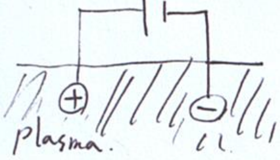
Definition of Plasma

A *plasma* is a "quasineutral" gas of charged and neutral particles which exhibits "collective behaviour".

What is the plasma?

$\left|\frac{e\phi}{kT_e}\right| \ll 1$

* Gedank Experiment



$$\varepsilon_0 \nabla \cdot \vec{E} = \rho$$

$$* \; n_e = n \exp\left(\frac{e\phi}{kT_e}\right) \approx n\left(1 + \frac{e\phi}{kT_e} + \cdots\right)$$

$$\nabla^2 \phi = \frac{-\rho}{\varepsilon_0}$$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{n e^2}}$$

$$\frac{d^2\phi}{dx^2} = \frac{e}{\varepsilon_0}(n_e - n_i) = \frac{e}{\varepsilon_0}\left[n\left(1 + \frac{e\phi}{kT_e}\right) - n\right] = \frac{ne^2\phi}{\varepsilon_0 k T_e} = \frac{1}{\lambda_D^2}\phi$$

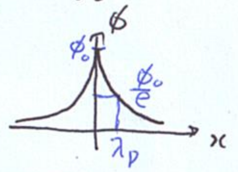
$$\frac{\phi}{L^2} = \frac{e}{\varepsilon_0}(n_e - n_i) \rightarrow \left(e\phi \ll kT_e\right) \rightarrow \frac{e^2 L^2}{\varepsilon_0}(n_e - n_i) \ll kT_e$$

$$\rightarrow \frac{e^2 L^2}{\varepsilon_0}(n_e - n_i) = \frac{e^2 n}{\varepsilon_0 k T_e} \cdot \frac{(n_e - n_i)}{n} kT_e L^2 = \frac{L^2}{\lambda_D^2} kT_e \frac{(n_e - n_i)}{n} \ll kT_e$$

$$\rightarrow \frac{n_e - n_i}{n} \ll \frac{\lambda_D^2}{L^2} \quad \text{if } \lambda_D \ll L \Rightarrow \left(\frac{\lambda_D^2}{L^2} \rightarrow 0\right) \Rightarrow \boxed{n_e \approx n_i}$$

quasi-neutrality

$$\boxed{\frac{d^2\phi}{dx^2} = \frac{1}{\lambda_D^2}\phi \quad \begin{pmatrix} x=0, \ \phi = \phi_0 \\ x \to \infty, \ \phi = 0 \end{pmatrix} \Downarrow \quad \phi = \phi_0 e^{-\frac{|x|}{\lambda_D}}}$$



collective behaviour

$$\boxed{N_D = \frac{4}{3}\pi \lambda_D^3 n \gg 1}$$

charged particle number; plasma density

$\omega\tau \gg 1$

ω : plasma frequency

τ : collision time.

Single Particle Motion (Assumption : ① uniform $\vec{E}, \vec{B}$, ② $\vec{F} = m\vec{a} = q(\vec{E} + \vec{v}\times\vec{B})$)

1) $\vec{E} = E_x \hat{x} + E_z \hat{z}$

$$\begin{aligned} m\dot{v}_x &= qE_x \\ m\dot{v}_y &= 0 \\ m\dot{v}_z &= qE_z \end{aligned} \Rightarrow \begin{aligned} v_x &= \frac{q}{m}E_x t + v_{x_0} \\ v_y &= v_{y_0} \\ v_z &= \frac{q}{m}E_z t + v_{z_0} \end{aligned} \Rightarrow \begin{aligned} x &= \frac{1}{2}\frac{q}{m}E_x t^2 + v_{x_0}t + x_0 \\ y &= v_{y_0} t + y_0 \\ z &= \frac{1}{2}\frac{q}{m}E_z t^2 + v_{z_0}t + z_0 \end{aligned}$$

2) $\vec{B} = B\hat{z}$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} = v_y B\hat{x} - v_x B\hat{y}$$

$$\begin{aligned} m\dot{v}_x &= qBv_y \\ m\dot{v}_y &= -qBv_x \\ m\dot{v}_z &= 0 \end{aligned} \Rightarrow \begin{aligned} m\ddot{v}_x &= qB\dot{v}_y = -\frac{(qB)^2}{m}v_x \\ m\ddot{v}_y &= -qB\dot{v}_x = -\frac{(qB)^2}{m}v_y \\ m\ddot{v}_z &= 0 \end{aligned} \Rightarrow \begin{aligned} \ddot{v}_x &= -\omega_c^2 v_x \\ \ddot{v}_y &= -\omega_c^2 v_y \\ \ddot{v}_z &= 0 \end{aligned} \quad \left(\omega_c = \frac{|q|B}{m}\right)$$

$$v_x = A\sin(\omega_c t) + B\cos(\omega_c t) = v_\perp \cos(\omega_c t + \delta)$$

$$m\dot{v}_x = qBv_y \rightarrow v_y = \frac{m}{qB}\left[-v_\perp \omega_c \sin(\omega_c t + \delta)\right] = \mp v_\perp \sin(\omega_c t + \delta) \quad \begin{pmatrix} - : \text{ion} \\ + : \text{electron} \end{pmatrix}$$

$$\begin{pmatrix} v_x^2 + v_y^2 = v_\perp^2 \\ v_{x_0} = v_\perp \cos\delta \\ v_{y_0} = \mp v_\perp \sin\delta \\ \tan\delta = \mp \frac{v_{y_0}}{v_{x_0}} \end{pmatrix}$$

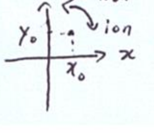
$$x = \frac{v_\perp}{\omega_c}\sin(\omega_c t + \delta) + x_0$$

$$y = \pm \frac{v_\perp}{\omega_c}\cos(\omega_c t + \delta) + y_0$$

$$z = v_{z_0} t + z_0$$

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2 = r_L^2 \quad (\text{Larmor radius})$$

$$r_L = \frac{m v_\perp}{|q|B}$$

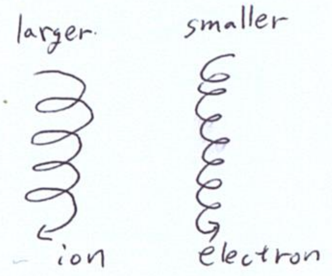


$\Rightarrow$ diamagnetic

Single Particle Motion

3) $\vec{E} = E_x\hat{x} + E_z\hat{z}$, $\vec{B} = B_z\hat{z}$

$$\vec{F} = m\vec{a} = q(\vec{E} + \vec{v}\times\vec{B}) = q\{(E_x + v_y B_z)\hat{x} - v_x B_z\hat{y} + E_z\hat{z}\}$$



$$\dot{v}_x = \frac{q}{m}(E_x + v_y B_z) \qquad v_x = v_\perp\cos(w_c t + \delta)$$

$$\dot{v}_y = -\frac{q}{m}B_z v_x \quad \Rightarrow \quad v_y = \mp v_\perp \sin(w_c t + \delta) - \frac{E_x}{B_z}$$

$$\dot{v}_z = \frac{q}{m}E_z \qquad v_z = \frac{q}{m}E_z t + v_{z0}$$

$$x = \frac{v_\perp}{w_c}\sin(w_c t + \delta) + x_0$$

$$\Rightarrow y = \pm\frac{v_\perp}{w_c}\cos(w_c t + \delta) - \frac{E_x}{B_z}t + y_0 \rightarrow (x - x_0)^2 + \left(y - y_0 + \frac{E_x}{B_z}t\right)^2 = r_L^2$$

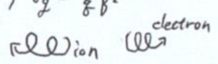
$$z = \frac{1}{2}\frac{q}{m}E_z t^2 + v_{z0}t + z_0$$

if) $\int_0^\tau m\frac{d\vec{v}}{dt} = 0 = q(\vec{E} + \vec{v}_\perp\times\vec{B}) \rightarrow q(\vec{E} + \vec{v}_\perp\times\vec{B})\times\vec{B}$ ~~steady state~~

$$\Rightarrow q(\vec{E}\times\vec{B}) + q\{\vec{B}(\vec{B}\cdot\vec{v}_\perp) - \vec{v}_\perp(\vec{B}\cdot\vec{B})\} = q(\vec{E}\times\vec{B}) + q(-B^2\vec{v}_\perp) = 0$$

$$\therefore v_\perp = \frac{\vec{E}\times\vec{B}}{B^2} = \vec{v}_E \text{ (guiding center)}$$

$\left(\vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{q\vec{E}\times\vec{B}}{qB^2} = \frac{\vec{F}_E\times\vec{B}}{qB^2}\right)$ 유추 ← general force. $\boxed{\vec{v}_f = \frac{\vec{F}\times\vec{B}}{qB^2}}$ $\rightarrow \vec{v}_g = \frac{m\vec{g}\times\vec{B}}{qB^2}$

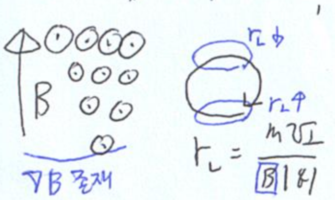


Non-uniform $\vec{B}$

$\boxed{* \; r_L << L_B}$ → r_L가 일정하다 → Talyor 전개 때메

$\vec{B} = B(y)\hat{z}$, $\vec{F} = q(\vec{v}\times\vec{B})$

assumption $\delta = 0$; $B(y) = B_0 + y\frac{\partial B}{\partial y}\Big|_{y=0} + \cdots$ assumption $\frac{\partial B}{\partial y}\Big|_{y=0} = \frac{\partial B}{\partial y}$



$r_L = \frac{mv_\perp}{|B||q|}$

$F_x = 0$ (∇B가 y 방향만 있어서)

$$F_y = -q\underbrace{v_x B}_{\text{외적계산 결과}} = -q(v_\perp\cos w_c t)B = -q(v_\perp\cos w_c t)\left[B_0 + y\frac{\partial B}{\partial y}\right]$$

$$= -qv_\perp\cos w_c t\left[B_0 \pm r_L\cos w_c t\frac{\partial B}{\partial y}\right] = -qv_\perp B_0\cos w_c t \mp qv_\perp r_L\cos^2 w_c t\frac{\partial B}{\partial y}$$

$$\bar{F}_y = \frac{1}{\tau}\int_0^\tau F_y\,dt = \cancel{\frac{1}{\tau}\int_0^\tau -qv_\perp B_0 \underbrace{\cos w_c t}_{\text{주기함수}}\,dt} \mp \frac{qv_\perp r_L}{\tau}\int_0^\tau\cos^2 w_c t\,dt\frac{\partial B}{\partial y}$$

$$\boxed{\bar{F}_x = \frac{1}{\tau}\int_0^\tau \mp qv_\perp B\sin w_c t\,dt = 0}$$

(τ : 주기, 한바퀴 돌때)

$$= \mp\frac{qv_\perp r_L}{\tau}\frac{\tau}{2}\frac{\partial B}{\partial y} \qquad \left(\because \cos^2 w_c t = \frac{1}{2}(1 + \underbrace{\cancel{\cos 2w_c t}}_{\text{주기함수}})\right)$$

$$= \mp\frac{qv_\perp r_L}{2}\frac{\partial B}{\partial y}$$

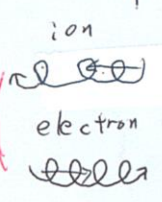
drift velocity

$\vec{F} = q(\vec{v}\times\vec{B}) = q(\vec{v}\times\vec{B}_0) + q(\vec{v}\times\vec{B}_1)$

$$\vec{v}_{\nabla B} = \frac{\vec{F}\times\vec{B}}{qB^2} = \frac{\mp qv_\perp r_L\frac{1}{2}\frac{\partial B}{\partial y}\hat{y}\times\vec{B}}{qB^2} = \frac{\mp v_\perp r_L\frac{1}{2}\nabla B\times\vec{B}}{B^2} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\nabla B}{B^2}$$

수학적 결과

물리적 결과

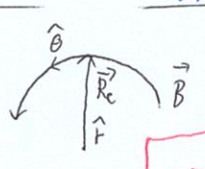


r_L의 변화로 인한 drift

Curvature

$\vec{B} = B_\theta(r)\hat{\theta}$

$$\vec{F} = \underbrace{\frac{mv_\parallel^2}{R_c}\hat{r}}_{\text{원심력}} = \frac{mv_\parallel^2}{R_c}\frac{\vec{R}_c}{R_c} = \frac{mv_\parallel^2}{R_c^2}\vec{R}_c$$

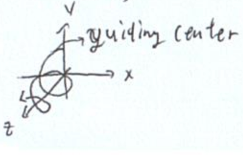


$\vec{v}_\parallel$, $\vec{B}$ → $\hat{\theta}$방향

$$\vec{v}_c = \frac{\vec{F}\times\vec{B}}{qB^2} = \frac{1}{qB^2}\frac{mv_\parallel^2}{R_c^2}\vec{R}_c\times\vec{B}$$

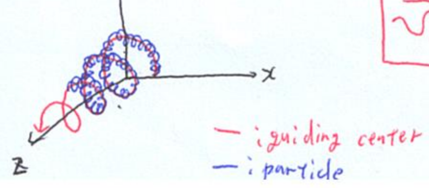
$$\boxed{\vec{v}_c = \frac{\vec{F}\times\vec{B}}{qB^2}}$$

$\vec{F}$ = 로렌츠힘 or 원심력 등등



HW

$\vec{B}$ 일때 ∇B존재하는 이유



* Drift Motions (guiding center)

- $\vec{E}$: $\vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2}$

A+B : $\vec{v}_{c+\nabla B} = \frac{m\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)}{qB^2R_c^2}\left(\vec{R}_c\times\vec{B}\right)$

- $\vec{F}_\perp$: $\vec{v}_F = \frac{\vec{F}\times\vec{B}}{qB^2}$
- $\vec{g}$: $\vec{v}_g = \frac{m\vec{g}\times\vec{B}}{qB^2}$

$$\frac{\nabla B}{B} = -\frac{\vec{R}_c}{R_c^2}$$

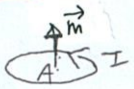
A

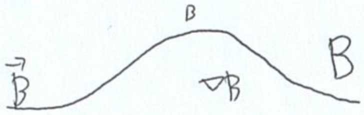
- ∇B : $\vec{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\nabla B}{B^2} = \pm\frac{1}{2}v_\perp\frac{mv_\perp}{B|q|}\frac{\vec{B}\times\nabla B}{B^2} \underset{\pm|q|=q}{=} -\frac{\frac{1}{2}mv_\perp^2}{qB^2}\frac{\vec{B}\times\vec{R}_c}{R_c^2} = \boxed{\frac{\frac{1}{2}mv_\perp^2}{qB^2}\frac{\vec{R}_c\times\vec{B}}{R_c^2}}$
- curvature : $\vec{v}_c = \boxed{\frac{mv_\parallel^2\vec{R}_c\times\vec{B}}{qB^2R_c^2}}$ B

magnetic moment

$$\vec{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\nabla B}{B^2} = \frac{\vec{F}\times\vec{B}}{qB^2} \rightarrow -\frac{1}{2}v_\perp\frac{mv_\perp}{Bq}\frac{\nabla B\times\vec{B}}{B^2} = \frac{\vec{F}\times\vec{B}}{qB^2} \rightarrow \vec{F}_\perp = -\frac{\frac{1}{2}mv_\perp^2}{B}\nabla B = -\mu\nabla_\perp B$$

$$\boxed{|\vec{m}|} = IA = \frac{|q|}{\tau}\pi r_L^2 = \frac{|q|}{2\pi}\omega_c\pi r_L^2 = \frac{|q|}{2}\frac{B|q|}{m}\frac{m^2v_\perp^2}{B^2|q|^2} = \boxed{\frac{\frac{1}{2}mv_\perp^2}{B}}$$ 증명완료





$\vec{F} = -\mu\nabla_\parallel B$

$\vec{F}_\parallel = m\frac{d\vec{v}_\parallel}{dt} = -\mu\nabla_\parallel B$

$$\rightarrow v_\parallel m\frac{dv_\parallel}{dt} = \boxed{\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right)} = -\mu\overbrace{\frac{\partial B}{\partial s}}^{\nabla_\parallel B}\overbrace{\frac{\partial s}{\partial t}}^{v_\parallel} \simeq \boxed{-\mu\frac{dB}{dt}}$$

Energy conservation

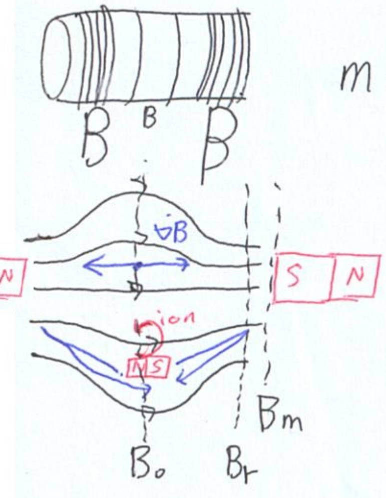
$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) + \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2\right) = 0$$

$$\rightarrow -\mu\frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

$$-\mu\frac{dB}{dt} + \mu\frac{dB}{dt} + B\frac{d\mu}{dt} = 0$$

$$\boxed{\therefore \frac{d\mu}{dt} = 0}$$ μ : adiabatic invariant



(E. Fermi) guiding center

magnetic mirror (Hannes Alfvén)

($\frac{1}{2}mv_\parallel^2$: kinetic energy

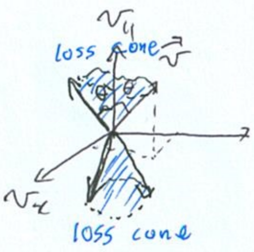
$\frac{1}{2}mv_\perp^2$: potential energy)

trapping condition : $B_r \le B_m$

① $\frac{1}{2}mv_{\parallel 0}^2 + \frac{1}{2}mv_{\perp 0}^2 = \cancel{\frac{1}{2}mv_{\parallel r}^2}^{\,0} + \frac{1}{2}mv_{\perp r}^2$

② μ : $\frac{\frac{1}{2}mv_{\perp 0}^2}{B_0} = \frac{\frac{1}{2}mv_{\perp r}^2}{B_r}$

$$\Rightarrow \frac{v_{\perp 0}^2}{v_{\perp r}^2} = \frac{B_0}{B_r} = \frac{v_{\perp 0}^2}{v_{\parallel 0}^2 + v_{\perp 0}^2} = \sin^2\theta$$



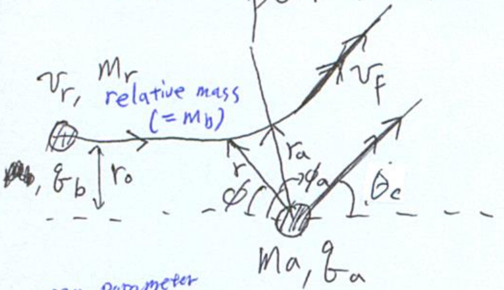
$v\sin\theta = v_\perp$

$v\cos\theta = v_\parallel$

$$\therefore \sin^2\theta = \frac{B_0}{B_r} \ge \frac{B_0}{B_m} = \frac{1}{R_m}$$

(R_m : mirror ratio)

Two particle motion (Rutherford scattering)



