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457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 06
Elements of Probability Theory – Part I (A&T: 2.3)

1. Four approaches for assigning "probabilities" to events (or four definitions)

Approach	Description	Example: Prob (a "Yut" stick shows the flat side) 
Notion of **Relative Frequency**	Relative frequency based on empirical data, Prob = (# of occurrences)/(# of observations)	
On a **Priori** Basis	Derived based on elementary assumptions on likelihood of events	
On **Subjective** Basis	Expert opinion ("degree of belief")	
Based on **Mixed** Information	Mix the information above to assign probability	

* An article on the probability and the "Yut" game: Weekly Donga (2014.11.3)

2. **Axioms*** of probability – foundation of probability theory

- $P(E) \geq 0$: the probability of an event is (   )
- $P(S) = 1$: the probability of the (   ) is equal to unity.
- For mutually exclusive events E_1 and E_2, $P(E_1 \cup E_2) =$

From these axioms, the following properties of probability have been derived.

- $0 \leq P(E) \leq 1$
  hint 1: $P(E \cup \overline{E}) = P(S) = 1,\ P(E \cup \overline{E}) = P(E) + P(\overline{E})$
- $P(\phi) = 0$
  hint 2: $P(\phi \cup S) = P(S) = 1,\ P(\phi \cup S) = P(\phi) + P(S) = P(\phi) + 1$
- $P(\overline{E}) = 1 - P(E)$
  hint 3: See hint 1

* Axiom: statement or idea which people accept as being true.

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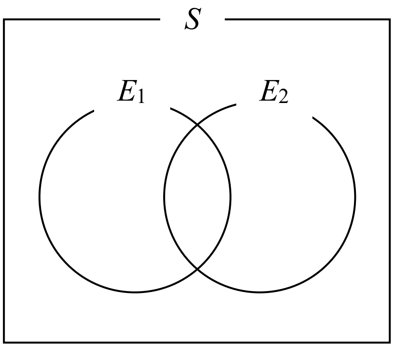
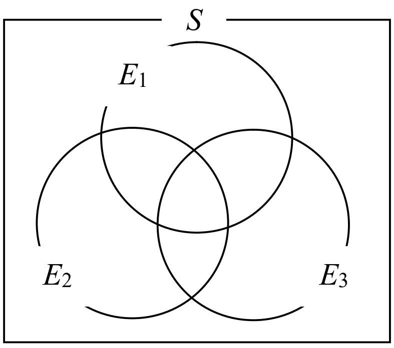
- "**Addition rule**": $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 E_2)$
- **"Inclusion-exclusion rule"**: $P(E_1 \cup E_2 \cup E_3) =$

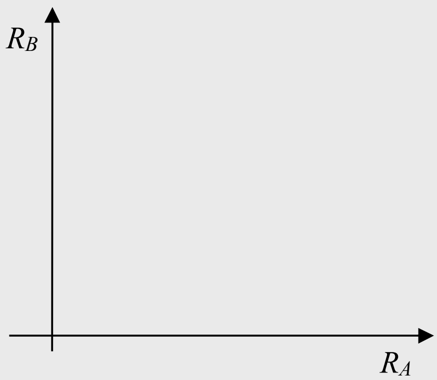




Example 1 (A&T 2.15): For the sample space of A&T 2.8(e), consider two events

$A = \{(R_A, R_B) \mid R_A > 100\}$
$B = \{(R_A, R_B) \mid R_B > 100\}$

Assuming each case of (R_A, R_B) has the same likelihood (i.e. probability proportional to the area of the event), compute the probability of the events A, B, AB and $A \cup B$.



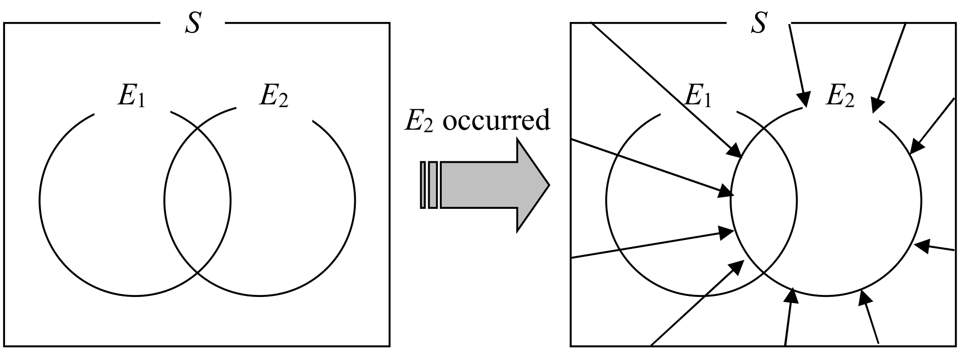
(Continued: Properties of probability)

