Optimal Design of Energy Systems

Chapter 6 System Simulation

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Chapter 6. System Simulation

6.1 Introduction

calculation of operating variables

┌ characteristics of all components

└ thermodynamic properties

→ a set of simultaneous equations

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6.2 Some use of simulation

- Design stage – to achieve improved design
- Existing system – to explore prospective modifications
  (operating stage)

- Design condition
- Off design condition (part load / over load) ← normal operation

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6.3 Classes of simulation

System
- continuous – water(or air) flow
- discrete – human(or car) flow

- deterministic – input variables are precisely specified
- stochastic – input conditions are uncertain
  - Random
  - Probability distribution

- steady state
- dynamic – changes with respect to time

  (transient analysis)

  (dynamic simulation)

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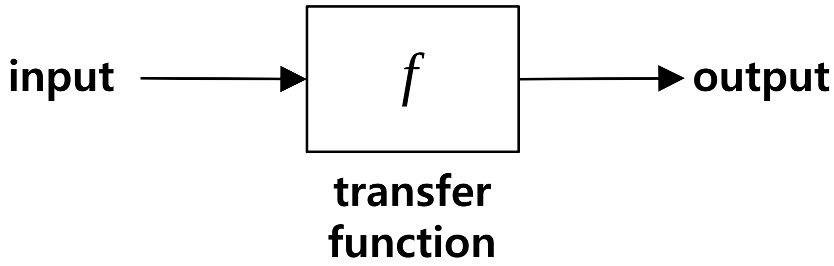
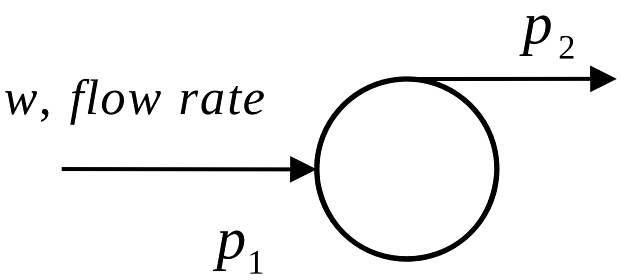
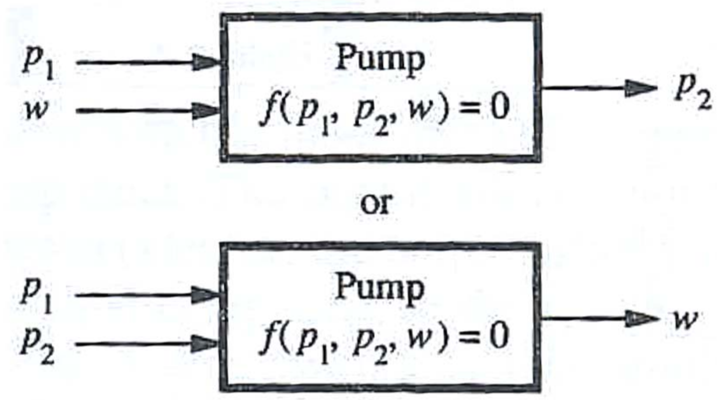
6.4 Information Flow diagram

