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Chapter 1 Tension, Compression, and Shear

1.1 Introduction to Mechanics of Materials

⦿ Objectives of M.M

Determine s_______, s_______ and d________ in (simple) s________ and their c_______ (or m________)

⦿ Basic Procedure of M.M

1. Define loads acting on the body and support conditions

2. Determine r________ forces at supports and i________ forces using static e____________ and f______ b______ d_________ (FBD) – possible for so-called statically ___________ structures (cf. statically ____________ structures)

※ **"Statics" and "Dynamics" stop here!** – forces and motions associated with p_________ and r________ bodies, i.e. no d_________

3. Study s________ and s_________ inside *real* (i.e. d_______) bodies by using physical properties of the materials as well as theoretical laws and concepts

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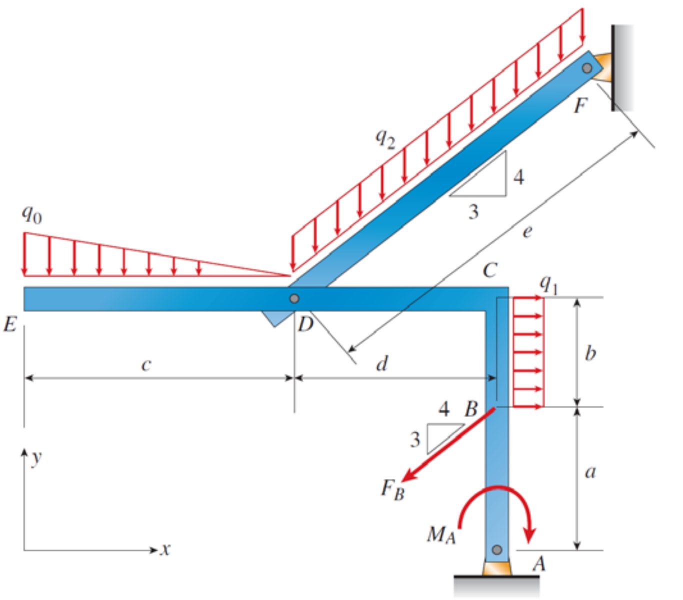
1.2 Statics Review

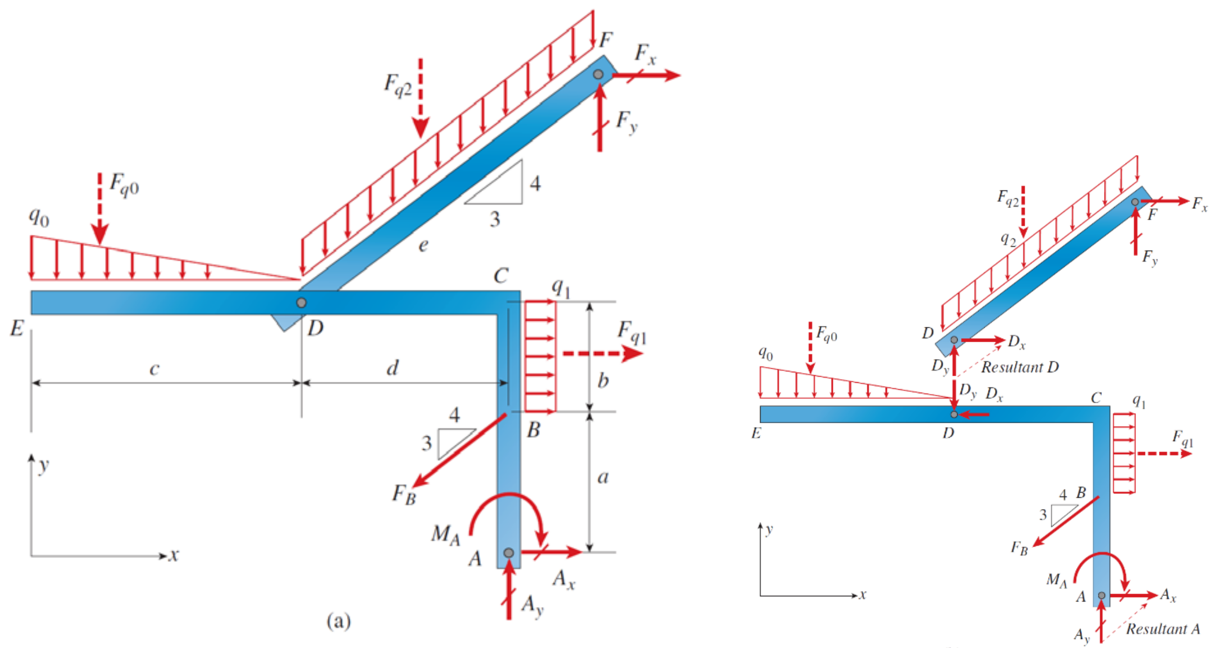
⦿ Equilibrium Equations

1. Two-dimensional problems:

2. Three-dimensional problems:

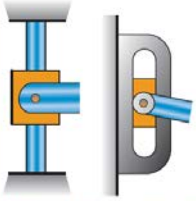
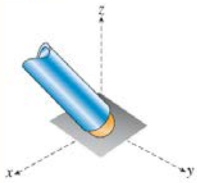
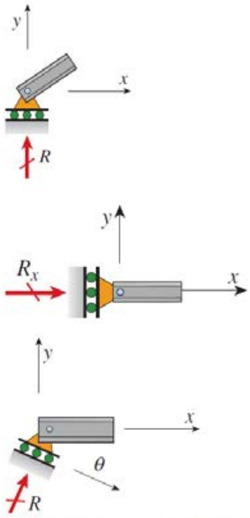
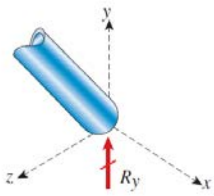
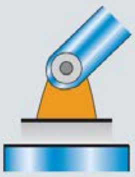
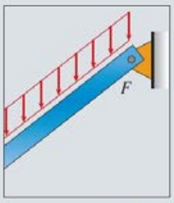
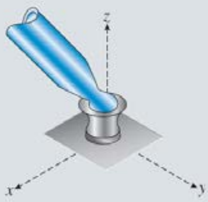
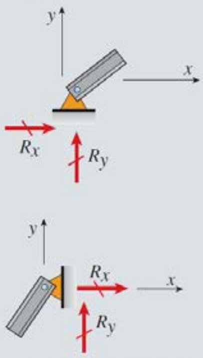
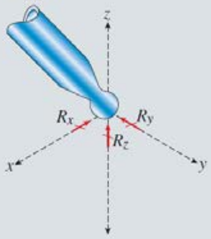
⦿ Free-Body Diagrams (FBD)

