- The variation of α with formal concentration is shown in Figure 9-2.

1) Weak electrolytes (compounds that are only partially dissociated) dissociate more as they are diluted.

2) o-Hydroxybenzoic acid is more dissociated than p-hydroxybenzoic acid at the same formal concentration
→ because the ortho isomer is a stronger acid.

See Fig 9-2

9.4 Weak-Base Equilibria

- The treatment of weak bases is almost the same as that of weak acids.

$$B + H_2O \underset{}{\overset{K_b}{\rightleftharpoons}} BH^+ + OH^- \qquad K_b = \frac{[BH^+][OH^-]}{[B]}$$

- We suppose that

  i) nearly all OH^- comes from the reaction of B + H_2O,

  ii) and little comes from dissociation of H_2O.

- Setting $[OH^-] = x$, we must also set $[BH^+] = x$,

  → because one BH^+ is produced for each OH^-.

- Calling the formal concentration of base F (= [B] + $[BH^+]$), we write:

$$[B] = F - [BH^+] = F - x$$

- Plugging these values into the K_b equilibrium expression, we get

Equation for weak base:

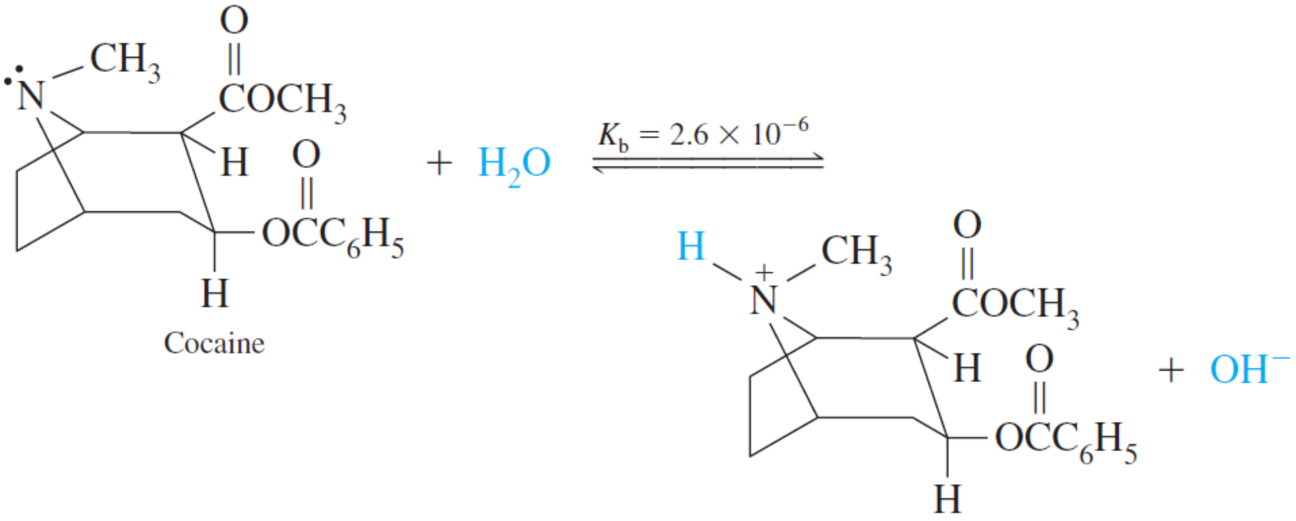
$$\boxed{\frac{[BH^+][OH^-]}{[B]} = \frac{x^2}{F - x} = K_b}$$

→ which looks a lot like a weak-acid problem, except that now $x = [OH^-]$.

$$K_a = \frac{[H^+][A^-]}{[HA]} = \frac{(x)(x)}{F - x}$$

A Typical Weak-Base Problem

- Consider the commonly occurring weak base, cocaine.



