Chapter 4: Bayesian Filtering for State Estimation of the Environment

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Course: Autonomous Machine Learning

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Outline

- Introduction
- Bayesian Filter
- Conclusion

Outline

- **Introduction**
  - **What is Bayesian ?**
  - **Problem Statement**
- Bayesian Filter
- Conclusion

Introduction [1/3]

- What is Bayesian theory ?

"In probability theory and statistics, Bayes' theorem describes the probability of an event, based on conditions that might be related to the event".

- use Bayes' Theorem to find the conditional probability of an event $P(A \mid B)$, when the "reverse" conditional probability $P(B \mid A)$ is the probability that is known

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

- Where A and B are events and $P(B) \neq 0$
- $P(A \mid B)$, a conditional probability, is the probability of observing event *A* given that *B* is true.
- $P(A)$ and $P(B)$ are the probabilities of observing *A* and *B* without regard to each other
- $P(B \mid A)$ is the probability of observing event *B* given that *A* is true.

Introduction [2/3]

- What is Bayesian ?

Posterior — Likelihood — Prior — Evidence

$$P(A|B) = \frac{P(B|A)\ P(A)}{P(B)}$$

- Given:
  - **Likelihood**: A doctor knows that flu causes stiff neck 50% of the time
  - **Prior** : Probability of any patient having flu is 1/50000
  - **Evidence**: Probability of any patient having stiff neck is 1/20
- if a patient has stiff neck
- What is the probability he/she has flu ?

$$P(M|S) = \frac{P(S|M)\ *\ P(M)}{P(S)} = \frac{0.5\ *\ 1/50000}{1/20} = 0.0002$$

Introduction [3/3]

- Given a state-space model of the environment
  - A system equation
  - A measurement equation
- Practical issues:
  - The state of the environment is hidden from the observer
  - Evolution of the state across time and measurements on the environment are both corrupted by the unavoidable presence of physical *uncertainties* in the environment.
- Solutions:
  - Bayesian Framework
- Goal :
  - Develop algorithms for solving the state-estimation problem.

Outline

- Introduction
- **Bayesian Filter**
  - **State-Space Model**
  - **Sequential**
  - **Bayesian Filter**
  - **Extended Kalman Filter**
- Conclusion

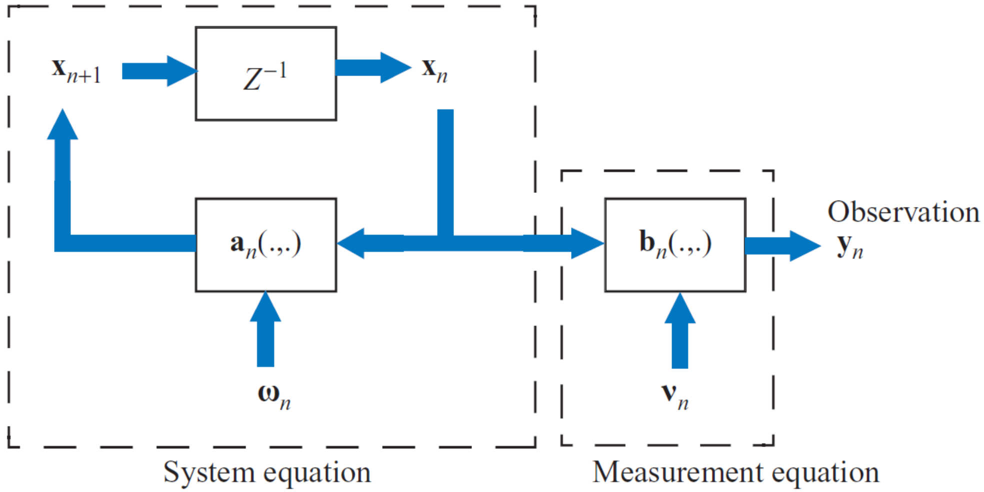
Bayesian Filter [1/8]