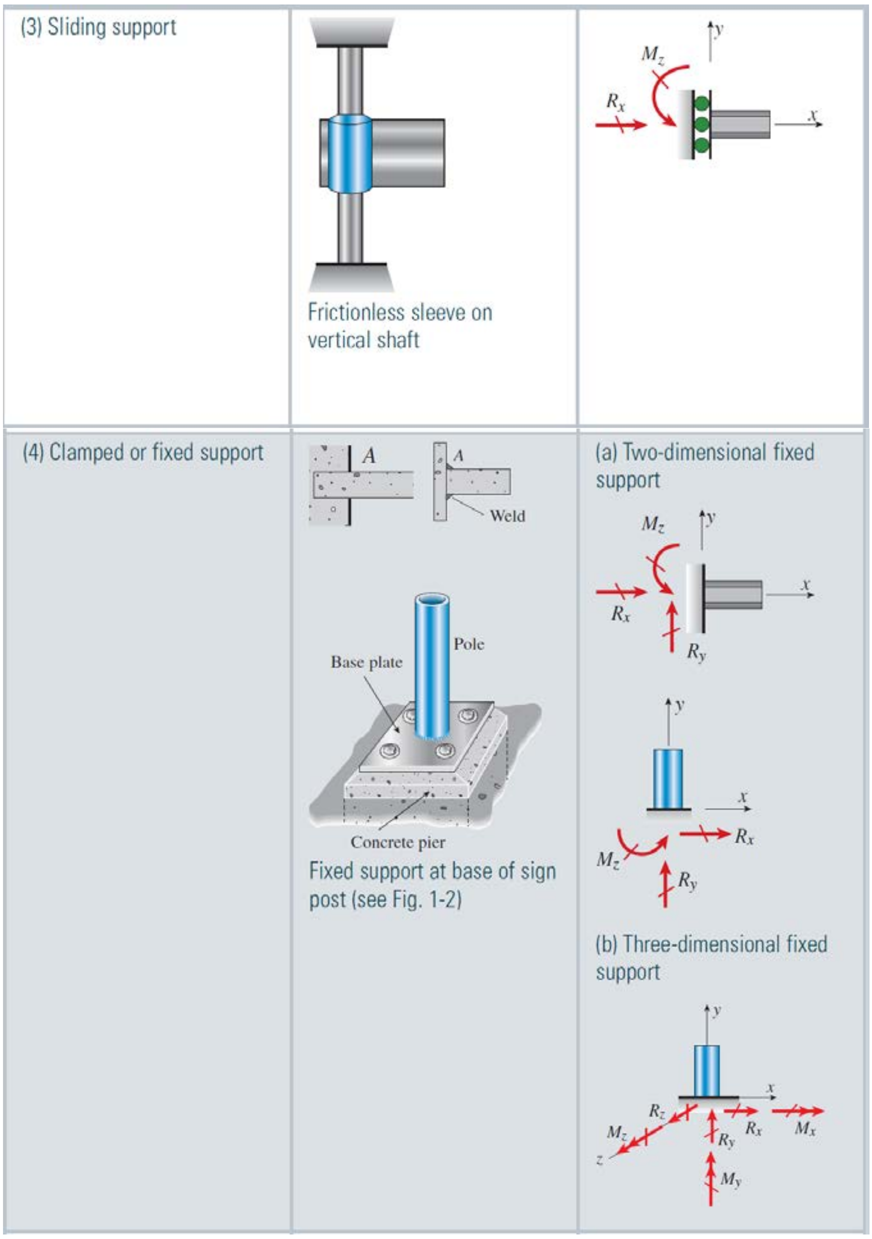




(a) (b)

⦿ Reaction Forces and Support Conditions

Type of support or connection	Simplified sketch of support or connection	Display of restraint forces and moments, or connection forces
(1) Roller support—horizontal, vertical, or inclined  Bridge with roller support (The Earthquake Engineering Online Archive)	 Horizontal roller support (constrains motion in both $+y$ and $-y$ directions)  Vertical roller restraints  Rotated or inclined roller support 	(a) Two-dimensional roller support  (b) Three-dimensional roller support 
(2) Pin support  Bridge with pin support (Courtesy of Joel Kerkhoff, P.Eng.)  Pin support on old truss bridge (© Barry Goodno)	  Pin support at F in Fig. 1-1 	(a) Two-dimensional pin support  (b) Three-dimensional pin support 



⊙ Internal Forces (Stress R_________)

To find member deformation, we must find i_________ forces and moments, i.e. internal stress r__________, e.g.