- Fluid
- Energy



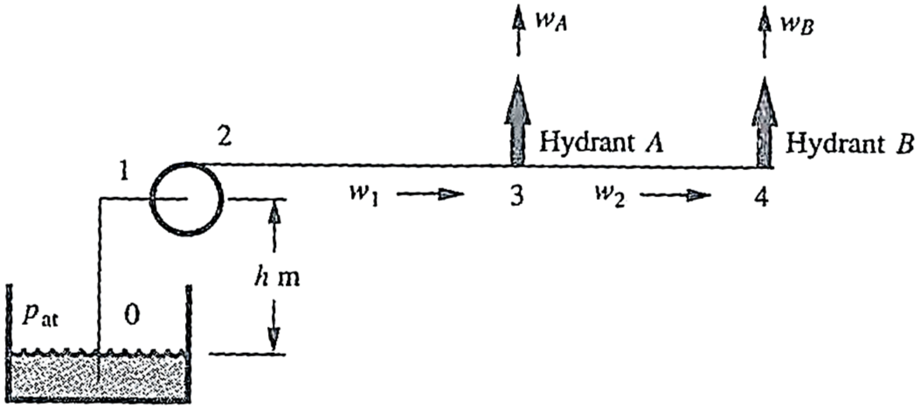
Ex>





6.4 Information Flow diagram

$$w_A = C_A \sqrt{p_3 - p_{atm}}$$
$$w_B = C_B \sqrt{p_4 - p_{atm}}$$
$$P_2 - P_3 = C_2 w_1^{\;2}$$
$$P_3 - P_4 = C_3 w_2^{\;2}$$
$$p_{atm} = p_1 + C_2 w_1^{\;2} + \rho g h$$



pump $w_1 = f(p_1, p_2)$

mass balance $w_1 = w_A + w_2$

$w_2 = w_B$

# of equations = 8

# of variables = 8

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6.4 Information Flow diagram

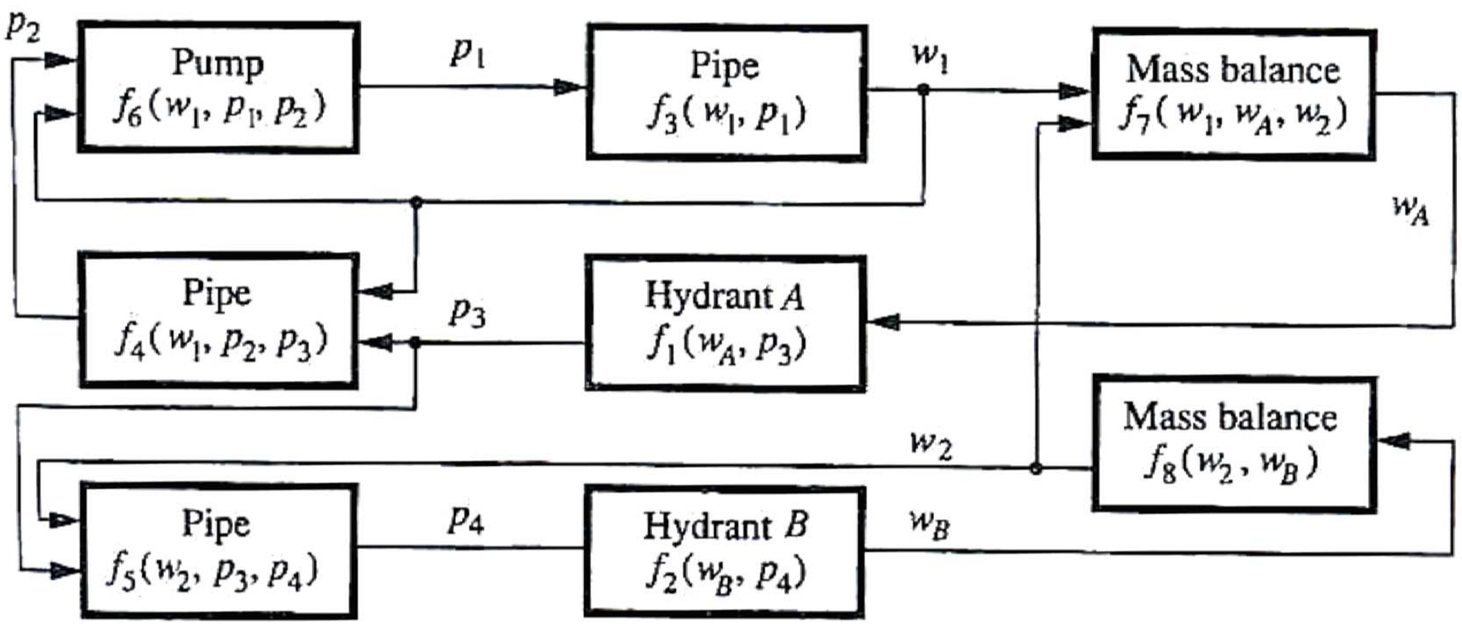

FIGURE 6-3
Information-flow diagram for fire-water system.