Cocaine

- If the formal concentration is 0.037 2 M, the problem is formulated as follows:

$$\underset{0.037\,2 - x}{\mathrm{B}} + \mathrm{H_2O} \rightleftharpoons \underset{x}{\mathrm{BH^+}} + \underset{x}{\mathrm{OH^-}}$$

$$\frac{x^2}{0.037\,2 - x} = 2.6 \times 10^{-6} \Rightarrow x = 3.1 \times 10^{-4}\,\mathrm{M}$$

- Because x = [OH$^-$], we can write

$$[\mathrm{H}^+] = K_\mathrm{w}/[\mathrm{OH}^-] = 1.0 \times 10^{-14}/3.1 \times 10^{-4} = 3.2 \times 10^{-11}\ \mathrm{M}$$

$$\mathrm{pH} = -\log[\mathrm{H}^+] = 10.49$$

→ This is a reasonable pH for a weak base.

- What fraction of cocaine has reacted with water?
- We can formulate α for a base,

→ called the fraction of association:

Fraction of association of a base:

$$\alpha = \frac{[\mathrm{BH}^+]}{[\mathrm{BH}^+] + [\mathrm{B}]} = \frac{x}{\mathrm{F}} = 0.008\ 3$$

→ Only 0.83% of the base has reacted.

9.5 Buffers

- Figure 9-3 shows the pH dependence of an enzyme-catalyzed reaction that is fastest near pH 8.0.

See Fig 9-3

Buffers

- A buffered solution resists changes in pH

  → when acids or bases are added

  → or when dilution occurs.

- The buffer is a mixture of an acid and its conjugate base.

  → There must be comparable amounts of the conjugate acid and base (within a factor of ~10) to exert significant buffering

Mixing a Weak Acid and Its Conjugate Base

- If you mix A moles of a weak acid with B moles of its conjugate base,

  → the moles of acid remain close to A and the moles of base remain close to B.

  → Little reaction occurs to change either concentration.

- To understand why this should be so,

  → look at the K_a and K_b reactions in terms of Le Chatelier's principle.

- Consider an acid with $pK_a = 4.00$ and its conjugate base with $pK_b = 10.00$.
- Let's calculate the fraction of acid that dissociates in a 0.10 M solution of HA.

$$\underset{0.10 - x}{\mathrm{HA}} \rightleftharpoons \underset{x}{\mathrm{H}^+} + \underset{x}{\mathrm{A}^-} \qquad pK_a = 4.00$$

$$\frac{x^2}{\mathrm{F} - x} = K_a \Rightarrow x = 3.1 \times 10^{-3}\ \mathrm{M}$$

$$\text{Fraction of dissociation} = \alpha = \frac{x}{\mathrm{F}} = 0.031$$

$$\text{Fraction of dissociation} = \alpha = \frac{x}{\text{F}} = 0.031$$

- The acid is only 3.1% dissociated under these conditions.

- In a solution containing 0.10 mol of A^- dissolved in 1.00 L,

  → the extent of reaction of A^- with water is even smaller.

$$\underset{0.10 - x}{A^-} + H_2O \rightleftharpoons \underset{x}{HA} + \underset{x}{OH^-} \qquad pK_b = 10.00$$

$$\frac{x^2}{\text{F} - x} = K_b \Rightarrow x = 3.2 \times 10^{-6}$$

$$\text{Fraction of association} = \alpha = \frac{x}{\text{F}} = 3.2 \times 10^{-5}$$

- HA dissociates very little,

  → and adding extra A^- to the solution makes HA dissociate even less.

- Similarly, A^- does not react much with water,

  → and adding extra HA makes A^- react even less.

- If 0.050 mol of A^- plus 0.036 mol of HA are added to water,

  → there will be close to 0.050 mol of A^- and close to 0.036 mol of HA in the solution at equilibrium.

Henderson-Hasselbalch Equation

- The central equation for buffers is the **Henderson-Hasselbalch** equation,

  → which is merely a rearranged form of the K_a equilibrium expression.

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

$$\log K_a = \log \frac{[H^+][A^-]}{[HA]} = \log[H^+] + \log \frac{[A^-]}{[HA]}$$

$$\underbrace{-\log[H^+]}_{pH} = \underbrace{-\log K_a}_{pK_a} + \log \frac{[A^-]}{[HA]}$$

Henderson-Hasselbalch equation for an acid:

$$HA \xrightleftharpoons{K_a} H^+ + A^-$$

$$pH = pK_a + \log \frac{[A^-]}{[HA]}$$

- The Henderson-Hasselbalch equation tells us the pH of a solution,

  → provided we know i) the ratio of the concentrations of conjugate acid and base, as well as ii) pK_a for the acid.