1. Axial forces (N)
2. Transverse shear forces (V)
3. Bending moments (M)

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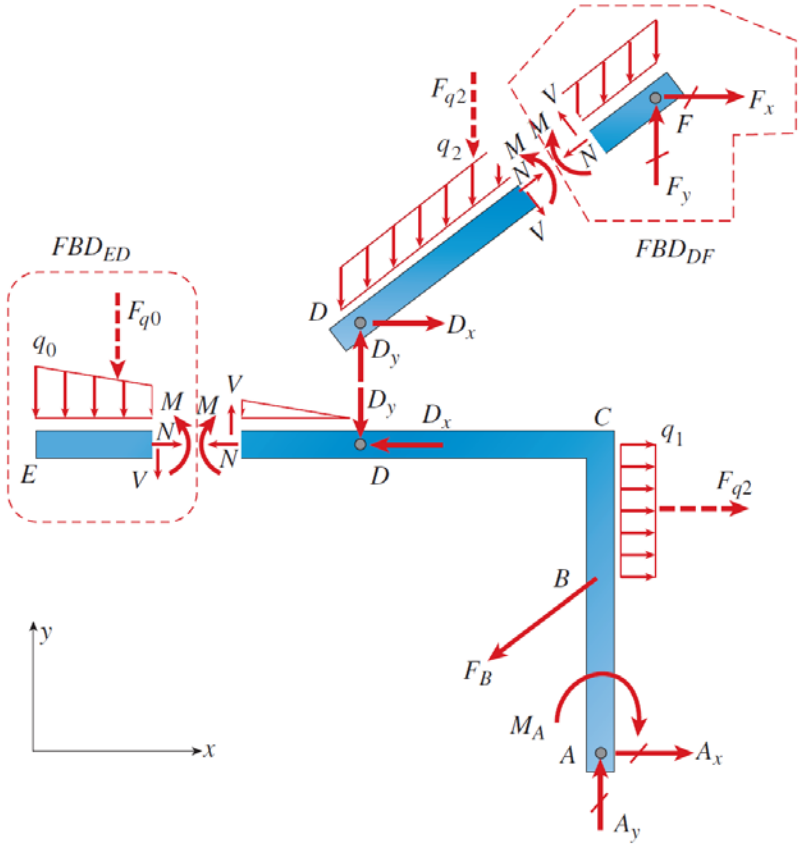
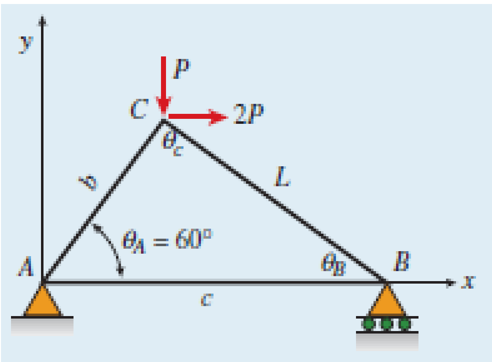
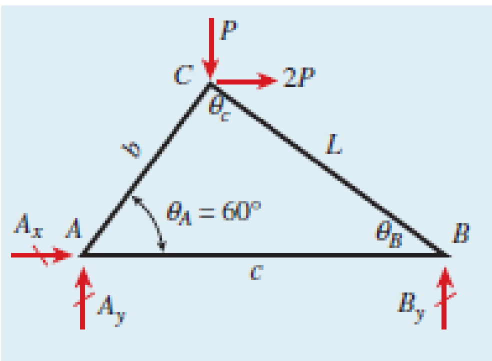


⊙ **Example 1-1:** Find reaction forces and member forces of the plane truss using the method of joints, i.e. applying equilibrium conditions to each joint





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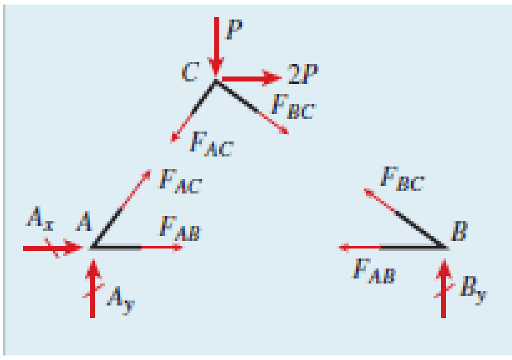
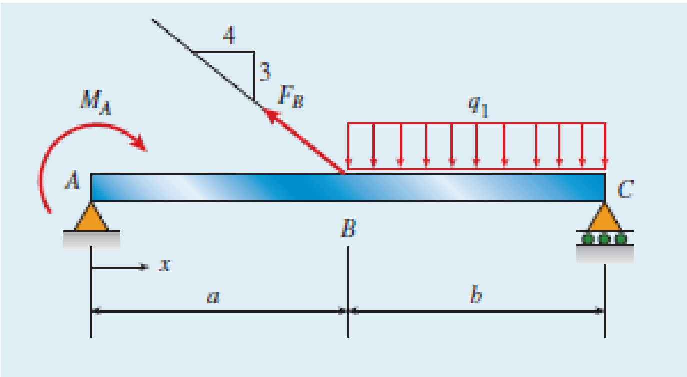
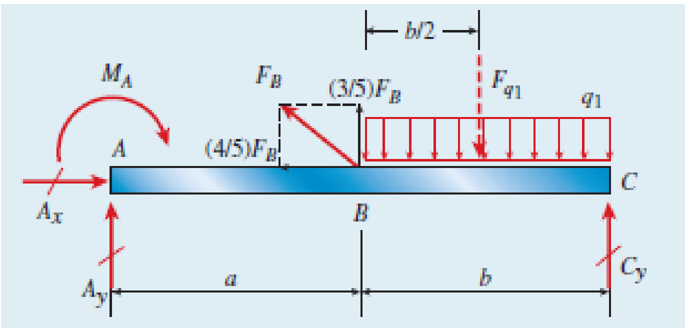
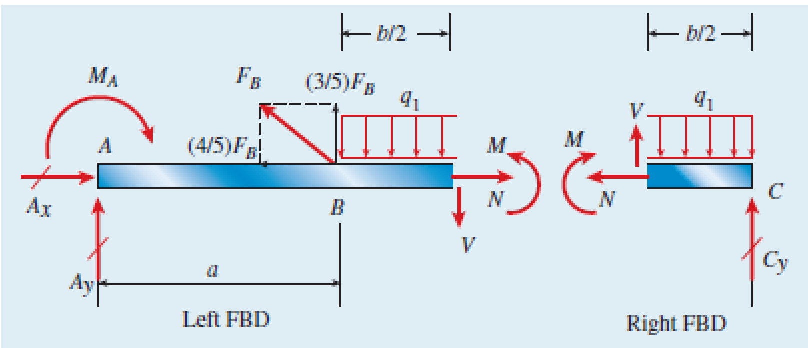


