2. Description of block geometry and stability using vector methods

1) Description of orientation

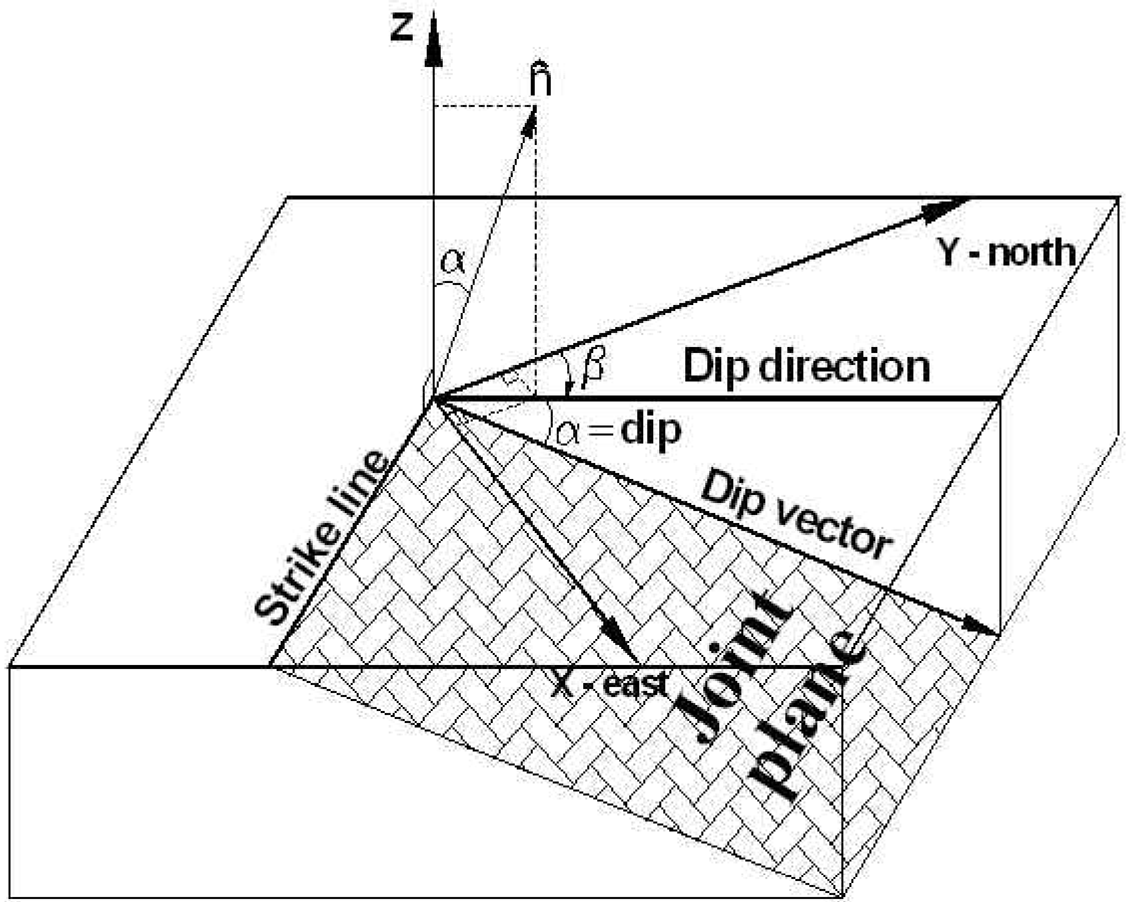


Fig. Dip direction, dip, strike and normal vector of a joint plane.

