CHAPTER 9. HEALTH DIAGNOSTICS AND PROGNOSTICS

9.1 Introduction

Last several decades, tremendous advance has been made on the physics-based analysis and design under uncertainties. However, it is still difficult for the physics-based analysis and design to deal with system failures with multiple failure mechanisms, complex physics-of-failure, and/or complicated joint behaviors. To overcome the difficulties of physics-based approaches, **sensor-based approach** has been emerged and actively engaged to promote life prediction, reliability analysis, and maintenance. Basic elements of sensor-based approach are shown in Figure 29.

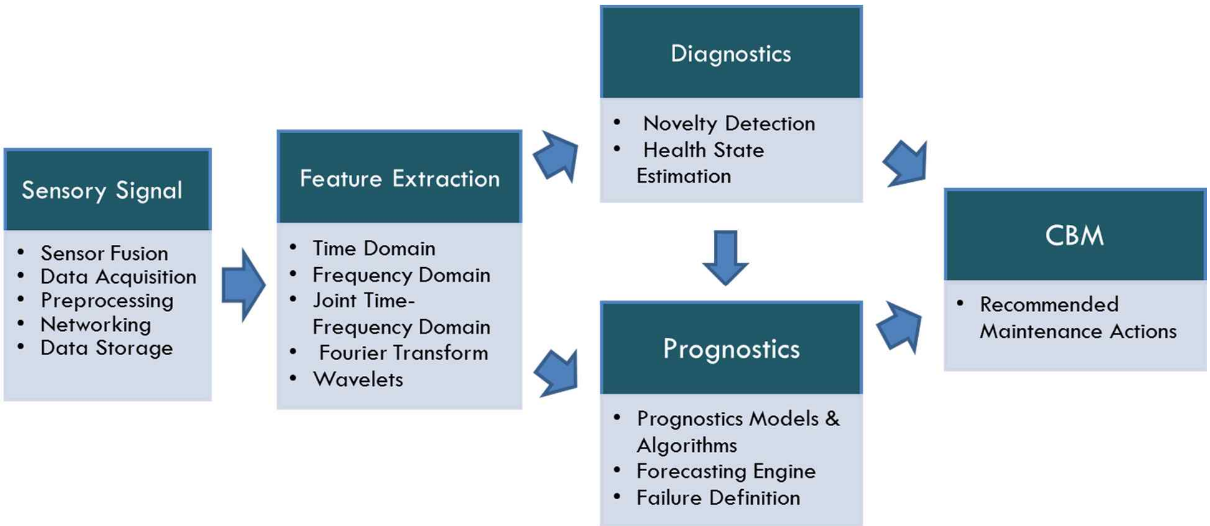


Figure 29: Basic Elements of Sensor-Based Risk Management

Diagnostics – The ability to detect and classify fault conditions.

Prognostics – The capability to provide early detection of a possible failure condition and to manage and predict the progression of this fault condition to component failure.

Maintenance:

Corrective Maintenance (CM): Action after failure
Preventive Maintenance (PM): Time-based action
Condition-Based Maintenance (CBM): Action if needed