Energy conservation

$$E=\underbrace{\frac{1}{2}m_r v_r^2}_{A}=\frac{1}{2}m_r v_f^2=\underbrace{\frac{1}{2}m_r\left(\dot r^2+r^2\dot\phi^2\right)+\frac{q_a q_b}{4\pi\varepsilon_0 r}}_{B}$$

$$\therefore v_r = v_f$$

new parameter

$$2K \equiv \frac{q_a q_b/4\pi\varepsilon_0}{\frac{1}{2}m_r v_f^2}=\text{const}$$

Angular momentum conservation $\Rightarrow \underbrace{\dot\phi=\frac{v_r r_0}{r^2}}_{C}$ (1)

$$m_r r^2\dot\phi = m_r v_r r_0 = m_r v_r r_0' \qquad \therefore r_0'=r_0$$

A, B, C에 의해 $\dot r = \mp v_r\left(1-\frac{2K}{r}-\frac{r_0^2}{r^2}\right)^{\frac{1}{2}}$

↳ −는 가까워지는것, +는 멀어지는 것

$$\frac{d\phi}{dr}=\frac{\dot\phi}{\dot r}=\mp\frac{r_0}{r^2}\left(1-\frac{2K}{r}-\frac{r_0^2}{r^2}\right)^{-\frac{1}{2}}$$

$$\boxed{\text{HW}\quad \phi=\cos^{-1}\left[\frac{r_0}{\sqrt{r_0^2+K^2}}\left(1-\frac{2K}{r}-\frac{r_0^2}{r^2}\right)^{\frac{1}{2}}\right]-\cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+K^2}}\right)}$$

$r=r_a$, $\dot r_a=0$, $1-\frac{K}{r_a}-\frac{r_0^2}{r_a^2}=0$, $\phi_a=\cos^{-1}0-\cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+K^2}}\right)$

$\pi-2\phi_a=\theta_c$ $\quad\therefore \theta_c = 2\cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+K^2}}\right) \Rightarrow \begin{cases}\frac{r_0}{\sqrt{r_0^2+K^2}}=\cos\frac{\theta_c}{2}\\ \frac{K}{\sqrt{r_0^2+K^2}}=\sin\frac{\theta_c}{2}\end{cases}$

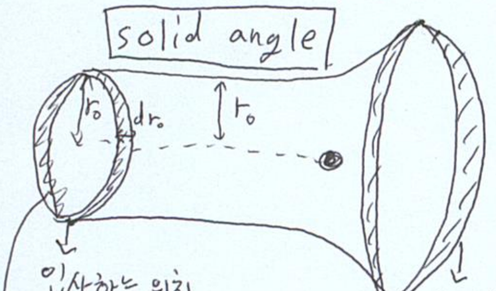
$$K=r_0\tan\frac{\theta_c}{2} \Leftarrow \frac{K}{r_0}=\tan\frac{\theta_c}{2}$$

$$2K\equiv\frac{q_a q_b/4\pi\varepsilon_0}{\frac{1}{2}m_r v_r^2}$$

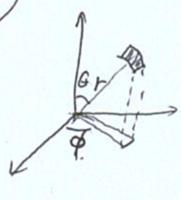
$\Downarrow$

$\theta_c=\frac{\pi}{2}\rightarrow K=r_0$ $\quad\therefore$ K는 scattering 각도가 $\frac{\pi}{2}$가 될때의 r_0 값

solid angle



$$ds = r^2\underbrace{d\Omega}_{\text{solid angle}}$$



$$d\Omega=\frac{dA}{r^2}=\frac{r^2\sin\theta\, d\theta\, d\phi}{r^2}=\sin\theta\, d\theta\, d\phi$$

$dA\equiv d\sigma_s = 2\pi r_0 dr_0$

$d\Omega = 2\pi\sin\theta_c\, d\theta_c$

$(d\Omega=\sin\theta\, d\theta\, \underbrace{d\phi}_{\text{적분}\rightarrow 2\pi})$

The differential scattering cross section for scattering into $(\theta_c, \theta_c+d\theta_c)\equiv d\sigma_s$

$$d\sigma_s=\boxed{\frac{d\sigma_s}{d\Omega}}d\Omega=\boxed{\sigma_s'(\theta_c)}d\Omega$$

$$=2\pi\sigma_s'(\theta_c)\sin\theta_c\, d\theta_c=2\pi r_0 dr_0$$

↳ Calomb cross section

(Rutherford scattering cross section)

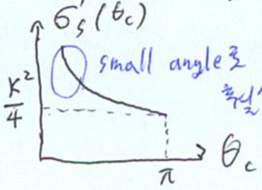
$\sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$\sigma_s'(\theta_c)=\frac{r_0 dr_0}{\sin\theta_c d\theta_c}=\frac{r_0}{\sin\theta_c}\left(\frac{r_0^2\sec^2\frac{\theta_c}{2}}{2K}\right)=\frac{K^2}{4\sin^4\left(\frac{\theta_c}{2}\right)}=\frac{1}{4}\left(\frac{q_a q_b}{4\pi\varepsilon_0 m_r v_r^2}\right)^2\frac{1}{\sin^4\frac{\theta_c}{2}}$$

$\frac{K}{r_0}=\tan\frac{\theta_c}{2}\rightarrow -\frac{K}{r_0^2}dr_0=\frac{1}{2}\sec^2\left(\frac{\theta_c}{2}\right)d\theta_c$

$\therefore\left|\frac{dr_0}{d\theta_c}\right|=\frac{r_0^2}{2K}\sec^2\left(\frac{\theta_c}{2}\right)$

HW : $\sigma_s'(\theta_c)$의 physics meaning



교재 : J.A. Bittencourt Fundamentals of Plasma Physics

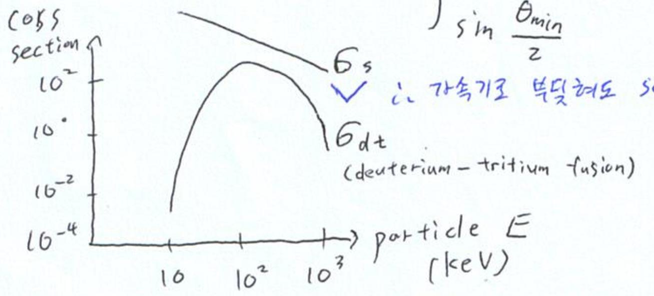
Total scattering cross section

$$\sigma_s = \int \sigma_s'(\theta_c)\, d\Omega = 2\pi \int_{\theta_{min}}^{\pi} \left[\frac{K^2}{4\sin^4(\frac{\theta_c}{2})} \right] \sin\theta_c\, d\theta_c$$

$$= \pi K^2 \int_{\theta_{min}}^{\pi} \frac{\cos(\theta_c/2)}{\sin^3(\theta_c/2)}\, d\theta_c = 2\pi K^2 \int_{\sin\frac{\theta_{min}}{2}}^{\sin\frac{\pi}{2}} \frac{d(\sin\theta)}{\sin^3\theta} = \pi K^2 \left[\sin^{-2}\frac{\theta_{min}}{2} - 1 \right]$$

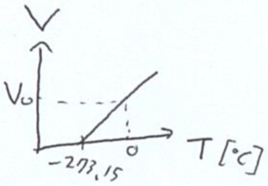
$r_0^{max} = \lambda_D$

$\theta_{min} = 2\tan^{-1}\left(\frac{K}{\lambda_D}\right)$



Kinetic Theory of gases

- Boyle's law $PV = const$ at const T
- Charles's law $\frac{V}{T} = \frac{V_0}{273.15°C}$ at const P

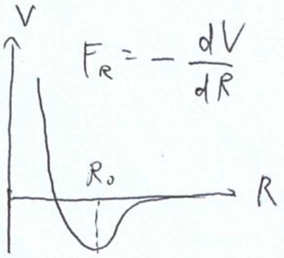


- Equation of state for a gas $PV = qRT$ (q : mole, R : 8.31 J/K·mole)
- Joule's law : The energy content of a gas is independent of its volume.
- Dolton's law of Partial Pressures : $P = P_1 + P_2$ at const V
- Avogadro's hypothesis : at the same P, V, T ⇒ same # of molecules.

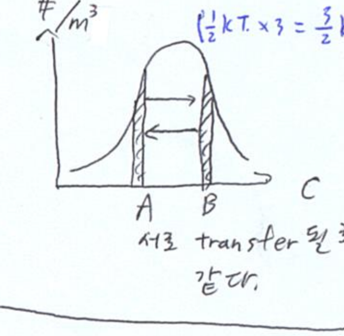
Kinetic Theory

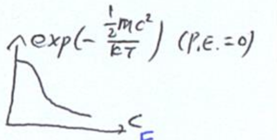
* Assumptions

1. Nature of the molecules
   - identical molecules (mass)
   - spherical molecules
   - negligible intermolecular forces



2. uniform distribution of the molecules in space. (continuous density)
3. Continuous motion of the molecules.
4. Isotropy of velocities (mechanical equilibrium)
5. Maxwell-Boltzmann distribution of velocities in thermal equilibrium.
6. Equipartition of energy degree of freedom. ($\frac{1}{2}kT \times 3 = \frac{3}{2}kT$)
7. Detailed balancing

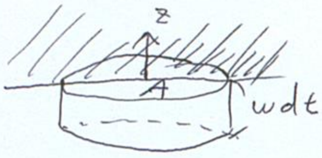




$\exp(-\frac{E}{kT}) = \exp(-\beta E)$ Boltzmann factor

$\left(dn = A\exp\left(-\frac{\boxed{\frac{1}{2}mc^2 + P.E}}{kT}\right) du\, dv\, dw \right)$ E

$c^2 = u^2 + v^2 + w^2$



$\vec{c} = (u, v, w)$

\# of collisions on A

$= n_0 A w\, dt$

rate of collisions on A

$$= \frac{n_0 A w\, dt}{dt} = n_0 A w \ [\#/s]$$

$$P = (\dot{P}_+ + \dot{P}_-)/A = \frac{1}{3} n m \overline{c^2}$$

$$PV = \frac{1}{3} n V m \overline{c^2} = \frac{1}{3} N m \overline{c^2}$$

$$\boxed{\frac{1}{2} m \overline{c^2}} = \bar{E}_k = \frac{3}{2N} \frac{1}{3} N m \overline{c^2} = \frac{3}{2N} \varrho R T$$

$$= \frac{3}{2N} \frac{N}{N_A} R T = \frac{3}{2} \frac{R}{N_A} T = \boxed{\frac{3}{2} k T}$$

$$\left(\begin{array}{l} R = 8.31\ J/mole, \ N_A = 6.02 \times 10^{23}\ /mole \\ k = 1.38 \times 10^{-23}\ J/K \end{array} \right)$$

* molecule partial current

$$J_+ = (n_1 w_1 + n_2 w_2 + \cdots) A = n_+ \bar{w}_+ A$$

$$\circledast\ \bar{w}_+ = \frac{n_1 w_1 + n_2 w_2 + \cdots}{n_1 + n_2 + \cdots} = \frac{n_1 w_1 + n_2 w_2 + \cdots}{n_+}$$

$$\circledast\ n_+ = n_- = \frac{1}{2} n, \quad \bar{w}_+ = \frac{1}{2} \bar{c}$$

$$\therefore J_+ = \frac{1}{4} n \bar{c} A \ [\#/s]$$

* momentum partial current $[(kg \cdot m/s)/s]$

$$n_w A w m w = n_w A m w^2$$

$$\dot{P}_+ = (n_1 w_1^2 + n_2 w_2^2 + \cdots) A m = n_+ \overline{w_+^2} A m = \frac{1}{6} n m \overline{c^2} A$$

$$\overline{w_+^2} = \frac{n_1 w_1^2 + n_2 w_2^2 + \cdots}{n_1 + n_2 + \cdots} = \frac{n_1 w_1^2 + n_2 w_2^2 + \cdots}{n_+}$$

$$c^2 = u^2 + v^2 + w^2, \quad \overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3} \overline{c^2}$$

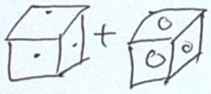
$$\overline{w^2} = \overline{w_+^2} = \overline{w_-^2}$$

$$\therefore \dot{P}_+ = \frac{1}{6} n m \overline{c^2} A \ [kg \cdot m/s^2]$$

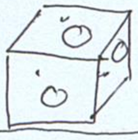
$$PV = \frac{1}{3} N m \overline{c^2} = \frac{1}{3} N 3 k T = N k T$$

$$\therefore \frac{PV}{T} = N k$$

$$PV = NkT \rightarrow P = \frac{N}{V} kT = n k T$$



$$= P_1 + P_2 = n_1 k T + n_2 k T = (n_1 + n_2) k T$$



$$= P = \frac{N_1 + N_2}{V} k T = (n_1 + n_2) k T = P_1 + P_2$$

* number density

$$P = n k T \qquad \text{space } 10^4\ \#/m^3$$

$$n = \frac{P}{kT} \qquad \text{KSTAR } 10^{20}\ \#/m^3$$

$$n\ [\#/m^3] = 2.45 \times 10^{25} P\ [atm]$$

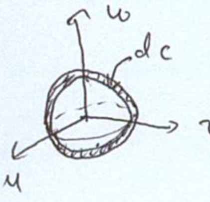
at 300K

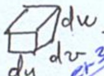
* molecular speed

$$\frac{1}{2} m \overline{c^2} = \frac{3}{2} k T$$

H_2 : $1.93 \times 10^3\ m/s$ (at 300K)

Maxwell's speed distribution





$$\boxed{dn} = A \exp\left(-\frac{\frac{1}{2} m c^2 + P.E.}{kT}\right) du\, dv\, dw$$

큐빅 미소

다름

구각 미소

이 안에 n이 포함

$$\boxed{dn} = \boxed{A} \exp\left(-\frac{\frac{1}{2} m c^2 + P.E.}{kT}\right) 4\pi c^2 dc$$

충돌 있다.
P.E. 일정
전체 E 일정

$$= n B c^2 \exp\left(-\frac{\frac{1}{2} m c^2}{kT}\right) dc$$

밀도 (공간 적분안함)

$$= n f(c)\, dc$$

$$\int_0^\infty f(c)\, dc = 1$$

$$\Downarrow$$

$$B = \sqrt{\frac{2}{\pi}} \left(\frac{m}{kT}\right)^{\frac{3}{2}}$$

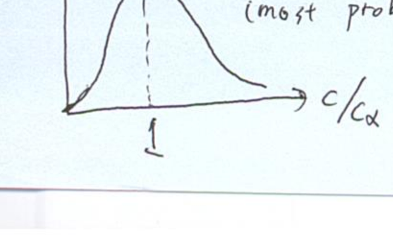
f(c) 미분해서 0

$$C_\alpha = \sqrt{\frac{2kT}{m}}$$

(most probable speed)

$$\bar{c} = \int_0^\infty c f(c)\, dc = \sqrt{\frac{8kT}{\pi m}}$$

$$\sqrt{\overline{c^2}} = \left\{\int_0^\infty c^2 f(c)\, dc\right\}^{\frac{1}{2}} = \sqrt{\frac{3kT}{m}} \Rightarrow \frac{1}{2} m \overline{c^2} = \frac{3}{2} k T$$



$$\left(* \int_0^\infty c^{2n} e^{-ac^2} dc = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}, \quad \int_0^\infty c^{2n+1} e^{-ac^2} dc = \frac{n!}{2 a^{n+1}} \ (a>0) \right)$$

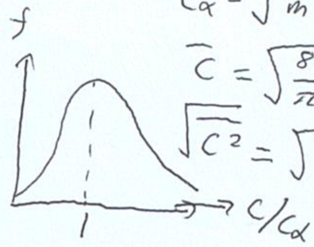
$$P = \frac{1}{3} n m \overline{c^2}$$

$$PV = \frac{1}{3} N m \overline{c^2} = \mathcal{g} RT$$

$$\frac{1}{2} m \overline{c^2} = \frac{3}{2} kT$$

$$\boxed{dn = A \exp\left(-\frac{\frac{1}{2}mc^2 + P.E.}{kT}\right) 4\pi c^2 dc} \quad ①$$

$$= n \beta c^2 \exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right) dc = n f(c) dc$$

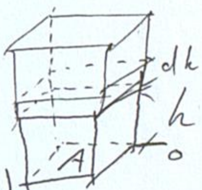


$$c_\alpha = \sqrt{\frac{2kT}{m}}$$

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}}$$

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}}$$

※ Effect of potential energy - Isothermal atmosphere

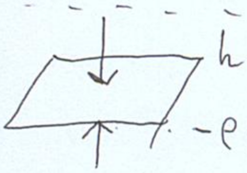


$$① \quad 4\pi A \exp\left(-\frac{P.E.}{kT}\right) c^2 \exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right) dc$$

$$= n \quad B \quad c^2 \exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right) dc$$

$$\Rightarrow n = \frac{4\pi A}{B} \exp\left(-\frac{P.E.}{kT}\right)$$

$$= \frac{4\pi A}{B} \exp\left(-\frac{mgh}{kT}\right) = n_0 \exp\left(-\frac{mgh}{kT}\right)$$



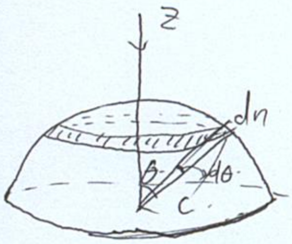
$$-\rho g\, dh A = A dP$$

$$-n m g\, dh A = A kT dn$$

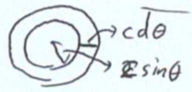
$$\frac{dn}{n} = -\frac{mg}{kT} dh$$

$$\int_{n_0}^{n} \frac{dn}{n} = -\int_0^h \frac{mg}{kT} dh \Rightarrow \ln\frac{n}{n_0} = -\frac{mgh}{kT}$$

$$\therefore n = n_0 \exp\left(-\frac{mgh}{kT}\right)$$



$$dn = n f(c) dc \frac{d\Omega}{4\pi}$$



$$dS = 2\pi c \sin\theta \, c\, d\theta = c^2 d\Omega$$

dS: 띠의 넓이

$$\therefore d\Omega = 2\pi \sin\theta \, d\theta$$

$$dJ_+ = dn\, c \cos\theta A$$

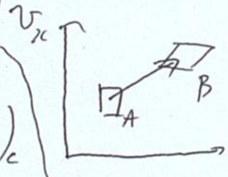
$$= n f(c) dc \frac{2\pi}{4\pi} \sin\theta \, d\theta\, c \cos\theta A$$

$$= \frac{1}{2} \sin\theta \cos\theta \, d\theta A\, c n f(c) dc$$

$$\therefore J_+ = \int_0^{\frac{\pi}{2}} \frac{A}{2} \sin\theta \cos\theta \int_0^\infty c n f(c) dc \, d\theta = \frac{A}{2} n \bar{c} \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta = \frac{A}{4} n \bar{c} \left[-\frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} n \bar{c} A$$

※ Boltzmann equation



phase space $f(x, v_x, t)$

$$0 = \frac{dN}{dt} = \frac{d}{dt} \int f(x, v_x, t)\, dx\, dv_x = \int \frac{df}{dt} dx\, dv_x$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_x}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{\alpha} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$= \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{\mathcal{g}(\vec{E} + \vec{v} \times \vec{B})}{m} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

if there is collision

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{\alpha} \cdot \frac{\partial f}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