- Trend/Plunge (선주향/선경사)

- Trend: An angle in the horizontal plane measured in clockwise from the north to the vertical plane containing the given line
- Plunge: An acute angle measured in a vertical plane between the given line and the horizontal plane

Ex.) 320/23, 012/56

- Dip direction/Dip (경사방향/경사)

- Dip direction: Trend of the maximum dip line (dip vector) of the given plane
- Dip: Plunge of the maximum dip line of the given plane

Ex.) 320/23, 012/56

- Strike/Dip (주향/경사)

- Strike: An angle measured in the horizontal plane from the north to the given plane
- Dip: Plunge of the maximum dip line of the given plane

Ex.) N15°E/30°SE, N25°W/56°SW

- Conversion of Strike/Dip to Dip direction/Dip

- N60˚E/30˚SE:

- N60˚E/30˚NW:

- N60˚W/30˚NE:

- N60˚W/30˚SW:

- Normal vector of a plane

3D Cartesian coordinates of normal vector of a joint whose dip direction (β) and dip (α) are given (when +Z axis points vertical up).

$$N_x = \sin\alpha \sin\beta$$
$$N_y = \sin\alpha \cos\beta$$
$$N_z = \cos\alpha$$

● Normal vector of a plane

Ex.1) Calculate the normal vector of a joint whose dip direction (β) and dip (α) are 060° and 30°, respectively (when +Z axis points vertical up).

Ex.2) Obtain the normal vector components of a joint using dip direction and dip when X is north, Y is upward and Z is east.

2) Equations of lines and planes

- Line

$$\vec{x}_0 = \left(x_0, y_0, z_0 \right)$$

$$\vec{x}_1 = \left(x_1, y_1, z_1 \right)$$

$x = x_0 + lt, \quad y = y_0 + mt, \quad z = z_0 + nt \ldots\ldots\ldots$ parametric form (equation)

$\left(l = x_1 - x_0, m = y_1 - y_0, n = z_1 - z_0, \right)$

$\dfrac{x - x_0}{x_1 - x_0} = \dfrac{y - y_0}{y_1 - y_0} = \dfrac{z - z_0}{z_1 - z_0} \quad \left(= t \right) \ldots\ldots\ldots\ldots\ldots$ standard (Cartesian) form

$\vec{x} = \vec{x}_0 + \left(\vec{x}_1 - \vec{x}_0 \right) t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ vector equation

2) Equations of lines and planes

- Plane

$\hat{n}:$ normal vector, ex) $\hat{n} = (A, B, C)$

$D:$ distance from an origin to the plane in direcion of normal vector (A, B, C)

$Ax + By + Cz = D$…………… Cartesian form

$\rightarrow \hat{n} \cdot \vec{p} = D$………………………vector form

2) Equations of lines and planes

- Half-space

$Ax + By + Cz \geq D$…………… upper half at C > 0

$Ax + By + Cz \leq D$…………… lower half at C > 0

2) Equations of lines and planes

- Intersection of a plane and a line

$$Ax + By + Cz = D$$

$$x = x_0 + lt, \quad y = y_0 + mt, \quad z = z_0 + nt$$

$$A(x_0 + lt) + B(y_0 + mt) + C(z_0 + nt) = D$$

$$\rightarrow t = \frac{D - (Ax_0 + By_0 + Cz_0)}{Al + Bm + Cn}$$

2) Equations of lines and planes

- Intersection of two planes

$$\hat{n}_1 = (A_1, B_1, C_1)$$

$$\hat{n}_2 = (A_2, B_2, C_2)$$

$$\vec{I}_{12} = \hat{n}_1 \times \hat{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

$$= (B_1C_2 - C_1B_2)\hat{i} + (C_1A_2 - A_1C_2)\hat{j} + (A_1B_2 - B_1A_2)\hat{k}$$

$$= (B_1C_2 - C_1B_2, \quad C_1A_2 - A_1C_2, \quad A_1B_2 - B_1A_2)$$

$$\hat{I}_{12} = \frac{\hat{n}_1 \times \hat{n}_2}{\|\hat{n}_1 \times \hat{n}_2\|}$$

2) Equations of lines and planes

- Intersection of three planes

$$A_1x + B_1y + C_1z = D_1$$
$$A_2x + B_2y + C_2z = D_2$$
$$A_3x + B_3y + C_3z = D_3$$

2) Equations of lines and planes

● Angles between lines and planes

- Angle between two lines

$$\hat{a}\cdot\hat{b}=\cos\theta \quad \rightarrow \quad \theta=\cos^{-1}\left(\hat{a}\cdot\hat{b}\right)$$