⊙ **Example 1-2:** Find the reactive forces, and compute internal forces and moment at the midpoint of member segment BC in the simple beam under applied loads







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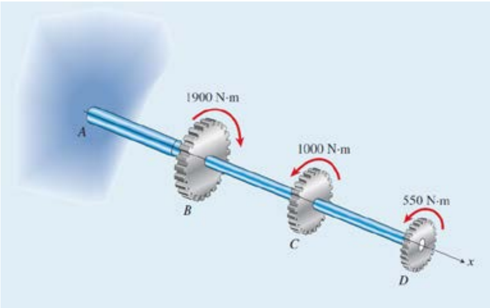
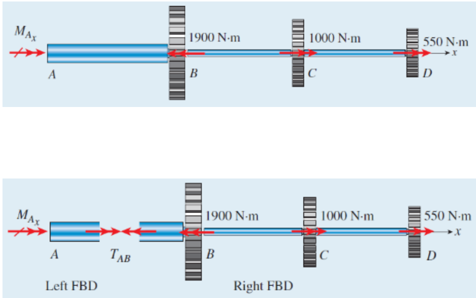
⦿ Straight structural members and their internal forces:

1. Bars (truss structures, Ex 1-1): axial (normal) force
2. Beams and columns (frame structures, Ex 1-2, 1-4): bending moment, shear force, and axial (normal) force
3. Shafts (Ex 1-3): torsional moment



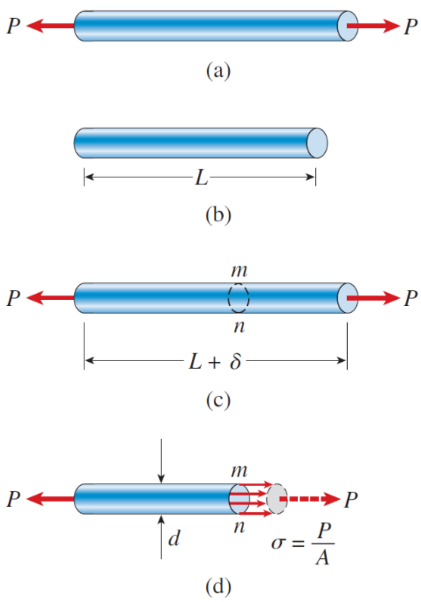


1.3 Normal Stress and Strain

⦿ Prismatic Bar

1. Straight structural member having the same _______ __________ throughout its lengths
2. An _________ __________ is a load directed along the axis of the member
3. Resulting in either __________ or __________
4. Examples: tow bar (Fig. 1-19), truss, connecting rods in engines, etc.

⦿ Normal stress in a bar
(Orthogonal to the surface, cf. shear stress)



1. Internal axial force P is the r_________ of the s_______ (σ) acting over the entire cross section
   i.e. $\int \sigma(x) dA = P$
2. Unit of normal stress (σ):
   ( )/unit ( )
   e.g. $N/m^2 = Pa$, $lb/in^2 = psi$, MPa, ksi
3. If the stress is distributed uniformly over the cross section, $\sigma = \frac{\quad}{\quad}$

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4. Stretched → 'tensile' stress (sign:)
   Compressed → 'compressive' stress (sign:)

5. It is noted that $\sigma = P/A$ works only when the stress is distributed uniformly over the cross section → This condition is satisfied if the force acts at the c__________ of the cross section (proof available at pp. 31-32) and the location of interest is far enough from the end of the member (Saint-Venant's principle, See figure from the course materials by Dr. N.A. Libre)

⊙ Normal strains in a bar

1. Consider a bar with the total length L and the elongation δ
   - Elongation of the half of the bar:
   - Elongation of the quarter of the bar:

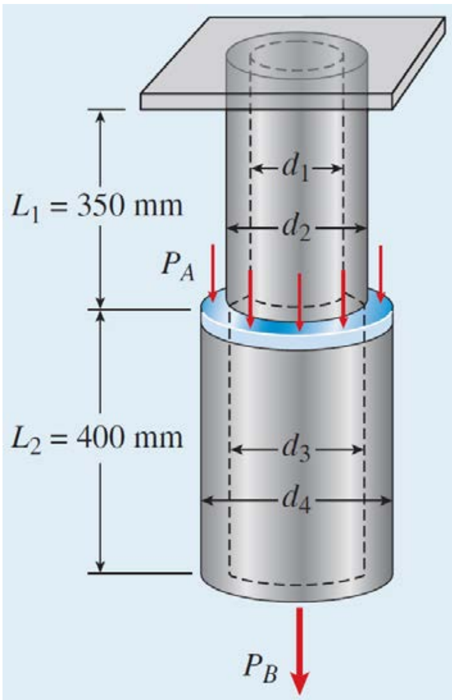
2. Thus, the enlogation of the unit length of the bar, i.e. (normal) strain, is

$$\varepsilon = \frac{\quad}{\quad}$$

3. Tensile (positive) and compressive strains (negative)

4. Unit of the strain: dimension_____, i.e. no units

⊙ **Example 1-5**: $P_A = 7{,}800\ \mathrm{N}$ (uniformly distributed around a cap plate), $d_1 = 51\ \mathrm{mm}$, $d_2 = 60\ \mathrm{mm}$, $d_3 = 57\ \mathrm{mm}$, and $d_4 = 63\ \mathrm{mm}$. $L_1 = 350\ \mathrm{mm}$ and $L_2 = 400\ \mathrm{mm}$. Neglect the self-weight of the pipes.



(a) Find P_B so that the tensile stress in upper part is $14.5\ \mathrm{MPa}$

(b) Find P_B so that upper and lower pars have same tensile stress

(c) For **(b)**, it is known that the elongations of the upper and lower pipe segments are $3.56\ \mathrm{mm}$ and $7.63\ \mathrm{mm}$, respectively. Tensile strains in the upper and lower pipe segments?

