - \frac{1}{W_2}\right)$$

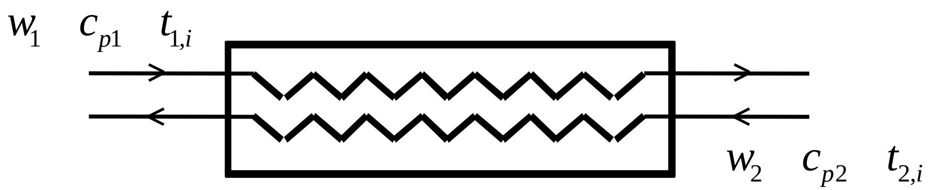
$$W_1 \rightarrow W_{\min}$$

5.9 Number of Transfer Unit (NTU)

$$q = UA\Delta t_{lm}$$

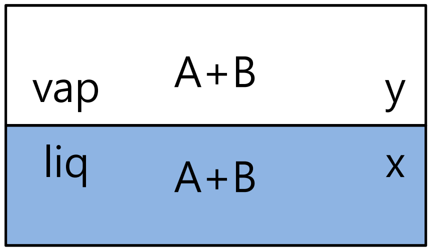


$$q = \varepsilon \ q_{\max}$$



$$NTU = \frac{UA}{W_{\min}}$$

5.10 Binary Solutions, P-T-x diagram



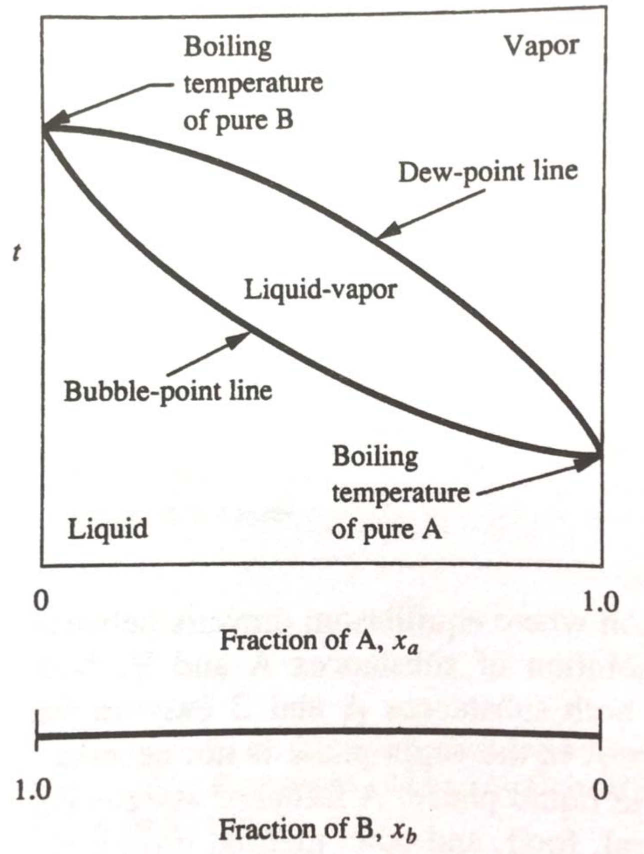
$$mass\ \ fraction\ of\ \ A = \frac{mass\ of\ \ A}{mass\ of\ \ A + mass\ of\ \ B}$$

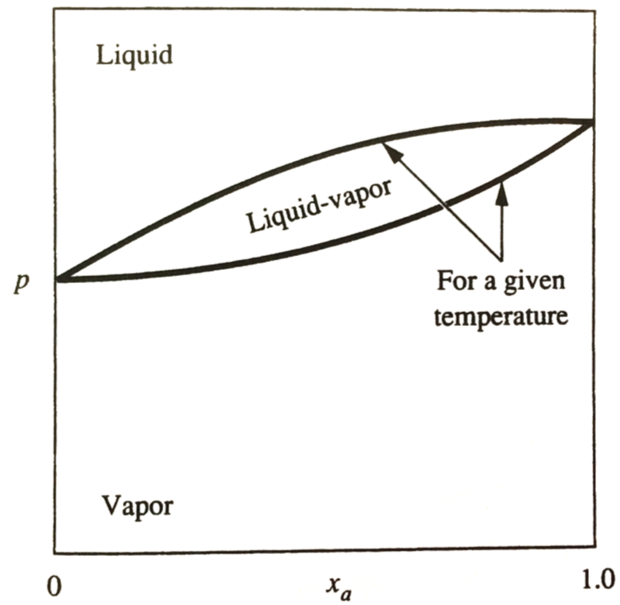
$$mole\ \ fraction\ of\ \ A = \frac{\dfrac{m_a}{M_a}}{\dfrac{m_a}{M_a} + \dfrac{m_b}{M_b}}$$

molecular mass weight

ex> petroleum
cryogenic separation
food

5.10 Binary Solutions, P-T-x diagram





5.12 Developing T-x diagram

Sat. P – Sat. T relation

$$\ln P = C + \frac{D}{T}$$

$$Tds = dh - vdP$$
$$-sdT = dg - vdP$$
$$\left.\begin{aligned} -s_f dT + v_f dP &= dg_f \\ -s_g dT + v_g dP &= dg_g \end{aligned}\right\} \text{equal}$$
$$\frac{dP}{dT} = \frac{s_f - s_g}{v_f - v_g} = \frac{h_{fg}}{T(v_f - v_g)}$$
$$\approx \frac{h_{fg}}{Tv_f} = \frac{h_{fg}P}{RT^2}$$