$\left(\frac{\partial f}{\partial t}\right)_c = 0$; Vlasov equation

$\left(\frac{\partial f}{\partial t}\right)_c = \frac{f_n - f}{\tau}$; Krook collision term

f_n: neutral atoms의 분포함수

τ: collision time

$$\sigma = \sum_j n_j \mathcal{g}_j = \sum_j \mathcal{g}_j \int f_j \, dv_j$$

$$\vec{j} = \sum_j n_j \mathcal{g}_j \vec{u}_j = \sum_j \mathcal{g}_j \int \vec{v}_j f_j \, d\vec{v}_j$$

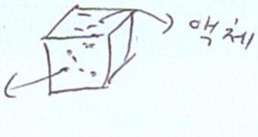
$$(\vec{v} = \vec{u} + \vec{w})$$

$\vec{u}$: 단체속도, $\vec{w}$: 열운동

$$\varepsilon_0 \nabla \cdot \vec{E} = \sigma \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Perrin의 실험



$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{q(\vec{E}+\vec{v}\times\vec{B})}{m}\cdot\frac{\partial f}{\partial \vec{v}} = 0$$

$f(x, y, z, v_x, v_y, v_z, t)$

~~$\bar{v} = \bar{c} = \bar{u} + \bar{w} \approx \bar{w}$~~

$\vec{v}$의 평균값

$$\int_{\vec{v}} f\, d\vec{v} = n \quad\Rightarrow\quad \int_{\vec{v}} \vec{v} f\, d\vec{v} = n\vec{u}$$

$$\int_{\vec{v}}\left[\frac{df}{dt} = \underbrace{\frac{\partial f}{\partial t}}_{①} + \underbrace{\vec{v}\cdot\nabla f}_{②} + \frac{\overbrace{q\vec{E}}^{③} + \overbrace{q\vec{v}\times\vec{B}}^{④}}{m}\cdot\frac{\partial f}{\partial \vec{v}}\right] d\vec{v} \rightarrow \underline{\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{u}) = 0}$$

continuity equation

$$\boxed{*\ \nabla\cdot(\phi\vec{A}) = \phi\nabla\cdot\vec{A} + \vec{A}\cdot\nabla\phi}$$

① $\displaystyle\int_{\vec{v}} \frac{\partial f}{\partial t} d\vec{v} = \frac{\partial}{\partial t}\int f\, d\vec{v} = \frac{\partial n}{\partial t}$

② $\displaystyle\int_{\vec{v}} \vec{v}\cdot\nabla f\, d\vec{v} = \int_{\vec{v}} \nabla\cdot(f\vec{v})\, d\vec{v} - \int_{\vec{v}} f\, \cancel{\nabla\cdot\vec{v}}^{\,0}\, d\vec{v}$

$$= \nabla\cdot\int_{\vec{v}} f\vec{v}\, d\vec{v} = \nabla\cdot(n\vec{u})$$

velocity space 적분

③ $\displaystyle\int_{\vec{v}} \frac{q}{m}\vec{E}\cdot\frac{\partial f}{\partial \vec{v}} d\vec{v} = \frac{q}{m}\int_{\vec{v}} \vec{E}\cdot\frac{\partial f}{\partial \vec{v}} d\vec{v} = \frac{q}{m}\left\{\int_{\vec{v}} \underbrace{\frac{\partial}{\partial \vec{v}}\cdot}_{} (f\vec{E})\, d\vec{v} - \int_{\vec{v}} f\, \cancel{\frac{\partial \vec{E}}{\partial \vec{v}}}^{\,0}\, d\vec{v}\right.$

velocity 에 대한 divergence

→ divergence theorem.

$$= \frac{q}{m}\int_{S^\infty} \cancel{f}^{\,0}\,\vec{E}\cdot d\vec{s} = 0$$

↳ velocity space 에서의 무한 폐곡면 ⇒ ($v \to \infty$) ⇒ ($f \to 0$)

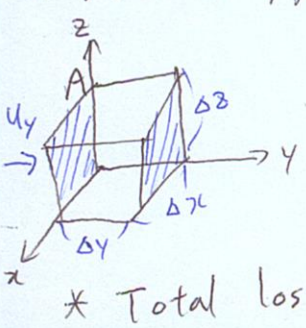
④ $\displaystyle\int_{\vec{v}} \frac{q}{m}(\vec{v}\times\vec{B})\cdot\frac{\partial f}{\partial \vec{v}} d\vec{v} = \frac{q}{m}\int_{\vec{v}} (\vec{v}\times\vec{B})\cdot\frac{\partial f}{\partial \vec{v}} d\vec{v} = \frac{q}{m}\left\{\int_{\vec{v}} \frac{\partial}{\partial \vec{v}}\cdot(f\vec{v}\times\vec{B})\, d\vec{v}\right.$

$$\left. - \int_{\vec{v}} f\, \cancel{\frac{\partial}{\partial \vec{v}}\cdot(\vec{v}\times\vec{B})}^{\,0}\, d\vec{v}\right\}$$

$$= \frac{q}{m}\int_{S^\infty} \cancel{f}^{\,0}\,\vec{v}\times\vec{B}\cdot d\vec{s} = 0$$

$$\begin{vmatrix} \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial v_z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = 0$$

Fluid Approach



$\Delta V = \Delta x \Delta y \Delta z$

ρ ; mass density [kg/m³]

$\vec{u} = (u_x, u_y, u_z)$

* y-direction

mass rate of fluid leaving A

$$\left[(\rho u_y)_{y+\Delta y} - (\rho u_y)_y\right]\Delta x \Delta z = \frac{\Delta(\rho u_y)}{\Delta y}\Delta y \Delta x \Delta z$$

$$= \frac{\Delta(\rho u_y)}{\Delta y}\Delta V$$

* Total loss rate of mass density in A

빠져나가는것

$$\left[\frac{\Delta(\rho u_x)}{\Delta x} + \frac{\Delta(\rho u_y)}{\Delta y} + \frac{\Delta(\rho u_z)}{\Delta z}\right]\Delta V = \underline{-\frac{\partial \rho}{\partial t}\Delta V} \xrightarrow{\Delta x,\,\Delta y,\,\Delta z \to 0} \nabla\cdot(\rho\vec{u}) = -\frac{\partial \rho}{\partial t}$$

loss rate of mass density in A with the time rate of change of the mass density

$$\therefore \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

Fluid Approach

if steady $\frac{\partial \rho}{\partial t} = 0 \rightarrow \nabla \cdot (\rho \vec{u}) = 0$

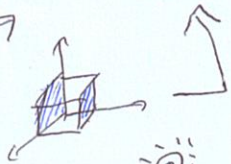
if uniform density $\nabla \cdot \vec{u} = 0$ incompressibility

fluid equation of motion (momentum conservation equation)

$$m \frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \rightarrow nm \frac{d\vec{u}}{dt} = nq(\vec{E} + \vec{u} \times \vec{B}) - \underset{\text{압력}}{\nabla p} - nm \frac{\vec{u} - \vec{u}_0}{\tau}$$

neutral fluid velocity

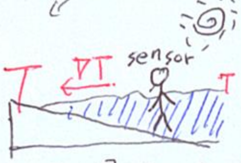
$$\int_{\vec{v}} m\vec{v} \left(\frac{df}{dt}\right) d\vec{v} \quad \text{HW}$$



$$nm\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right]$$

Convective derivative

$$\underbrace{\frac{dT}{dt}}_{\text{Total derivative}} = \underbrace{\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T}_{\text{Convective derivative}}$$



① $\vec{u} = 0 \rightarrow \frac{dT}{dt} = \frac{\partial T}{\partial t}$

② $\frac{\partial T}{\partial t} = 0 \rightarrow \frac{dT}{dt} = \vec{u} \cdot \nabla T$

Lagragian

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial T}{\partial z}\frac{\partial z}{\partial t}$$

$$= \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}u_x + \frac{\partial T}{\partial y}u_y + \frac{\partial T}{\partial z}u_z$$

$$= \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T$$

Eulerian

Lagrangian description

$$\vec{X}(\vec{x}_0, t), \quad \vec{u}(\vec{X}(\vec{x}_0, t), t) = \frac{d\vec{X}}{dt}(\vec{x}_0, t)$$

---

equation of continuity

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$$

equation of motion

$$nm \frac{d\vec{u}}{dt} = nq(\vec{E} + \vec{u} \times \vec{B}) - \nabla \cdot \overleftrightarrow{P} - nm \frac{\vec{u} - \vec{u}_0}{\tau}$$

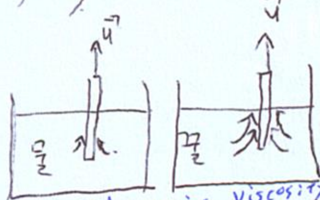
$$= nm\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right)$$

Navier-Stokes Equation

$-\nabla \cdot \overleftrightarrow{P}$ 같은의미

$$\rho\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right] = \boxed{\underbrace{-\nabla P}_{\text{diagonal term}} + \underbrace{\rho \nu \nabla^2 \vec{u}}_{\text{off-diagonal term (viscosity)}}}$$

isotropic한 fluid의 경우

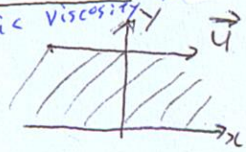


kinematic viscosity (momentum diffusivity) ν

dynamic viscosity

$$\tau = \mu \frac{\partial u}{\partial y}$$

shear stress



equation of state

$$P = c\rho^\gamma \rightarrow \nabla p = \nabla(c\rho^\gamma) = \nabla(c(nm)^\gamma) = cm^\gamma \nabla n^\gamma = cm^\gamma \gamma n^{\gamma-1} \nabla n$$

$$\left(c = \text{const}, \quad \gamma = \frac{C_p \text{ 정압비열}}{C_v \text{ 정적비열}}\right)$$

$$\frac{\nabla P}{P} = \gamma \frac{\nabla n}{n} \rightarrow \text{if isothermal } P = nkT \rightarrow \frac{\nabla P}{P} = \frac{kT\nabla n}{kTn} = \frac{\nabla n}{n}$$

$$\gamma = \frac{2+N}{N} \quad (N: \text{degree of freedom})$$

$$\begin{cases} \frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0 \\ nm\frac{d\vec{u}}{dt} = nm\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right) = nq(\vec{E} + \vec{u} \times \vec{B}) - \nabla \cdot \overleftrightarrow{P} - nm\frac{\vec{u} - \vec{u}_0}{\tau} \\ \frac{\nabla P}{P} = \gamma \frac{\nabla n}{n} \end{cases}$$

→ 변수 $n, \vec{u}, P$, 식 세개

$$\begin{cases} \varepsilon_0 \nabla \cdot \vec{E} = \sum nq \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \sum nq\vec{u} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

plasma와 관련됨.

$\vec{E}, \vec{B}$ 구하는 용도

$P = c\rho^\gamma$ → 굉장한 가정

Fick's law

$$\vec{\Gamma} = n\vec{u} = -D\nabla n$$

$$\Rightarrow \frac{\partial n}{\partial t} - D\nabla^2 n = 0$$

* Fluid drift perpendicular to $\vec{B}$

$$nm\left[\cancel{\frac{\partial \vec{u}}{\partial t}} + \cancel{(\vec{u}\cdot\nabla)\vec{u}}\right] = nq(\vec{E}+\vec{u}\times\vec{B}) - \nabla P$$

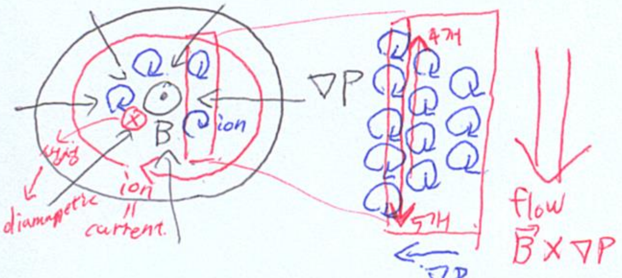
assumption

$$nq(\vec{E}+\vec{u}\times\vec{B}) - \nabla P = 0 \xrightarrow{(\times\vec{B})} nq(\vec{E}\times\vec{B}) + nq\boxed{(\vec{u}\times\vec{B})\times\vec{B}} - \nabla P\times\vec{B} = 0$$

$$\vec{B}\times(\vec{B}\times\vec{u}) = \vec{B}(\vec{B}\cdot\vec{u}) - \vec{u}(\vec{B}\cdot\vec{B}) = \underbrace{\vec{B}(\vec{B}\cdot\vec{u})}_{\hat{z}\cdot\vec{u}_\perp \text{ 만 고려시 } 0} - \vec{B}\vec{u}$$

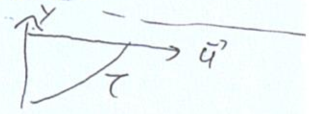
$$\rightarrow nq(\vec{E}\times\vec{B}) - nqB^2\vec{u}_\perp - \nabla P\times\vec{B} = 0 \rightarrow \vec{u}_\perp = \underbrace{\frac{\vec{E}\times\vec{B}}{B^2}}_{\vec{v}_E} + \underbrace{\frac{\vec{B}\times\nabla P}{nqB^2}}_{\vec{v}_D}$$

diamagnetic drift



Shear stress $\tau = \underset{\text{dynamic viscosity}}{\mu}\dfrac{du}{dy}$

$\nu = \dfrac{\mu}{\rho}$: kinetic viscosity, momentum diffusivity



$\tau = \mu\dfrac{d\vec{u}}{dy} = \mu\nabla\vec{u}$ (P와 단위 같다.)

$\nabla P \longleftrightarrow \mu\nabla^2\vec{u} = \rho\nu\nabla^2\vec{u}$

$P = C\rho^\gamma$ $\gamma = 1$: isothermal

$\gamma = \frac{2+N}{N}$: adiabatic compression

* Complete set of fluid equations

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\vec{u}_j) = 0 \quad (j: \text{ion, electron})$$

$$n_jm_j\left[\frac{\partial \vec{u}_j}{\partial t} + (\vec{u}_j\cdot\nabla)\vec{u}_j\right] = n_jq_j(\vec{E}+\vec{u}_j\times\vec{B}) - \nabla P_j + \bar{R}_{ji}$$

$$P_j = C_j(n_jm_j)^{\gamma_j}$$

$$\varepsilon_0\nabla\cdot\vec{E} = \cancel{\sum_j n_jq_j} \quad \nabla\cdot\vec{B} = 0$$

0 아니다! $\phi^{n_i+n_e}$ plasma approximation

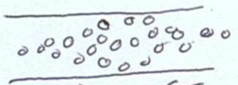
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \nabla\times\vec{B} = \mu_0\sum_j n_jq_j\vec{u}_j + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

* plasma approximation

$n_i \approx n_e \approx n$

$n_iq_i + n_eq_e \approx n(e)+n(-e) \neq 0$

* Fluid drift parallel to $\vec{B}$



T = constant

∇n ∇n

$-\nabla P = F$ $\vec{F} = -\nabla P$

isothermal, $\vec{B} = B_0\hat{z}$ for electron

ϕ −   +   −

$\vec{E}$ ←   →

$\vec{F}_E$ (for electron)   →   ←

$$nm\left[\frac{\partial \vec{u}}{\partial t} + \cancel{(\vec{u}\cdot\nabla)\vec{u}}\right] = nq(\vec{E}+\cancel{\vec{u}\times\vec{B}}) - \nabla P + \bar{R}$$

무시   0

$$\rightarrow nm\frac{\partial u}{\partial t} = -neE_z - kT\frac{\partial n}{\partial z}$$

$$\frac{\partial u_z}{\partial t} = -\frac{e}{m}E_z - \frac{kT}{nm}\frac{\partial n}{\partial z} = 0$$

$\frac{\partial u}{\partial t} = 0$ (neglect electron inertia. $\approx m$ 매우 작다)

$$+\frac{e}{m}\frac{\partial\phi}{\partial z} - \frac{kT}{nm}\frac{\partial n}{\partial z} = 0$$

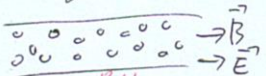
$$\frac{e}{kT}\frac{\partial\phi}{\partial z} - \frac{1}{n}\frac{\partial n}{\partial z} = 0 \rightarrow \frac{e\phi}{kT} - \ln n = \text{constant}$$

$$\rightarrow n = n_0\exp\left(\frac{e\phi}{kT}\right)$$

($n = n_0$ with $\phi = 0$)

(Boltzmann relation) response

homogeneous



$$nm\left[\frac{\partial \vec{u}}{\partial t} + \cancel{(\vec{u}\cdot\nabla)\vec{u}}\right] = nq(\vec{E}+\cancel{\vec{u}\times\vec{B}}) - \cancel{\nabla P} + \bar{R}$$

무시   0   저항

$$nm\frac{\partial u}{\partial t} = -neE_z + \bar{R}_{ei}$$

$$\bar{R}_{ei} = -n_e m\nu_{ei}(\vec{u}_e - \vec{u}_i)$$

$$nm\cancel{\frac{\partial \vec{u}}{\partial t}} = -neE_z - nm\nu(\vec{u}_e - \vec{u}_i) = 0$$

neglect inertia

$$E_z = -\frac{m\nu}{e}(\vec{u}_e - \vec{u}_i)$$

$$J_z = ne(\vec{u}_i - \vec{u}_e)$$

$$\rightarrow E_z = \frac{m\nu}{ne^2}J_z = \eta J_z \quad \text{Ohm's Law (simplified)}$$

$\boxed{\eta = \frac{m\nu}{ne^2}}$ resistivity

$$\therefore \bar{R}_{ei} = -nm\nu_{ei}(\vec{u}_e - \vec{u}_i) = -nm\frac{ne^2}{m}\eta(\vec{u}_e - \vec{u}_i) = ne\eta\vec{J}$$

*Single Fluid Equation $\quad (n_i \approx n_e \approx n)$

$\rho \equiv n_i M_i + n_e m_e \approx n(M+m)$; mass density

$\sigma \equiv e(n_i - n_e)$; charge density $\neq 0$ (plasma approximation)

$$\vec{v} \equiv \frac{n_i M \vec{u}_i + n_e m \vec{u}_e}{n_i M + n_e m} = \frac{n_i M \vec{u}_i + n_e m \vec{u}_e}{\rho} \approx \frac{M\vec{u}_i + m\vec{u}_e}{M+m} \text{ ; mass velocity}$$

$\vec{J} = n_i e \vec{u}_i - n_e e \vec{u}_e \approx ne(\vec{u}_i - \vec{u}_e)$; current density

$\vec{u}_i = \vec{u}_e + \frac{\vec{j}}{ne}$, $(M+m)\vec{v} = M\vec{u}_i + m\vec{u}_e \rightarrow \vec{u}_e = -\frac{M}{m}\vec{u}_i + \frac{M+m}{m}\vec{v}$

$\therefore \vec{u}_i = -\frac{M}{m}\vec{u}_i + \frac{M+m}{m}\vec{v} + \frac{\vec{j}}{ne} \Rightarrow \vec{u}_i = \vec{v} + \frac{m}{m+M}\frac{\vec{j}}{ne} \approx \vec{v} + \frac{m}{M}\frac{\vec{j}}{ne}$

as the same way $\vec{u}_e = \vec{v} - \frac{M}{M+m}\frac{\vec{j}}{ne} \approx \vec{v} - \frac{\vec{j}}{ne}$

$$\frac{\partial}{\partial t} n_i + \nabla\cdot(n_i \vec{u}_i) = 0 \quad \times M$$
$$\frac{\partial}{\partial t} n_e + \nabla\cdot(n_e \vec{u}_e) = 0 \quad \times m$$

$\rightarrow \frac{\partial}{\partial t}(Mn_i + mn_e) + \nabla\cdot(Mn_i\vec{u}_i + mn_e\vec{u}_e) = 0$

$\Rightarrow \boxed{\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0}$

$\times e$, $\times(-e)$ $\rightarrow \frac{\partial}{\partial t}[e(n_i - n_e)] + \nabla\cdot(en_i\vec{u}_i - en_e\vec{u}_e) = 0 \Rightarrow \boxed{\frac{\partial}{\partial t}\sigma + \nabla\cdot\vec{j} = 0}$ charge continuity equation

$$nM\frac{d\vec{u}_i}{dt} = ne(\vec{E} + \vec{u}_i\times\vec{B}) - \nabla P_i + \vec{R}_{ie}$$
$$nm\frac{d\vec{u}_e}{dt} = -ne(\vec{E} + \vec{u}_e\times\vec{B}) - \nabla P_e + \vec{R}_{ei}$$

$\rightarrow n\frac{d}{dt}(M\vec{u}_i + m\vec{u}_e) = e(n_i - n_e)\vec{E} + e(n_i\vec{u}_i - n_e\vec{u}_e)\times\vec{B} - \nabla P$

$\Rightarrow n(M+m)\frac{d\vec{v}}{dt} = \boxed{\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{j}\times\vec{B} - \nabla P}$

*Electron inertia neglected.