- Angle between two planes

$$\hat{n}_1\cdot\hat{n}_2=\cos\theta \quad \rightarrow \quad \theta=\cos^{-1}\left(\hat{n}_1\cdot\hat{n}_2\right)$$

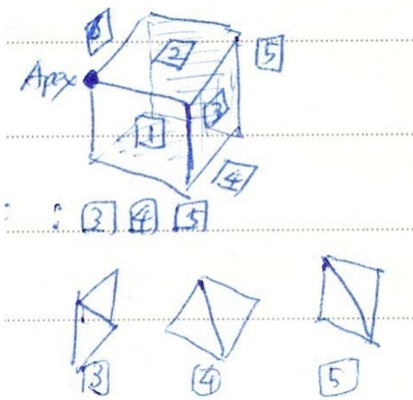
-Angle between a line and a plane

$$\left|\hat{n}_1\cdot\hat{a}\right|=\cos\left(90-\theta\right)=\sin\theta \quad \rightarrow \quad \theta=\sin^{-1}\left(\left|\hat{n}_1\cdot\hat{a}\right|\right)$$

3) Description of a block

- Calculation of volume

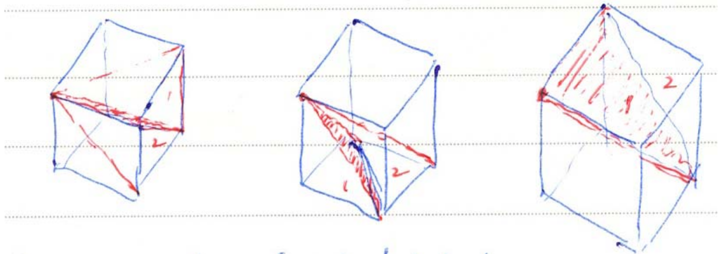
①Select an apex

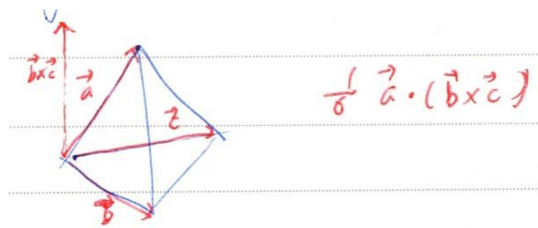


②Take faces not including the apex

③Divide each face into triangles

④Make tetrahedrons with the apex and each triangle





⑤Summing up the volumes of all tetrahedrons

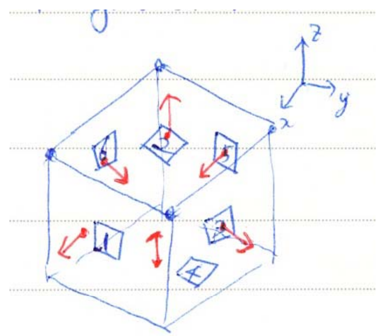
3) Description of a block

● Defining a block

Adjusting the signs of normal vectors so that $C_i \geq 0 \quad \left(A_i \geq 0 \text{ if } C_i = 0\right)$

① Finding vertices

- Defining a block inner space



$$A_1 x + B_1 y + C_1 z \leq D_1 \quad : \quad L_1$$
$$A_2 x + B_2 y + C_2 z \leq D_2 \quad : \quad L_2$$
$$A_3 x + B_3 y + C_3 z \leq D_3 \quad : \quad L_3$$
$$A_4 x + B_4 y + C_4 z \geq D_4 \quad : \quad U_4$$
$$A_5 x + B_5 y + C_5 z \geq D_5 \quad : \quad U_5$$
$$A_6 x + B_6 y + C_6 z \geq D_6 \quad : \quad U_6$$

3) Description of a block

- Finding candidate vertices

$${}_6C_3 = \frac{6!}{3!3!} = 20 \quad : \quad C_{123}, C_{124}, C_{125}, C_{126}, C_{134}, \ldots, C_{456}$$