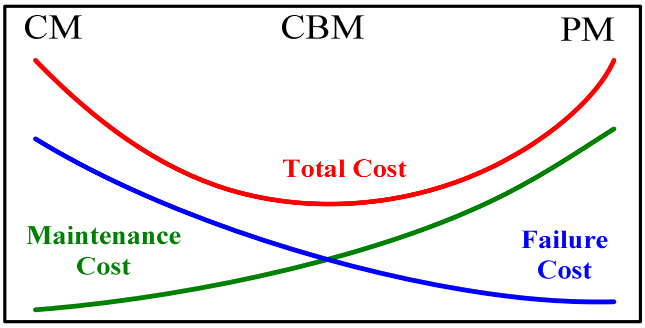
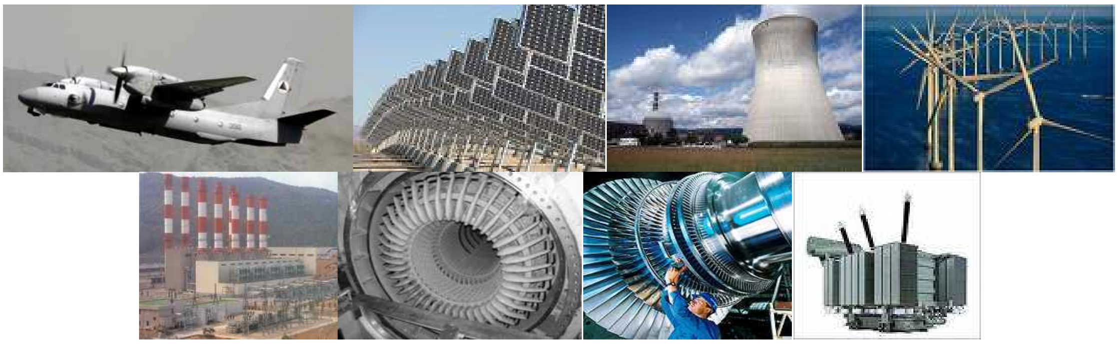


Figure 30: Cost Associated to Maintenance Strategies



Health diagnostics and prognostics are very useful to analyze health condition, to predict remaining useful life (RUL), and to make cost-effective maintenance action for large-scale systems, such as power plants, nuclear reactors, wind turbine generators, solar power systems, etc.




9.2 Signal Processing

Signal Processing can be any computer operation or series of operations performed on data to get insightful information. Usually, sensory data will be processed in either time domain, or frequency domain, and sometime in joint time-frequency domain to show extract the data feature.

9.2.1 Matlab Signal Processing Block-Set

The Signal Processing Blockset is a tool for digital signal processing algorithm simulation and code generation. It enables you to design and prototype signal processing systems using key signal processing algorithms and components in the Simulink® block format. Figure 31 shows the Library contained for signal processing block-set library (Type "***dsplib***" in the Matlab command window to open this library).

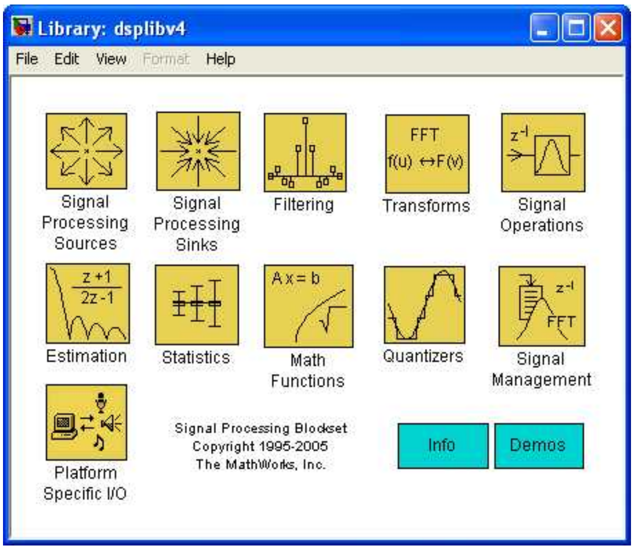


Figure 31: Matlab Signal Processing Block-Set Library

9.2.2 Time Domain Data Processing

Different statistics tools can be applied to time domain sensory signals to acquire data features, for example:
Distribution Fitting: find the best distribution fit of the input data
Histogram: generate histogram of input or sequence of inputs
Autocorrelation: compute autocorrelation of vector inputs
Correlation: compute cross-correlation of two inputs
Max. /Min.: find max. /Min. values in an input or sequence of inputs
Mean: find mean value of an input or sequence of inputs
RMS: compute root-mean-square (RMS) value of an input or sequence of inputs
Sort: Sort input elements by value
Standard Deviation: find standard deviation of an input or sequence of inputs
Variance: Compute variance of an input or sequence of inputs

Example: Building the following data processing block diagram as shown in Fig. 32, when the sinusoid signal, as shown in Fig. 33(a) is being processed, the RMS signal in Fig. 33(b) can be obtained. RMS signals are usually used to detect the changes of machine vibrations.

