Master equations:

$$S = K\frac{1+b\theta}{Y\hat{q}\theta-(1+b\theta)}$$

$$X_a = Y\frac{S^0-S}{1+b\theta}$$

<u>Special cases</u>

1. When $\Theta = \Theta_x$ is very small, $S=S_0$ and $X_a=0$: washout; not substrate removal and accumulation of active biomass

The active biomass is washed out before substantial growth occurs to show observable substrate utilization

Denote the Θ_x at washout as $\Theta_x^{\min}$, then, using the equation for S:

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x-(1+b\theta_x)}$$

$$S^0 = K\frac{1+b\theta_x^{min}}{Y\hat{q}\theta_x^{min}-\left(1+b\theta_x^{min}\right)}$$

This gives

$$\theta_x^{min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK}$$

2. For $\Theta > \Theta_x$, S declines with the increase in Θ_x. X_a initially increases, but reaches a maximum and then decreases as decay becomes dominant.

3. When $\Theta = \Theta_x$ is very large, the decay predominates and substrate concentration is not further reduced (S reaches S_{min})

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x-(1+b\theta_x)}$$

$$\theta_x^{min} \to \infty$$

$$S_{min} = K\frac{b}{Y\hat{q} - b}$$

<u>(3) Bacterial growth following Monod kinetics including inert biomass</u>

Mass balance for inert biomass (assume steady state):

$$0 = QX_i^0 - QX_i + (1 - f_d)bX_aV$$

$$X_i = X_i^0 + X_a(1 - f_d)b\theta$$

(This shows that operating at large Θ results in large accumulation of inert biomass)

The total volatile suspended solids (VSS) concentration, X_v is calculated as (used Θ instead of Θ_x)

$$X_v = X_i + X_a = X_i^0 + X_a(1 - f_d)b\theta_x + X_a = X_i^0 + Y(S^0 - S)\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$