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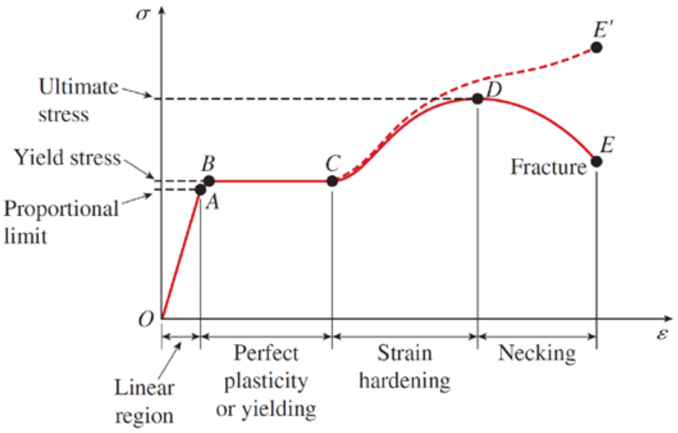
1.4 Mechanical Properties of Materials

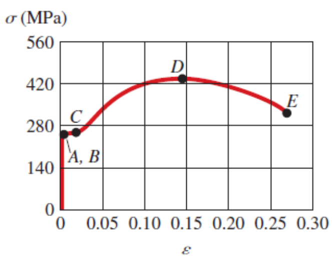
⦿ Laboratory tests to understand the mechanical behavior of materials

1. Tensile-test (Figs. 1-25, 26)
2. Compression test (Fig. 1-27)
3. Youtube video on tensile tests:
   https://www.youtube.com/watch?v=D8U4G5kcpcM#t=53.1643371

⦿ Stress-strain diagrams from tensile-tests

1. Diagram on the relationship between stress(σ) and strain(ε) – obtained by lab tests
2. "True" stress vs "nominal" stress: divided by actual (deformed) area or original
3. "True" strain vs "nominal" strain: divided by actual (deformed) length or original
4. Structural steel (also known as mild steel or low-carbon steel)

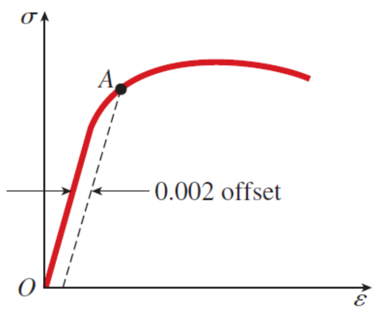




✓ Modulus of elasticity(E): the s_______ of the curve in the linear region

✓ Yield stress and ultimate stress are also called yield "strength" and ultimate "strength," respectively to refer to the capacity of a structure to resist loads

5. "Ductile" material: the material undergoes large p_________ strains before failures (↔ "brittle" material), e.g. low-carbon steel, aluminum, copper, magnesium, lead, etc.
6. Determining yield point when it is not obvious (e.g. aluminum alloy in Fig. 1-31): Use the offset method (the intersection with the offset line at a standard strain, say 0.002)



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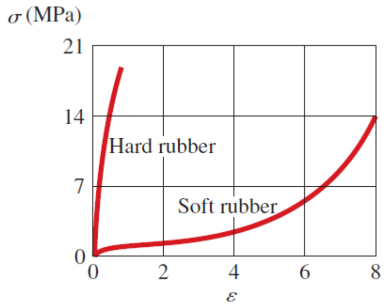
7. Rubber: stays elastic even after exhibiting nonlinear relationship

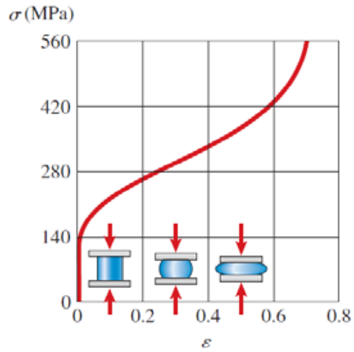


8. Measures of ductility:

   ✓ Percent elongation: $\frac{L_1 - L_0}{L_0}(100)$

   ✓ Percent reduction in area: $\frac{A_0 - A_1}{A_0}(100)$

9. "Brittle" materials: fails with only little elongation after the proportional limit, e.g. concrete, stone, cast iron, glass, ceramics

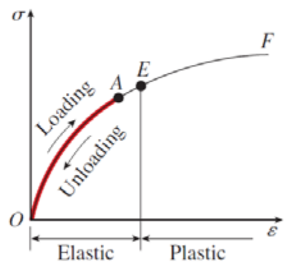


⊙ Stress-strain diagrams from compression-tests

1. Before yielding: similar behavior to that from tensile-tests

2. After yielding: bulges outward and shows increased resistance to further shortening

1.5 Elasticity, Plasticity, and Creep



(a)

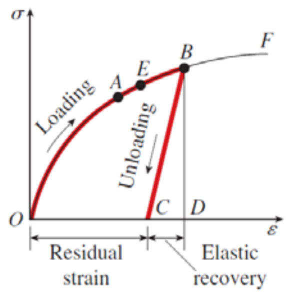
⊙ Behavior when the applied load is "unloaded"

1. "Elastic" behavior: follows the exactly same curve to return to the original dimension

2. "Partially elastic" behavior: follows a new unloading line that is parallel to the initial loading slope → creates "residual strain" (results in an elongation "permanent set")

3. Elastic limit: the upper limit of the elastic region (stress)

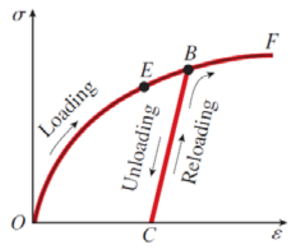
4. Plasticity: the characteristic of a material by which it undergoes inelastic strains beyond the strain at the elastic limit



(b)

⊙ Reloading (See figure → for the re-loading behavior)

After unloading (beyond the elastic limit) and re-loading, the properties of the material have changed, i.e. "new material"