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6.6 Two methods of simulation

- Successive substitution
- Newton-Raphson

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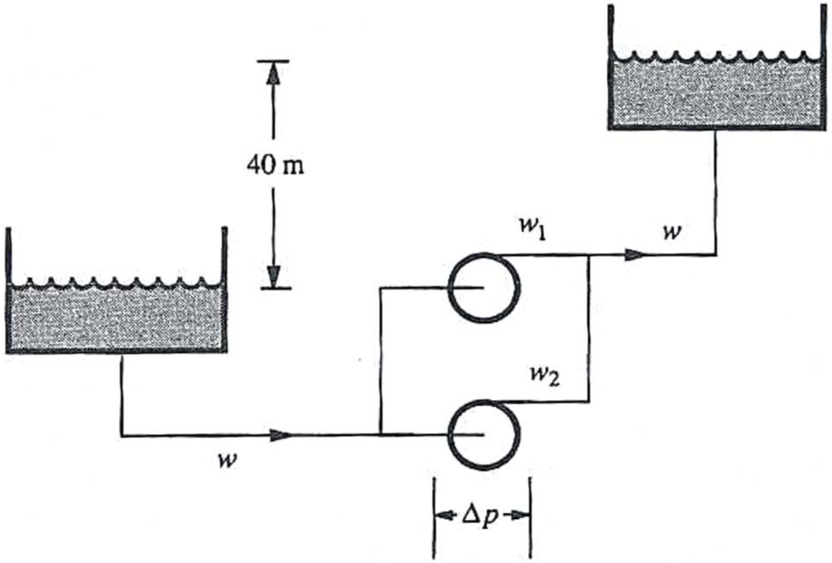
6.7 Successive substitution

<Example 6.1> What are w_1, w_2 , w and ΔP ?



- Pump 1 : $\Delta p = 810 - 25w_1 - 3.75{w_1}^2$
- Pump 2 : $\Delta p = 900 - 65w_2 - 30{w_2}^2$

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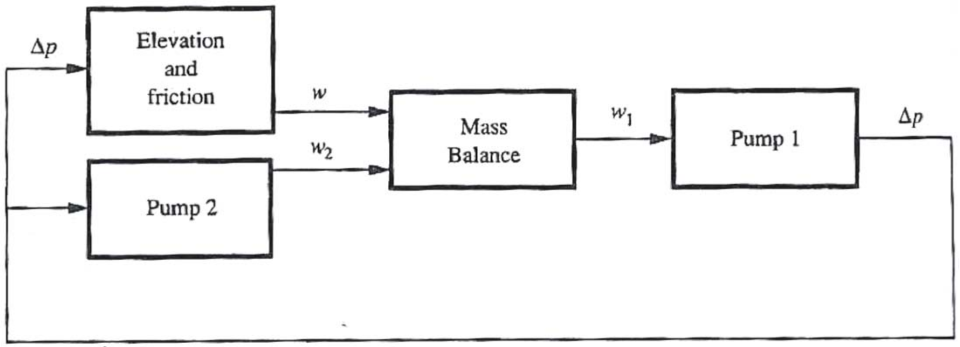
6.7 Successive substitution

<Solution>

- ✓ Pressure difference due to elevation and friction :

$$\Delta p = 7.2w^2 + \frac{(40\ m)(1000\ kg/m^3)(9.807\ m/s^2)}{1000\ Pa/kPa}$$

- ✓ Pump 1 : $\Delta p = 810 - 25w_1 - 3.75{w_1}^2$
- ✓ Pump 2 : $\Delta p = 900 - 65w_2 - 30{w_2}^2$
- ✓ Mass balance : $w = w_1 + w_2$



6.7 Successive substitution

<Solution>

✓ Iteration with initial assumption : $w_1 = 4.2$

$$\Delta p = f_1(w_1)$$

$$w_2 = f_2(\Delta p)$$

$$w = f_3(\Delta p)$$

$$w_1 = f_2(w, w_2)$$

Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]
1	**4.200**	2.060	5.852	638.85
2	3.792	1.939	6.112	661.26
3	4.174	2.052	5.870	640.34
4	3.818	1.946	6.097	659.90
5	4.151	2.045	5.885	641.63
⋮	⋮	⋮	⋮	⋮
∞	3.991	1.997	5.988	650.49