$nm\frac{d\vec{u}_e}{dt} = 0 = -ne(\vec{E} + \vec{u}_e\times\vec{B}) - \nabla P_e + \vec{R}_{ei} \Rightarrow \vec{E} + \vec{u}_e\times\vec{B} = -\frac{\nabla P_e}{ne} + \frac{\vec{R}_{ei}}{ne}$

$\Rightarrow \vec{E} + (\vec{v} - \frac{\vec{j}}{ne})\times\vec{B} = -\frac{\nabla P_e}{ne} - \frac{nm\nu_{ei}(\vec{u}_e - \vec{u}_i)}{ne}$

$$\left(\begin{aligned} &\vec{u}_e = \vec{v} - \frac{\vec{j}}{ne} \\ &\vec{R}_{ei} = -nm\nu_{ei}(\vec{u}_e - \vec{u}_i) \\ &\quad = -n\eta ne^2(\vec{u}_e - \vec{u}_i) = \eta ne\vec{j} \end{aligned}\right)$$

$\Rightarrow \boxed{\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \frac{\vec{j}\times\vec{B} - \nabla P_e}{ne}}$: generalized Ohm's Law

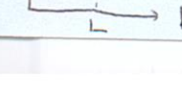
$$\frac{\nabla P_e/ne}{\vec{v}\times\vec{B}} \approx \frac{\frac{P_e}{Lne}}{v_{ti}B} \approx \frac{\frac{nT_e}{Lne}}{v_{ti}B} \approx \frac{\frac{Mv_{ti}^2}{Le}}{v_{ti}B} \approx \frac{\frac{Mv_{ti}}{Be}}{L} \approx \frac{\rho_i}{L} \ll 1$$

ion의 Larmor radius

assumption)

$\vec{v} = \vec{v}_{ti}$ (thermal ion)

$T_i = T_e$ (temperature)



characteristic Length $/ \nabla P_e \sim +\frac{P_e}{L}$

$$\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \cancel{\frac{\vec{j}\times\vec{B} - \nabla P_e}{ne}}$$

$\vec{j}\times\vec{B} \rightarrow 0$, $\nabla P_e \rightarrow 0$ order낮음

∇Pe의 order 비슷

$\varepsilon_0 = 0 \Leftrightarrow c \rightarrow \infty$

$\eta \rightarrow 0$ 인경우 = ideal MHD eq-n

Single Fluid Equation Complete Form (MHD eq-n)

Magneto hydro dynamics (자기유체역학)

$\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0$, $\cancel{\frac{\partial}{\partial t}\sigma} + \nabla\cdot\vec{j} = 0$

$\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{j}\times\vec{B} - \nabla P$

$\vec{E} + \vec{v}\times\vec{B} = \cancel{\eta\vec{j}}$, $\frac{d}{dt}\left(\frac{\rho^\gamma}{P}\right) = 0$

$\cancel{\varepsilon_0}\nabla\cdot\vec{E} = \cancel{\sigma}$, $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\nabla\cdot\vec{B} = 0$, $\nabla\times\vec{B} = \mu_0\vec{j} + \cancel{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}$

Waves in Plasmas

$c = \frac{\omega}{k}$

wave eqn $\frac{\partial^2 u}{\partial t^2} - v^2 \nabla^2 u = 0 \rightarrow u = A\cos\left(\frac{t}{c} + \delta\right) \rightarrow \vec{u} = \bar{u}\exp i(\vec{k}\vec{r} - \omega t)$

($v = c$)

$\Rightarrow u = \bar{u}\cos(kx - \omega t)$

$\vec{k}$: wave vector

$|\vec{k}|$: wave number $= \frac{2\pi}{\lambda}$

* phase velocity :

velocity of a point of constant phase on the wave

$$\frac{d}{dt}(kx - \omega t) = k\frac{dx}{dt} - \omega = 0$$

$$\rightarrow v_\varphi = \frac{dx}{dt} = \frac{\omega}{k}$$

* Group velocity

$E_1 = E_0 \cos[(k+\Delta k)x - (\omega + \Delta\omega)t]$

$E_2 = E_0 \cos[(k-\Delta k)x - (\omega - \Delta\omega)t]$

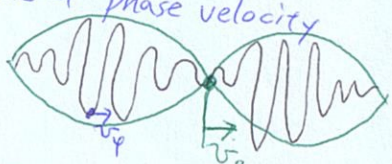
$kx - \omega t = a$, $\Delta k x - \Delta\omega t = b$

$E = E_1 + E_2 = E_0[\cos(a+b) + \cos(a-b)]$

$= 2E_0 \cos a \cos b = 2E_0 \cos(\Delta k x - \Delta\omega t) \cos(kx - \omega t)$

group velocity = amplitude 의 phase velocity

$$v_g = \frac{\Delta\omega}{\Delta k} \rightarrow \frac{d\omega}{dk}$$



Plasmas

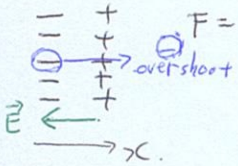
Assumption

① $\vec{B} = 0$ ② no thermal motion $T_0 = 0$

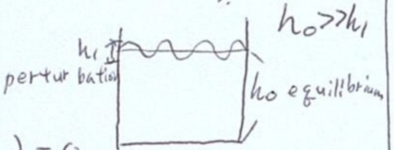
③ Fixed ion uniformly distributed

④ infinite plasma ⑤ only x-direction

$F = -\nabla P = 0$ ($\because$ ②)



$h_0 \gg h_1$



$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\vec{u}_e) = 0$

$n_e m\left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e\cdot\nabla)\vec{u}_e\right] = -n_e e(\vec{E} + \cancel{\vec{u}_e\times\vec{B}}) \cancel{-\nabla P_e} + R_{ei}$ (① $\to 0$, ② $\to 0$, R_{ei} 무시)

$\varepsilon_0 \nabla\cdot\vec{E} = e(n_i - n_e)$

$\nabla\times\vec{E} = -\cancel{\frac{\partial\vec{B}}{\partial t}}$ (0)

$\cancel{P_e} = C(n_e m_e)^\gamma$ (0)

$\cancel{\nabla\cdot\vec{B}} = 0$ (0)

$\cancel{\nabla\times\vec{B}} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ (0)

$\begin{pmatrix}E_1\\u_1\\n_1\end{pmatrix} = \begin{pmatrix}\bar{E}\\\bar{u}\\\bar{n}\end{pmatrix}\exp i(kx - \omega t)$

$n_e = n_0 + n_1$ (equilibrium + perturbation) ($\nabla n_0 = 0$, $\frac{\partial n_0}{\partial t} = 0$)

linearization

$\vec{u}_e = \vec{u}_0 + \vec{u}_1$ ($\vec{u}_0 = 0$, $\frac{\partial \vec{u}_0}{\partial t} = 0$)

$\vec{E} = \vec{E}_0 + \vec{E}_1$ ($\vec{E}_0 = 0$, $\frac{\partial\vec{E}_0}{\partial t} = 0$)

$\frac{\partial}{\partial t}(\cancel{n_0} + n_1) + \nabla[(n_0 + n_1)(\cancel{\vec{u}_0} + \vec{u}_1)] = 0$

$\cancel{(n_0 + n_1)} m\left[\frac{d(\cancel{\vec{u}_0} + \vec{u}_1)}{dt} + [(\cancel{\vec{u}_0} + \vec{u}_1)\cdot\nabla](\cancel{\vec{u}_0} + \vec{u}_1)\right] = -e\cancel{(n_0+n_1)}(\cancel{\vec{E}_0} + \vec{E}_1)$

$\Rightarrow \varepsilon_0\nabla\cdot(\cancel{\vec{E}_0} + \vec{E}_1) = e(\cancel{n_{i0}} + \cancel{n_{i1}} - (\cancel{n_0} + n_1))$

perturbation의 곱

$$\Downarrow$$

$n_1 = \bar{n}\exp i(kx - \omega t)$

$\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0\vec{u}_1 + \cancel{n_1\vec{u}_1}) = 0$ (perturbation의 곱 $\to 0$)

$m\left[\frac{\partial\vec{u}_1}{\partial t} + \cancel{(\vec{u}_1\cdot\nabla)\vec{u}_1}\right] = -e\vec{E}_1$

$\frac{\partial}{\partial t} = -i\omega$, $\frac{\partial}{\partial x} = ik$

$\varepsilon_0\nabla\vec{E}_1 = -en_1$

$$\Downarrow$$

$$\begin{cases} -i\omega n_1 + ikn_0u_1 = 0 \\ -i\omega m u_1 = -eE_1 \\ ik\varepsilon_0 E_1 = -en_1 \end{cases}$$

연립

$\cancel{-i}\omega m\cancel{u_x} = -e\left(\frac{-e}{\cancel{ik}\varepsilon_0}\right)\left(\frac{\cancel{ik}n_0}{\cancel{i}\omega}\right)\cancel{u_1}$

$\omega m = \frac{e^2}{\varepsilon_0\omega}n_0 \rightarrow \omega^2 = \frac{n_0e^2}{m\varepsilon_0} = \boxed{\omega_p^2}$

plasma frequency

$f_p = \frac{\omega_p}{2\pi} \sim 9\sqrt{n_0(\#/m^3)}$

$f_c \cong \frac{28}{B(T)}$ (GHz) (cyclotron)

KSTAR $f_p = 90$GHz ($n_0 = 10^{20}$) $\gtrsim f_c$ (when $B \sim 0.3T$)

HW : 왜 k는 나타나지 않는가?

k, ω가 다 있어야 wave 이다.

Plasma Waves

* plasma oscillations

$$\frac{\partial n_e}{\partial t} + \nabla(n_e \vec{u}_e) = 0$$

$$n_e m\left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e \cdot \nabla)\vec{u}_e\right] = -n_e e \vec{E} \quad -\nabla P_e$$

$$\varepsilon_0 \nabla \cdot \vec{E} = (n_i - n_e) e$$

($f(\omega, k) = 0$ dispersion relation)

$n = n_0 + n_1$, $n_1 = \bar{n}_1 \exp i(kx - \omega t)$

$\vec{u} = \cancel{\vec{u}_0} + \vec{u}_1$, $\vec{u}_1 = \bar{u}_1 \exp i(kx - \omega t)$

$\vec{E} = \cancel{\vec{E}_0} + \vec{E}_1$, $\vec{E}_1 = \bar{E}_1 \exp i(kx - \omega t)$

$a_0 \gg a_1$

$$\omega^2 = \frac{n e^2}{m \varepsilon_0} = \omega_p^2 \Leftarrow \begin{cases} -i\omega \bar{n}_1 + n_0 i k \bar{u}_1 = 0 \\ -i\omega m \bar{u}_1 = -e\bar{E}_1 - ik\gamma k T_e \bar{n}_1 \\ ik\varepsilon_0 \bar{E}_1 = -e\bar{n}_1 \end{cases}$$

$$\left(\frac{\nabla P_e}{P_e} = \gamma \frac{\nabla n_e}{n_e} \right) \qquad \gamma k T_e \nabla n_e$$

$$\Rightarrow \nabla P_e = \gamma P_e \frac{\nabla n_e}{n_e} = \gamma n_e k T_e \frac{\nabla n_e}{n_e}$$

$\left(\frac{1}{2} m v_{th}^2 = k T_e\right)$

*Electron plasma wave

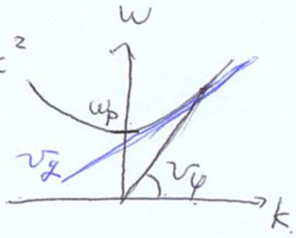
볼츠만 상수

$$\omega^2 = \omega_p^2 + \frac{\gamma k T_e}{m} \boxed{k}^2 = \frac{n_0 e^2}{m \varepsilon_0} + \frac{\gamma k T_e}{m} k^2$$

파수

$$\Rightarrow \omega^2 = \omega_p^2 + \frac{1}{2}\gamma v_{th}^2 k^2$$

$$v_g = \frac{d\omega}{dk} = \frac{\gamma v_{th}^2}{2} \frac{k}{\omega} = \frac{\gamma v_{th}^2}{2 v_\varphi} \qquad \left(v_\varphi = \sqrt{\frac{1}{2}\gamma}\, v_{th}\right)$$



$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}\right] = -\underbrace{\nabla p}_{= \gamma k T \nabla \rho} + \overbrace{\cancel{\rho \nu \nabla^2 u}}^{\text{무시}}$$

$(\rho = \rho_0 + \rho_1 , \ \vec{v} = \vec{v}_1)$

$$\nabla p = \gamma p \frac{\nabla \rho}{\rho}$$

$$\Rightarrow \frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \vec{v}_1 = 0 \qquad \Rightarrow \quad -i\omega \rho_1 + \rho_0 i k v_1 = 0$$

$$\rho_0 \frac{d\vec{v}_1}{dt} = -\gamma \frac{p \nabla \rho}{\rho} \qquad\quad -i\omega \rho_0 v_1 = -\gamma \frac{P_0 i k \rho_1}{\rho_0}$$

$$\Rightarrow +\cancel{i}\omega \rho_0 \cancel{v_1} = +\frac{ikP_0\gamma}{\rho_0}\left(\frac{\rho_0 i k}{\cancel{i}\omega}\right)\cancel{v_1}$$

$$\therefore \omega^2 = \frac{\gamma P_0}{\rho_0} k^2$$

$$\left(\frac{\omega}{k}\right)^2 = \frac{\gamma P_0}{\rho_0} = \frac{\gamma n_0 k T}{n_0 M} = \frac{\gamma k T}{M} = C_s^2$$

sound speed

We assumed 1-D

Electron Plasma Wave Derivation

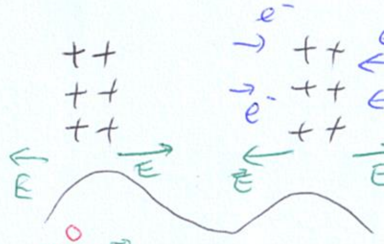
$n = n_0 + n_1 \qquad \frac{\partial n_0}{\partial t} = 0 \qquad \nabla n_0 = 0 \qquad \frac{\nabla P_e}{P_e} = \gamma \frac{\nabla n_e}{n_e} \qquad \nabla P_e = \gamma P_e \frac{\nabla n_e}{n_e} = \gamma n_e k_b T_e \frac{\nabla n_e}{n_e} = \gamma k_b T_e \nabla n_e$

$\vec{u} = \cancel{\vec{u}_0} + \vec{u}_1 \qquad \vec{E} = \cancel{\vec{E}_0} + \vec{E}_1 \qquad \frac{\partial}{\partial t} = -i\omega \qquad \nabla = ik$

$n_1 = n_1 \exp[i(kx - \omega t)] \qquad \vec{u}_1 = u_1 \exp[i(kx - \omega t)] \qquad \vec{E}_1 = E_1 \exp[i(kx - \omega t)]$

~~~ : we used phasor for convenience of calculation

$$\begin{cases} \frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{u}_e) = 0 \\ m n_e \left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e \cdot \nabla) \vec{u}_e \right] = -n_e e \vec{E} \underbrace{\boxed{-\nabla P_e}}_{= -\gamma k_b T_e \nabla n_e} = -n_e e \vec{E} - \gamma k_b T_e \nabla n_e \\ \varepsilon_0 \nabla \cdot \vec{E} = (n_i - n_e) e \end{cases}$$

$\frac{\partial}{\partial t}(\cancel{n_0} + n_1) + \nabla \cdot [(n_0 + \cancel{n_1}) \vec{u}_1] = 0 \Rightarrow \frac{\partial}{\partial t} n_1 + \nabla \cdot (n_0 \vec{u}_1) = 0$

$\frac{\partial n_0}{\partial t} = 0 \qquad n_0 \vec{u}_1 \gg n_1 \vec{u}_1$

$\Rightarrow \frac{\partial}{\partial t} n_1 + n_0 \nabla \cdot \vec{u}_1 + \vec{u}_1 \cdot \cancelto{\nabla n_0 = 0}{\nabla n_0} = 0$

$\Rightarrow \frac{\partial}{\partial t} n_1 + n_0 \nabla \cdot \vec{u}_1 = 0$

$n_0 \frac{\partial \vec{u}_1}{\partial t} \gg n_1 \frac{\partial \vec{u}_1}{\partial t}$

$\underline{-i\omega n_1 + n_0 i k u_1 = 0}$ (1)

$m(n_0 + \cancel{n_1}) \left[\frac{\partial \vec{u}_1}{\partial t} + \cancelto{\text{too small}}{(\vec{u}_1 \cdot \nabla) \vec{u}_1} \right] = -(n_0 + \cancel{n_1}) e \vec{E}_1 - \gamma k_b T_e \nabla (\cancelto{\nabla n_0 = 0}{n_0} + n_1)$

$n_0 \vec{E}_1 \gg n_1 \vec{E}_1$

$\Rightarrow m n_0 \frac{\partial \vec{u}_1}{\partial t} = -n_0 e \vec{E}_1 - \gamma k_b T_e \nabla n_1 \Rightarrow \underline{-i\omega m n_0 u_1 = -n_0 e E_1 - \gamma k_b T_e i k n_1}$ (2)

$\varepsilon_0 \vec{\nabla} \cdot \vec{E}_1 = [n_0 - (n_0 + n_1)] e \Rightarrow \underline{\varepsilon_0 i k E_1 = -n_1 e}$ (3)

ion fixed

From (1) $n_1 = \frac{n_0 \cancel{i} k u_1}{\cancel{i} \omega} = \frac{n_0 k u_1}{\omega}$

From (3) $E_1 = \frac{-n_1 e}{\varepsilon_0 i k} = \frac{n_1 e}{\varepsilon_0 k} i = \frac{e i}{\varepsilon_0 k} \left(\frac{n_0 k}{\omega} u_1 \right) = i \frac{e n_0}{\varepsilon_0 \omega} u_1$

Insert n_1, E_1 to (2)

$-\cancel{i} \omega m \cancel{n_0} u_1 = -\cancel{i} \cancel{n_0} e \frac{e n_0}{\varepsilon_0 \omega} u_1 - \cancel{i} \gamma k_b T_e \frac{\cancel{n_0} k^2 u_1}{\omega}$

$\omega m u_1 = \frac{e^2 n_0}{\varepsilon_0 \omega} u_1 + \frac{\gamma k_b T_e}{\omega} k^2 u_1 \Rightarrow (\omega m) u_1 = \left(\frac{e^2 n_0}{\varepsilon_0 \omega} + \frac{\gamma k_b T_e}{\omega} k^2 \right) u_1$

$$\omega^2 = \frac{e^2 n_0}{\varepsilon_0 m} + \frac{\gamma k_b T_e}{m} k^2$$

$$\omega^2 = \omega_p^2 + \frac{\gamma k_b T_e}{m} k^2$$
~~~

