Optimal Design of Energy Systems

Chapter 14 Steady-State Simulation

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Chapter 14. Steady-State Simulation

Performance prediction at { off-design / design } conditions

- Performance of components
- Properties
- conditions

} should be known

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14.2 Convergence and Divergence in successive substitution

<Example 14.1> Using the Gauss-Seidel method, solve for the x's in the following set of simultaneous linear equations:

$$\text{A:} \quad 4x_1 - 3x_2 + x_3 = 12 \quad \rightarrow x_1$$

$$\text{B:} \quad x_1 - 2x_2 + 2x_3 = 6 \quad \rightarrow x_2$$

$$\text{C:} \quad 2x_1 + x_2 + 3x_3 = 6 \quad \rightarrow x_3$$

Trial value
x_2=0, x_3=0

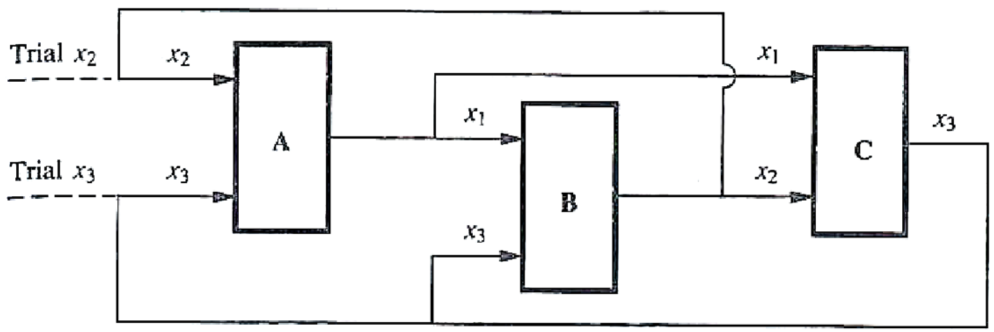

FIGURE 14-1
The Gauss-Seidel method as a successive substitution process.

14.2 Convergence and Divergence in successive substitution

<Example 14.1>

TABLE 14.1
Gauss-Seidel Solution of Example 14.1

Cycle	x_1	x_2	x_3
1	3.0	−1.5	0.5
2	1.75	−1.625	1.375
. . .	. . .	. . .	. . .
10	2.045	−1.021	0.977
∞	2	−1	1

convergent

$$x_1 = 2, \quad x_2 = -1, \quad x_3 = 1$$

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14.2 Convergence and Divergence in successive substitution

<Example 14.1>

with different order

A: $4x_1 - 3x_2 + x_3 = 12 \quad \rightarrow x_1$

C: $2x_1 + x_2 + 3x_3 = 6 \quad \rightarrow x_2$

B: $x_1 - 2x_2 + 2x_3 = 6 \quad \rightarrow x_3$

TABLE 14.2
Example 14.1 with equations solved in the A-C-B sequence

Cycle	x_1	x_2	x_3
1	3	0	1.5
2	2.625	− 3.75	− 2.0625
3	0.703	10.78	13.43
4	7.29	−49.75	−50.61

divergent

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14.2 Convergence and Divergence in successive substitution

A-B-C, convergent

$$\begin{bmatrix} 4 & -3 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

A-C-B, divergent

$$\begin{bmatrix} 4 & -3 & 1 \\ 2 & 1 & 3 \\ 1 & -2 & 2 \end{bmatrix}$$

For convergent case, large-magnitude coefficient in diagonal position

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14.2 Convergence and Divergence in successive substitution

A-B-C, convergent

A-C-B, divergent

Multiply lower-triangular by λ

$$\begin{vmatrix} 4\lambda & -3 & 1 \\ 1\lambda & -2\lambda & 2 \\ 2\lambda & 1\lambda & 3\lambda \end{vmatrix} = 0 \qquad \begin{vmatrix} 4\lambda & -3 & 1 \\ 2\lambda & 1\lambda & 3 \\ 1\lambda & -2\lambda & 2\lambda \end{vmatrix} = 0$$

