9. The kinematics and stability of removable blocks

1) Introduction

- Analysis
  - Mode analysis: to distinguish keys + p. keys from stable blocks (R.F.)
  - Stability analysis: to distinguish key blocks from potential key blocks (S.F.)

- Failure mode
  - Lifting: move to free space without sliding
  - Single face sliding: move to free space with sliding on a plane (joint)
  - Double face sliding: move to free space with sliding on two adjacent planes

- Nomenclature

  $\vec{r}$: resultant force

  $\vec{d}$: driving force ($\hat{d} = \vec{d} \big/ |\vec{d}|$)

  $\hat{n}_i$: upward normal vector of plane i

  $\hat{v}_i$: normal vector of plane i pointing the inside of a block

2) Failure mode and driving force

- If $\vec{r} \cdot \hat{v}_l > 0$ for all planes, the block is lifting

  $\rightarrow \ \vec{d} = \vec{r}$, no supporting plane

- If $\vec{r} \cdot \hat{v}_l \leq 0$ only for the plane $l = i$, the block is of single face sliding on the plane i

$$\rightarrow \ \hat{d} = \frac{(\hat{v}_i \times \vec{r}) \times \hat{v}_i}{|\hat{v}_i \times \vec{r}|} \quad (= \hat{s}_i)$$

$$\vec{d} = \left(|\hat{v}_i \times \vec{r}| + (\hat{v}_i \cdot \vec{r}) \tan \phi_i \right) \hat{d}$$

- If $\vec{r} \cdot \hat{v}_l \leq 0$ only for the plane i and j, the block may be of double face sliding on the plane i and j

$$\rightarrow \ \hat{s}_i = \frac{(\hat{v}_i \times \vec{r}) \times \hat{v}_i}{|\hat{v}_i \times \vec{r}|}, \quad \hat{s}_j = \frac{(\hat{v}_j \times \vec{r}) \times \hat{v}_j}{|\hat{v}_j \times \vec{r}|}$$

If $\hat{s}_i \cdot \hat{v}_j > 0$ and $\hat{s}_j \cdot \hat{v}_i < 0 \ \rightarrow$ i th plane is a supporting plane (single face sliding)

If $\hat{s}_i \cdot \hat{v}_j < 0$ and $\hat{s}_j \cdot \hat{v}_i > 0 \ \rightarrow$ j th plane is a supporting plane (single face sliding)

If $\hat{s}_i \cdot \hat{v}_j < 0$ and $\hat{s}_j \cdot \hat{v}_i < 0 \ \rightarrow$ i th and i th planes are supporting planes (double face sliding)

2) Failure mode and driving force

$$\rightarrow \quad \hat{d} \;=\; \frac{\left(\hat{v}_i \times \hat{v}_j\right)}{\left|\hat{v}_i \times \hat{v}_j\right|}\operatorname{sign}\left[\hat{v}_i \times \hat{v}_j \cdot \vec{r}\right] \quad \left(= \hat{s}_{ij}\right)$$

$$\vec{d} \;= \left(\vec{r} \cdot \hat{s}_{ij} - N_i \tan\phi_i - N_j \tan\phi_j\right)\hat{s}_{ij}$$

$$\text{where } N_i = \frac{\left|\left(\vec{r} \times \hat{v}_j\right) \cdot \left(\hat{v}_i \times \hat{v}_j\right)\right|}{\left|\hat{v}_i \times \hat{v}_j\right|^2} \quad \text{and} \quad N_j = \frac{\left|\left(\vec{r} \times \hat{v}_i\right) \cdot \left(\hat{v}_i \times \hat{v}_j\right)\right|}{\left|\hat{v}_i \times \hat{v}_j\right|^2}$$

3) JP of each failure mode for a given resultant force

- Lifting

$$\hat{v}_i = \text{sign}\left[\vec{r}\cdot\hat{n}_i\right]\hat{n}_i \text{ for all } i \quad \left(\rightarrow \vec{r}\cdot\hat{v}_i \geq 0\right)$$