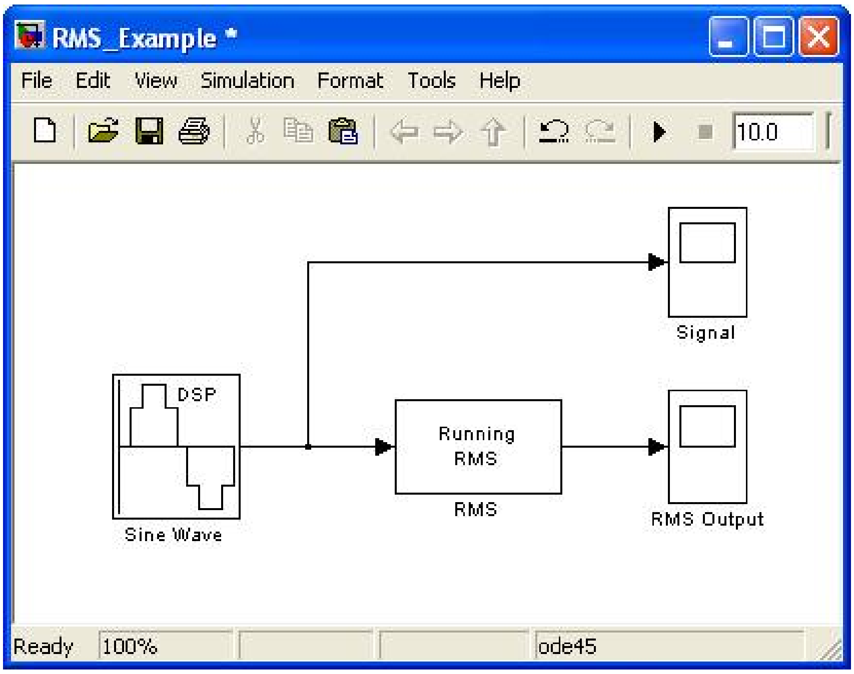
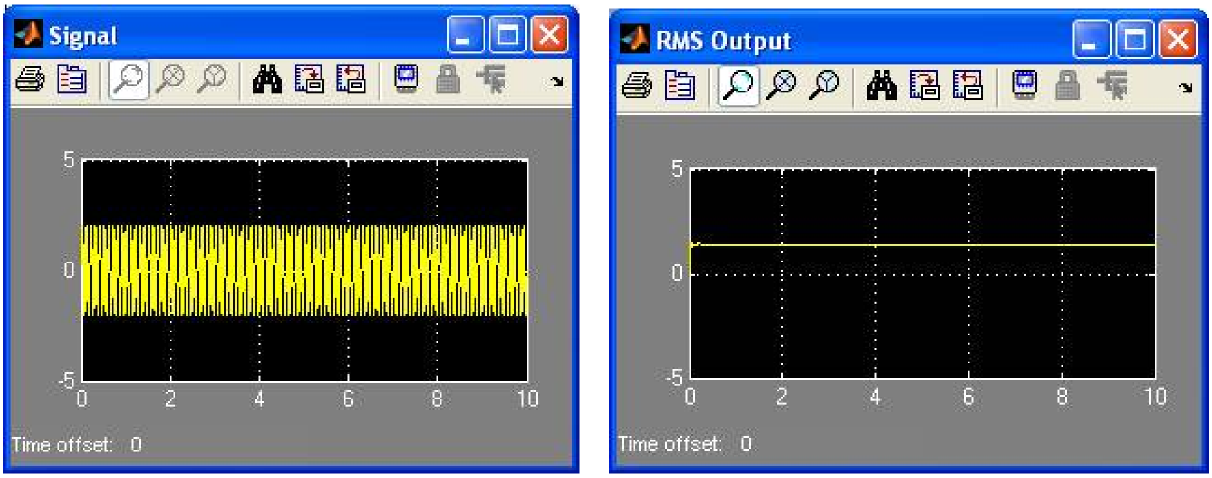


Figure 32: Signal Processing Example: RMS block diagram



(a) **(b)**

Figure 33: Sample Sinusoid Signal (a) and the RMS signal (b)

9.2.3 Frequency Domain Data Processing

When it is not clear of the data feature in time domain, usually we will transform the signal into frequency domain.