$\lambda = 0,\ 0.125 + 0.696i,\ 0.125 - 0.696i$

$\lambda = 0,\ 0.2713,\ -4.146$

$|\lambda| < 1$ convergent

$|\lambda| > 1$ divergent

14.3 Partial substitution in successive substitution

$$x_{j,i+1} = \beta x_{j,i+1^*} + (1-\beta) x_{j,i}$$

new — $x_{j,i+1}$

new value computed from the eq. — $x_{j,i+1^*}$

previous — $x_{j,i}$

β → Partial substitution factor

$\beta = 1$ Successive substitution (Gauss-Seidel method)

$\beta \downarrow$ Toward a more convergent process

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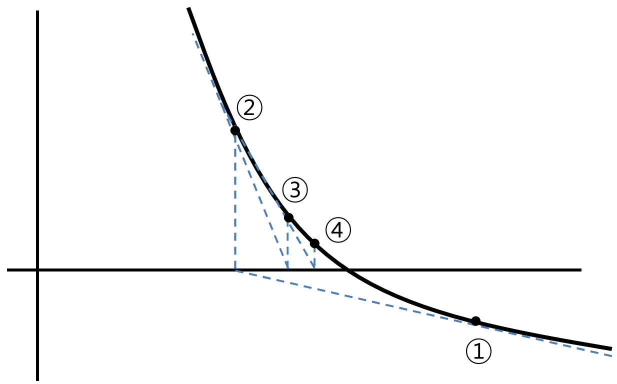
14.4 Evaluation of Newton-Raphson technique

Successive Substitution	Newton-Raphson
① straightforward to program	① more reliable
② sparing computer memory	② more rapidly convergent
	③ not necessary to list the eq. in any special order

For small system



14.5 Some characteristics of the Newton-Raphson technique

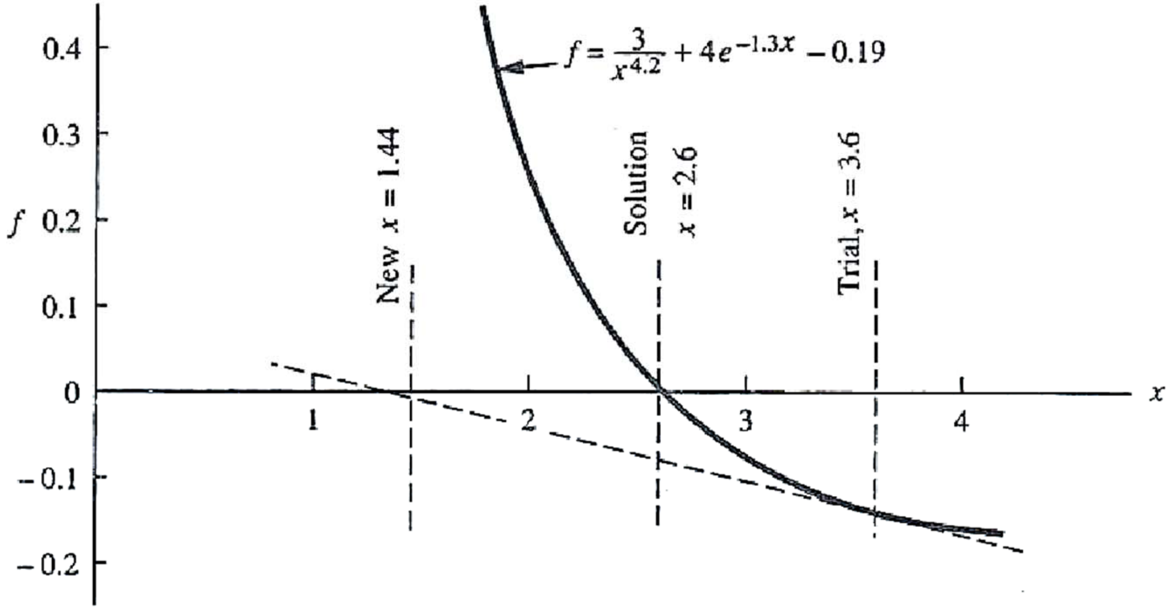


FIGURE 14-6
First Newton-Raphson iteration may move the values of the variables further from the solution than the trial values.