Henderson-Hasselbalch Equation

- If a solution is prepared from the weak base B and its conjugate acid,

  → the analogous equation is

Henderson-Hasselbalch equation for a base:

$$BH^+ \xrightleftharpoons{K_a = K_w/K_b} B + H^+$$

$$\boxed{pH = pK_a + \log \frac{[B]}{[BH^+]}}$$

pK_a applies to *this* acid

- where pK_a is the acid dissociation constant of the weak acid BH^+.

- The important features of the equations above are that

  i) the base (A^- or B) appears in the numerator of both equations,

  ii) and the equilibrium constant is K_a of the acid in the denominator.

Properties of the Henderson-Hasselbalch Equation

- If $[A^-]$ = [HA], then pH = pK_a.

$$\mathrm{pH} = \mathrm{p}K_\mathrm{a} + \log\frac{[\mathrm{A}^-]}{[\mathrm{HA}]} = \mathrm{p}K_\mathrm{a} + \log 1 = \mathrm{p}K_\mathrm{a}$$

- Regardless of how complex a solution may be, whenever pH = pK_a, for a particular acid,

  → $[A^-]$ must equal [HA] for that acid.

- All equilibria must be satisfied simultaneously in any solution at equilibrium.

- If there are 10 different acids and bases in the solution,

  → the 10 forms of Equation 9-16 will have 10 different quotients $[A^-]/[HA]$,

  → but all 10 equations must give the same pH,

  → because there can be only one concentration of H^+ in a solution.

Henderson-Hasselbalch equation for an acid:

$$\mathrm{HA} \xrightleftharpoons{K_a} \mathrm{H^+} + \mathrm{A^-}$$

$$\mathrm{pH} = \mathrm{p}K_a + \log \frac{[\mathrm{A^-}]}{[\mathrm{HA}]}$$

- Another feature of the Henderson-Hasselbalch equation

- For every power-of-10 change in the ratio $[A^-]/[HA]$,

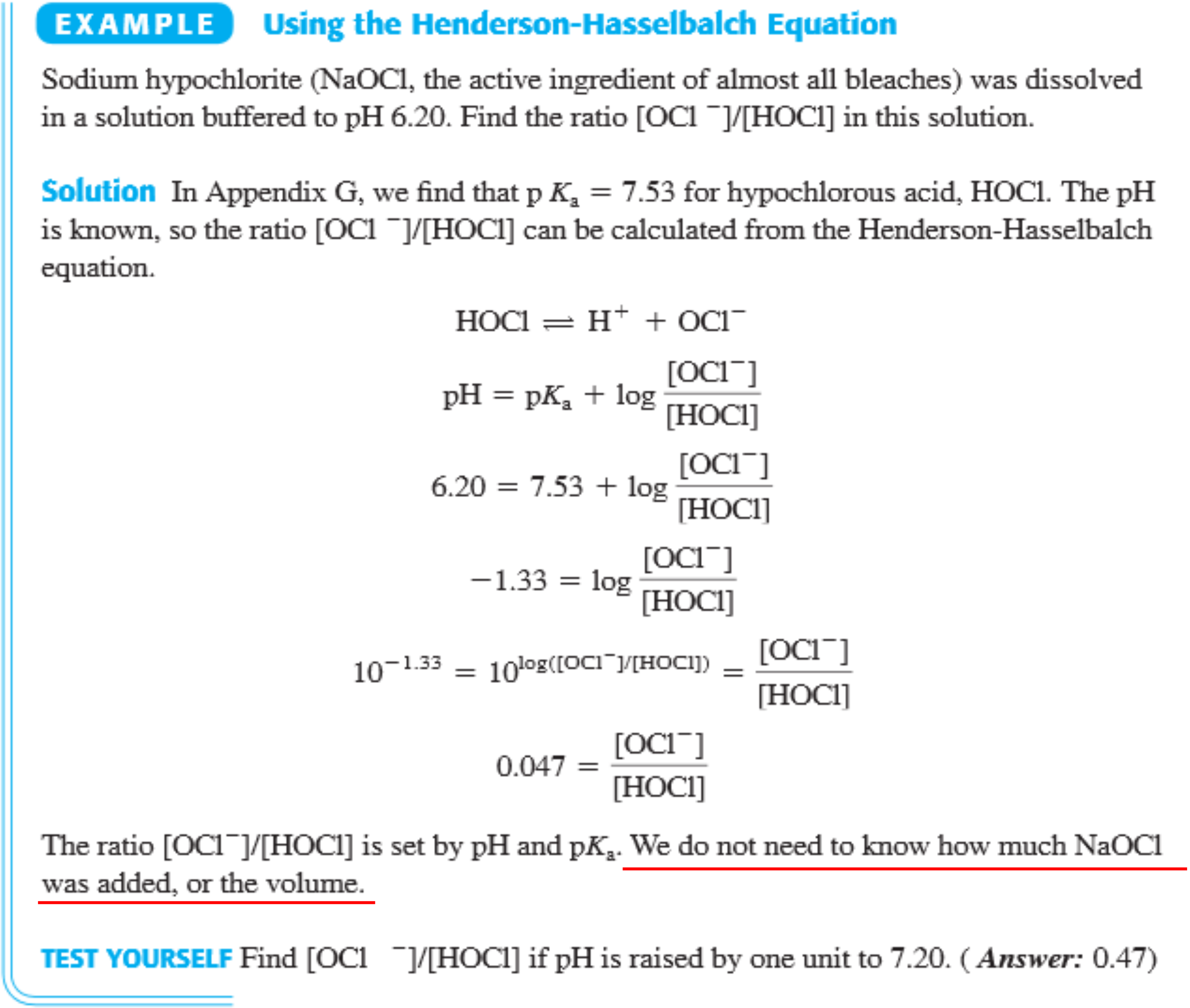
→ the pH changes by one unit (Table 9-1).