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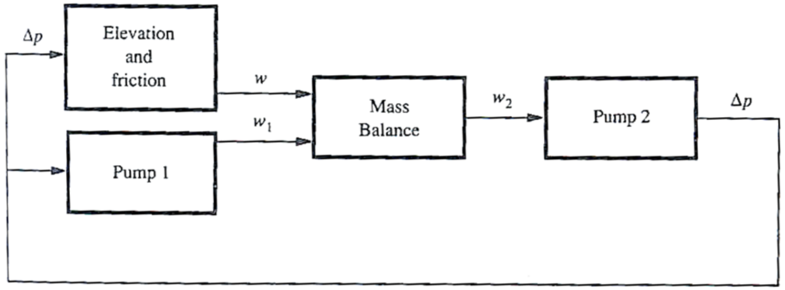
6.8 Pitfalls in successive substitution

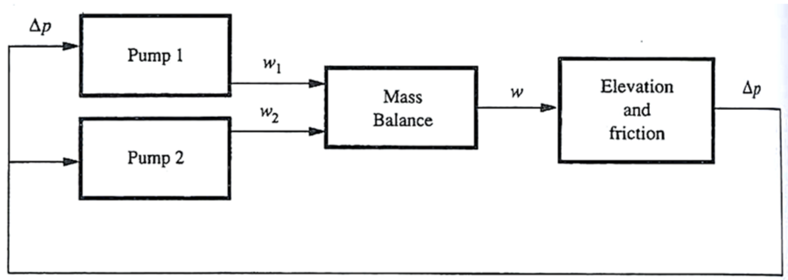
Diagram 2



Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]
1	4.000	**2.000**	5.983	650.00
2	3.942	1.983	6.019	653.16
3	4.258	2.077	5.812	635.53
4	2.443	1.554	6.814	726.54
5	11.353	4.371	**divergence**	42.87

✓ Divergence occurs

Diagram 3



Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]
1	3.973	1.992	**6.000**	651.48
2	4.028	2.008	5.965	648.47
3	3.916	1.975	6.036	654.61
⋮	⋮	⋮	⋮	⋮
9	**divergence**		8.811	951.23

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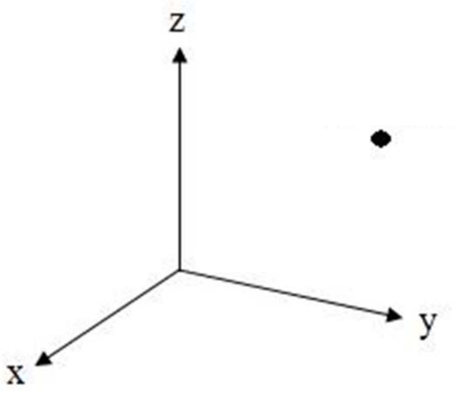
6.9 Taylor series expansion

$$z = z(x, y)$$

Near the point $(a,\ b,\ z(a,b))$



$$z = c_0 + c_1(x-a) + c_2(y-b)$$
$$+ c_3(x-a)^2 + c_4(x-a)(y-b) + c_5(y-b)^2 + \cdots$$

$$c_0 = z(a,b)$$

$$c_1 = \frac{\partial z(a,b)}{\partial x}$$

$$c_2 = \frac{\partial z(a,b)}{\partial y}$$

$$c_3 = \frac{1}{2}\frac{\partial^2 z(a,b)}{\partial x^2}$$

$$c_4 = \frac{\partial^2 z(a,b)}{\partial x \partial y}$$

$$c_5 = \frac{1}{2}\frac{\partial^2 z(a,b)}{\partial y^2}$$

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6.9 Taylor series expansion

- $y = y(x)$

  $y = d_0 + d_1(x-a) + d_2(x-a)^2 + \cdots$

  $$d_0 = y(a) \qquad d_1 = \frac{dy(a)}{dx} \qquad d_2 = \frac{1}{2}\frac{d^2y(a)}{dx^2}$$