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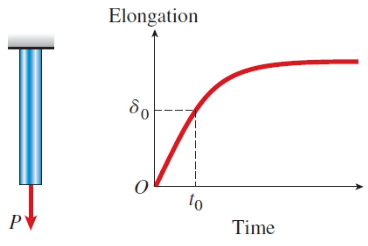
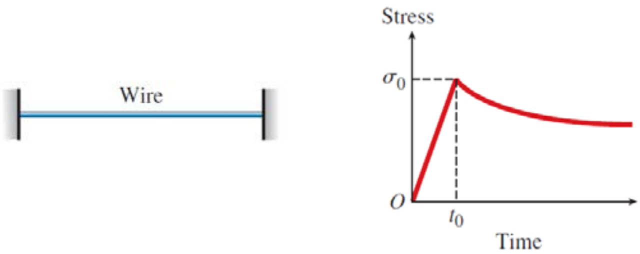
⦿ Creep

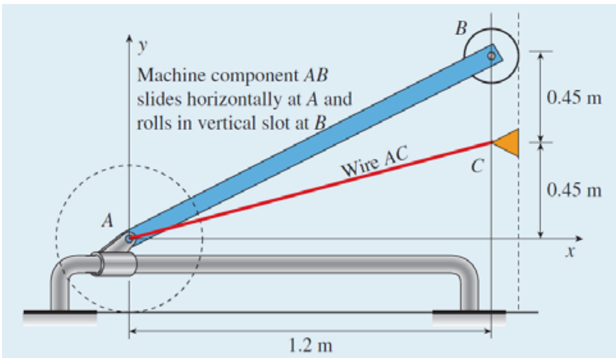
1. **Creep**: if the applied static load stays for long periods, the material can develop additional strains



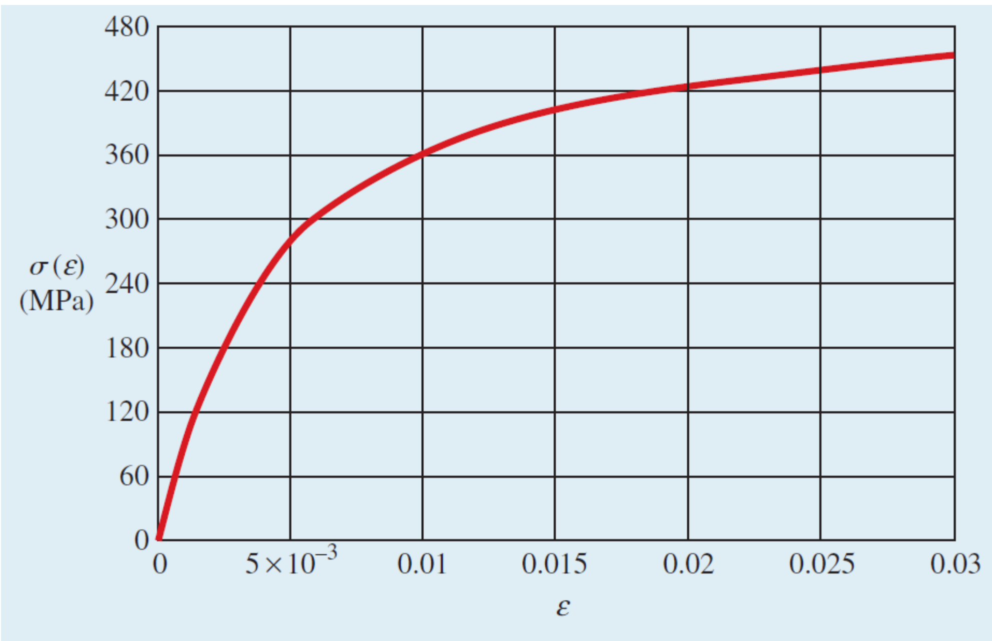
2. **Relaxation**: if a wire is stretched under a constant strain (e.g. stretched between two immovable supports), the stress diminishes as time goes on



⦿ **Example 1-7**: Rigid bar *AB* (length $L = 1.5\text{ m}$ and weight $W = 4.5\text{ kN}$) has roller supports at *A* and *B*. The machine component is supported by a single wire (diameter $d = 3.5\text{ mm}$). According to lab tests, stress-strain relationship for the wire (copper alloy) is $\sigma(\varepsilon) = \frac{124{,}000\varepsilon}{1+240\varepsilon}$ $0 \le \varepsilon \le 0.03$ (σ in MPa)



(a) Plot a stress-strain diagram for the material; what is the modulus of elasticity E (GPa)? What is the 0.2% offset yield stress (MPa)?



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(b) Find the tensile force T (kN) in the wire

(c) Find the normal axial strain ε and elongation δ (mm) of the wire.

(d) Find the permanent set of the wire if all forces are removed.

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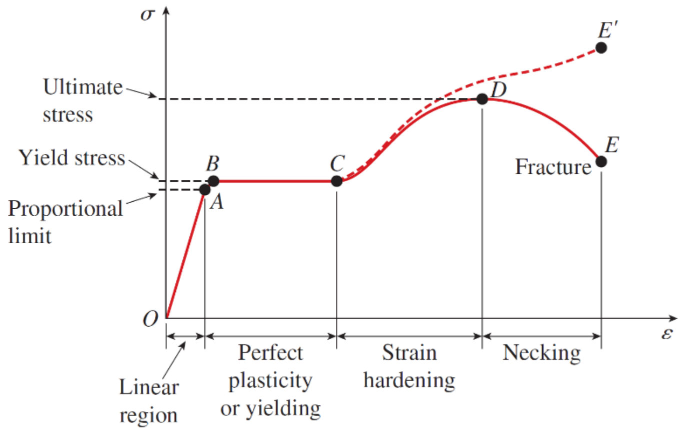
1.6 Linear Elasticity, Hooke's Law, and Poisson's Ratio

⦿ Linear elasticity

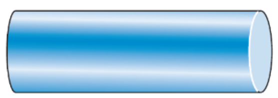


1. When a material behaves e__________ and also exhibits a l_________ relationship between s________ and s_________, it is said to be **linearly elastic**
2. This type of behavior is important because under normal conditions we want to design structures such that they avoid p__________ deformation due to yielding.

