

5. Discontinuity spacing

► Introduction

-Definition: Distance between two joint intersections in a scanline

-Spacing and frequency: The frequency is the reciprocal of a mean spacing. The reciprocal of a single spacing is called repetition (or repetition value).

- Spacing
 - Total spacing: Distance between two adjacent joint intersections regardless of joint sets (X_t)
 - Set spacing: Distance between two adjacent joint intersections belonging to the same joint set (X_d)
 - Normal set spacing: Set spacing measured along the joint normal (X_n)

- Expression: $X_n = X_d \cos \theta$ cf. $\lambda_L = \frac{\lambda_t}{\cos \theta}$
(θ : acute angle between a scanline and joint normal)

$$\lambda_S = \frac{1}{X_t} \quad (\text{total linear frequency is the reciprocal of the mean total spacing})$$

$$\lambda_t = \frac{1}{X_d} \quad ((\text{set}) \text{ linear frequency is the reciprocal of the mean set spacing})$$

$$\lambda_L = \frac{1}{X_n} \quad (\text{normal linear frequency is the reciprocal of the mean normal set spacing})$$

► Discontinuity spacing distributions

- Poisson distribution of joint intersections: Combined joint intersections of all joint sets are randomly located along scanlines on the whole. (refer to Fig. 5.4, p.127)

- Spacing follows negative exponential: (total) Spacing turned out by theoretical and observational approach to follow a negative exponential distribution when the joint intersections are randomly located.

Theoretical approach: When the number of joint intersections obeys Poisson distribution the probability that k of the intersections are located in a scanline whose length is x is as follow (λ is a linear frequency).

$$P(k, x) = \frac{e^{-\lambda x} (\lambda x)^k}{k!}$$

Then, $f(x)$, the probability density function of $P'(x)$ indicating a probability that total spacing X_t becomes x is as follow.

$$f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} P'(X_t \leq x) = \frac{d}{dx} (1 - P(0, x)) = \frac{d}{dx} (1 - e^{-\lambda x}) = \lambda e^{-\lambda x}$$

: Negative exponential distribution (mean = $\int_0^{\infty} \lambda x e^{-\lambda x} dx = \frac{1}{\lambda}$)

► Rock Quality Designation

-Definition: Percentage of the summed length of core pieces which are longer than 10cm.

- RQD and spacing/frequency: total spacing and total frequency are closely related with RQD.

-TRQD (Theoretical RQD)

Let the random variable of total spacing be x and PDF of x be $f(x)$. When the length of rock core is L ,

① Total frequency: $\lambda = 1/\bar{x}$

No. of joint intersections: λL

No. of joint pairs whose spacing is x : $\lambda L f(x) dx$

② No. of joint pairs whose spacing $x > t$: $\int_t^L \lambda L f(x) dx$

Sum of spacing $x > t$ $\int_t^L \lambda L x f(x) dx$

③ Let threshold of RQD be t . Then $TRQD_t$ becomes:

$$TRQD_t = \frac{100}{L} \int_t^L \lambda L x f(x) dx = 100 \lambda \int_t^L x f(x) dx$$

④ When the spacing obeys negative exponential distribution:

$$\begin{aligned} TRQD_t &= 100 \lambda^2 \int_t^L x e^{-\lambda x} dx && (L \text{ is large enough}) \\ &= 100 [e^{-\lambda t} (1 + \lambda t) - e^{-\lambda L} (1 + \lambda L)] \\ &= 100 e^{-\lambda t} (1 + \lambda t) \\ &= 100 e^{-t/\bar{x}} (1 + t/\bar{x}) \end{aligned}$$

- $TRQD_t$ vs. Mean spacing: refer to Fig. 5.5 (p.130).

- $TRQD_t$ vs. Frequency: refer to Fig. 5.6 (p.131).

► Accuracy and precision of discontinuity spacing estimates

- Error in spacing measurement:

Inaccuracy: Consistent error caused by persistent factor. ex) Spacing greater than L cannot be measured by a scanline of which length is L.

Imprecision: Inconsistent error caused by sampling size. As the sampling size increases the variance of mean spacing becomes smaller.

① Inaccuracy caused by short sampling lines

- Curtailment: Ignoring some samples whose values are greater than a certain level. It is classified into two groups according to whether the samples are counted/recorded or not: truncated (not counted) and censored (counted). Spacing greater than L is always truncated by a scanline of which length is L.

- Mathematical expression of curtailment

$$f(x) \rightarrow f_L(x): f_L(x) = \frac{f(x)}{F(L)}$$

$$\text{When } f(x) = \lambda e^{-\lambda x} \quad f_L(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda L}}$$

$$\begin{aligned} \bar{x} = \mu_{xL} &= \frac{\lambda}{(1 - e^{-\lambda L})} \int_0^L x e^{-\lambda x} dx \\ &= \frac{1 - e^{-\lambda L} - \lambda L e^{-\lambda L}}{\lambda(1 - e^{-\lambda L})} \end{aligned}$$

Eqn.(5.13) p.135

(As $L \rightarrow \infty$, \bar{x} becomes $\frac{1}{\lambda}$)

$$\begin{aligned} \text{Var}(x) &= E(x^2) - (E(x))^2 \\ &= M_{x^2} - \mu_{xL}^2 \\ &= \frac{\lambda}{(1 - e^{-\lambda L})} \int_0^L x^2 e^{-\lambda x} dx - \mu_{xL}^2 \\ &= \frac{2 - e^{-\lambda L}(2 + 2\lambda L + \lambda^2 L^2)}{\lambda^2(1 - e^{-\lambda L})} - \mu_{xL}^2 \end{aligned}$$

(As $L \rightarrow \infty$, $\text{Var}(x)$ becomes $\frac{1}{\lambda^2}$)

Eqn.(5.14) p.135

$$\begin{aligned} TRQD_t &= 100\lambda' \int_t^L x f_L(x) dx \\ &= \frac{100}{x} \int_t^L x f_L(x) dx \\ &= \frac{100}{1 - e^{-\lambda L} - \lambda L e^{-\lambda L}} (e^{-\lambda t}(1 + \lambda t) - e^{-\lambda L}(1 + \lambda L)) \end{aligned}$$

Eqn.(5.15) p.135

Fig. 5.7 p.136, Fig.5.8, Fig. 5.9

② Imprecision caused by small sample sizes

-Table 5.1 (p.141), Fig. 5.10 (p.142)

-As the scanline length increases the sample size of spacing also increases addressing the imprecision problem

-Application of the central limit theorem to mean spacing enables us to estimate the confidence range of the mean spacing by using the standard normal distribution.