Ion Waves (Acoustic Wave)



set①

$$\begin{cases} \frac{\partial n_i}{\partial t} + \nabla\cdot(n_i \vec{u}_i) = 0 \\ n_i M\left[\frac{\partial \vec{u}_i}{\partial t} + (\vec{u}_i\cdot\nabla)\vec{u}_i\right] = n_i e\vec{E} - \nabla P_i \\ \varepsilon_0 \nabla\cdot\vec{E} = e(n_i - n_e) \simeq 0 \end{cases}$$

ion 관점에선 전자가 바로바로 따라와서

$$n_e \overset{0}{\cancel{m}}\left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e\cdot\nabla)\vec{u}_e\right]$$

neglect electron inertia

$$= -n_e e\vec{E} - \underbrace{\nabla P_e}_{\gamma k T_e \nabla n_e}$$

$$\Rightarrow e\nabla\phi = \gamma k T_e \frac{\nabla n_e}{n_e}$$

$$\vec{E} = -\nabla\phi,\quad n_i = n_0 + n_1,\quad \vec{u}_i = \vec{u}_1,\quad \phi = \phi_1$$

$$n_e = n_0 \exp\left(\frac{e\phi_1}{kT}\right) \approx n_0 + \boxed{n_0\frac{e\phi_1}{kT_e}} = n_1$$

↳ ϕ을 다 없애지 않고 일부분만 없앤다.

set① $\Rightarrow -i\omega n_1 + n_0 i k u_1 = 0 \Rightarrow n_1 = \frac{n_0 k}{\omega} u_1$ (k: 파수)

$$-n_0 M i\omega u_1 = -n_0 e i k \phi_1 - \gamma k T_i i k n_1$$

$$n_0 \frac{e\phi_1}{kT_e} = n_1 \Rightarrow \phi_1 = \frac{kT_e}{e n_0} n_1$$

중성 기체

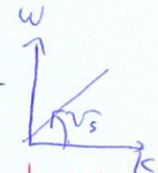
$$\left(\frac{\omega}{k}\right)^2 = \frac{\gamma k T}{M} = C_s^2$$

$$\therefore +\cancel{n_0}M\cancel{i}\omega \cancel{u_1} = +\cancel{n_0}\cancel{e}\cancel{i}k\frac{kT_e}{\cancel{e}\cancel{n_0}}\frac{\cancel{n_0}k}{\omega}\cancel{u_1} + \gamma k T_i \cancel{i}k\frac{\cancel{n_0}k}{\omega}\cancel{u_1}$$

$$M\omega = \frac{kT_e}{\omega}k^2 + \frac{\gamma k T_i}{\omega}k^2 \Rightarrow \left(\frac{\omega}{k}\right)^2 = \frac{kT_e + \gamma k T_i}{M} = v_s^2$$

plasma

↳ electron의 thermal motion



electron thermal motion
→ electric field sheld 덜됨
→ $\vec{E}$에 의해 전파

① $T_e = 0,\ T_i = 0 \Rightarrow$ 전파 불가

② $T_e = 0,\ T_i \neq 0 \Rightarrow$ ion의 thermal motion에 의해 전파

③ $T_i = 0,\ T_e \neq 0 \Rightarrow \vec{E}$에 의해서 전파

④ no electron $\Rightarrow$?? HW

*Electromagnetic Waves with $B_0 = 0,\ B_1 \neq 0$

$$\nabla\times\vec{E}_1 = -\dot{\vec{B}}_1 \quad \text{(진공)}$$

$$c^2\nabla\times\vec{B}_1 = \dot{\vec{E}}_1 \rightarrow c^2\nabla\times(\nabla\times\vec{B}) = c^2[\nabla(\overset{0}{\cancel{\nabla\cdot\vec{B}}}) - \nabla^2\vec{B}] = \nabla\times\dot{\vec{E}} = -\ddot{\vec{B}}$$

$$\Rightarrow +c^2(\cancel{i}k)^2\cancel{B_1} = +(\cancel{i}\omega)^2\cancel{B_1} \Rightarrow c^2k^2 = \omega^2 \Rightarrow \left(\frac{\omega}{k}\right)^2 = c^2$$

(플라즈마 있을 때)

$$\vec{j} = \sum_j n_j q_j \vec{u}_j = n_i e\vec{u}_i - n_e e\vec{u}_e = -n_0 e\vec{u}_1$$

ion fixed

$k\to 0$ cut off

$k\to\infty$ resonance

$$\left.\begin{aligned} &c^2\nabla\times\vec{B}_1 = c^2\mu_0\vec{j} + \dot{\vec{E}} \\ &n_0 m\left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e\cdot\nabla)\vec{u}_e\right] = -n_e e(\vec{E} + \cancel{\vec{u}_e\times\vec{B}}) - \cancel{\nabla P_e} \end{aligned}\right) \Rightarrow \omega^2 = c^2k^2 + \omega_p^2$$

perturbation의 곱 / 무시

Waves in plasma.

electron waves

$$\omega^2 = \omega_p^2 + \frac{\gamma k T_e}{m} k^2$$

ion waves

$$\omega^2 = k^2 \frac{kT_e + \gamma k T_i}{M}$$

electromagnetic waves

$B_0 = 0$, $B_1 \neq 0$ $\qquad \omega^2 = c^2 k^2$

$$\nabla \times \vec{E}_1 = -\dot{\vec{B}}_1$$

$$c^2 \nabla \times \vec{B}_1 = +\dot{\vec{E}}_1$$

$$\Downarrow$$

변수3개

$$\nabla \times \vec{E}_1 = -\dot{B}_1$$

$$\nabla \times \vec{B}_1 = \mu_0 \vec{J}_1 + \frac{1}{c^2}\dot{E}_1$$

$$\left(\begin{array}{l} n_e = n_0 + n_1 \\ \vec{u}_e = \cancel{\vec{u}_0} + \vec{u}_1 \\ \vec{E} = \cancel{\vec{E}_0} + \vec{E}_1 \\ \vec{B} = \cancel{\vec{B}_0} + \vec{B}_1 \end{array}\right)$$

$$\vec{J}_1 = (n_i e \vec{u}_i - n_e e \vec{u}_e)_1 = -(n_0 + n_1) e \vec{u}_1 \approx -n_0 e \vec{u}_1$$

$$m n_e \left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e \cdot \nabla)\vec{u}_e\right] = -\underbrace{\cancel{\nabla P_e}}_{\text{무시}} - n_e e(\vec{E} + \vec{u}_e \times \vec{B})$$

$$\Rightarrow m n_0 \frac{\partial \vec{u}_1}{\partial t} = -n_0 e \vec{E}_1$$

$$-i\omega m n_0 u_1 = -n_0 e E_1$$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \dot{\vec{J}} - \frac{1}{c^2}\ddot{\vec{E}}$$

$$\nabla(\nabla \cdot \vec{E}_1) - \nabla^2 \vec{E}_1 = \mu_0 n_0 e \dot{\vec{u}}_1 - \frac{1}{c^2}\ddot{\vec{E}}_1$$

$$i\vec{k}(i\vec{k} \cdot \vec{E}_1) - (ik \cdot ik)\vec{E}_1 = -i\omega \mu_0 n_0 e \vec{u}_1 - \frac{1}{c^2}(-i\omega)(-i\omega)\vec{E}_1$$

$\rightarrow \vec{k} \parallel \vec{E}_1 \rightarrow ik(ik\vec{E}_1) - (ik\,ik\,E_1) = 0$

$$0 = -i\omega \mu_0 n_0 e \vec{u}_1 + \frac{1}{c^2}\omega^2 \vec{E}_1$$

$$\cancel{i\omega}\mu_0 n_0 e \frac{e}{\cancel{i\omega} m}\cancel{\vec{E}_1} = \frac{1}{c^2}\omega^2 \cancel{\vec{E}_1}$$

$$\therefore \omega^2 = c^2 \mu_0 \frac{n_0 e^2}{m} = \frac{n_0 e^2}{m \varepsilon_0} = \omega_p^2$$

$\rightarrow \vec{k} \perp \vec{E} \rightarrow i\vec{k} \cdot \vec{E} = 0$

$$k^2 \cancel{\vec{E}_1} = \frac{-\mu_0 n_0 e^2}{m}\cancel{\vec{E}_1} + \frac{\omega^2}{c^2}\cancel{\vec{E}_1} \qquad \left(c^2 = \frac{1}{\mu_0 \varepsilon_0}\right)$$

$$\therefore \omega^2 = \omega_p^2 + c^2 k^2$$

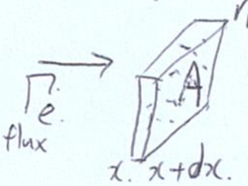
$k = 0 \Rightarrow$ cutoff ($\omega = \omega_p$)

$k \rightarrow \infty \Rightarrow$ resonance. ($\vec{k} \parallel \vec{B}$ 하게 넣어줄 때)

$\vec{B} \leftarrow \vec{E} \; \vec{k}$

Diffusion and mobility in weakly ionized gases

collision parameters



단위면적당 충돌확률

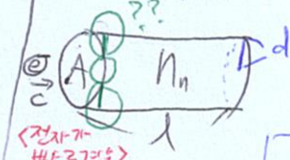
$$\Gamma(x+dx) = \Gamma(x) - \Gamma(x)\,\boxed{n_n A\,dx}\,\frac{\sigma}{A}$$

target atoms

$$\frac{\Gamma(x+dx) - \Gamma(x)}{dx} = -\Gamma(x)\,n_n \sigma$$

$$\lambda = \frac{1}{n_n \sigma} \text{ (Mean Free Path)}$$

$$\Gamma(x) = \Gamma_0 e^{-n_n \sigma x} = \Gamma_0 e^{-\frac{x}{\lambda}}$$



〈전자가 빠른경우〉

$$\text{MFP} = \boxed{\frac{l}{\text{충돌 횟수}}} = \frac{l}{n_n A l} = \frac{1}{n_n A} = \frac{1}{n_n \sigma}$$

$$\frac{ct}{n_n \pi d^2 c t}$$

$$\tau = \frac{\lambda}{c} = \frac{1}{n_n \sigma c}$$

〈중성입자가 빠른경우〉

$$\text{MFP} = \frac{c' t}{\frac{1}{4} n \bar{c}\, 4\pi d^2 t} = \frac{c'}{n(\pi d^2)\bar{c}} = \frac{c'}{n \sigma \bar{c}}$$

$$\left(J = \frac{1}{4} n \bar{c} A\right)$$

$$\tau = \frac{\lambda}{c'} = \frac{1}{\sqrt{2}\, n \sigma c'} = \frac{kT}{\sqrt{2}\, n k T \sigma c'} = \frac{kT}{\sqrt{2}\, P \sigma c'} \qquad (\bar{c} \sim \sqrt{2} c') \qquad \sim \frac{1}{\sqrt{2}\, n\sigma}$$

(1 atm, 20°C, $\tau \sim 100$ps)

$$\cancel{nm\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right]}^{\,0\text{ 가정}} = \pm n e \vec{E} - \nabla P - m n \nu \vec{u}$$

$$m n \nu \vec{u} = \pm n e \vec{E} - \nabla P \qquad (P = nkT, \text{ isothermal})$$

$$\vec{u} = \pm \frac{\cancel{n} e \vec{E}}{m \cancel{n} \nu} - \frac{kT \nabla n}{m n \nu} = \pm \frac{e \vec{E}}{m \nu} - \frac{kT}{m \nu}\frac{\nabla n}{n}$$

$$= \pm \mu \vec{E} - D \frac{\nabla n}{n}$$

mobility $\qquad$ diffusion coefficient

$$\therefore \vec{\Gamma} = n\vec{u} = \pm \mu n \vec{E} - D \nabla n$$

Fick's Law

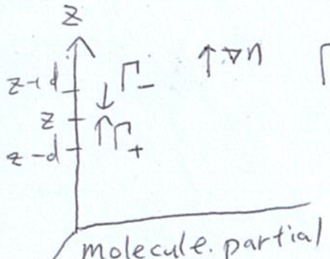
$$\mu = \frac{De}{kT} \text{ Einstein relation}$$

i) $\vec{E} = 0$, $\vec{\Gamma} = -D\nabla n$

ii) $\nabla n = 0$, $\vec{\Gamma} = \pm \mu n \vec{E}$

iii) $\nu = 0$, Boltzmann relation

Plasma Diffusion



$$\Gamma_{net} = \Gamma_+ - \Gamma_- = \frac{J_+ - J_-}{A} = \left\{ \frac{1}{4} n(z-d)\bar{c}A - \frac{1}{4} n(z+d)\bar{c}A \right\} / A$$

$$= \frac{1}{4}\bar{c}\left\{ \left[\cancel{n(z)} - d\frac{dn}{dz}\right] - \left[\cancel{n(z)} + d\frac{dn}{dz}\right]\right\} = -\frac{1}{2}\bar{c}d\frac{dn}{dz} = -\underbrace{\boxed{\tfrac{1}{2}\bar{c}d}}_{D}\nabla n$$

molecule partial current $J = \frac{n\bar{c}A}{4}$

$$\left(v_{th} = \frac{\lambda}{\tau} = \lambda\nu\right)$$

$$D = \frac{kT}{m\nu} \cong \frac{\frac{1}{2}m v_{th}^2}{m\frac{v_{th}}{\lambda}} = \frac{v_{th}\boxed{\lambda}}{2} \sim \frac{1}{2}\bar{c}\boxed{d} \sim \frac{(\Delta x)^2}{\Delta t} \sim \frac{\lambda^2}{\tau} = \lambda^2\nu$$

* ambipolar diffusion

$$\vec{\Gamma}_e = \vec{\Gamma}_i = \vec{\Gamma}$$

$$n = n_i = n_e$$

$$\left.\begin{aligned}\vec{\Gamma}_i &= \mu_i n\vec{E} - D_i\nabla n \\ \vec{\Gamma}_e &= -\mu_e n\vec{E} - D_e\nabla n\end{aligned}\right] =$$

$$\mu = \frac{e}{m\nu} \rightarrow \mu_i << \mu_e$$

$$\mu_i n\vec{E} - D_i\nabla n = -\mu_e n\vec{E} - D_e\nabla n$$

$$(\mu_i n + \mu_e n)\vec{E} = (D_i - D_e)\nabla n$$

$$\vec{E} = \frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{n}$$

$$D_a \equiv \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \sim D_i + \frac{\mu_i}{\mu_e}D_e$$

$$= D_i + \frac{\frac{D_i \cancel{e}}{kT_i}}{\frac{D_e\cancel{e}}{kT_e}}\cancel{D_e}$$

$$= D_i + \frac{T_e}{T_i}D_i \sim 2D_i \quad (T_i = T_e)$$

$$\vec{\Gamma}_i = \mu_i \cancel{n}\frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{\cancel{n}} - D_i\nabla n = \left(\frac{\mu_i D_i - \mu_i D_e}{\mu_i + \mu_e} - D_i\right)\nabla n$$

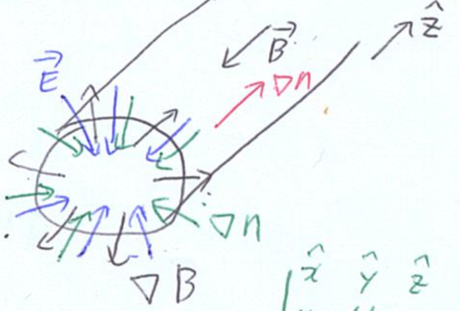
$$= \left(\frac{\cancel{\mu_i D_i} - \mu_i D_e - \cancel{\mu_i D_i} - \mu_e D_i}{\mu_i + \mu_e}\right)\nabla n = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}\nabla n = -D_a\nabla n$$

$$\left(D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}\right)$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(\underbrace{n\vec{u}}_{\vec{\Gamma}}) = S_{source} - S_{sink}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(\overbrace{-D_a\nabla n}^{\Gamma}) = \frac{\partial n}{\partial t} - D_a\nabla^2 n = 0$$

$\vec{B} \neq 0$ Diffusion across $\vec{B}$



Assume the magnetic field is uniform (no gradient) unlike the figure.

$$0 = \pm ne(\vec{E} + \vec{u}\times\vec{B}) - kT\nabla n - mn\nu\vec{u}$$

$$\vec{u} = \pm\frac{e}{m\nu}\vec{E} \pm \frac{ne}{mn\nu}\vec{u}\times\vec{B} - \frac{kT}{m\nu}\frac{\nabla n}{n} = \pm\mu\vec{E} \pm \frac{e}{m\nu}\vec{u}\times\vec{B} - D\frac{\nabla n}{n}$$

$$\begin{vmatrix}\hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ 0 & 0 & B\end{vmatrix}$$

$$\left.\begin{aligned}u_x &= \pm\mu E_x \pm \frac{e}{m\nu}u_y B - \frac{D}{n}\frac{\partial n}{\partial x} \\ u_y &= \pm\mu E_y \mp \frac{e}{m\nu}u_x B - \frac{D}{n}\frac{\partial n}{\partial y}\end{aligned}\right) \Rightarrow u_x = \pm\mu E_x \pm \overbrace{\frac{eB}{m\nu}}^{\omega_c}\left(\pm\mu E_y \mp \frac{e}{m\nu}u_x B - \frac{D}{n}\frac{\partial n}{\partial y}\right) - \frac{D}{n}\frac{\partial n}{\partial x}$$

$$= \pm\mu E_x + \frac{\omega_c}{\nu}\mu E_y - \frac{\omega_c^2}{\nu^2}u_x \mp \frac{\omega_c}{\nu}\frac{D}{n}\frac{\partial n}{\partial y} - \frac{D}{n}\frac{\partial n}{\partial x}$$

$$\mu = \frac{e}{m\nu} = \frac{eB}{m\nu B} = \frac{\omega_c}{\nu B}$$

$$D = \frac{kT}{m\nu} = \frac{eB\,kT}{m\nu\,Be} = \frac{\omega_c kT}{\nu eB}$$

$$\Rightarrow \left(1 + \frac{\omega_c^2}{\nu^2}\right)u_x \doteq \pm\mu E_x - D\frac{1}{n}\frac{\partial n}{\partial x} + \frac{\omega_c^2}{\nu^2}\left(\frac{E_y}{B} \mp \frac{kT}{neB}\frac{\partial n}{\partial y}\right)$$

$$\therefore u_x = \underbrace{\frac{\pm\mu}{1 + \frac{\omega_c^2}{\nu^2}}}_{\pm\mu_\perp}E_x - \underbrace{\frac{D}{1 + \frac{\omega_c^2}{\nu^2}}}_{D_\perp}\frac{1}{n}\frac{\partial n}{\partial x} + \frac{\omega_c^2/\nu^2}{1 + \frac{\omega_c^2}{\nu^2}}\left(\underbrace{\frac{E_y}{B}}_{\frac{E_y B}{B^2}} \mp \underbrace{\frac{kT}{neB}\frac{\partial n}{\partial y}}_{\frac{B\times\nabla P}{nqB^2}}\right)$$