- Single face sliding (sliding plane i)

$$\hat{v}_i = \text{-sign}\left[\vec{r}\cdot\hat{n}_i\right]\hat{n}_i$$

$$\hat{v}_l = \text{sign}\left[\hat{s}\cdot\hat{n}_l\right]\hat{n}_l \quad \text{for } l \neq i$$

- Double face sliding (adjacent sliding planes i, j)

$$\hat{v}_i = \text{-sign}\left[\hat{s}_j\cdot\hat{n}_i\right]\hat{n}_i$$

$$\hat{v}_j = \text{-sign}\left[\hat{s}_i\cdot\hat{n}_j\right]\hat{n}_j$$

$$\hat{v}_l = \text{sign}\left[\hat{s}_{ij}\cdot\hat{n}_l\right]\hat{n}_l \quad \text{for } l \neq i, j$$

4) Failure mode analysis by using stereographic projection

- Failure mode of JPs with a given $\hat{r}$

1) Locate $\hat{r}$, $\hat{s}_i$, and $\hat{s}_{ij}$

$\hat{s}_i$: one of two intersections made by intersection of the great circle i and $\hat{n}_i - \pm\hat{r}$ passing great circle (closer to $\hat{r}$ between the two points)

$\hat{s}_{ij}$: intersection of the ith one of the i th and j th great circles (closer to $\hat{r}$)

2) Analyze the failure mode of each JP

Lifting: JP including $\hat{r}$

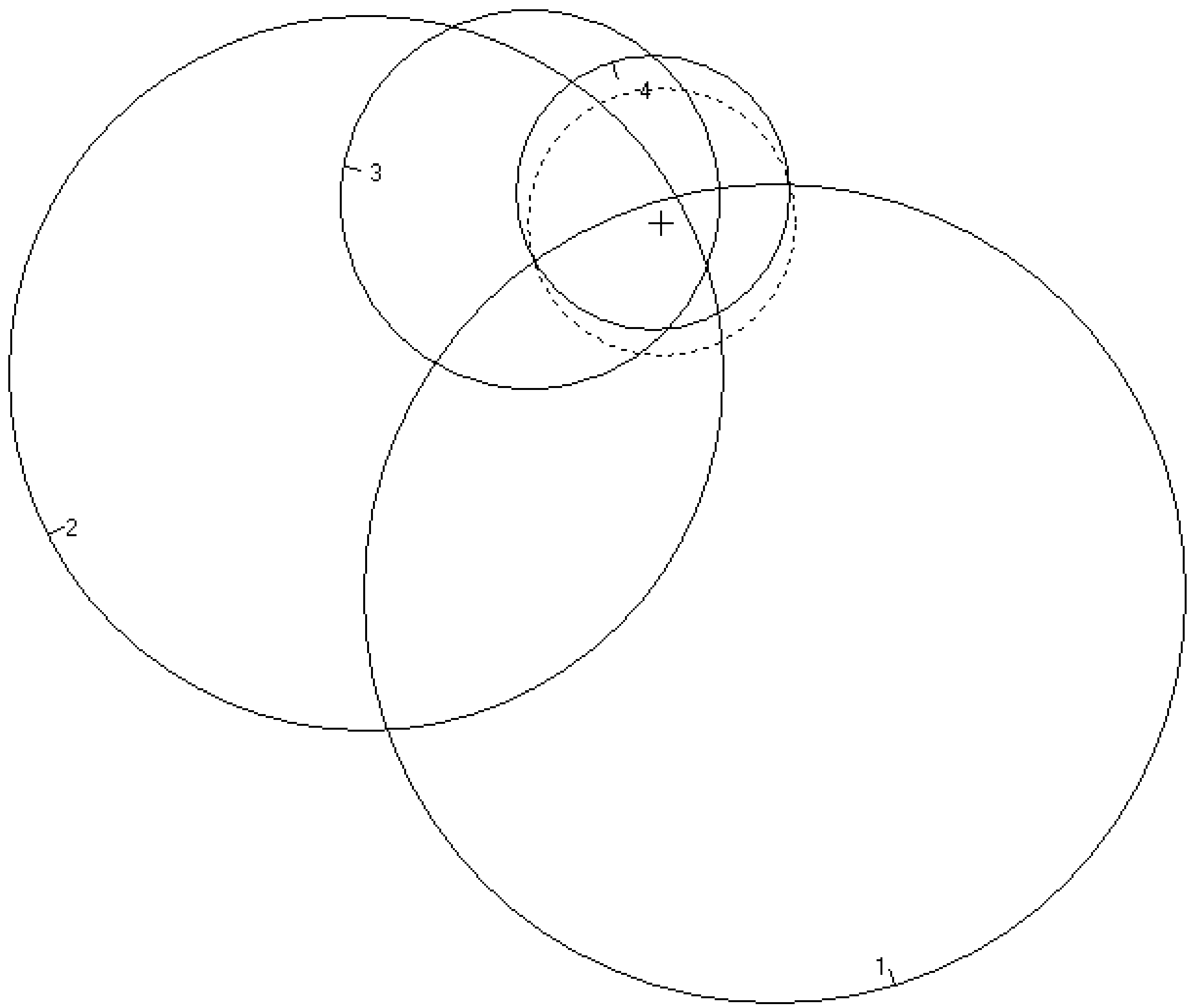
Single face sliding: JP including $\hat{s}_i$ and excluding $\hat{r}$ ($\hat{r} \cdot \hat{v}_i < 0$)