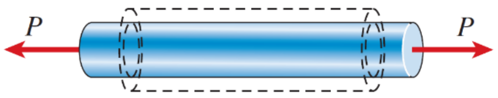
⦿ Hooke's law: l_________ relationship between stress and strain for a bar

1. Hooke's law: $\sigma = E\varepsilon$
2. E: **Modulus of elasticity** (or **Young's modulus**)
   - slope of the stress-strain curve in the linear elastic region
   - the unit is the same as that of ___________
3. "S_____" materials: large modulus of elasticity, e.g. steel (~30,000 ksi, 210 GPa), aluminum (~10,600 ksi, 73 GPa)
4. "F________" materials: small modulus of elasticity, e.g. plastics (100~2,000 ksi, 0.7~14 GPa)

⦿ Poisson's ratio



(a)



(b)

1. When a prismatic bar is loaded in tension, the axial elongation is accompanied by l________ c___________.
2. To characterize this, **Poisson's ratio** is defined as

$$\nu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon'}{\varepsilon}$$

3. Can obtain the lateral strain from the axial strain: $\varepsilon' = -\nu\varepsilon$
4. For most metals and many other materials, the range of Poisson's ratio is 0.25~0.35.
5. Theoretical upper limit: 0.5 (See Section 7.5, e.g. rubber's Poisson's ratio $\cong 0.5$)

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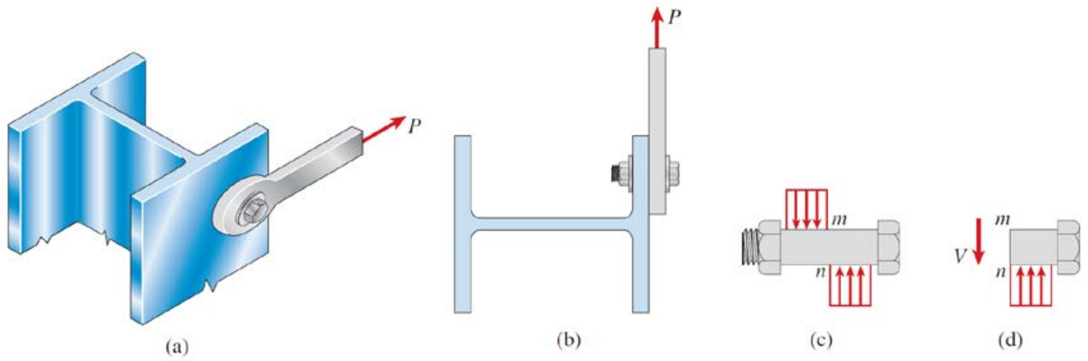
⦿ Other important properties

1. **Homogeneous**: the same material properties at every p________
2. **Isotropic**: the same material properties in all d__________, e.g. axial, lateral and others
3. **Anisotropic**: the material properties differ in various d_________

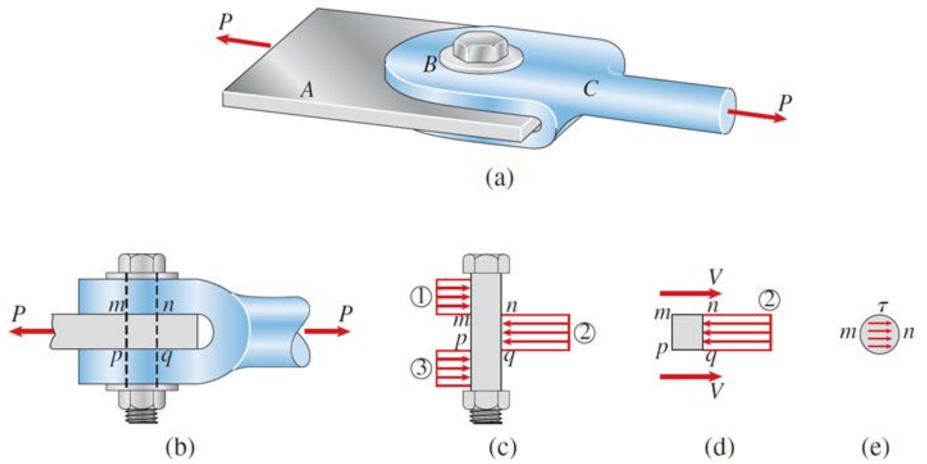
1.7 Shear Stress and Strain

⦿ "Shear" stress

1. Stresses that act t___________ to the surface of the material (vs "normal" stress acting perpendicular to the surface)
2. "Single shear" example: metal bar connected with the flange of a beam through a bolt ~ distortion and angle changes caused by the shear force $V = P$



3. "Double shear" example: flat bar (A) connected with clevis (C) through a bolt connection (B) ~ shear forces $V = P/2$



4. The shear force is resultant of shear s__________ (τ) over the entire cross section as shown in Figure (e) above, i.e. $V = \int \tau(\boldsymbol{x})dA$

5. Average shear stress: $\tau_{\text{avg}} = \frac{\quad}{\quad}$

6. When the shear stress is u_________ distributed over the cross section, the shear stress is $\tau = V\backslash A$

7. Shear "failure" examples:

Complete shear failure of a shear wall in Maipú (Source: AIR)

Shear failure of a reinforced concrete beam (Source: Lehigh University)

https://www.youtube.com/watch?v=GHMCG4fUUpM

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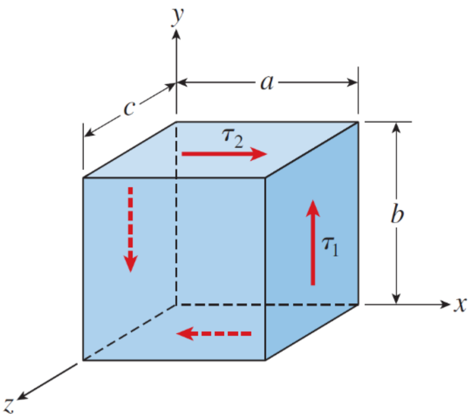
⊙ Equality of shear stresses on perpendicular planes