State-space Model

- the state of a dynamic system.
- System equation: $$\mathbf{x}_{n+1} = \mathbf{a}_n(\mathbf{x}_n, \boldsymbol{\omega}_n)$$
  - *n* denotes discrete time.
  - $\mathbf{x}_n$ the vector denotes the current value of the state.
  - $\mathbf{x}_{n+1}$ denotes its immediate future value.
  - vector ω_n denotes *system noise*
  - $\mathbf{a}_n$(.,.) is a vector function of its two arguments, representing transition from state $\mathbf{x}_n$ to state $\mathbf{x}_{n+1}$
- Measurement equation: $$\mathbf{y}_n = \mathbf{b}_n(\mathbf{x}_n, \boldsymbol{\nu}_n)$$
  - vector $\mathbf{y}_n$ denotes a set of measurements (observables)
  - vector v_n denotes *measurement noise.*
  - and $\mathbf{b}_n$(.,.) denotes another vector function.

Bayesian Filter [2/8]

State-space Model



Generic state-space model of a time-varying, nonlinear dynamic system, where Z^{-1} denotes a block of time-unit delays.

- **Assumptions:**
  - The initial state $\mathbf{x}_0$ is uncorrelated with the system noise ω_n for all *n*
  - The two sources of noise, $\mathbf{v}_n$ and v_n, are statistically independent of each other

$$\mathbb{E}[\boldsymbol{\omega}_n \mathbf{v}_k^T] = \mathbf{0} \qquad \text{for all } n \text{ and } k.$$

This equation is a sufficient condition for independence when $\mathbf{v}_n$ and ω_n are *jointly Gaussian.*

Bayesian Filter [3/8]

- Sequential State Estimation problem
- The State-estimation problem
  - Prediction : k > n
  - Filtering :   k=n
  - Smoothing : k <n

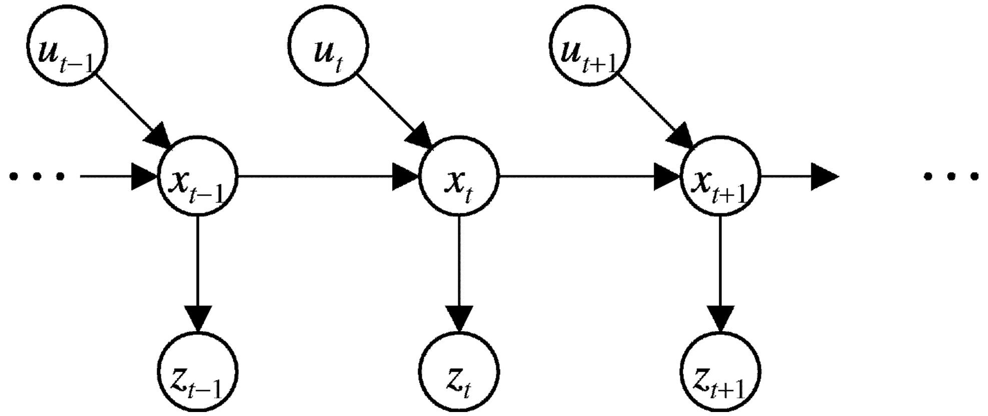
Bayesian Filters [4/8]

- Framework
  - **Given:**
    - Stream of observations z and action data u:
    - Sensor model P(z|x).
    - Action model P(x|u,x').
    - Prior probability of the system state P(x).
  - **Wanted:**
    - Estimate of the state *X* of a dynamical system.
    - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Bayesian Filters [5/8]

Markov Assumption



$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

Bayesian Filters [6/8]

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Bayes

$$= \eta \; P(z_t \mid x_t, u_1, z_2, \ldots, u_{t-1}) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$$

Markov

$$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$$

Total prob.