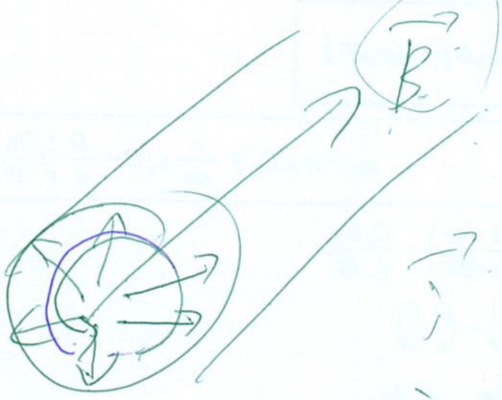
$\frac{E_y B}{B^2} \Rightarrow \vec{u}_E$ E×B drift

$\frac{B\times\nabla P}{nqB^2} = \vec{u}_D$ diamagnetic drift

$\oplus \longleftrightarrow \ominus$ → 궤적 → diffusion → $\vec{E}$ → 전자 붙어

그대로

↳ $\vec{E}$ → diffusion → $\vec{E} \times \vec{B}$ → 회전 → dift



ion.

$$j \times B = \nabla P$$

Solving the Equation with Cylindrical Coordination.

$$0 = \pm ne(\vec{E} + \vec{u} \times \vec{B}) - \nabla P - mn\nu\vec{u}$$

$$\vec{u} = \pm \frac{e}{m\nu}\vec{E} \pm \frac{ne}{mn\nu}\vec{u} \times \vec{B} - \frac{k_B T}{m\nu}\frac{\nabla n}{n}$$

$$\boxed{\text{cross product} \quad \vec{u} \times \vec{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ u_r & u_\theta & u_z \\ 0 & 0 & B \end{vmatrix} = (u_\theta B)\hat{r} - (u_r B)\hat{\theta}}$$

$$\boxed{u_r = \pm \mu E_r \pm \frac{e}{m\nu} u_\theta B - \frac{D}{n}\frac{\partial n}{\partial r} \qquad u_\theta = \pm \mu E_\theta \mp \frac{e}{m\nu} u_r B - \frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial \theta}}$$

$$u_r = \pm \mu E_r \pm \frac{eB}{m\nu}\left\{ \pm \mu E_\theta \mp \frac{e}{m\nu} u_r B - \frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial \theta} \right\} - \frac{D}{n}\frac{\partial n}{\partial r}$$

$$u_r = \pm \mu E_r + \frac{\omega_c}{\nu}\left\{ \mu E_\theta - \frac{\omega_c}{\nu} u_r \mp \frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial \theta} \right\} - \frac{D}{n}\frac{\partial n}{\partial r}$$

$$u_r = \pm \mu E_r + \frac{\omega_c}{\nu}\mu E_\theta - \frac{\omega_c^2}{\nu^2} u_r \mp \frac{\omega_c}{\nu}\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial \theta} - \frac{D}{n}\frac{\partial n}{\partial r}$$

$$\left(1 + \frac{\omega_c^2}{\nu^2}\right) u_r = \pm \mu E_r + \frac{\omega_c}{\nu}\mu E_\theta \mp \frac{\omega_c}{\nu}\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial \theta} - \frac{D}{n}\frac{\partial n}{\partial r}$$

$$\boxed{\mu = \frac{e}{m\nu} = \frac{eB}{m\nu B} = \frac{\omega_c}{B\nu} \qquad D = \frac{k_B T}{m\nu} = \frac{k_B T e B}{m\nu e B} = \frac{k_B T}{eB}\frac{\omega_c}{\nu}}$$

$$u_r = \frac{\pm \mu}{\left(1 + \omega_c^2/\nu^2\right)} E_r - \frac{D}{\left(1 + \omega_c^2/\nu^2\right)}\frac{1}{n}\frac{\partial n}{\partial r} + \frac{\omega_c^2/\nu^2}{\left(1 + \omega_c^2/\nu^2\right)}\left(\frac{E_\theta}{B} \mp \frac{k_B T}{neB}\frac{1}{r}\frac{\partial n}{\partial \theta}\right)$$

$$u_\theta = \frac{\pm \mu}{\left(1 + \omega_c^2/\nu^2\right)} E_\theta - \frac{D}{\left(1 + \omega_c^2/\nu^2\right)}\frac{1}{n}\frac{1}{r}\frac{\partial n}{\partial \theta} - \frac{\omega_c^2/\nu^2}{\left(1 + \omega_c^2/\nu^2\right)}\left(\frac{E_r}{B} \mp \frac{k_B T}{neB}\frac{\partial n}{\partial r}\right)$$

$$\boxed{\mu_\perp = \frac{\mu}{1 + \omega_c^2/\nu^2} \qquad D_\perp = \frac{D}{1 + \omega_c^2/\nu^2}}$$

$$\vec{u} = \pm \mu_\perp \vec{E} - D_\perp \frac{\nabla n}{n} + \frac{\omega_c^2/\nu^2}{\left(1 + \omega_c^2/\nu^2\right)}\left(\frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \nabla P}{nqB^2}\right)$$

$$\therefore \vec{u} = \pm \mu_\perp \vec{E} - D_\perp \frac{\nabla n}{n} + \frac{\omega_c^2/\nu^2}{\left(1 + \omega_c^2/\nu^2\right)}\left(\overrightarrow{u_E} + \overrightarrow{u_D}\right)$$

Diffusion in Plasmas

$$0 = \pm ne(\vec{E} + \vec{u}\times\vec{B}) - k_B T \nabla n - mn\nu\vec{u}$$

i) $B = 0$, weakly ionized $\qquad \vec{\Gamma} = n\vec{u}$

$$\vec{u} = \pm\mu\vec{E} - D\frac{\nabla n}{n}, \quad \mu = \frac{e}{m\nu}, \quad D = \frac{k_B T}{m\nu} \sim \frac{(\Delta x)^2}{\Delta t} = \nu\lambda^2$$

ii) $B \neq 0$, weakly ionized

$$\vec{u}_\perp = \pm\frac{\mu}{1+\omega_c^2/\nu^2}\vec{E} - \frac{D}{1+\omega_c^2/\nu^2}\frac{\nabla n}{n} + \frac{(\vec{u}_E + \vec{u}_D)\,\omega_c^2/\nu^2}{1+\omega_c^2/\nu^2}$$

① $\omega_c^2/\nu^2 \ll 1 \longrightarrow \mu_\perp \sim \mu, \quad D_\perp \sim D$ $\qquad$ (질량 $m\uparrow \rightarrow r_c = \frac{mv_\perp}{B|q|}\uparrow$ → 이동이 더 많다)

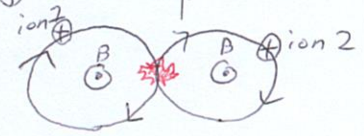
② $\omega_c^2/\nu^2 \gg 1 \rightarrow \mu_\perp \simeq \frac{\mu}{\omega_c^2/\nu^2}, \quad D_\perp \sim \frac{D}{\omega_c^2/\nu^2} = \frac{k_B T\nu}{\omega_c^2 m} \propto \nu$

$$\left(\omega_c = \frac{Be}{m} = \frac{v_\perp}{\frac{mv_\perp}{Be}} = \frac{v_\perp}{r_L}\right)$$

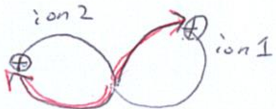
$$\frac{k_B T\nu}{\omega_c^2 m} \approx \frac{mv_\perp^2}{\left(\frac{v_\perp}{r_L}\right)^2 m} = \nu r_L^2 = \frac{(\Delta x)^2}{\Delta t}$$

iii) $B \neq 0$, fully ionized

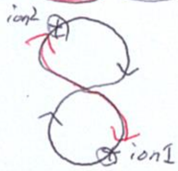
① like-particle collision



→ 180° collision

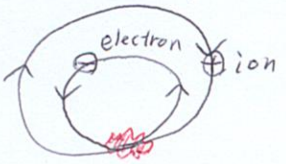


→ 90° collision

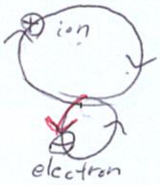


⇒ 거의 영향 X

② unlike-particle collision



→



($\because M \gg m$)

$B\uparrow \Rightarrow D\downarrow$

$T\downarrow \Rightarrow \nu_{ei}\uparrow \Rightarrow D\uparrow$

$n\uparrow \Rightarrow D\uparrow$

$\left(D_\perp = \frac{k_B T}{m\nu}\frac{\nu_D^2}{\omega_c^2} \propto T\right)$ 다른 경향

* Set of MHD equations

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

$$\frac{\partial\sigma}{\partial t} + \nabla\cdot\vec{J} = 0$$

$$\boxed{\rho\cancel{\frac{d\vec{v}}{dt}} = \cancel{\sigma\vec{E}} + \vec{j}\times\vec{B} - \nabla P}$$

0 steady state 작다.

$$\boxed{\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \cancel{\frac{\vec{j}\times\vec{B} - \nabla P}{ne}}}$$

작다.

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0$$

$$\varepsilon_0\nabla\cdot\vec{E} = \sigma$$

steady state plasma

둘이 같다. → $\vec{j}\times\vec{B} = \nabla P = k_B T\nabla n$

$$(\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j})\times\vec{B}$$

$$\vec{E}\times\vec{B} + \vec{B}\times(\vec{B}\times\vec{v}) = \eta\vec{j}\times\vec{B}$$

$$\vec{E}\times\vec{B} - B^2 v_\perp = \eta_\perp k_B T\nabla n$$

$$\vec{v}_\perp = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\eta_\perp k_B T\nabla n}{B^2}$$

$$\vec{\Gamma}_\perp = n\vec{v}_\perp = -\frac{n\eta_\perp k_B T}{B^2}\nabla n = -D_\perp\nabla n$$

$$D_\perp = \frac{n\eta_\perp k_B T}{B^2} \propto T_e^{-\frac{1}{2}}$$

$$\eta_\perp = \frac{m\nu_{ei}}{ne^2} \propto T_e^{-\frac{3}{2}}$$

$B\uparrow \Rightarrow D\downarrow$

$\nu\uparrow \Rightarrow D\uparrow$

$n\uparrow \Rightarrow D\uparrow$

실험식

$$D_\perp = \frac{1}{16}\frac{k_B T}{eB}$$

* Diffusion in Plasma

gas $\quad \vec{\Gamma} = -D\nabla n, \quad D = \frac{(\Delta x)^2}{\Delta t} = \lambda^2 \nu$ (a)

weakly ionized gas $B=0$ $\quad \vec{\Gamma} = \pm n\mu\vec{E} - D\nabla n, \quad D = \frac{k_B T}{m\nu}$ (b)

weakly ionized gas $B \neq 0$ $\quad \vec{\Gamma}_\perp = \pm n\mu_\perp \vec{E} - D_\perp \nabla n + \frac{n\omega_c^2/\nu^2}{1+\omega_c^2/\nu^2}(\vec{u}_E + \vec{u}_D), \quad D_\perp = \frac{k_B T/m\nu}{1+\omega_c^2/\nu^2}$ (c)

fully ionized plasma $B \neq 0$ $\quad \vec{\Gamma}_\perp = \frac{\vec{E}\times\vec{B}}{B^2} - D_\perp \nabla n; \quad D_\perp = \frac{n\eta\Sigma k_B T}{B^2}$ (d) $\quad \eta = \frac{m\nu_{ei}}{ne^2}$

① ν

(a) $D \sim \frac{v^2}{\nu^2}\nu = \frac{v^2}{\nu} \propto \frac{1}{\nu}$

(b) $D = \frac{k_B T}{m\nu} \propto \frac{1}{\nu}$

(c) $D_\perp \sim \frac{k_B T/m\nu}{\omega_c^2/\nu^2} \propto \nu$

(d) $D_\perp = \frac{n \frac{m\nu_{ei}}{ne^2} k_B T}{B^2} \propto \nu_{ei}$

② n $\quad (\nu = n\sigma v)$ $\rightarrow$ cross section $= \pi d^2$

(a) $D \sim \frac{v^2}{n\sigma v} = \frac{v}{n\sigma} \propto \frac{1}{n}$

(b) $D \propto \frac{1}{\nu} \propto \frac{1}{n}$

(c) $D_\perp \propto \nu \propto n$

(d) $D_\perp \propto n\eta = n\frac{m\nu_{ei}}{ne^2} \propto \nu_{ei} = \frac{ne^4}{16\pi\varepsilon_0^2 m^2 v^3} \propto \frac{n}{v^3} \propto nT^{-\frac{3}{2}}$

③ T

(a) $D \sim \frac{v}{n\sigma} \propto v \propto \sqrt{T}$

(b) $D = \frac{k_B T}{mn\sigma v} \propto \frac{T}{v} \propto \sqrt{T}$

(c) $D_\perp \propto k_B T \nu \propto T v \propto T^{\frac{3}{2}}$

(d) $D_\perp \propto \eta T = \frac{m\nu_{ei}}{ne^2} T \propto T^{-\frac{3}{2}} T \propto T^{-\frac{1}{2}}$

④ m $\quad (k_B T \doteqdot \frac{1}{2}mv^2 = \text{const})$

(a) $D \propto v \propto \frac{1}{\sqrt{m}}$

(b) $D \propto v \propto \frac{1}{\sqrt{m}}$

(c) $D_\perp \sim \frac{mv^2/mn\sigma v}{B^2e^2/m^2n^2\sigma^2v^2} \propto m^2 v^3 \propto \sqrt{m}$ $\quad r_L$ 때문

(d) $D_\perp \propto \eta T \propto m\nu_{ei} m v^2 \propto \frac{m^2 v^2}{m^2 v^3} \propto \sqrt{m}$

⑤ B

(a) X

(b) X

(c) $D_\perp \propto \frac{1}{\omega_c^2} \propto \frac{1}{B^2}$

(d) $D_\perp \propto \frac{1}{B^2}$

예측

$D_i \sim 40 D_e$

$D \sim 10^{-4}\,m^2/s$

$D_\perp \sim \frac{T^{-\frac{1}{2}}}{B^2}$

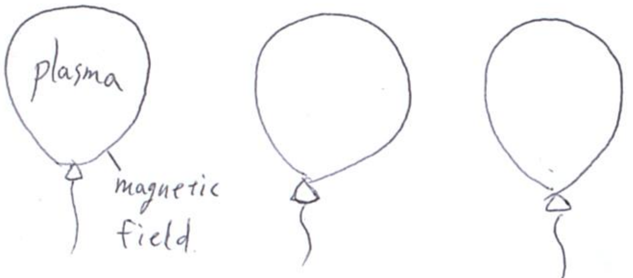
실험

$D_i \sim D_e$

$D \sim 1\,m^2/s$

$D_B = \frac{1}{16}\frac{k_B T}{eB}$

Anomalous transport



1. equilibrium — force balance
2. Stability
3. transport (기공들을 막는다)

equilibrium (평형)

non-stable (perturbation시 되돌아 갈수 없다.)

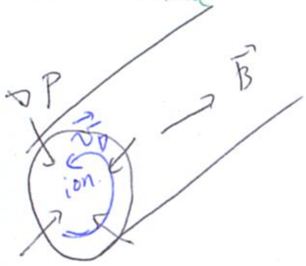
$$\underline{w = w_r + i w_i}$$

$n = n_0 + n_1$, $n_1 = \bar{n}_1 \exp i(kx - \omega t)$

$\propto \exp w_i t$

$w_i > 0 \rightarrow$ unstable

$w_i < 0 \rightarrow$ stable



$$\vec{v}_D = \frac{\vec{B} \times \nabla P}{n q B^2}$$

$$\vec{j}_D = n_i q_i \vec{u}_D^i + n_e q_e \vec{u}_D^e$$

$n_i = n_e$ $\qquad (\nabla P = \nabla P_i + \nabla P_e)$

$$\vec{j}_D = \frac{\vec{B} \times \nabla P_i}{B^2} + \frac{\vec{B} \times \nabla P_e}{B^2} = \frac{\vec{B} \times \nabla P}{B^2}$$

MHD Eq.

$$\cancel{\frac{\partial \rho}{\partial t}}^{\,0} + \cancel{\nabla \cdot (\rho \vec{v})}^{\,0} = 0$$

$$\cancel{\frac{\partial \sigma}{\partial t}}^{\,0} + \nabla \cdot \vec{j} = 0$$

$$\cancel{\rho \frac{d\vec{v}}{dt}}^{\,0} = \cancel{\sigma \vec{E}}^{\text{order 낮음}} + \vec{j} \times \vec{B} - \nabla P$$

$$\vec{E} + \cancel{\vec{v} \times \vec{B}}^{\,0} = \eta \vec{j} + \cancel{\frac{\vec{j} \times \vec{B} - \nabla P_e}{ne}}^{\text{order 낮음}}$$

$$\varepsilon_0 \nabla \cdot \vec{E} = \cancel{\sigma}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = \cancel{-\frac{\partial \vec{B}}{\partial t}}^{\,0}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \cancel{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}^{\,0}$$

Assuption $\frac{\partial}{\partial t} = 0$

$\vec{v} = 0$

$$\Rightarrow \boxed{\vec{j} \times \vec{B} = \nabla P}$$

$$\nabla \cdot \vec{B} = 0$$

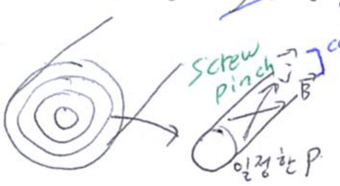
$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\vec{E} = \eta \vec{j}$$

$$\vec{B} \times \boxed{(\vec{j} \times \vec{B})} = \vec{B} \times \boxed{\nabla P}$$

$$\parallel$$

$$\vec{j}(\vec{B} \cdot \vec{B}) - \vec{B}(\vec{B} \cdot \vec{j}) \Rightarrow \vec{j}_\perp B^2 = \vec{B} \times \nabla P \quad \therefore \vec{j}_\perp = \frac{\vec{B} \times \nabla P}{B^2} = \vec{j}_D$$



$\nabla P \perp \vec{j}$

$\nabla P \perp \vec{B}$

Concept of β

$$\nabla P = \vec{j} \times \vec{B} = \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B} = \frac{1}{\mu_0} \left[(\vec{B} \cdot \nabla) \vec{B} - \frac{1}{2} \nabla B^2 \right] \Rightarrow \nabla \left(P + \frac{B^2}{2\mu_0} \right) = \cancel{\frac{1}{\mu_0} (\vec{B} \cdot \nabla) \vec{B}}$$

if $\nabla B = 0$

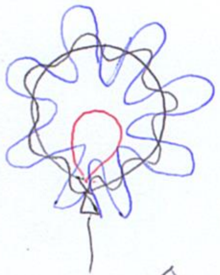
actually order 낮음

$$\nabla \left(P + \frac{B^2}{2\mu_0} \right) = 0 \rightarrow P + \boxed{\frac{B^2}{2\mu_0}} = \text{constant}$$

magnetic field pressure

$$\beta \equiv \frac{P}{B^2/2\mu_0} = \frac{\text{Plasma particle } P}{\text{magnetic field } P}$$

