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Chapter 8 Applications of Plane Stress

8.2 Spherical Pressure Vessels

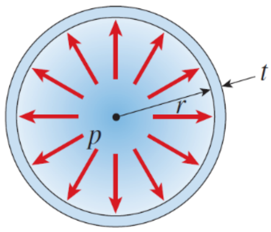
⦿ Spherical Pressure Vessels

1. Pressure vessels: c_______ structures containing liquids or gases under pressure

Thin-walled spherical pressure vessel used for storage of propane in this oil refinery

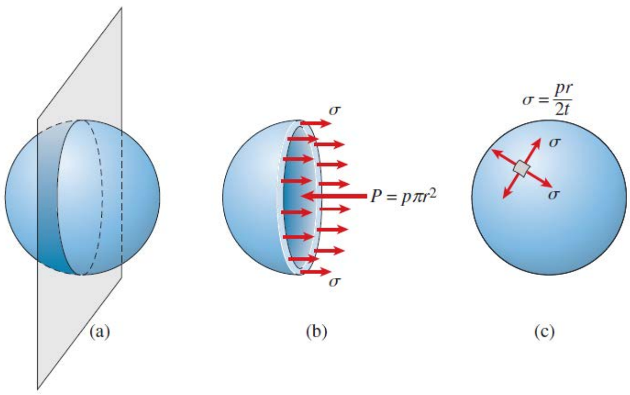
   1) Examples: tanks, pipes, pressurized cabins in aircraft and space vehicles
   2) More general category: s______ structures (other examples: roof domes, airplane wings, submarine hulls)

2. "Thin-walled" structure: r/t 10



   → Can determine the stresses in the walls with reasonable accuracy using statics alone

3. Horizontal resultant force by the pressure:
   $P = p\pi r^2$
   Note: Internal pressure ___ external pressure (otherwise, buckling)



4. The resultant of the tensile stresses σ: $\sigma(2\pi r_m t)$

   where $r_m = r + t/2$

5. From the equilibrium of horizontal forces, the tensile stresses in the wall of the vessel is

$$\sigma = \frac{pr^2}{2r_m t}$$

6. For thin-walled vessel, $r_m \cong$, thus

$$\sigma = \frac{pr}{2t}$$

7. From (rotational) symmetry, it is found that the wall of a pressurized spherical vessel is subjected to uniform tensile stresses σ *in all directions*.

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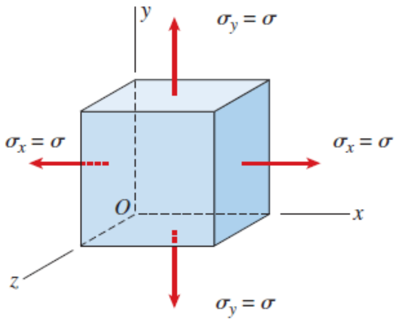
⦿ Stresses at the Outer Surface

1. Usually free of any external loads and rotational symmetry:

   $\sigma_x = \sigma_y = \quad , \ \sigma_z = \quad , \tau_{xy} =$



2. Using the transformation formula

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta =$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta =$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta =$$

   → Every plane and direction is a p_______ plane and direction with

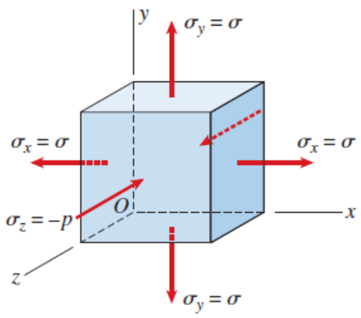
   $\sigma_1 = \sigma_2 =$ ____ and $\sigma_3 =$

   This means there is z______ shear stress in any direction

3. Maximum shear stress: occurs when rotated $45°$ around x or y axes (i.e. out-of-plane)

$$\tau_{\max} = \frac{\sigma}{2} = \frac{pr}{4t}$$

⦿ Stresses at the Inner Surface



1. In addition to the stresses at the outer surface, there is a compressive stress $\sigma_z = -p$

2. In any direction (within xy plane), we see principal stresses, i.e.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \quad \sigma_3 = -p$$

3. Maximum (out-of-plane) shear stress (when rotated $45°$ around x or y axis):

$$\tau_{\max} = \frac{\sigma + p}{2} = \frac{pr}{4t} + \frac{p}{2} = \frac{p}{2}\left(\frac{r}{2t} + 1\right)$$

4. When the ratio $\frac{r}{t} \gg 1$ (thin-walled), $\tau_{\max} \cong \frac{pr}{4t}$

5. Thus, for a thin-walled spherical shell structure, the stress state at the inner surface can be considered as the same as at the outer surface (under biaxial stress).

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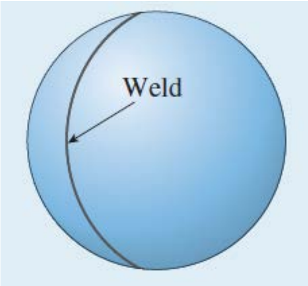

- **Example 8-1**: Consider a compressed-air tank with inner diameter $5.5\ \mathrm{m}$ and wall thickness $t = 45\ \mathrm{mm}$, formed by welding two steel hemispheres

  (a) Maximum permissible air pressure p_a when the allowable tensile stress of the steel is $\sigma_{\mathrm{allow}} = 93\ \mathrm{MPa}$.

  (b) Maximum permissible air pressure p_b when the allowable shear stress of the steel is $\tau_{\mathrm{allow}} = 42\ \mathrm{MPa}$.