- Select block vertices satisfying all the inequalities

$$C_{123}, C_{134}, C_{146}, C_{126}, C_{235}, C_{345}, C_{256}, C_{456}$$

② Defining faces with vertices

$[1]: \quad C_{123} - C_{134} - C_{146} - C_{126}$

$[2]: \quad C_{123} - C_{126} - C_{256} - C_{235}$

$[3]: \quad C_{123} - C_{134} - C_{345} - C_{235}$

$[4]: \quad C_{134} - C_{146} - C_{456} - C_{345}$

$[5]: \quad C_{235} - C_{345} - C_{456} - C_{256}$

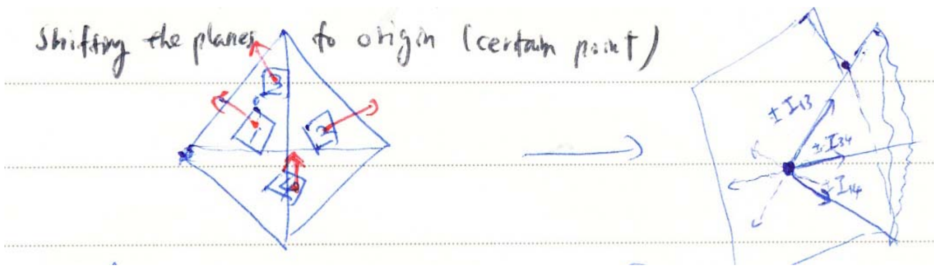
$[6]: \quad C_{146} - C_{126} - C_{256} - C_{456}$

4) Block pyramid

- Equations

Each plane is shifted to intersect the origin

$$A_1x + B_1y + C_1z \le 0 \quad : \quad L_1^0$$
$$A_2x + B_2y + C_2z \le 0 \quad : \quad L_2^0$$
$$A_3x + B_3y + C_3z \le 0 \quad : \quad L_3^0$$
$$A_4x + B_4y + C_4z \ge 0 \quad : \quad U_4^0$$



- Finding edges

$\vec{I}_{12} = \hat{n}_1 \times \hat{n}_2 \quad \rightarrow \quad \pm \vec{I}_{12}$ (The sign depends on the order of two vectors cross-producted)

No. of intersection vectors (edges): $2 \times {}_4C_2$

No. of intersection vectors which satisfy all the inequalities defining the half-spaces: 0

5) Equations of forces (p.43)

Force: vector $F = (X, Y, Z)$

Resultant force: obtained by vector summation

$$R = \sum_{i=1}^{n} F_i = \left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} Y_i, \sum_{i=1}^{n} Z_i \right)$$

Friction force: Normal load x friction coefficient

$$R_f = -\sum_{i=1}^{n} \left(N_i \tan \phi_i \right) \hat{s}$$

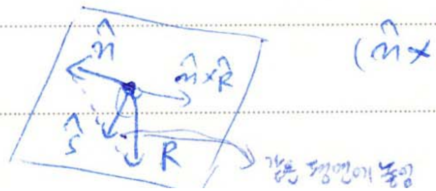
6) Computation of sliding directions

① Single face sliding

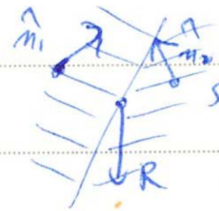
$$\hat{s} \, // \left(\hat{n} \times \vec{R} \right) \times \hat{n}$$

② Sliding on two planes

$$\hat{s} \, // \, \mathrm{sign}\left[\left(\hat{n}_1 \times \hat{n}_2 \right) \cdot \vec{R} \right] \left(\hat{n}_1 \times \hat{n}_2 \right)$$



$(\hat{n} \times \hat{R}) \times \hat{n} \, // \, \hat{s}$



$\mathrm{sign}\left[(\hat{n}_1 \times \hat{n}_2) \cdot \vec{R} \right] (\hat{n}_1 \times \hat{n}_2) \, // \, \hat{s}$

H.W.) Make a computer code with which you can do below work.

1) If you type in dip direction and dip of a plane it calculates the normal vector of the plane (z coordinate is always positive value).

2) If you give x, y and z coordinates of vertices of a hexahedron it calculates the volume of the hexahedron.

Show your code and its performance to others within 5 min. in next class.