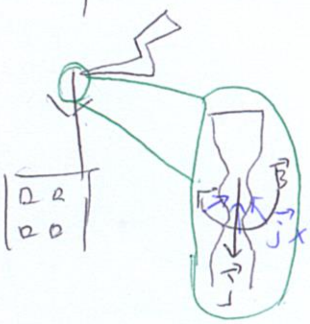
Plasma Equilibrium

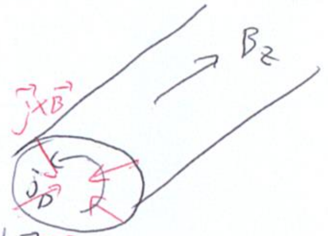


force balance
= equilibrium

perturbation ↓
= stable

transport
= diffusion



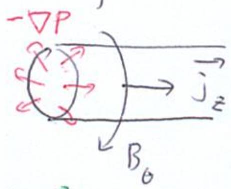


$$\cancel{\rho \frac{d\vec{v}}{dt}} = \vec{j}\times\vec{B} - \nabla P$$

$$\vec{v}_D = \frac{\vec{B}\times\nabla P}{n q B^2}$$

$$\vec{j}_D = \frac{\vec{B}\times\nabla P}{B^2}$$

* z - pinch



$$\begin{cases}\nabla\times\vec{B} = \mu_0\vec{j} \\ \vec{j}\times\vec{B} = \nabla P\end{cases}$$

$$\begin{cases}\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 j_z \\ -j_z B_\theta = \frac{\partial P}{\partial r}\end{cases}$$

$$\nabla\times\vec{B} = \frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & rB_\theta & 0\end{vmatrix}$$

$$\vec{j}\times\vec{B} = \begin{vmatrix}\hat{r} & \hat{\theta} & \hat{z} \\ 0 & 0 & j_z \\ 0 & B_\theta & 0\end{vmatrix}$$

$$-\frac{B_\theta}{r\mu_0}\frac{\partial}{\partial r}(rB_\theta) = \frac{\partial P}{\partial r}$$

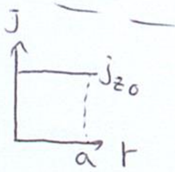
$$-\frac{B_\theta}{r\mu_0}\left(B_\theta + r\frac{\partial B_\theta}{\partial r}\right) = \frac{\partial P}{\partial r}$$

$$-\frac{B_\theta^2}{\mu_0 r} - \frac{B_\theta}{\mu_0}\frac{\partial B_\theta}{\partial r} = \frac{\partial P}{\partial r}$$

$$-\frac{B_\theta^2}{\mu_0 r} - \frac{1}{2\mu_0}\frac{\partial B_\theta^2}{\partial r} = \frac{\partial P}{\partial r}$$

$$[N/m^3] \quad \frac{\partial}{\partial r}\left(P + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

magnetic tension force



$$I = \pi a^2 j_{z0} \qquad P(a) = 0$$

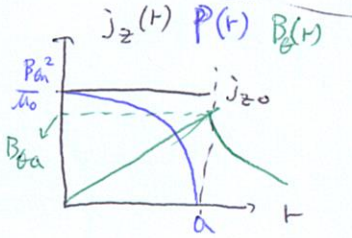
$$B_{\theta a} = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{2} j_{z0}\, a$$

$$B_{\theta r} = \frac{\mu_0 I_r}{2\pi r} = \frac{\mu_0}{2} j_{z0}\, r = \frac{r}{a} B_{\theta a}$$

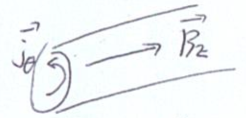
$$I_r = \pi r^2 j_{z0}$$

$$P_r = \frac{B_{\theta a}^2}{\mu_0}\left(1 - \frac{r^2}{a^2}\right) \qquad \therefore \beta = 2\left(\frac{a^2}{r^2} - 1\right)$$

$$\langle \beta_\theta \rangle = \frac{\langle P \rangle}{B_{\theta a}^2/2\mu_0}, \quad \langle P \rangle = \frac{\int_0^a 2\pi r P(r)\, dr}{\pi a^2} = \frac{B_{\theta a}^2}{2\mu_0}$$



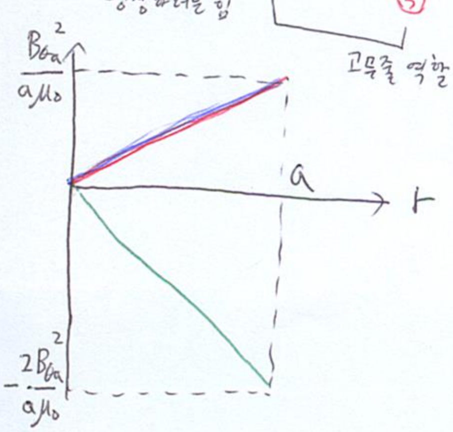
* θ - pinch



$$\underbrace{\frac{\partial}{\partial r}\Big(\underbrace{P}_{①} + \frac{B_\theta^2}{2\mu_0}\Big)}_{②} + \underbrace{\frac{B_\theta^2}{\mu_0 r}}_{③} = 0$$

① 팽창하려는 힘

고무줄 역할

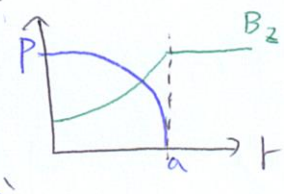


$$\rho \frac{d\vec{v}}{dt} = \vec{j}\times\vec{B} - \nabla p$$

$$\frac{d\vec{v}}{dt} = \cancel{\frac{\partial \vec{v}}{\partial t}} + \cancel{(\vec{v}\cdot\nabla)\vec{v}}$$

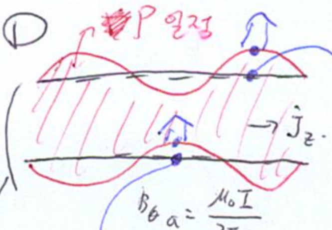
steady state

속도가 느릴 때만



①

P 일정



$\frac{\partial}{\partial r}\frac{B_\theta^2}{2\mu_0}$ 작고 $\frac{\partial P}{\partial r}$는 크다 → 더 늘린다.

$$B_{\theta a} = \frac{\mu_0 I}{2\pi a}$$

sausage instability

$\frac{\partial}{\partial r}\frac{B_\theta^2}{2\mu_0}$ 크고, $\frac{\partial P}{\partial r}$는 작다. → 더 조인다.

$$\begin{cases}\nabla\times\vec{B} = \mu_0\vec{j} \\ \vec{j}\times\vec{B} = \nabla P\end{cases}$$

$$\begin{cases}-\frac{\partial B_z}{\partial r} = \mu_0 j_\theta \\ j_\theta B_z = \frac{\partial P}{\partial r}\end{cases}$$

$$-\frac{B_z}{\mu_0}\frac{\partial B_z}{\partial r} = \frac{\partial P}{\partial r}$$

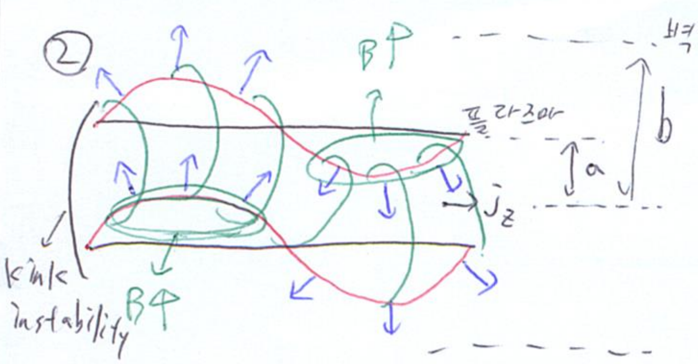
$$\Downarrow$$

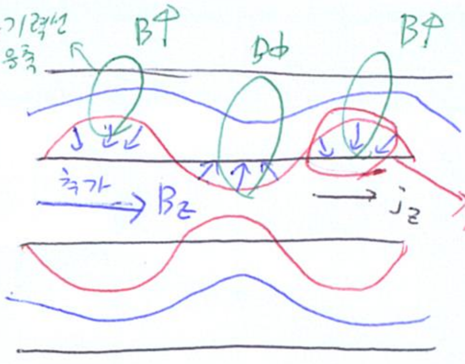
$$-\frac{\partial}{\partial r}\left(\frac{B_z^2}{2\mu_0}\right) = \frac{\partial P}{\partial r}$$

$$\Downarrow$$

$$\frac{\partial}{\partial r}\left(P + \frac{B_z^2}{2\mu_0}\right) = 0$$

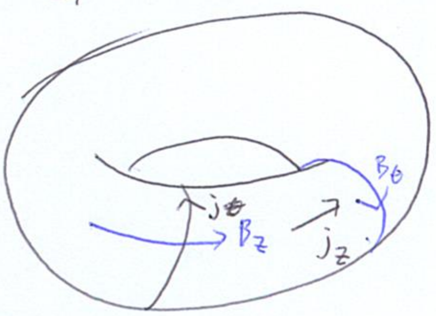
②

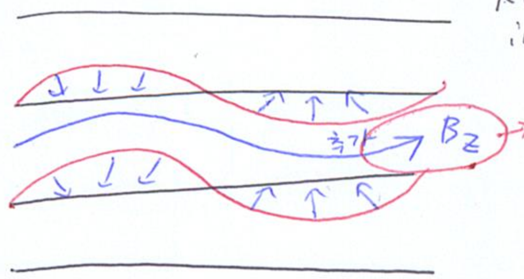




sausage instability 해결
magnetic pressure

Grad-Shafranov Equation



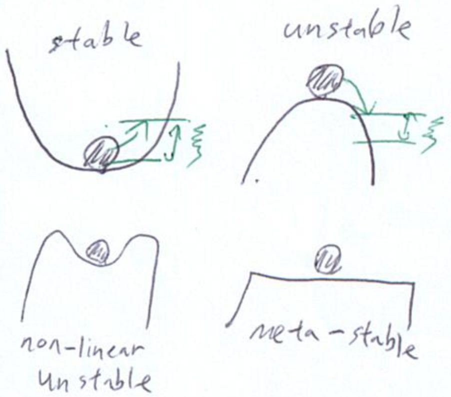


kink instability 해결
magnetic tension

Plasma Instability

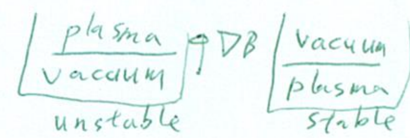
non-uniformity $\xrightarrow[\text{perturbation}]{}$ Instability

$\nabla P, j$

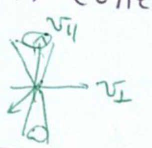


- streaming instability ; $\vec{j}$, fast particle
- Rayleigh-Taylor instability ; external force
  ↳ ∇p에 의해 생김.

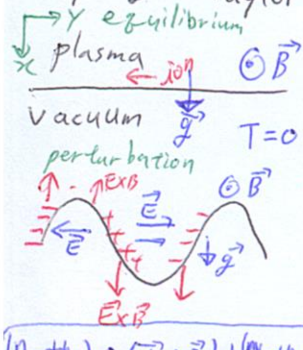
- Universal instability ; ∇n, ∇p → drift wave
  약간 차이 → oscilliation → wave



- Velocity space instability ; loss cone instability
  (kinetic instability)



Rayleigh-Taylor instability



$$\vec{u}_{i0} = \frac{M\vec{g}\times\vec{B}}{eB^2} \qquad \vec{u}_{e0} \simeq 0$$

equilibrium

$$n_0 M\left[\cancel{\frac{\partial \vec{u}_0}{\partial t}}_{\text{초기속도}} + \cancel{(\vec{u}_0\cdot\nabla)}_{\text{공간일정}}\vec{u}_0\right] = n_0 e(\cancel{\vec{E}_0}_{\text{초기엔 0}} + \vec{u}_0\times\vec{B}) + n_0 M\vec{g}$$

$$\Rightarrow e(\vec{u}_0\times\vec{B}) + M\vec{g} = 0$$

$$\boxed{(n_0+n_1)e(\vec{u}_0\times\vec{B}) + (n_0+n_1)M\vec{g} = 0}$$

$$nM\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = ne(\vec{E}+\vec{u}\times\vec{B}) + nM\vec{g}$$

$u_1 \propto \exp i(ky-\omega t)$

$$n = n_0 + n_1, \quad \vec{u} = \vec{u}_0 + \vec{u}_1, \quad \vec{E} = \cancel{\vec{E}_0} + \vec{E}_1$$

$\nabla\cdot\vec{u}_0 = 0$, $\frac{\partial \vec{u}_0}{\partial t} = 0$

$$\Rightarrow n_0 M\frac{\partial \vec{u}_1}{\partial t} + n_0 M\vec{u}_0\cdot\nabla\vec{u}_1 = n_0 e\vec{E}_1 + n_0 e\vec{u}_1\times\vec{B}$$

Electron plasma wave

$$\frac{\partial n_0}{\partial t} + \nabla\cdot(n_e\vec{u}_e) = 0$$

$$n_e m\left[\frac{\partial\vec{u}_e}{\partial t} + (\vec{u}_e\cdot\nabla)\vec{u}_e\right] = -n_e e(\vec{E} + \cancel{\vec{u}_e\times\vec{B}}) - \gamma k_B T\nabla n_e$$

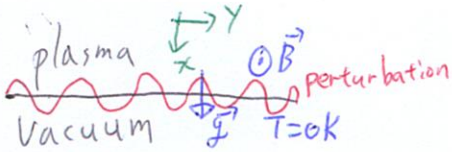
$$\varepsilon_0\nabla\cdot\vec{E} = e(n_i - n_e)$$

$$\hookrightarrow \omega^2 = \omega_p^2 + \frac{\gamma k_B T}{m}k^2 \quad \left(\omega_p^{-2} = \frac{m\varepsilon_0}{ne^2}\right)$$

$$n_1 = \bar{n}_1 \exp i(kx - \omega t)$$
$$= \bar{n}_1 \exp i(kx - \omega_r t - i\gamma t)$$
$$= \bar{n}_1 \exp i(kx - \omega_r t)\exp(\gamma t)$$

- $\gamma > 0$; unstable
  growth rate
- $\gamma < 0$; stable

Rayleigh - Taylor instability



$$\vec{u}_{i0} = \frac{M\vec{g}\times\vec{B}}{eB^2}, \quad \vec{u}_{e0} \simeq 0$$

* equilibrium $\frac{\partial \vec{u}_0}{\partial t} = 0,\ \nabla\cdot\vec{u}_0 = 0,\ \vec{E}_0 = 0$

Eq. ①

$$n_0 M\left[\cancel{\frac{\partial \vec{u}_0}{\partial t}} + \cancel{(\vec{u}_0\cdot\nabla)\vec{u}_0}\right] = n_0 e\left(\cancel{\vec{E}_0} + \vec{u}_0\times\vec{B}\right) + n_0 M\vec{g} \Rightarrow \boxed{e(\vec{u}_0\times\vec{B}) + M\vec{g} = 0}$$

* perturbation $\left(\frac{\partial \vec{u}_0}{\partial t} = 0\right)$ $(\vec{u}_1\cdot\nabla\vec{u}_1 = 0)$ $(\nabla\cdot\vec{u}_0 = 0)$ $(n_1\vec{E}_1 = 0)$ $(\vec{E}_0 = 0)$

$$(n_0 + \cancel{n_1})M\left[\frac{\partial}{\partial t}(\cancel{\vec{u}_0} + \vec{u}_1) + \left((\vec{u}_0 + \cancel{\vec{u}_1})\cdot\nabla\right)(\cancel{\vec{u}_0} + \vec{u}_1)\right] = (n_0 + \cancel{n_1})e\left(\cancel{\vec{E}_0} + \vec{E}_1 + (\vec{u}_0 + \vec{u}_1)\times\vec{B}\right) + (n_0 + n_1)M\vec{g}$$

$\left(n_1 \frac{\partial \vec{u}_1}{\partial t} = 0\right)$

$$\Rightarrow n_0 M\frac{\partial \vec{u}_1}{\partial t} + n_0 M(\vec{u}_0\cdot\nabla)\vec{u}_1 = n_0 e\vec{E}_1 + \cancel{(n_0+n_1)e\vec{u}_0\times\vec{B} + (n_0+n_1)M\vec{g}} + (n_0 + n_1)e\vec{u}_1\times\vec{B}$$

$(\because \text{Eq. ①})$

$(n_1 u_1 = 0)$

$$\Rightarrow n_0 M\frac{\partial \vec{u}_1}{\partial t} + n_0 M(\vec{u}_0\cdot\nabla)\vec{u}_1 = n_0 e\vec{E}_1 + (n_0 + \cancel{n_1})e\vec{u}_1\times\vec{B} \quad (u_i \propto \exp i(ky - \omega t))$$

$$\Rightarrow \cancel{n_0} M(-i\omega\vec{u}_1) + \cancel{n_0} M u_0(ik\vec{u}_1) = \cancel{n_0} e\,\vec{u}_1\times\vec{B} + \cancel{n_0} e\vec{E}_1$$

$$\left(\vec{u}_1\times\vec{B} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ u_{1x} & u_{1y} & 0\\ 0 & 0 & B\end{vmatrix}\right)$$

$$\hat{x}: -iM(\omega - ku_0)u_{1x} = e\cancel{E_{1x}} + eu_{1y}B$$

$$\Rightarrow \hat{y}: -iM(\omega - ku_0)u_{1y} = eE_{1y} - eu_{1x}B$$

$$\Rightarrow \begin{bmatrix}-iM(\omega - ku_0) & -eB\\ eB & -iM(\omega - ku_0)\end{bmatrix}\begin{bmatrix}u_{1x}\\ u_{1y}\end{bmatrix} = \begin{bmatrix}0\\ eE_{1y}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}u_{1x}\\ u_{1y}\end{bmatrix} = \frac{1}{e^2B^2 - \cancel{M^2(\omega - ku_0)^2}}\begin{bmatrix}-iM(\omega - ku_0) & eB\\ -eB & -iM(\omega - ku_0)\end{bmatrix}\begin{bmatrix}0\\ eE_{1y}\end{bmatrix} \simeq \begin{bmatrix}\frac{\cancel{e^2}B}{\cancel{e^2}B^2}E_{1y}\\ -\frac{i\cancel{e}M}{e^{\cancel{2}}B^2}(\omega - ku_0)E_{1y}\end{bmatrix}$$

Assumption : $e^2B^2 \gg M^2(\omega - ku_0)^2$

$\left(\Omega_c = \frac{eB}{M}\right)$ cyclotron frequency $\quad (\Omega_c)^2 \gg (\omega - ku_0)^2$

$$\Rightarrow \begin{aligned} u^i_{1x} &= \frac{E_{1y}}{B}\\ u^i_{1y} &= -i\frac{\omega - ku_0}{\Omega_c B}E_{1y}\end{aligned}$$

as the same way

$$\Rightarrow \quad u^e_{1x} = \frac{E_{1y}}{B} \quad \left(\Omega_c = \frac{eB}{M}\right)$$

$$u^e_{1y} \simeq 0 \quad \left(\omega_c = \frac{eB}{m}\right)$$

Continuity equation

$\star$ equilibrium

$$\cancel{\frac{\partial n_0}{\partial t}}^{\,0} + \boxed{\nabla \cdot (n_0 \vec{u}_0) = 0} \quad (\text{Equ ②})$$