FFT

The Fourier transform transforms a time domain signal into a frequency domain representation of that signal. This means that it generates a description of the distribution of the energy in the signal as a function of frequency. This is normally displayed as a plot of frequency (x-axis) against amplitude (y-axis) called a *spectrum*. In signal processing the Fourier transform is almost always performed using an algorithm called the ***Fast Fourier Transform (FFT)***.

Example: FFT

```
t = 0:0.001:0.6;
x = sin(2*pi*50*t)+sin(2*pi*120*t)+sin(2*pi*200*t);
y = x + randn(size(t));
figure(1)
subplot(2,1,1)
plot(1000*t(1:50),y(1:50))
xlabel('Time (Milli-Seconds)')
ylabel('Signal with Random Noise')

subplot(2, 1, 2)
Y = fft(y, 512);
Fy = Y.* conj(Y) / 512;
f = 1000*(0:256)/512;
plot(f, Fy(1:257))
xlabel('frequency (Hz)');
ylabel('Frequency Content of Signal');
```

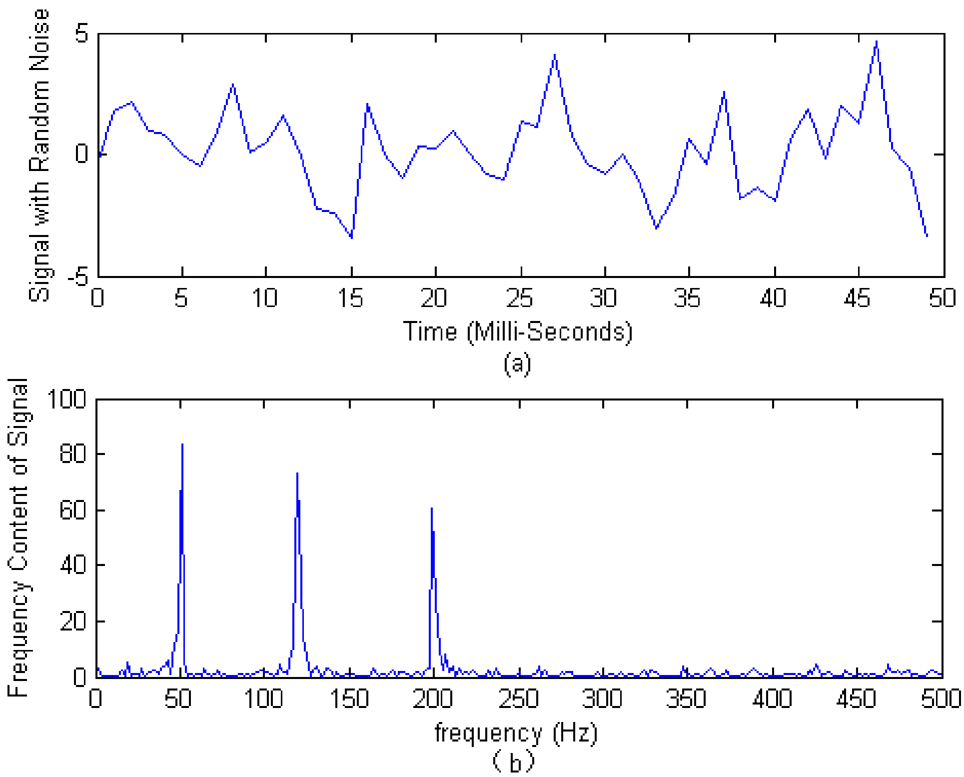



Figure 34: Sample Time Doman Signal (a) and Frequency Doman Signal (b)

* *You can also build a FFT block Diagram to do this example.*

9.2.4 Joint Time-Frequency Domain Analysis

There is a tradeoff between resolution in frequency and resolution in time. Good frequency resolution implies poor time resolution and good time resolution implies poor frequency resolution. Although frequency-domain representations such as the power spectrum of a signal often show useful information, the representations don't show how the frequency content of a signal evolves over time.