  (c) If the normal strain at the outer surface of the tank is not to exceed 0.0003, what is the maximum permissible pressure p_c? (Note: Hooke's law is valid and $E = 210\ \mathrm{GPa}$ and $\nu = 0.28$)

  (d) The welded seam can sustain the tenslie load up to $7.5\ \mathrm{MN}$ per meter. If the required factor of safety is 2.5, what is the maximum permissible pressure p_d?

  (e) Considering the four preceding factors, what is the allowabl pressure p_{allow}?

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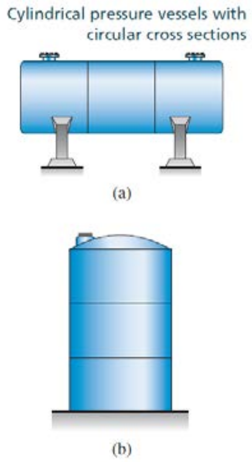
8.3 Cylindrical Pressure Vessels



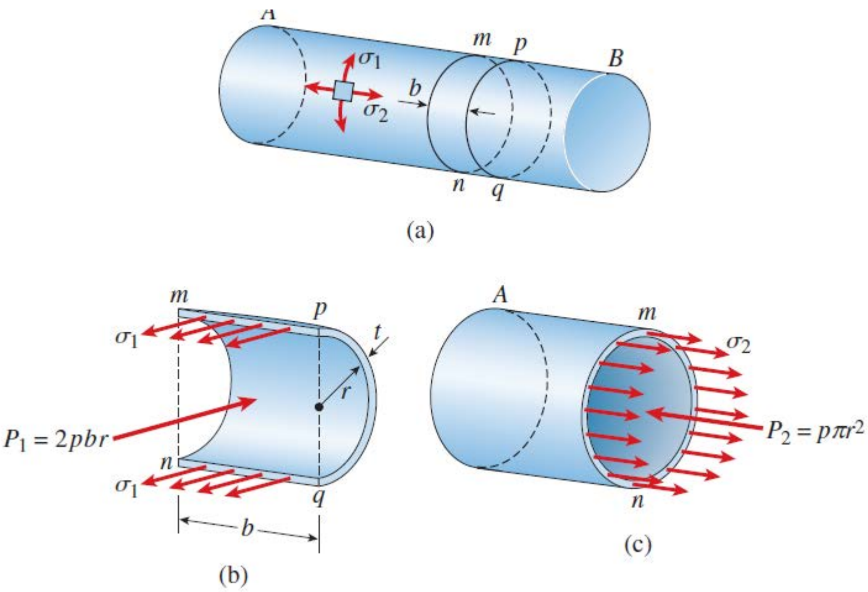
⦿ Cylindrical Pressure Vessels

1. Examples: compressed air tanks, fire extinguishers, propane tanks, grain silos

2. Stresses:

   1) σ_1: circumferential stress or h______ stress

   2) σ_2: longitudinal stress or axial stress



⦿ Circumferential (Hoop) Stress

1. Resultant forces

   $P_1 = 2pbr$ and $\sigma_1(2bt)$

2. From the equilibrium,

   $$\sigma_1 = \frac{pr}{t}$$

⦿ Longitudinal (Axial) Stress

1. Resultant forces: $P_2 = p\pi r^2$ and $\sigma_2(2\pi rt)$

2. From the equilibrium, $\sigma_2 = \frac{pr}{2t}$

3. It is noted that

   $\sigma_1 = \qquad \sigma_2$

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- Stresses at the Outer Surface

1. Maximum in-plane shear stress (when rotated by $45°$)

$$(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1}{4} = \frac{pr}{4t}$$

2. Maximum out-of-plane shear stress (when rotated by $45°$ around x and y axes)

$$(\tau_{\max})_x = \frac{\sigma_1 - 0}{2} = \frac{pr}{2t} \quad (\tau_{\max})_y = \frac{\sigma_2 - 0}{2} = \frac{pr}{4t}$$

- Stresses at the Inner Surface

1. Stresses: $\sigma_1 = \frac{pr}{t}$, $\sigma_2 = \frac{pr}{2t}$ and $\sigma_3 = -p$

2. Three maximum shear stresses

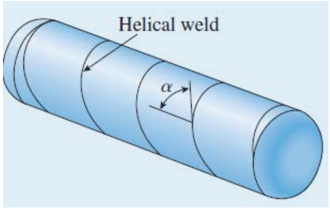
   1) $(\tau_{\max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{2t} + \frac{p}{2} \cong \frac{pr}{2t}$

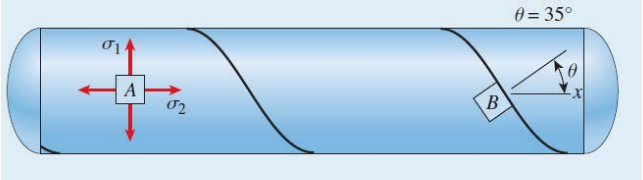
   2) $(\tau_{\max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{pr}{4t} + \frac{p}{2} \cong \frac{pr}{4t}$

   3) $(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t}$

3. It is noted once again that we can disregard the presence of the compressive stress $-p$ for a thin-walled cylindrical vessels

- **Example 8-2**: Cylindrical pressure vessel welded following a helical joint with angle $\alpha = 55°$ with the longitudinal axis. The vessel has $r = 1.8\text{ m}$, $t = 20\text{ mm}$, $E = 200\text{ GPa}$, $\nu = 0.30$ and $p = 800\text{ kPa}$. Calculate the followings: (a) σ_1 and σ_2, (b) the maximum in-plane and out-of-plane shear stresses, (c) circumferential and longitudinal strains ε_1 and ε_2; and (d) the normal stress σ_w and shear stress τ_w acting perpendicular and parallel, respectively to the welded seam.





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