$\star$ perturbation

$$\frac{\partial}{\partial t}(\cancel{n_0}^{\,0} + n_1) + \nabla \cdot \left[(n_0 + n_1)(\vec{u}_0 + \vec{u}_1) \right] = 0$$

$$\Rightarrow \frac{\partial n_1}{\partial t} + \underbrace{\cancel{\nabla \cdot (n_0 \vec{u}_0)}}_{(\because \text{Eq. ②})\ 0} + \nabla \cdot (n_0 \vec{u}_1) + \nabla \cdot (\vec{u}_0 n_1) + \cancel{\nabla \cdot (n_1 \vec{u}_1)}^{\,0} = 0$$

$$\Rightarrow \frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \vec{u}_1 + (\vec{u}_1 \cdot \nabla) n_0 + n_1 \cancel{\nabla \cdot \vec{u}_0}^{\,0} + (\vec{u}_0 \cdot \nabla) n_1 = 0$$

$\frac{\partial n_0}{\partial x} + \cancel{\frac{\partial n_0}{\partial y}}^{\,0}$ y축 isotropic

$$\Rightarrow \left(-i\omega n_1 + i k n_0 u^i_{1y} + u^i_{1x} n_0' + i k u^i_0 n_1 = 0 \right) \ (\text{ion})$$

$(n_i = n_e)$

$$\left(-i\omega n_1 + \underbrace{\cancel{i k n_0 u^e_{1y}}}_{u^e_{1y} \simeq 0} + n_0' u^e_{1x} + \underbrace{\cancel{i k n_1 u^e_0}}_{u^e_0 \simeq 0} = 0 \right) \ (\text{electron})$$

$$\Rightarrow -i\omega n_1 + n_0' \frac{E_{1y}}{B} = 0 \longrightarrow \boxed{n_1 = \frac{E_{1y}}{i\omega B} n_0'} \quad \text{Eq. ③}$$

$$\Rightarrow -\cancel{i}\cancel{\omega} \underbrace{\left(\frac{E_{1y}}{\cancel{i\omega} B} n_0' \right)}_{\text{Eq. ③}} + \cancel{i} k n_0 \underbrace{\left(\cancel{+i} \frac{\omega - k u_0}{\Omega_c B} E_{1y} \right)}_{u^i_{1y}} + \underbrace{n_0' \frac{E_{1y}}{B}}_{u^i_{1x}} + \cancel{i} k u^i_0 \underbrace{\left(\frac{E_{1y}}{\cancel{i}\omega B} n_0' \right)}_{\text{Eq. ③}} = 0$$

양변에 곱해서 약분

$$\Rightarrow -\cancel{\frac{E_{1y}}{B}} n_0' + k n_0 \frac{\omega - k u_0}{\Omega_c \cancel{B}} \cancel{E_{1y}} + \cancel{\frac{E_{1y}}{B} n_0'} + k u^i_0 \frac{\cancel{E_{1y}}}{\omega \cancel{B}} n_0' = 0$$

$$\vec{g} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ g & 0 & 0 \\ 0 & 0 & B \end{vmatrix}$$

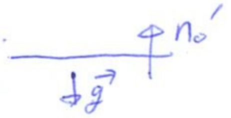
$$\Rightarrow k n_0 \frac{\omega - k u_0}{\Omega_c} + k u^i_0 \frac{n_0'}{\omega} = 0 \qquad \left(u^i_0 = \frac{M \vec{g} \times \vec{B}}{e B^2} = -\frac{g}{\Omega_c} \hat{y} \right)$$

$$\Rightarrow \cancel{k} n_0 \frac{\omega - k u_0}{\cancel{\Omega_c}} - \frac{\cancel{k}}{\omega} \frac{g}{\cancel{\Omega_c}} n_0' = 0 \Rightarrow n_0 (\omega - k u_0) - \frac{g n_0'}{\omega} = 0$$

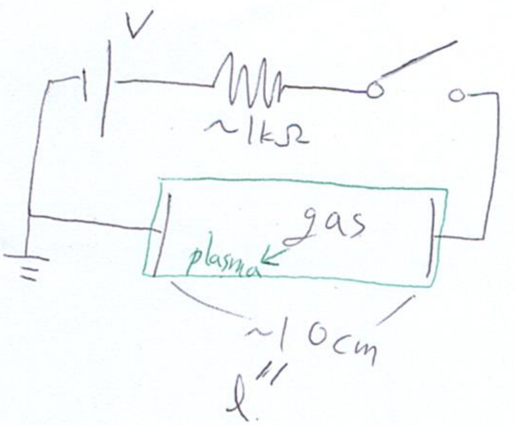
$$\Rightarrow \omega^2 - k u_0 \omega - g \frac{n_0'}{n_0} = 0 \Rightarrow \mathrm{Im} \left[\omega = \frac{k u_0 \pm \sqrt{k^2 u_0^2 + 4 g \frac{n_0'}{n_0}}}{2} \right] \begin{matrix} > 0 \Rightarrow \text{unstable} \\ < 0 \Rightarrow \text{stable} \end{matrix}$$

must be $\ominus$

$$k^2 u_0^2 + 4 g \frac{n_0'}{n_0} < 0 \longrightarrow \text{imaginary number}$$



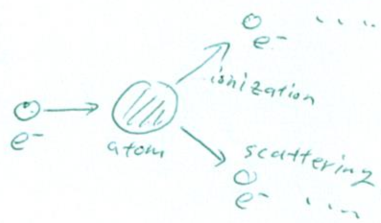
Plasma Breakdown



$V = V_s \rightarrow$ breakdown
(방전 개시 전압) (방전, 절연파괴)

seed electron
- cosmic ray
- radiation
- UV-waves

avalanche



$$\begin{cases} I = I_0 e^{\alpha l} \\ \dfrac{\alpha}{p} = A \exp\left(-\dfrac{B}{E/p}\right) \end{cases}$$

pressure

1) α-작용 (e)

α: 단위 길이당 이온화 수 ($n\alpha dl = dn$)

$$\Downarrow$$

$$n = n_0 e^{\alpha l}$$

$$\Downarrow$$

$$I = I_0 e^{\alpha l}$$

δ: 전자가 이온화 에너지를 얻을 만큼 이동한 거리

$(V_I = E\delta)$ $\lambda > \delta \Rightarrow$ avalanche 가능

$$\frac{n}{N} = \exp\left(-\frac{\delta}{\lambda}\right) \quad (n = N\lambda\alpha)$$

(n: 이온화 개수, N: 전체 개수)

$$\Downarrow$$

$$\lambda\alpha = \exp\left(-\frac{\delta}{\lambda}\right)$$

$$\Downarrow$$

$$\frac{\lambda\alpha}{p} = \frac{1}{p}\exp\left(-\frac{V_I/\lambda p}{E/p}\right)$$

$$\therefore \frac{\alpha}{p} = \frac{1}{p\lambda}\exp\left(-\frac{V_I/\lambda p}{E/p}\right)$$

$$A = \frac{1}{p\lambda}, \quad B = \frac{V_I}{p\lambda}$$

↳ constant

2) β-작용 (i)

$$X^+ + Y \rightarrow X^+ + Y^+ + e^-$$

Head-on collision

$m_1 \quad m_2$

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$
$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

$$v_2' = \frac{2m_1 v_1}{m_1 + m_2}$$

energy loss factor

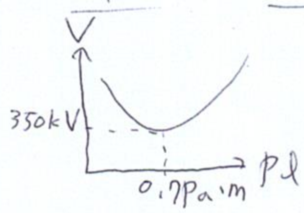
$$K = \frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_1 v_1^2} = \frac{4m_1 m_2}{(m_1+m_2)^2}$$

다른 충돌 고려하면

$$\bar{K} = \frac{2m_1 m_2}{(m_1+m_2)^2} \quad (m_1 \gg m_2)$$

$$\simeq \frac{2m_2}{m_1} \sim 10^{-4} \quad (\because β작용은 매우 작다.)$$

3) γ-작용 (secondary e^-)
(plasma가 유지되는 현상 설명)

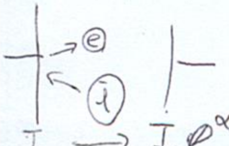


$$\frac{\alpha}{p} = A\exp\left(-\frac{B}{E/p}\right) \quad (V_s = El)$$

$$\alpha l = Apl \exp\left(-\frac{B}{E/p}\right) = \Phi = \ln\left(\frac{1}{\gamma}+1\right)$$

$$-\frac{B}{V_s/pl} = \ln\frac{\Phi}{Apl}$$

$$\boxed{V_s} = \frac{B\boxed{pl}}{\ln\frac{A\boxed{pl}}{\Phi}}$$



$$\eta = \gamma(e^{\alpha l} - 1)$$

$I_0 \rightarrow I_0 e^{\alpha l}$

$$I = I_0 e^{\alpha l} + \eta I_0 e^{\alpha l} + \eta^2 I_0 e^{\alpha l} + \cdots = \frac{I_0 e^{\alpha l}}{1-\eta} \quad (\eta < 1)$$

ion: $I_0 e^{\alpha l} - I_0$

if) $\eta = 1 \rightarrow$ breakdown condition $\rightarrow e^{\alpha l} = \frac{1}{\gamma} + 1 \rightarrow \alpha l = \Phi = \ln\left(\frac{1}{\gamma}+1\right)$

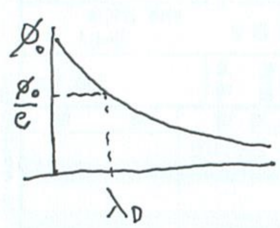
↓

electron: $\gamma(I_0 e^{\alpha l} - I_0) \rightarrow$ 다시 도달: $\gamma(I_0 e^{\alpha l} - I_0)e^{\alpha l} = \eta I_0 e^{\alpha l} \rightarrow \cdots$

Summary

* Plasma

: A plasma is a quasineutral gas of neutral and charged particles which exhibit collective behaviour.



① $L \gg \lambda_D$: quasineutrality

② $\frac{4}{3}\pi \lambda_D^3 n = N \gg 1$: collective behaviour

③ $\omega \tau \gg 1$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_b T_e}{n e^2}}$$

* Single ptl Approach

- uniform $\vec{E}$ $\leftarrow \ominus$, $\oplus \rightarrow$ $\vec{E} \rightarrow$

- uniform $\vec{B}$ $\odot \vec{B}$

$W_c = \frac{B|q|}{m}$

$r_L = \frac{m v_\perp}{B|q|}$

- $\vec{E} \times \vec{B}$ drift $\vec{v}_E = \frac{\vec{E} \times \vec{B}}{B^2}$

- general force $\vec{F}_E = q\vec{E}$ $\vec{v}_E = \frac{q\vec{E} \times \vec{B}}{qB^2} = \frac{\vec{F}_E \times \vec{B}}{qB^2}$

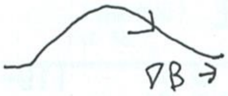
$\Rightarrow \vec{v}_F = \frac{\vec{F} \times \vec{B}}{qB^2}$

- gravitational drift $\vec{v}_g = \frac{M\vec{g} \times \vec{B}}{qB^2}$

- ∇B $\vec{v}_{\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\vec{B} \times \nabla B}{B^2}$

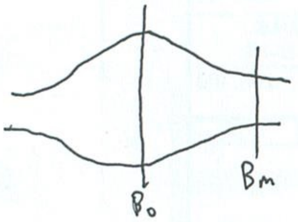
- Curvature $\vec{v}_R = \frac{m v_\parallel^2 \vec{R}_c \times \vec{B}}{R_c^2 \, qB^2}$

-



adiabatic invariant $\mu = \frac{\frac{1}{2} m v_\perp^2}{B}$

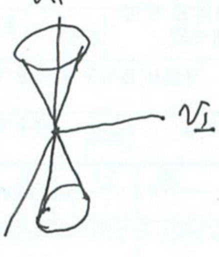
- magnetic mirror



$B_m > B_r$

$\sin^2\theta > \frac{1}{R_m}$

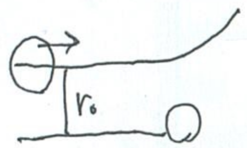
$\frac{1}{R_m} = \frac{B_0}{B_m}$

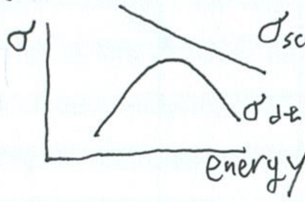


KSTAR $10^{20} [\#/m^3]$

3 GHz $\sim 10^3$ years/m^3

* Rutherford Scattering





* Kinetic Approach

molecule partial current $J_+ = \frac{1}{4} n \bar{c} A$ momentum partial current $\dot{p}_+ = \frac{1}{6} n m \bar{c^2} A$

$p = (\dot{p}_+ + \dot{p}_-)/A = \frac{1}{3} n m \bar{c^2}$ distribution function $f(x, y, z, v_x, v_y, v_z, t)$ $\bar{v^2} = \frac{\int v^2 f d\vec{v}}{\int f d\vec{v}}$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{q(\vec{E} + \vec{v} \times \vec{B})}{m} \cdot \frac{\partial f}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_c = \frac{df}{dt}$$

* Fluid Approach

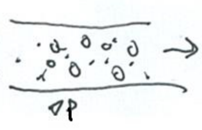
$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$: continuity eq.

$nm\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right] = nq(\vec{E} + \vec{u} \times \vec{B}) - \nabla p + R$

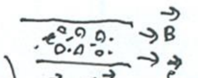
$p = C(nm)^\gamma$

$\perp$: $\vec{u}_D = \frac{\vec{B} \times \nabla P}{nqB^2}$

$\parallel$:



$n_e = n_0 \exp\left(\frac{e\phi}{kT}\right)$



$\vec{E} = \eta \vec{j}$ $\eta = \frac{m \nu_{ei}}{ne^2}$

Single Fluid (MHD) Eq.

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \vec{v}) = 0$$

$$\rho \frac{d\vec{v}}{dt} = \sigma \vec{E} + \vec{j}\times\vec{B} - \nabla p$$

$$\vec{E} + \vec{v}\times\vec{B} = \eta \vec{j} + \frac{\vec{j}\times\vec{B} - \nabla p_e}{ne}$$

Ideal MHD eq.

$$\frac{\partial \sigma}{\partial t} + \nabla\cdot\vec{j} = 0$$

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$

* Plasma Waves

- Plasma oscillations

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$T = 0$

$W_p = \sqrt{\frac{n_0 e^2}{m \varepsilon_0}}$

$n = n_0 + n_1$, $\vec{u} = \vec{u}_1$, $\vec{E} = \vec{E}_1$

$\frac{\partial n_0}{\partial t} = 0$

$n_1 = \bar{n}_1 \exp i(kx - wt)$

$D(w,k) = 0$ $\quad ( )u_1 = ( )n_1$

Dispersion relation

- Electron plasma wave

$T \neq 0$ $\quad w^2 = w_p^2 + \frac{\gamma k_b T_e}{m} k^2$

- ion acoustic (sound) wave

$w^2 = \frac{k_b T_e + \gamma k_b T_i}{M} k^2$

+ +
+ +
+ +

- EM wave

$w^2 = k^2 c^2 + w_p^2$

- resonance $\quad k \to \infty$

- cutoff $\quad k \to 0$

* Plasma Diffusion (Transport)

$D = \frac{(\Delta x)^2}{\Delta t}$ $\quad \vec{\Gamma} = -D\nabla n$ $\quad \frac{\partial n}{\partial t} + \nabla\cdot(n\vec{u}) = 0$ $\quad \vec{\Gamma}$

diffusion coefficient D

gas, weakly ionised ($B = 0$, $B \neq 0$), fully ionised ($B\neq 0$)

experimental

gas: $D = \frac{k_b T}{m\nu}$ $\quad$ weakly $B = 0$: $D = \frac{k_b T}{m\nu}$ $\quad$ weakly $B \neq 0$: $D_\perp$: $\frac{k_b T / m\nu}{1 + w_c^2/\nu^2}$

fully ionized ($B \neq 0$): $D_\perp = \frac{n\eta \varepsilon k_b T}{B^2}$ $\quad$ experimental : $D_\perp^{B} = \frac{1}{16}\frac{k_b T}{eB}$ ← Bohm diffusion

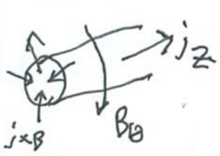
$\nu \propto T_e^{-3/2}$

 ← banana orbit

* Plasma Equilibrium & Stability

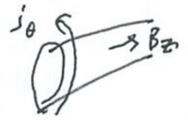
$\vec{j}\times\vec{B} = \nabla p$ $\quad \nabla\times\vec{B} = \mu_0 \vec{j}$ $\quad \nabla\cdot\vec{B} = 0$ $\quad \beta = \frac{p}{B^2/2\mu_0}$

- Z-pinch



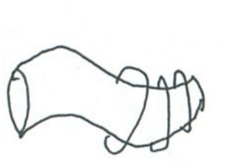
$$\frac{\partial}{\partial r}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \underbrace{\frac{B_\theta^2}{\mu_0 r}}_{\text{tension}} = 0$$

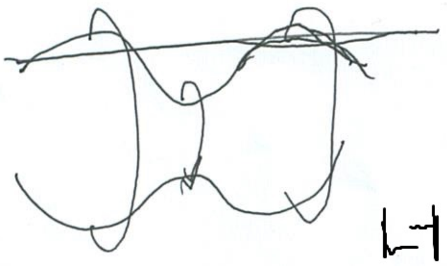
- θ-pinch



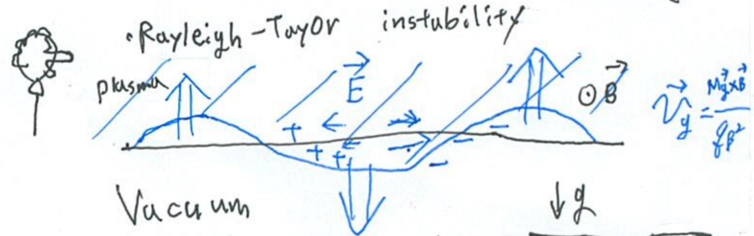
$$\frac{\partial}{\partial r}\left(p + \frac{B_z^2}{2\mu_0}\right) = 0$$

- Tokamak





- Rayleigh-Tayor instability



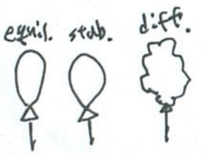
$\vec{v}_g = \frac{M\vec{g}\times\vec{B}}{qB^2}$

$w^2 - k u_{i0} w - \frac{n_0'}{n_0} g = 0$ $\quad w = \frac{k u_{i0} \pm \sqrt{k^2 u_{i0}^2 + 4\frac{n_0'}{n_0} g}}{2}$

$n_1 = \bar{n}_1 \exp i(ky - wt)$ $\quad \gamma > 0$ unstable

$w = w_r + i\gamma$ $\quad \gamma < 0$ stable

$n_1 = \bar{n}_1 \exp i(ky - w_r t) e^{\gamma t}$ $\quad k^2 u_{i0}^2 + 4\frac{n_0'}{n_0} g < 0$



wave.

* Plasma breakdown : Townsend theory $\left\{\begin{array}{l}\alpha\\ \beta\\ \gamma\end{array}\right.$ 가 (?)