See Table 9-1

- As the base (A^-) increases, the pH goes up.
- As the acid (HA) increases, the pH goes down.

- For any conjugate acid-base pair, if $pH = pK_a - 1$,

→ there must be 10 times as much HA as A^-.



EXAMPLE Using the Henderson-Hasselbalch Equation

Sodium hypochlorite (NaOCl, the active ingredient of almost all bleaches) was dissolved in a solution buffered to pH 6.20. Find the ratio [OCl $^-$]/[HOCl] in this solution.

Solution In Appendix G, we find that p K_a = 7.53 for hypochlorous acid, HOCl. The pH is known, so the ratio [OCl $^-$]/[HOCl] can be calculated from the Henderson-Hasselbalch equation.

$$\mathrm{HOCl} \rightleftharpoons \mathrm{H^+} + \mathrm{OCl^-}$$

$$\mathrm{pH} = \mathrm{p}K_a + \log\frac{[\mathrm{OCl^-}]}{[\mathrm{HOCl}]}$$

$$6.20 = 7.53 + \log\frac{[\mathrm{OCl^-}]}{[\mathrm{HOCl}]}$$

$$-1.33 = \log\frac{[\mathrm{OCl^-}]}{[\mathrm{HOCl}]}$$

$$10^{-1.33} = 10^{\log([\mathrm{OCl^-}]/[\mathrm{HOCl}])} = \frac{[\mathrm{OCl^-}]}{[\mathrm{HOCl}]}$$