14.8 Quasi-Newton Method

temporary value (trial) → $x_{1,t}$

corrected value → $x_{1,c}$

$$JX = F$$

$$J = \begin{bmatrix} \partial f_1 / \partial x_1 & \partial f_1 / \partial x_2 & \partial f_1 / \partial x_3 \\ \partial f_2 / \partial x_1 & \partial f_2 / \partial x_2 & \partial f_2 / \partial x_3 \\ \partial f_3 / \partial x_1 & \partial f_3 / \partial x_2 & \partial f_3 / \partial x_3 \end{bmatrix} \quad X = \begin{bmatrix} x_{1,t} - x_{1,c} \\ x_{2,t} - x_{2,c} \\ x_{3,t} - x_{3,c} \end{bmatrix} \quad F = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$\overset{③}{X} = -\overset{②①⑤}{HF}$$

$$H = J^{-1}$$

④ $$V_{k+1} = X_{k+1} + X_k$$

⑥ $$Y_k = F_{k+1} - F_k$$

⑦ $$H_{k+1} = H_k + \frac{(X_k - H_k Y_k) X_k^T H_k}{X_k^T H_k Y_k}$$

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14.8 Quasi-Newton Method

ex> Newton-Raphson vs. Quasi-Newton

duct $f_1 = 0.0625 + 0.653Q^{1.8} - P$

fan $f_2 = 0.3 - 0.2Q^2 - P$

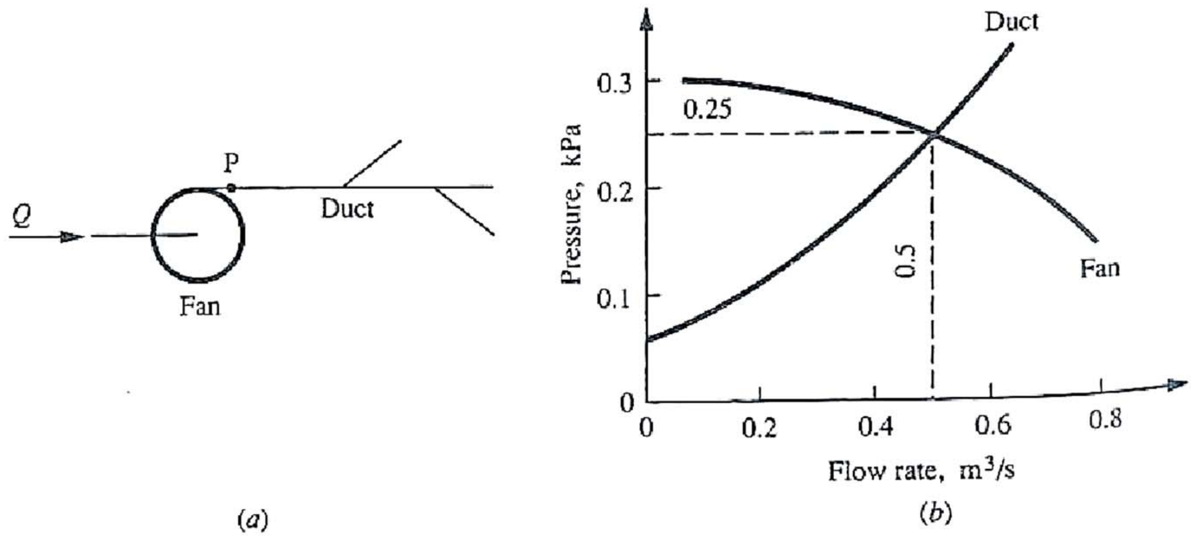

FIGURE 14-11
(*a*) A fan-duct system, and (*b*) the pressure-flow characteristics.