- $y = y(x_1, x_2, \cdots, x_n)$

  $$y = y(a_1, a_2, \cdots, a_n) + \sum_{j=1}^{n} \frac{\partial y(a_1, a_2, \cdots, a_n)}{\partial x_j}(x_j - a_j)$$

  $$+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 y(a_1, a_2, \cdots, a_n)}{\partial x_i \partial x_j}(x_j - a_i)(x_j - a_j)$$

  $$+ \cdots$$

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6.9 Taylor series expansion

<Example 6.2> Express $z = \ln(x^2 / y)$ as a Taylor-series expansion at (x=2,y=1)

<Solution>

$$z = \ln \frac{x^2}{y} = c_0 + c_1(x-2) + c_2(y-1) + c_3(x-2)^2 + c_4(x-2)(y-1) + c_5(y-1)^2 + \cdots$$

$$c_0 = \ln \frac{2^2}{1} = 1.39$$

$$c_1 = \frac{\partial z(2,1)}{\partial x} = \frac{2x/y}{x^2/y} = 1$$

$$c_2 = \frac{\partial z(2,1)}{\partial y} = -\frac{x^2/y^2}{x^2/y} = -1$$

$$c_3 = \frac{1}{2}\frac{\partial^2 z(2,1)}{\partial x^2} = \frac{1}{2}\left(-\frac{2}{x^2}\right) = -\frac{1}{4}$$

$$c_4 = \frac{\partial^2 z(2,1)}{\partial x \partial y} = 0$$

$$c_5 = \frac{1}{2}\frac{\partial^2 z(2,1)}{\partial y^2} = \frac{1}{2}\frac{1}{y^2} = \frac{1}{2}$$

$$\therefore \quad z = 1.39 + (x-2) - (y-1) - \left(\frac{1}{4}\right)(x-2)^2 + \left(\frac{1}{2}\right)(y-1)^2 + \cdots$$

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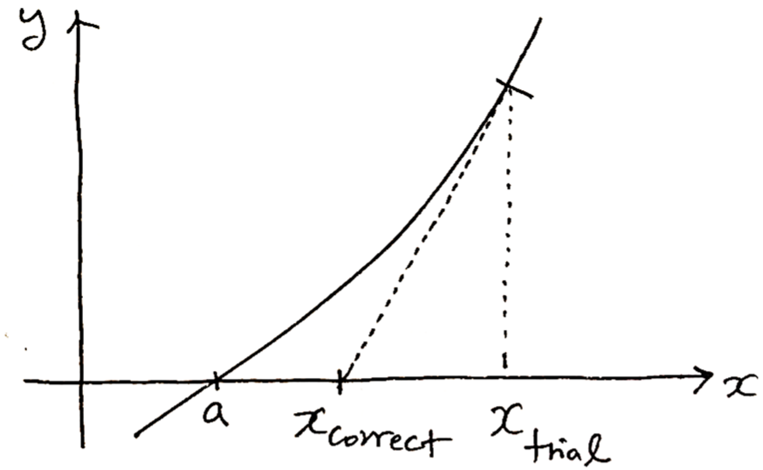
6.10 Newton-Raphson with one equation and one unknown

$$y = y(x) \qquad\qquad x \approx a$$

$$\rightarrow \quad y(x) \approx y(a) + y'(a)(x-a)$$



$$y(x_t) = y(x_c) + \frac{y(x_t) - y(x_c)}{x_t - x_c}(x_t - x_c)$$

$$x_c = x_t - \frac{y(x_t)}{y'(x_t)}$$

$$x_{new} = x_{old} - (x_{trial} - x_{correct})$$

6.10 Newton-Raphson with one equation and one unknown

Ex> $y(x) = x + 2 - e^{x}$, $y(x_c) = 0$

if $x_t = 2$ $y(x_t) = x_t + 2 - e^{x_t} = -3.39$

$$x_c = 2 - \frac{-3.39}{1 - e^{2}} = 1.469 \rightarrow x_{t,new}$$

Iteration	x_t	y(x)	y'(x)	x_c
1	2.000	-3.389	-6.389	1.470
2	1.470	-0.878	-3.347	1.207
3	1.207	-0.137	-2.345	1.149
4	1.149	-0.006	-2.154	1.146
5	**1.146**	0.000	-2.146	