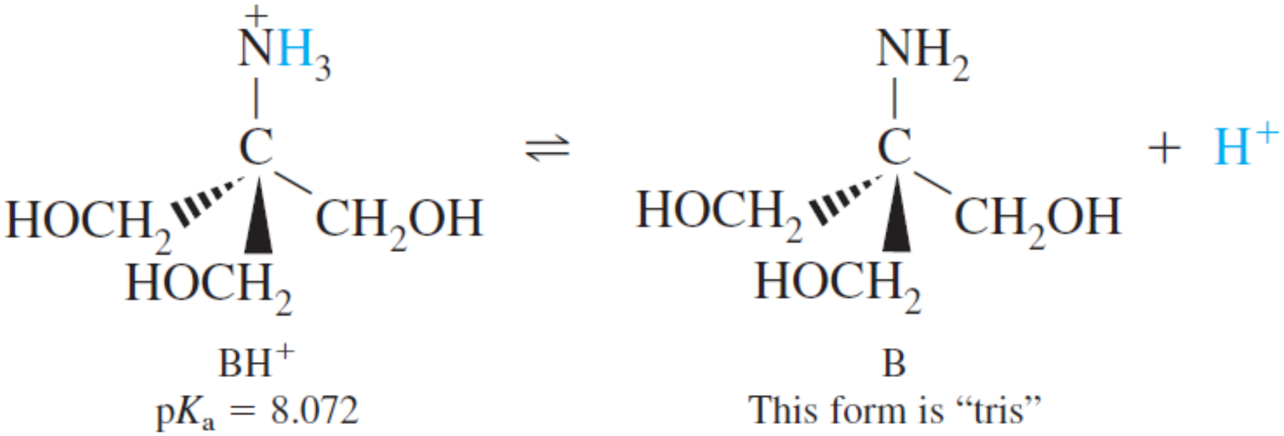
$$0.047 = \frac{[\mathrm{OCl^-}]}{[\mathrm{HOCl}]}$$

The ratio [OCl$^-$]/[HOCl] is set by pH and pK_a. We do not need to know how much NaOCl was added, or the volume.

TEST YOURSELF Find [OCl $^-$]/[HOCl] if pH is raised by one unit to 7.20. (***Answer:*** 0.47)

A Buffer in Action

- For illustration, we choose a widely used buffer called “tris,” which is short for tris(hydroxymethyl) aminomethane.



from tris hydrochloride tris(hydroxymethyl) aminomethane

- In Appendix G, we find pK_a = 8.072 for the conjugate acid of tris.
- An example of a salt containing the BH^+ cation is tris hydrochloride,
  → which is BH^+Cl^-.
- When BH^+Cl^- is dissolved in water,
  → it dissociates to BH^+ and Cl^-.

- Notice that the volume of solution is irrelevant,

  → because volume cancels in the numerator and denominator of the log term:

$$\begin{aligned}\mathrm{pH} &= \mathrm{p}K_\mathrm{a} + \log \frac{\text{moles of B}/\cancel{\text{L of solution}}}{\text{moles of BH}^+/\cancel{\text{L of solution}}} \\ &= \mathrm{p}K_\mathrm{a} + \log \frac{\text{moles of B}}{\text{moles of BH}^+}\end{aligned}$$

Strong Plus Weak Reacts Completely

- Strong acid reacts with a weak base essentially “completely”

  → because the equilibrium constant is large.

$$\underset{\substack{\text{Weak}\\\text{base}}}{\mathrm{B}} + \underset{\substack{\text{Strong}\\\text{acid}}}{\mathrm{H}^+} \rightleftharpoons \mathrm{BH}^+ \qquad K = \frac{1}{K_\mathrm{a}\,(\text{for BH}^+)}$$

- If B is tris(hydroxymethyl)aminomethane,

  → the equilibrium constant for reaction with HCl is

$$K = \frac{1}{K_\mathrm{a}} = \frac{1}{10^{-8.072}} = 1.2 \times 10^8$$

- Strong base reacts “completely” with a weak acid

→ because the equilibrium constant is, again, very large.

$$\underset{\substack{\text{Strong}\\\text{base}}}{OH^-} + \underset{\substack{\text{Weak}\\\text{acid}}}{HA} \rightleftharpoons A^- + H_2O \qquad K = \frac{1}{K_b\,(\text{for } A^-)}$$

- If HA is acetic acid,

→ then the equilibrium constant for reaction with NaOH is

$$K = \frac{1}{K_b} = \frac{K_a\,(\text{for HA})}{K_w} = 1.7 \times 10^9$$

- The reaction of a strong acid with a strong base is even more complete than a strong plus weak reaction:

$$\underset{\substack{\text{Strong}\\\text{acid}}}{H^+} + \underset{\substack{\text{Strong}\\\text{base}}}{OH^-} \rightleftharpoons H_2O \qquad K = \frac{1}{K_w} = 10^{14}$$

- If you mix a strong acid, a strong base, a weak acid, and a weak base,

  1) the strong acid and strong base will neutralize each other

     → until one is used up.

  2) The remaining strong acid or strong base will then react with the weak base or weak acid.