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14.8 Quasi-Newton Method

ex> Newton-Raphson vs. Quasi-Newton

✓ Newton-Raphson

$$JX = F$$

$$J = \begin{bmatrix} \partial f_1 / \partial P & \partial f_1 / \partial Q \\ \partial f_2 / \partial P & \partial f_2 / \partial Q \end{bmatrix} \quad X = \begin{bmatrix} P_t - P_c \\ Q_t - Q_c \end{bmatrix} \quad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -1 & 1.1759 \\ -1 & -0.4 \end{bmatrix} \quad F = \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix}$$

$$J^{-1} = \begin{bmatrix} -0.2539 & -0.7461 \\ 0.6345 & -0.6345 \end{bmatrix} \quad X = \begin{bmatrix} -0.156 \\ 0.392 \end{bmatrix} \quad \begin{matrix} P_t = 0.256 \\ Q_t = 0.608 \end{matrix}$$

14.8 Quasi-Newton Method

ex> Newton-Raphson vs. Quasi-Newton

✓ Newton-Raphson

TABLE 14.9
Newton-Raphson simulation of fan-duct system

Iteration	Variables	Matrix of Partial Derivatives		Inverse of Matrix	
At trial values	P = 0.1	−1.0	1.1759	−0.2539	−0.7461
	Q = 1.0	−1.0	−0.4000	0.6345	−0.6345
1	P = 0.256	−1.0	0.7902	−0.2356	−0.7644
	Q = 0.608	−1.0	−0.2435	0.9674	−0.9674
2	P = 0.250	−1.0	0.6836	−0.2291	−0.7709
	Q = 0.508	−1.0	−0.2032	1.1276	−1.1276
3	P = 0.250	−1.0	0.6748	−0.2285	−0.7715
	Q = 0.500	−1.0	−0.1999	1.1432	−1.1432
4	P = 0.250	−1.0			
	Q = 0.500	−1.0			

$$P = 0.25\ kPa$$

$$Q = 0.5\ m^3/s$$

14.8 Quasi-Newton Method

ex> Newton-Raphson vs. Quasi-Newton

✓ Quasi-Newton

$$F_k = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix}$$

$$X_k = \begin{bmatrix} P_{1,c} - P_{1,t} \\ Q_{1,c} - Q_{1,t} \end{bmatrix} = -\begin{bmatrix} -0.25392 & -0.74608 \\ 0.63449 & -0.63449 \end{bmatrix} \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.15629 \\ -0.39053 \end{bmatrix}$$

$$V_{k+1} = \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} + \begin{bmatrix} 0.15629 \\ -0.39053 \end{bmatrix} = \begin{bmatrix} 0.25629 \\ 0.60947 \end{bmatrix} = \textit{new values of the variables}$$

$$F_{k+1} = \begin{bmatrix} 0.07402 \\ -0.03058 \end{bmatrix}, \quad Y_k = F_{k+1} - F_k = \begin{bmatrix} 0.07402 \\ -0.03058 \end{bmatrix} - \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.54148 \\ -0.03058 \end{bmatrix}$$

$$H_{k+1} = H_k + \frac{(X_k - H_k Y_k) X_k^T H_k}{X_k^T H_k Y_k} = \begin{bmatrix} -0.24630 & -0.74956 \\ 0.76030 & -0.69190 \end{bmatrix}$$

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14.8 Quasi-Newton Method

ex> Newton-Raphson vs. Quasi-Newton

✓ Quasi-Newton

TABLE 14.10
Quasi-Newton simulation of the fan-duct system

Iteration	Variables, V	Functions, F	Inverse, H	
1	0.25629	0.074024	−0.24630	−0.74956
	0.60947	−0.030579	0.76030	−0.69190
2	0.25160	0.020603	−0.23200	−0.76370
	0.53203	−0.008211	1.04286	−0.97139
3	0.25011	0.001656	−0.22850	−0.76713
	0.50257	−0.000624	1.13130	−1.05802
4	0.25001	0.000042	−0.22744	−0.76816
	0.50004	−0.000016	1.16053	−1.08636

Newton-Raphson	Quasi-Newton
Fast convergence if the trial values are good	Wider convergence range than NR method