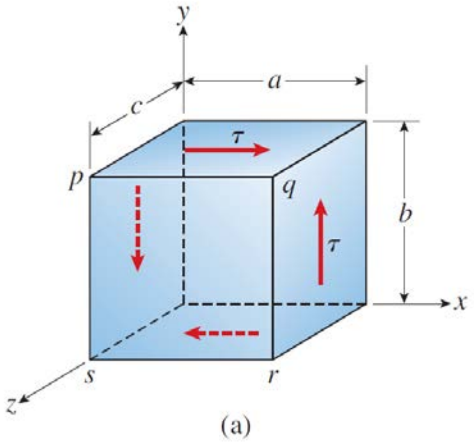
※ parallelepiped: prism whose bases are parallelograms

1. Proof of $\tau_1 = \tau_2$:



   - The total shear force acting on the right hand face:
   - The shear force acting on the opposite face:
   - A couple (moment force) generated by the two shear forces:
   - Similarly, the couple generated by the shear forces acting on the top and bottom faces:
   - From the moment equilibrium,

2. Summary of findings:

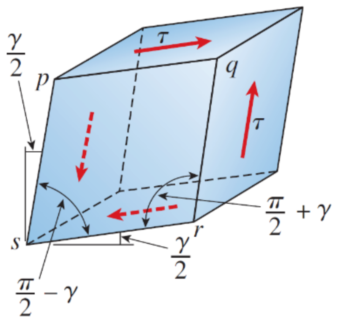


(a)

   a) Shear stresses on opposite faces are equal in magnitude and opposite in direction
   b) Shear stresses on adjacent (and perpendicular) faces are equal in magnitude and have directions such that both stresses point toward, or both point away from the line of intersection of the faces

3. The state of stress described above is called **"pure shear"** (Section 3.5)

4. The properties summarized above remain valid even when normal stresses act, i.e. not pure bending, because normal stresses acting on a small element are under equilibrium and thus do not alter the equilibrium equations used in the proof above.



⊙ Shear strain (γ)

1. Definition: change in the angle $\pi/2 \rightarrow \pi/2 \pm \gamma$

2. Unit: degrees or radians

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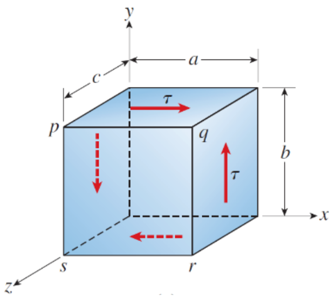
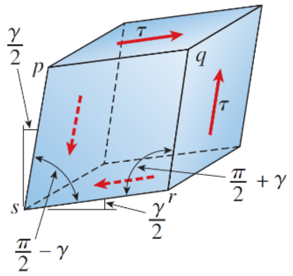
⊙ Sign conventions of shear stresses and strains

1. Stress: a shear stress acting on a positive face of an element is positive if it acts in the positive direction of one of the coordinate axes and negative if it acts in the negative direction of an axis

2. Strain: shear strain in an element is positive when the angle between two positive faces (or two negative faces) is reduced.





⊙ Hooke's law in shear

1. Diagram of shear stress and strain obtained by direct-shear tests or torsion tests

2. Hooke's law in shear: $\tau = G\gamma$

3. G: Shear modulus of elasticity (or modulus of rigidity) ~ same unit as E

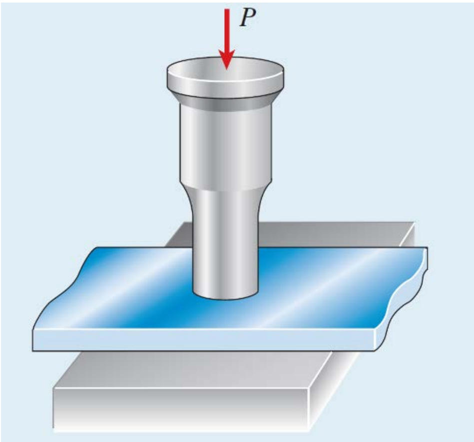
4. Mild steel: G = 11,000 ksi (75 GPa); Aluminum alloys: G =4,000 ksi (28 GPa)

5. Relationship between G and E (Section 3.6):

$$G = \frac{E}{2(1+\nu)}$$

6. Because ν is between 0 and 1/2, G is between one-half and one-third.

⊙ **Example 1-9**: Punch with a diameter $d = 3/4$ in. is used to make holes in a steel plates with a thickness $t = 3/10$ in. When the applied force is $P = 24$ kips, compute the average shear stress in the plate and the average compressive stress in the punch.



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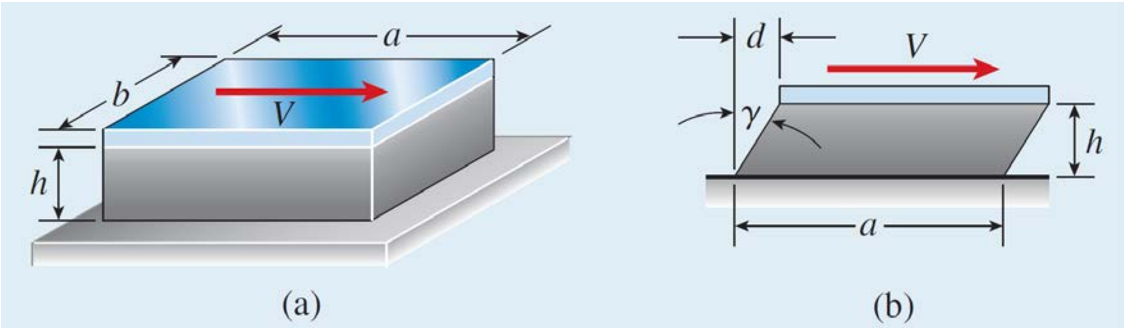
⊙ **Example 1-11**: Suppose a bearing pad in the figure is an elastomer (e.g. rubber) capped by a steel plate. When the pad is subjected to the shear force V, obtain formulas for the average shear stress τ_{aver} in the elastomer and the horizontal displacement d of the plate.



Bearing pad installed between bridge deck and pier (Source: mageba)