Optimal Design of Energy Systems

Chapter 7 Optimization

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Chapter 7. Optimization

7.1 Introduction

- role of engineers = optimization

  = finding the conditions of maximum or minimum

- difficult to optimize ← complex

- criterion required
  - initial cost
  - total cost

- component simulation + system simulation

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7.2 Levels of optimization

- comparison of alternate concepts
- optimization within a concept

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7.3 Mathematical Representation of Optimization Problems

- object(ive) function y : function to be optimized

$$y = y(x_1, \cdots, x_n)$$

↑

Independent variable

- constraints

equality constraint $\phi_i = \phi_i(x_1, \cdots, x_n) = 0$

inequality constraint $\psi_i = \psi_i(x_1, \cdots, x_n) \le L_j$

✓ $y = a + Y(x_1, \cdots, x_n)$

$\min y = a + \min Y$

$\max y = \min(-y)$

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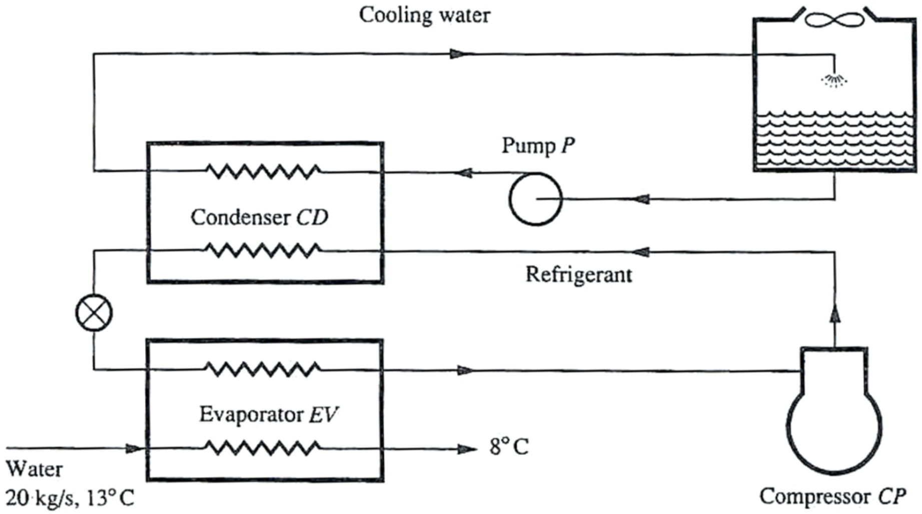
7.4 Water-chilling system



cost size

$$y(x_{comp}, x_{cond}, x_{evap}, x_{CT}, x_{pump}) \rightarrow \text{ minimize}$$

$$Q = \phi(x_{comp}, x_{cond}, x_{evap}, x_{CT}, x_{pump}) = \dot{m} c_p \Delta T$$

$$t_{evap}(x_{comp}, x_{cond}, x_{evap}, x_{CT}, x_{pump}) \geq 0\,^oC$$

$$t_{cond}(x_{comp}, x_{cond}, x_{evap}, x_{CT}, x_{pump}) \leq 100\,^oC$$

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7.5 Optimization procedures

- Calculus method (Chap.8) : Lagrange Multipliers
- Search method (Chap.9) → multivariable optimization

  exact optimum is approached not reached
- Dynamic programming (Chap.10) → optimum function
- Geometric programming (Chap.11)
- Linear programming (Chap.12)

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7.11 Setting up the Mathematical Statement of optimization Problem

- To translate physical situation into mathematical statement
- objective function – trivial
- constraints – difficult to establish