Double face sliding: JP including $\hat{s}_{ij}$ and not including $\hat{s}_i$ in the half space of the j th plane as well as not including $\hat{s}_j$ in the half space of the i th plane, which satisfies $\hat{s}_i \cdot \hat{v}_j < 0$ and $\hat{s}_j \cdot \hat{v}_i < 0$

Stables: JPs not belonging to above mentioned modes (there is no sliding vector in it)

(refer to p.309 of the textbook)

4) Failure mode analysis by using stereographic projection



4) Failure mode analysis by using stereographic projection

- Failure mode of a JP with variable $\hat{r}$

Equilibrium region: location of $\hat{r}$ where the same failure mode is applied

A JP consisting of a plane: Fig.9.9

A JP consisting of two or more planes: Fig.9.11

1) $\hat{w}_i$ is defined as a normal vector of the i th plane pointing outside of a block.

ex) JP = 1100 $\rightarrow$ $\hat{w}_1 = \hat{n}_1$, $\hat{w}_2 = \hat{n}_2$, $\hat{w}_3 = -\hat{n}_3$, $\hat{w}_4 = -\hat{n}_4$

2) Draw the JP and locate its $\hat{c}_{ij}$ and $\hat{w}_i$.

3) Draw the boundary lines of equilibrium regions.

$\hat{w}_i \times \hat{c}_{ij}$: upwards - $\hat{w}_i \rightarrow \hat{c}_{ij}$ in ccw

downwards - $\hat{w}_i \rightarrow \hat{c}_{ij}$ in cw

$\hat{w}_1 - \hat{w}_2$, $\hat{w}_2 - \hat{w}_4$ ... : draw great circles connecting $\hat{w}_i$ and $\hat{w}_j$ in the same order of JP forming planes.

4) Failure mode analysis by using stereographic projection

4) Failure mode labelling (Fig.9.11)

Lifting: inside of JP

Single face sliding: $\hat{c}_{ij} - \hat{c}_{ik} - \hat{w}_i$

Double face sliding: $\hat{w}_i - \hat{w}_j - \hat{c}_{ij}$

Stable: $\hat{w}_i - \hat{w}_j \ldots \hat{w}_i$

5) Drawing friction angle lines

$$\hat{t}_{ij} = \cos(\phi_i)\hat{w}_i + \sin(\phi_i)\hat{c}_{ij}$$