Joint Time-Frequency Analysis (JFTA) is a set of transforms that maps a one-dimensional time domain signal into a two-dimensional representation of energy versus time and frequency. JTFA shows the frequency content of a signal and the change in frequency with time.
There are a number of different transforms available for JTFA. Each transform type shows a different time-frequency representation. The Short Time Fourier Transform (STFT) is the simplest JTFA transform. For the STFT, you apply FFT repeatedly to short segments of a signal at ever-later positions in time. You can display the result on a 3-D graph or a so-called 2-D 1/2 representation (the energy is mapped to light intensity or color values).

The STFT technique uses FFT and suffers from an inherent coupling between time resolution and frequency resolution as mentioned earlier (increasing the first decreases the second, and vice versa). Other JTFA methods and transforms can yield a more precise estimate of the energy in a given Frequency-Time domain. Some options include:

- Gabor spectrogram
- Wavelet transform
- Wigner distribution
- Cohen class transforms

9.3 Health Monitoring

The process of implementing a damage detection strategy for engineering structures is referred to as **Structural Health Monitoring** (SHM). The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of system health. For long term SHM, the output of this process is periodically updated information regarding the ability of the structure to perform its intended function in light of the inevitable aging and degradation resulting from operational environments.

The SHM problem can be addressed in the context of a statistical pattern recognition paradigm, which includes four-step process: (i) Operational evaluation, (ii) Data acquisition, normalization and cleansing, (iii) Feature extraction and information condensation, and (iv) Statistical model development for feature discrimination.

- Operational evaluation

Operational evaluation attempts to answer the following four questions regarding the implementation of a damage identification capability.

a) What are the life-safety and/or economic justification for performing SHM?
b) How is damage defined for the system being investigated and, for multiple damage possibilities, which cases are of the most concern?
c) What are the conditions, both operational and environmental, under which the system to be monitored functions?
d) What are the limitations on acquiring data in the operational environment?

- Data acquisition, normalization and cleansing

The data acquisition portion of the SHM process involves selecting the excitation methods, the sensor types, number and locations, and the data acquisition/storage /transmittal hardware.

As data can be measured under varying conditions, the ability to normalize the data becomes very important to the damage identification process. One of the most common procedures is to normalize the measured responses by the measured inputs.

Data cleansing is the process of selectively choosing data to pass on to or reject from the feature extraction process. Signal processing techniques such as filtering and re-sampling can be used as data cleansing procedures.

- Feature extraction and information condensation

The area of the SHM process that receives the most attention in the technical literature is the **data feature extraction** that allows one to distinguish between the undamaged and damaged structure. Inherent in this feature selection process is the condensation of the data. The best features for damage identification are, again, application specific.

One of the most common feature extraction methods is based on correlating measured system response quantities, such a vibration amplitude or frequency, with the first-hand observations of the degrading system.

Another method of developing features for damage identification is to apply engineered flaws, similar to ones expected in actual operating conditions, to systems and develop an initial understanding of the parameters that are sensitive to the expected damage. The flawed system can also be used to validate that the diagnostic measurements are sensitive enough to distinguish between features identified from the undamaged and damaged system. The use of analytical tools such as experimentally-validated finite element models can be a great asset in this process. In many cases the analytical tools are used to perform numerical experiments where the flaws are introduced through computer simulation.

Damage accumulation testing, during which significant structural components of the system under study are degraded by subjecting them to realistic loading conditions, can also be used to identify appropriate features. This process may involve induced-damage testing, fatigue testing, corrosion growth, or temperature cycling to accumulate certain types of damage in an accelerated fashion. Insight into the appropriate features can be gained from several types of analytical and experimental studies as described above and is usually the result of information obtained from some combination of these studies.