EXAMPLE Effect of Adding Acid to a Buffer

If we add 12.0 mL of 1.00 M HCl to the solution in the previous example, what will be the new pH?

Solution The key is to realize that, *when a strong acid is added to a weak base, they react completely to give* BH^+ (Box 9-3). We are adding 12.0 mL of 1.00 M HCl, which contains (0.012 0 L)(1.00 mol/L) = 0.012 0 mol of H^+. This much H^+ consumes 0.012 0 mol of B to create 0.012 0 mol of BH^+:

	B Tris	$+$	H^+ From HCl	$\rightarrow BH^+$
Initial moles	0.102 6		0.012 0	0.029 6
Final moles	$\underbrace{0.090\,6}_{(0.102\,6\,-\,0.012\,0)}$		—	$\underbrace{0.041\,6}_{(0.029\,6\,+\,0.012\,0)}$

Information in the table allows us to calculate the pH:

$$\mathrm{pH} = \mathrm{p}K_a + \log \frac{\text{moles of B}}{\text{moles of BH}^+}$$

$$= 8.072 + \log \frac{0.090\,6}{0.041\,6} = 8.41$$

The volume of the solution is irrelevant.

TEST YOURSELF Find the pH if only 6.0 instead of 12.0 mL HCl were added. (***Answer:*** 8.51)

- We see that the pH of a buffer does not change very much
  → when a limited amount of strong acid or base is added.

- Addition of 12.0 mL of 1.00 M HCl changed the pH from 8.61 to 8.41.

- Addition of 12.0 mL of 1.00 M HCl to 1.00 L of unbuffered solution would have lowered the pH to 1.93.

- But why does a buffer resist changes in pH?
  → It does so because the strong acid or base is consumed by B or BH^+.

- If you add NaOH,

  → BH^+ is converted to B.

- As long as you don't use up B or BH^+ by adding too much HCl or NaOH,

  → the log term of the Henderson-Hasselbalch equation does not change much

  → and pH does not change much.

- The buffer has its maximum capacity to resist changes of pH when pH = pK_a.

How Buffers Work

- A buffer resists changes in pH because added acid or base is consumed by buffer.
- As buffer is used up,

  → it becomes less resistant to changes in pH.

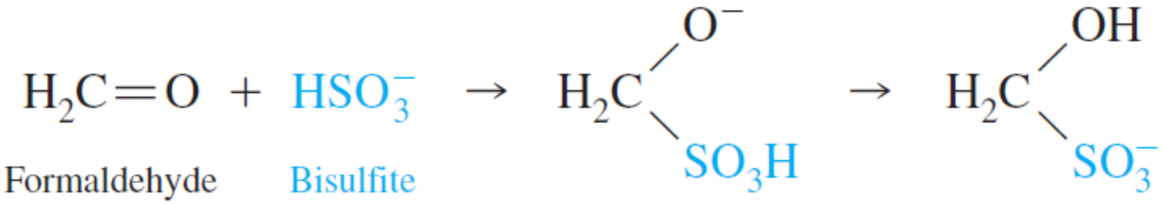
- A mixture containing approximately a 10:1 mole ratio of HSO_3^- : SO_3^{2-} is prepared.
- Because pK_a for HSO_3^- is 7.2,

  → the pH should be approximately.

$$\mathrm{pH} = \mathrm{p}K_\mathrm{a} + \log \frac{[\mathrm{SO_3^{2-}}]}{[\mathrm{HSO_3^-}]} = 7.2 + \log \frac{1}{10} = 6.2$$