Strategy

(1) Specify all direct constraints

(2) Component characteristics + properties

(3) Mass / energy balance

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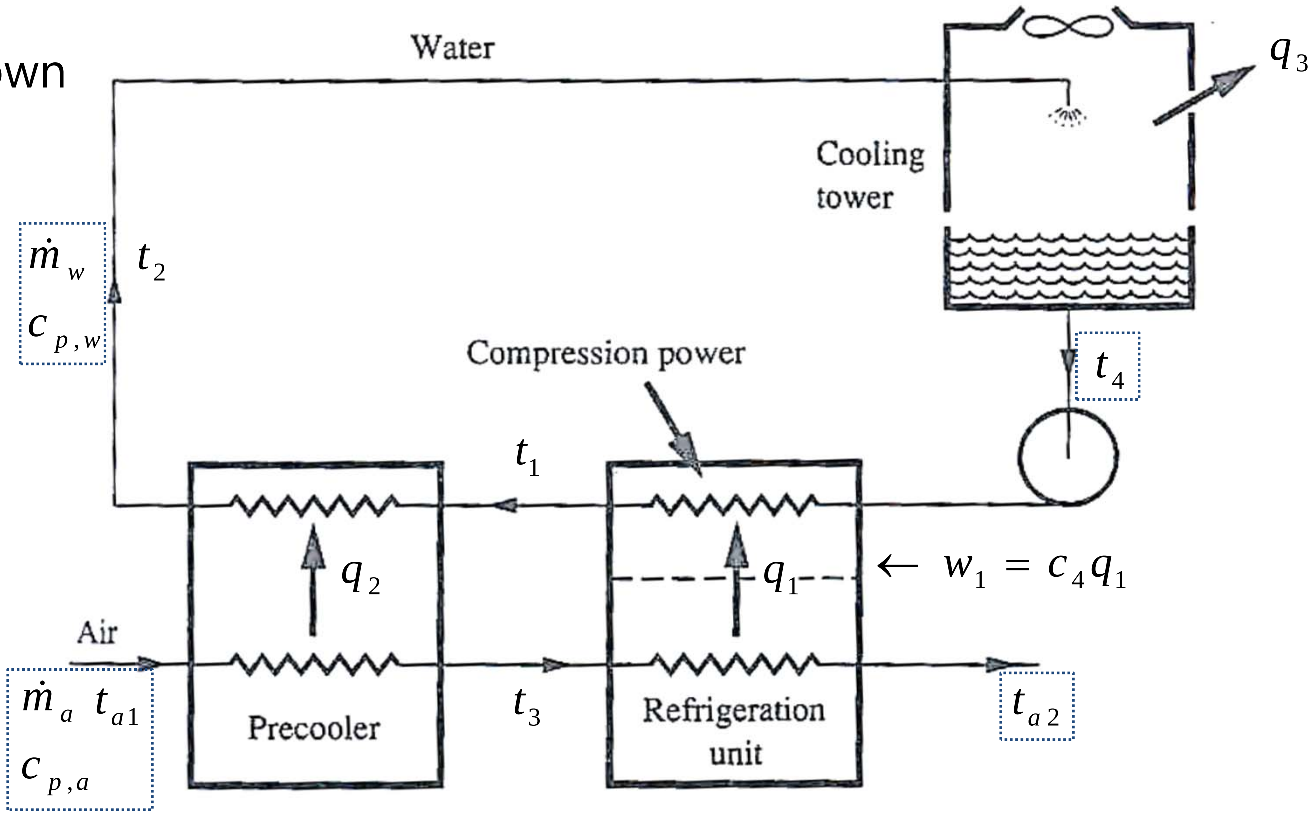
7.11 Setting up the Mathematical Statement of optimization Problem

<Example 7.1> Intercooling of Air

[dotted box] : known



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7.11 Setting up the Mathematical Statement of optimization Problem

<Example 7.1>

Minimize first cost

Refrigeration unit $x_1 = c_1 q_1$

Precooler $x_2 = \frac{c_2 q_2}{t_3 - t_1}$

Cooling tower $x_3 = c_3 q_3$

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7.11 Setting up the Mathematical Statement of optimization Problem

<Example 7.1>

Total cost (objective function) $y = x_1 + x_2 + x_3$

Constraints (1) $q_1 = \dot{m}_a c_{p,a}(t_3 - t_{a2})$

$q_2 = \dot{m}_a c_{p,a}(t_{a1} - t_3)$

(2) component

$x_1 = c_1 q_1$

$x_2 = \dfrac{c_2 q_2}{t_3 - t_1}$

$x_3 = c_3 q_3$

$w_1 = c_4 q_1$ (work)

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7.11 Setting up the Mathematical Statement of optimization Problem

<Example 7.1>

Constraints (3) balance

Refrigeration unit $q_1 + w_1 = \dot{m}_w c_{p,w}(t_1 - t_4)$

Precooler $\dot{m}_a c_{p,a}(t_{a1} - t_3) = \dot{m}_w c_{p,w}(t_2 - t_1)$

Cooling tower $\dot{m}_w c_{p,w}(t_2 - t_4) = q_3$

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7.11 Setting up the Mathematical Statement of optimization Problem

<Example 7.1>

# of equations 9 $\rightarrow$ 2

# of unknowns

$$\left.\begin{matrix} q_1, \ q_2, \ q_3 \\ t_1, \ t_2, \ t_3 \\ x_1, \ x_2, \ x_3 \\ w_1 \end{matrix}\right\} 10 \rightarrow 3 \quad x_1, \ x_2, \ x_3$$

Minimize $y = x_1 + x_2 + x_3$

Subject to

$$\phi_1(x_1, \ x_2, \ x_3) = 0$$

$$\phi_2(x_1, \ x_2, \ x_3) = 0$$