The operational implementation and diagnostic measurement technologies needed to perform SHM produce more data than traditional uses of structural dynamics information. A condensation of the data is advantageous and necessary when comparisons of many feature sets obtained over the lifetime of the structure are envisioned. Also, because data will be acquired from a structure over an extended period of time and in an operational environment, robust data reduction techniques must be developed to retain feature sensitivity to the structural changes of interest in the presence of environmental and operational variability. To further aid in the extraction and recording of quality data needed to perform SHM, the statistical significance of the features should be characterized and used in the condensation process.

- Statistical model development

Statistical model development is concerned with the implementation of the algorithms that operate on the extracted features to quantify the damage state of the structure. The algorithms used in statistical model development usually fall into two categories: ***supervised learning*** and ***unsupervised learning***. When data are available from both the undamaged and damaged structure, the statistical pattern recognition algorithms fall into the general classification referred to as supervised learning. Group classification and regression analysis are categories of the supervised learning algorithms. Unsupervised learning refers to algorithms that are applied to data not containing examples from the damaged structure. Outlier or novelty detection is the primary class of algorithms applied in unsupervised learning applications. All of the algorithms analyze statistical distributions of the measured or derived features to enhance the damage identification process.

9.4 Health Prognostics

Real-time diagnosis and prognosis which interprets data acquired by smart sensors and distributed sensor networks, and utilizes these data streams in making critical decisions provides significant advancements across a wide range of application. Figure 35 shows a typical paradigm of the sensor-based life and reliability prognostics, which utilizes the sensory signal to produce the system degradation signal through the signal processing, and then the diagnostics of the system current health condition and further predict the system Residual Useful Life (RUL) and reliability will be carried out based on the system degradation signals. Uncertainties for sensory signal noise, data processing error and prediction variability are considered in this process. Technical approaches to system sensor-based life and reliability prognostics can be categorized broadly into model-based approaches and data-driven approaches.