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_2, \ldots, u_{t-1}, x_{t-1}) \; P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \; dx_{t-1}$$

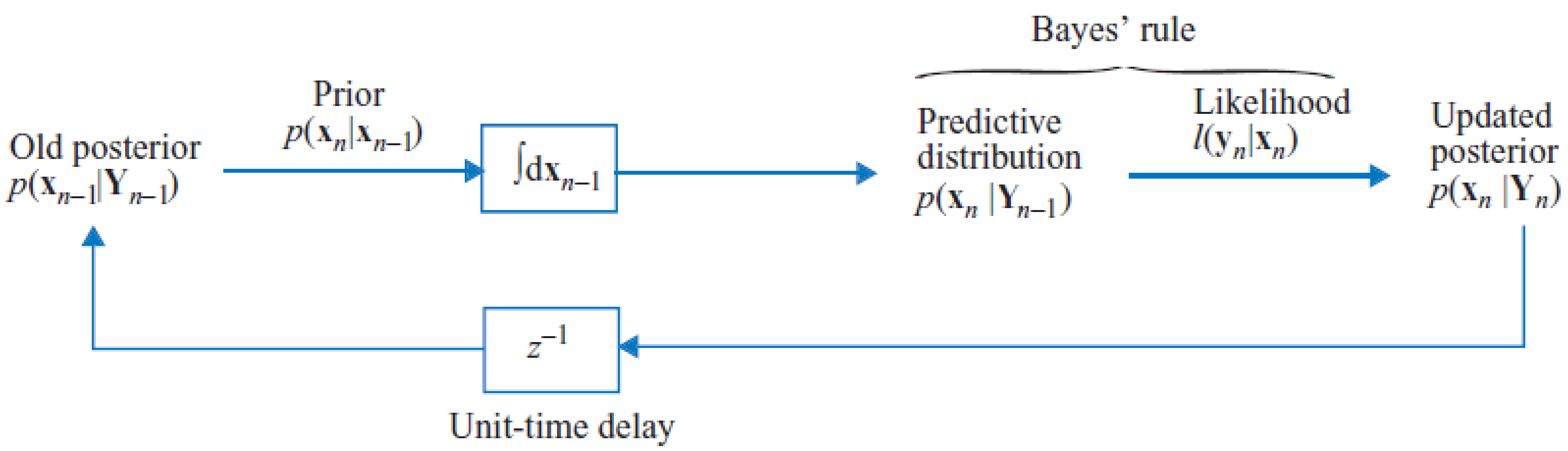
Markov

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_{t-1}, x_{t-1}) \; P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \; dx_{t-1}$$

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_{t-1}, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Bayesian Filter [7/8]

- The Bayesian Filter
  - Optimal of Bayesian Filter



Bayesian Filter [8/8]

- Approximation of the Bayesian Filter
  - *Direct numerical approximation of the posterior*
    - *Kalman Filter Theory*
  - *Indirect numerical approximation of the posterior*
    - *Monte Carlo*
  - *Particle filters*
    - *Monte Carlo*

Outline

- Introduction
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Conclusion [1/1]

- Overview of Bayesian Theorem
- State-Space Model
- Bayesian Filter for state estimation

THANK YOU

Q&A

Appendix

- Time update

$$\underbrace{p(\mathbf{x}_n \mid \mathbf{Y}_{n-1})}_{\substack{\text{predictive}\\ \text{distribution}}} = \int \underbrace{p(\mathbf{x}_n \mid \mathbf{x}_{n-1})}_{\text{prior}} \underbrace{p(\mathbf{x}_{n-1} \mid \mathbf{Y}_{n-1})}_{\text{old posterior}} \, d\mathbf{x}_{n-1}$$

- Measurement Update

$$\underbrace{p(\mathbf{x}_n \mid \mathbf{Y}_n)}_{\substack{\text{updated}\\ \text{posterior}}} = \frac{1}{Z_n} \underbrace{p(\mathbf{x}_n \mid \mathbf{Y}_{n-1})}_{\substack{\text{predictive}\\ \text{distribution}}} \underbrace{l(\mathbf{y}_n \mid \mathbf{x}_n)}_{\text{likelihood}}.$$