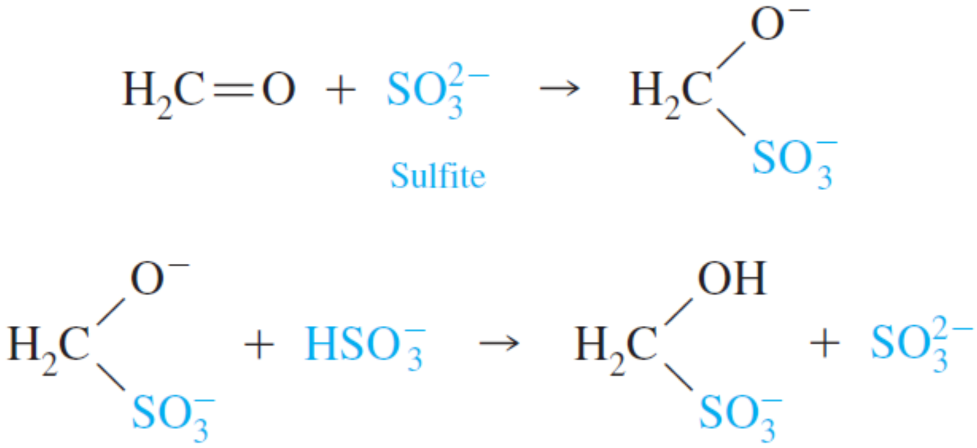
- When formaldehyde is added,

→ the net reaction is the consumption of HSO_3^-, but not of SO_3^{2-}:



$$H_2C{=}O + HSO_3^- \rightarrow H_2C(O^-)(SO_3H) \rightarrow H_2C(OH)(SO_3^-)$$

(A)



$$H_2C{=}O + SO_3^{2-} \rightarrow H_2C(O^-)(SO_3^-)$$

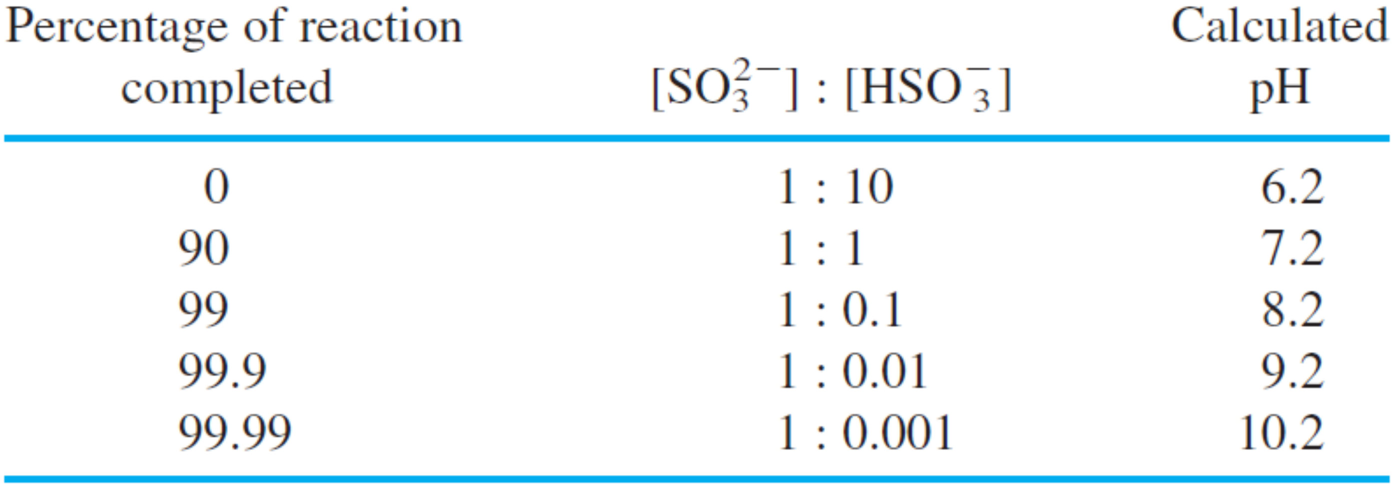
$$H_2C(O^-)(SO_3^-) + HSO_3^- \rightarrow H_2C(OH)(SO_3^-) + SO_3^{2-}$$

(B)

- In sequence A, bisulfite is consumed directly.
- In sequence B, the net reaction is destruction of HSO_3^-,

  with no change in the SO_3^{2-} concentration.

- We can prepare a table showing how the pH should change as the HSO_3^- reacts.



Percentage of reaction completed	$[SO_3^{2-}] : [HSO_3^-]$	Calculated pH
0	1 : 10	6.2
90	1 : 1	7.2
99	1 : 0.1	8.2
99.9	1 : 0.01	9.2
99.99	1 : 0.001	10.2

- Through 90% completion,

  → the pH should rise by just 1 unit.