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6.11 Newton-Raphson with multiple equations and unknowns

$$f_1(x_1, x_2, x_3) = 0$$
$$f_2(x_1, x_2, x_3) = 0$$
$$f_3(x_1, x_2, x_3) = 0$$

trial value : $x_{1t}, \quad x_{2t}, \quad x_{3t}$

$$f_1(x_{1t}, x_{2t}, x_{3t}) = f_1(x_{1c}, x_{2c}, x_{3c}) + \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_1}(x_{1t} - x_{1c})$$

$$+ \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_2}(x_{2t} - x_{2c})$$

$$+ \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_3}(x_{3t} - x_{3c})$$

$$+ \cdots$$

$$f_2 = \cdots$$

$$f_3 = \cdots$$

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6.11 Newton-Raphson with multiple equations and unknowns

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_{1t} - x_{1c} \\ x_{2t} - x_{2c} \\ x_{3t} - x_{3c} \end{bmatrix} = \begin{bmatrix} f_1(x_{1t}, x_{2t}, x_{3t}) \\ f_2(x_{1t}, x_{2t}, x_{3t}) \\ f_3(x_{1t}, x_{2t}, x_{3t}) \end{bmatrix}$$

$$\rightarrow \quad x_{i,new} = x_{i,old} - (x_{i,t} - x_{i,c})$$

6.11 Newton-Raphson with multiple equations and unknowns

<Example 6.3> Solve Example 6.1 by Newton-Raphson method

<Solution>

✓ requirement : $f_1 = \Delta p - 7.2w^2 - 392.28 = 0$

$$f_2 = \Delta p - 810 + 25w_1 + 3.75w_1^2 = 0$$

$$f_3 = \Delta p - 900 + 65w_2 + 30w_2^2 = 0$$

$$f_4 = w - w_1 + w_2 = 0$$

✓ trial value : $\Delta p = 750,\ w_1 = 3,\ w_2 = 1.5,\ w = 5$

$\rightarrow f_1 = 177.7,\ f_2 = 48.75,\ f_3 = 15.0,\ f_4 = 0.50$

	∂/∂Δp	∂/∂w_1	∂/∂w_2	∂/∂w
∂f_1/∂	1	0	0	-14.4w
∂f_2/∂	1	25+7.5w_1	0	0
∂f_3/∂	1	0	65+60w_2	0
∂f_4/∂	0	-1	-1	1

6.11 Newton-Raphson with multiple equations and unknowns

<Solution>

✓ $$\begin{bmatrix} 1.0 & 0.0 & 0.0 & -72.0 \\ 1.0 & 47.5 & 0.0 & 0.0 \\ 1.0 & 0.0 & 155.0 & 0.0 \\ 0.0 & -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} 177.7 \\ 48.75 \\ 15.0 \\ 0.50 \end{bmatrix} \qquad \Delta x_i = x_{i,t} - x_{i,c}$$

✓ $\Delta x_1 = 98.84,\ \Delta x_2 = -1.055,\ \Delta x_3 = -0.541\ \Delta x_4 = -1.096$

✓ corrected variable : $\Delta p = 750 - 98.84 = 651.16\ \ w_1 = 4.055,\ w_2 = 2.041,\ w = 6.096$

Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]	f_1	f_2	f_3	f_4
1	3.000	1.500	5.000	750.00	177.720	48.750	15.000	0.500
2	4.055	2.041	6.096	651.16	-8.641	4.171	8.778	0.000
3	3.992	1.998	5.989	650.48	-0.081	0.015	0.056	0.000
4	3.991	1.997	5.988	650.49	0.000	0.000	0.000	0.000

$$\therefore\ \Delta p = 650.49,\ w_1 = 3.991,\ w_2 = 1.997,\ w = 5.988$$