- $P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1 - P\left(\overline{E_1 \cup E_2 \cup \cdots \cup E_n}\right) = 1 - P(\overline{E}_1 \overline{E}_2 \cdots \overline{E}_n)$
- When $E_1, ..., E_n$ are mutually exclusive, $P\left(\bigcup_{i=1}^{n} E_i\right) =$

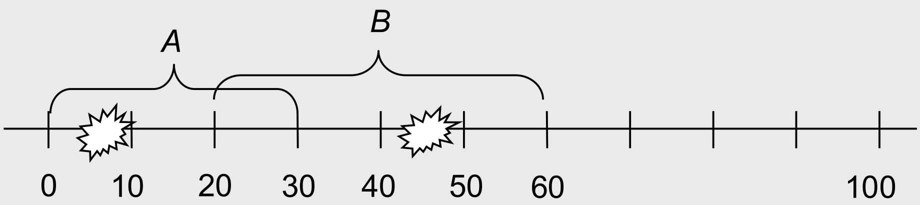
3. Conditional probability and statistical independence

   (a) Conditional probability: Conditional probability of E_1 given E_2 is defined as

$$P(E_1 \mid E_2) = \underline{\hspace{6cm}}$$



Example 2: Assume accidents are equally likely to occur anywhere on the highway.



P(*A*) =

P(*B*) =

If an accident occurs in the interval (20, 60), what is the probability of the event *A*?

Example 3: Consider batting stat split for Ah-seop Sohn (Lotte Giants)

Download the dataset 'SonAhseop.txt' from the eTL website.

Using R, complete the chart below.

	At Bats (AB)	Hits (H)	Batting Average (AVG)
vs Left Handed Pitcher			
vs Right Handed Pitcher			
vs Submarine Pitcher			
Total			

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son = read.table("SonAhseop.txt")
HH = which(son$Result=="H")
OO = which(son$Result=="O")

result_01 = rep(0,times=nrow(son))
result_01[HH] = 1;
son$Result_01 = result_01

Total_AB = length(son$Result_01)
Total_H = sum(son$Result_01)
Total_AVG = Total_H/Total_AB

son_LHP = son[son$Style=="L",]
son_RHP = son[son$Style=="R",]
son_SP = son[son$Style=="S",]

LHP_AB = length(son_LHP$Result_01)
RHP_AB = length(son_RHP$Result_01)
SP_AB = length(son_SP$Result_01)

LHP_H = sum(son_LHP$Result_01)
RHP_H = sum(son_RHP$Result_01)
SP_H = sum(son_SP$Result_01)

LHP_AVG = LHP_H/LHP_AB
RHP_AVG = RHP_H/RHP_AB
SP_AVG = SP_H/SP_AB
```

Batting average vs LHP and RHP using conditional probability definition:

$P(Hit|LHP) =$

$P(Hit|RHP) =$

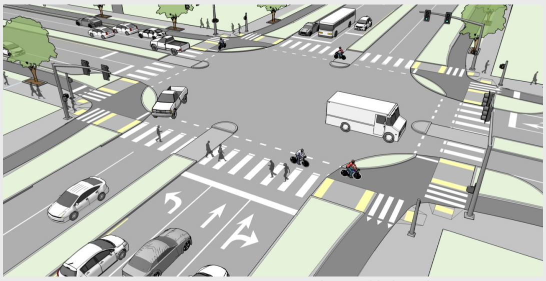
(b) $P(E \mid S)$?

(c) $P(\overline{E}_1 \mid E_2) = 1 - P(E_1 \mid E_2)$

hint: $P(E_1 \mid E_2) + P(\overline{E}_1 \mid E_2) =$

Note $P(E_1 \mid \overline{E}_2) \neq 1 - P(E_1 \mid E_2)$

Example 4 (A&T 2.18): Motor vehicles are approaching a certain intersection. According to the data collected from the intersection, the ratios of the three actions, i.e. going straight ahead, turning right and turning left are



http://altaplanning.com

(**S**traight Ahead): (Turning **R**ight): (Turning **L**eft) = 2: 1: 0.5 = 4: 2: 1

(a) Compute probabilities of the events S, R and L.

(b) If a car is making a turn, what is the probability that it will be a right turn?

(c) If a car is making a turn, what is the probability that it will not turn right?