Raoult's law

$$P_a = x_a P_{sat,a}$$

vapor pressure in mixture

sat. P of pure A

mole fraction of A in the liq. phase

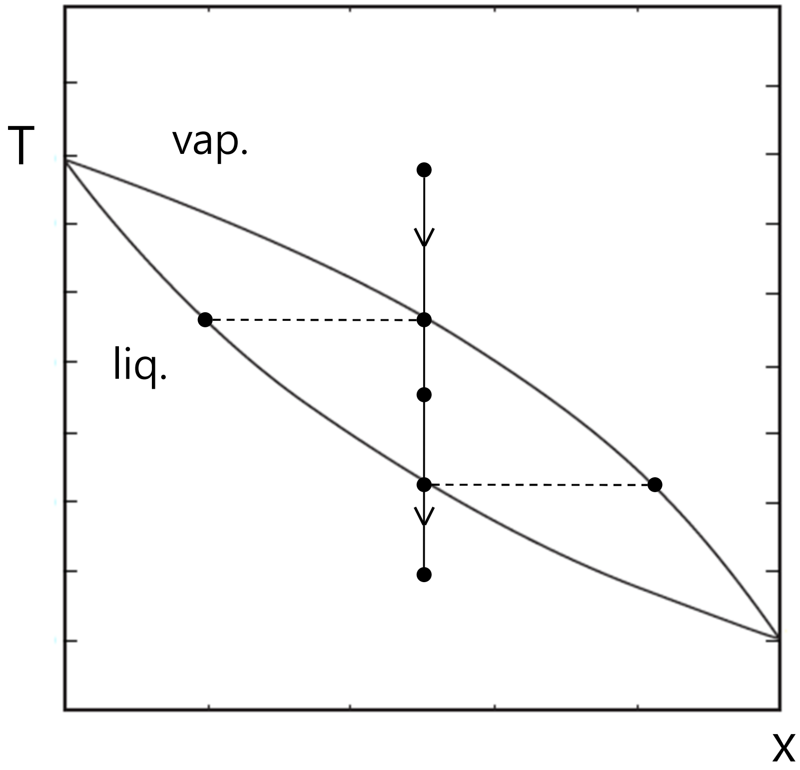
Dalton's law

$$P = P_a + P_b$$
$$P_a = y_a P$$
$$P_b = y_b P$$

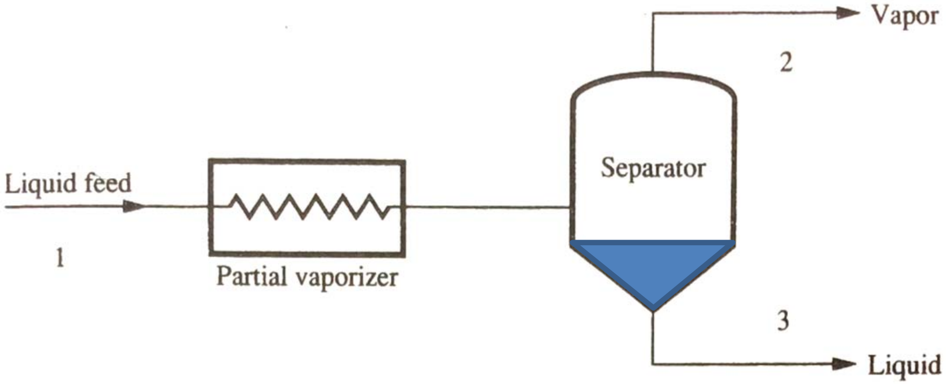
total P

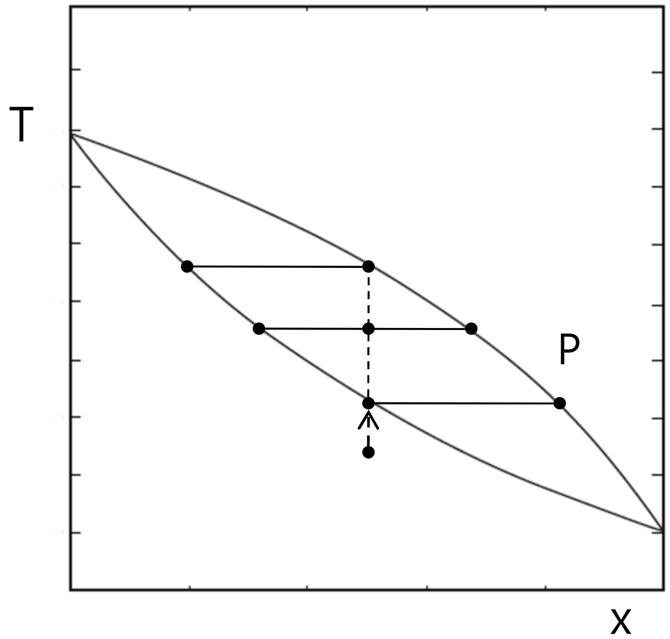
mole fraction of A in the vap. phase

5.13 Condensation of a binary mixture



5.14 Single stage distillation





5.14 Turbomachinery