- In the next 9% of the reaction,

  → the pH will rise by another unit.

- At the end of the reaction,

  → the change in pH is very abrupt.

Buffer pH Depends on Ionic Strength and Temperature

- The correct Henderson-Hasselbalch equation includes **activity coefficients**.
- The calculated pH of a buffer is not equal to the observed pH
  → because the ionic strength is not 0, so activity coefficients are not 1.

- Table 9-2 lists pK_a values for common buffers that are widely used in biochemistry.
- Values are listed for ionic strengths of 0 and 0.1 M.

- If a buffer solution has an ionic strength closer to 0.1 M than to 0,
  → it is sensible to use pK_a for $\mu = 0.1$ to obtain a more realistic calculation of pH.

See Table 9-2

Buffer pH Depends on Ionic Strength and Temperature

- Buffer pK_a depends on temperature,

  → as indicated in the last column of Table 9-2.

- Tris has an exceptionally large dependence,

  → 0.028 pK_a units per degree, near room temperature.

- A solution of tris with pH 8.07 at 25°C will have

  → pH 8.7 at 4°C

  → and pH 7.7 at 37°C.

When What You Mix Is Not What You Get

- In dilute solution or at extremes of pH,

  → the concentrations of HA and A^- are not equal to their formal concentrations.

- Suppose we mix F_{HA} moles of HA and F_{A^-} moles of the salt Na^+A^-.
- The mass and charge balances are.

$$\text{Mass balance:} \qquad F_{HA} + F_{A^-} = [HA] + [A^-]$$

$$\text{Charge balance:} \qquad [Na^+] + [H^+] = [OH^-] + [A^-]$$

- The substitution $[Na^+] = F_{A^-}$ and a little algebra leads to the equations.

$$[HA] = F_{HA} - [H^+] + [OH^-]$$

$$[A^-] = F_{A^-} + [H^+] - [OH^-]$$

- So far we have assumed that $[HA] \approx F_{HA}$ and $[A^-] \approx F_{A-}$,

  → we used these values in the Henderson-Hasselbalch equation.

- A more rigorous procedure is to use Equations 9-20 and 9-21.

$$[HA] = F_{HA} - [H^+] + [OH^-]$$

$$[A^-] = F_{A^-} + [H^+] - [OH^-]$$

- If F_{HA} or F_{A-} is small, or if $[H^+]$ or $[OH^-]$ is large,

  → then the approximations $[HA] \approx F_{HA}$ and $[A^-] \approx F_{A-}$ are not good.

- In acidic solutions, $[H^+] >> [OH^-]$, so $[OH^-]$ can be ignored
- In basic solutions, $[H^+]$ can be neglected.

EXAMPLE How Much Will the pH Change?

Suppose that a buffered solution contains 50 mmol of the buffer species HA and 50 mmol of the buffer species A^-. The pH will be equal to pK_a for the buffer. How much will the pH change if 20 mmol of some other acid is created by a chemical reaction?

Solution In the worst case, the acid generated by the reaction is strong enough to convert an equivalent quantity of A^- into HA. The moles of HA would then be $50 + 20 = 70$ mmol. The moles of A^- would then be $50 - 20 = 30$ mmol. The pH would be

$$\mathrm{pH} = \mathrm{p}K_\mathrm{a} + \log\frac{\text{mol A}^-}{\text{mol HA}} = \mathrm{p}K_\mathrm{a} + \log\frac{30\text{ mmol}}{70\text{ mmol}} = \mathrm{p}K_\mathrm{a} - 0.37$$

The pH would drop by 0.37 units. If this change is acceptable, you have enough buffer.

TEST YOURSELF If, instead, 15 mmol of strong base were generated, by how much would the pH of the original buffer rise? (***Answer:*** 0.27 pH units)

Analytical Chemistry

Chapter 10. Polyprotic Acid-Base Equilbria

10.1 Diprotic Acids and Bases

- **Amino acids**

  → have an acidic carboxylic acid group, a basic amino group, and a variable substituent designated R.

- The carboxyl group is a **stronger acid** than the ammonium group

  → so the nonionized form rearranges spontaneously to the **zwitterion**,

- **Zwitterion**

  → a molecule with both positive and negative charges