


457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 04
Elements of Set Theory – Part I (A&T: 2.1-2.2)

1. Why do we need 'set theory' in uncertainty analysis?

- **Uncertainty:** a multiplicity of possible ()
- **Probability:** numerical measure of the () of an event (i.e. a group of outcomes) relative to the other possible events

| | Possible outcomes | Event of Interest | Probability |
|--------------------------|-------------------|--|-------------|
| Tossing a dice | { } | "Even number" | |
| | | "Prime & odd" | |
| Watching a baseball game | { } |  | |

- Uncertainty analysis requires considering the collection of all () ().
- Principles of set theory provide essential tools for this task.

2. Definitions

(a) **Sample space** (S): the set of all possible outcomes, e.g. { }

Sample point (x): an individual outcome, e.g. " ", " "

Obviously, any sample point belongs to the sample space, that is, $x \in S$

Note that there is no sample point outside ()

| Criteria | Sample space | Examples |
|--------------------------|--------------|--|
| Continuous? | "Continuous" | Peak flow rate of Addison Creek (ft ³ /sec) |
| | "Discrete" | Number of tornadoes in U.S. in 2008 |
| Can count sample points? | "Finite" | () and () $S = \{1.2, 2.2, \dots, 1000.2\}$ |
| | "Infinite" | () or () $S = \{x \mid 2.73 \leq x \leq 5.25\}, S = \{0, 1, 2, \dots\}$ |

(b) **Event** (E): any collection of sample (); any subset of sample ()

Example 1: Sample space of annual peak flow rate of Addison Creek?

$$S = \{x \mid \quad \quad \quad \}$$

Event: $E_1 =$ "Peak flow rate of Addison creek is higher than 1000 ft³/sec"

$$E_1 = \{x \mid \quad \quad \quad \}$$

Example 3 (A&T 2.1): A contractor plans the acquisition of three bulldozers. After 6 months, each bulldozer can be *operational* (O) or *nonoperational* (N).

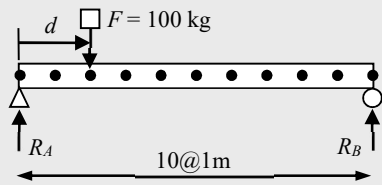
- (*) Sample space of the possible conditions of the three bulldozers after 6 months?
- (*) Event that the i -th bulldozer is *operational* after 6 months, $i = 1, 2, 3$?
- (*) Event there are two operational bulldozers after 6 months and its complementary event?
- (*) Venn diagrams of the sample space and the events above?

(e) “**Conditioned**” sample space

- A new sample space after observing/assuming the occurrence of a related event.
- In general, the sample space () when new information becomes available or assumed.

Example 3 (contd.): Venn diagram of the new sample space when we know the Bulldozer 2 survived six months?

Example 4 (A&T 2.5): A simply supported beam



- (a) 100 kg at any of the 1-meter interval points.
 The sample space of the reaction force R_A ?
 (hint: $R_A = 100 - 10d$ where d is distance from A)

- (b) Visualize the sample space of (R_A, R_B) ?
 (hint: $R_A + R_B = 100$)

- (c) Now the 100-kg load can be placed anywhere along the beam.

- Sample space of R_A
- Sample space of (R_A, R_B)
- Show the event " $20 < R_A < 40$ "

- (d) The load can be 100, 200, or 300 kg and can be placed anywhere along the beam.
 Sample space of (R_A, R_B) ?

- (e) The load can be any value between 100 and 300 kg and can be placed anywhere along the beam.

- Sample space of (R_A, R_B) ?
- Show the event " $R_A < 100$ and $R_B > 100$ "
- Show the event " $R_A > 200$ "