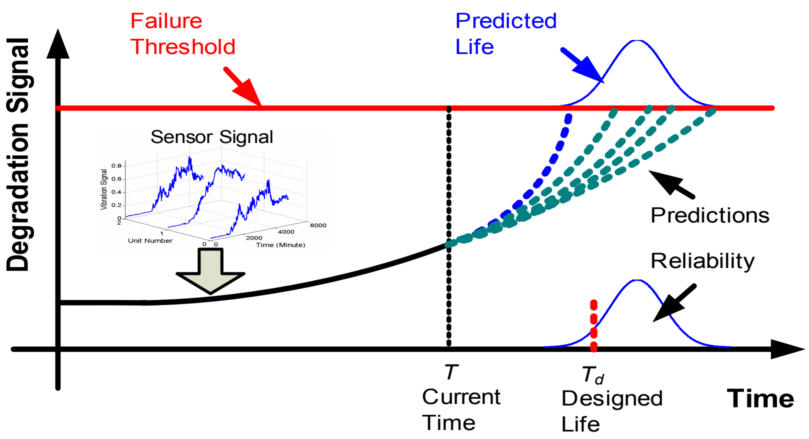


Figure 35: Sensor-Based Life and Reliability Prognostics

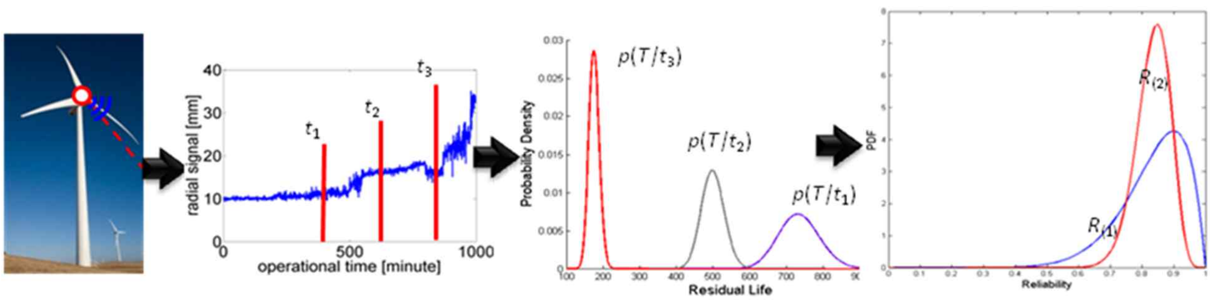


Figure 36: Procedures of Health Prognostics

- Model-Based Prognostics

Model-based prognostic approaches attempt to incorporate physical understanding (physical models) of the system into the estimation of remaining useful life (RUL). Different stochastic degradation models have been investigated in the literature, to model various degradation phenomena of systems or components.
Real-time degradation model parameters updating with evolving sensory signals is a challenge of model-based prognostic approaches. Bayesian updating techniques are commonly used for this purpose. Table 1 describes a Markov-Chain Monte Carlo (MCMC) method for non-conjugate Bayesian updating.

Example: An exponential degradation model

$$S(t_i) = S_0 + \delta \cdot \exp(\alpha t_i^2 + \beta t_i + \varepsilon(t_i) - \frac{\sigma^2}{2})$$

where $S(t_i)$ represents the degradation signal at time t_i; S_0 is a known constant; δ, α, **and** β **are stochastic model parameters** and ε represents the random error term which follows normal distribution with zero mean and σ^2 deviation. Figure 37 shows the updating of this model and corresponding *RUL*.

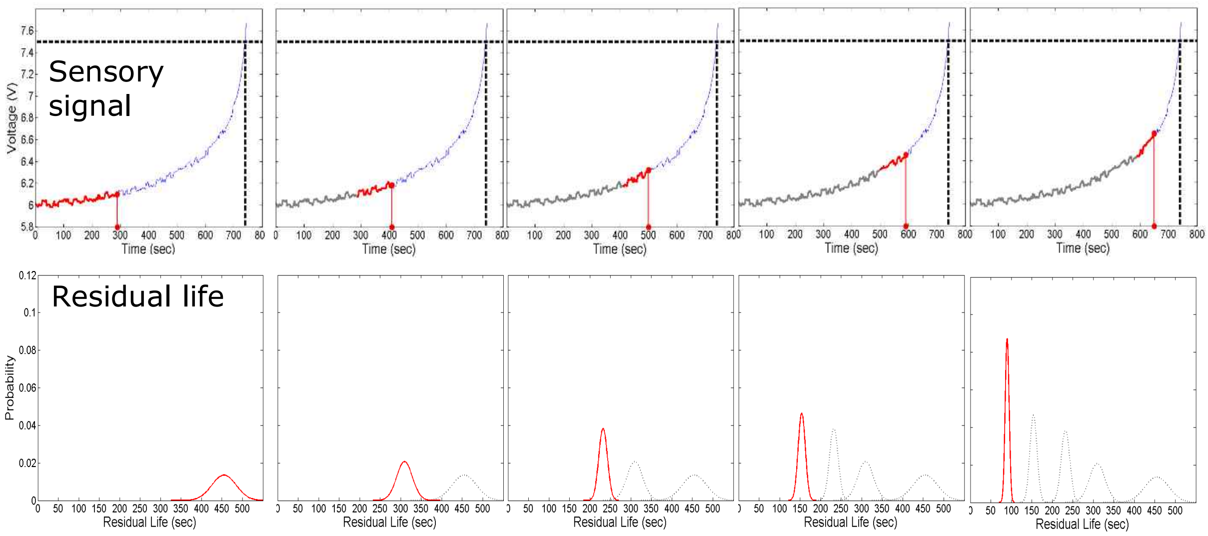


Figure 37: Model and RUL updating

- Data-Driven Prognostics

Data-driven prognostic techniques utilize monitored operational data related to system health. The major advantage of data driven approaches is that they can be deployed quicker and cheaper compared to other approaches, and that they can provide system-wide coverage. The principal disadvantage is that data driven approaches may have wider confidence intervals than other approaches and that they require a substantial amount of data for training.

Data and information updating schemes

a) Numerical Methods
   - Linear Regression
   - Kalman Filters
   - Particle Filters

Machine learning techniques

b) Artificial Intelligence Based Techniques
   - Artificial Neural Networks
   - Decision Tree Method
   - Novelty Detection Algorithms
   - Support Vector Machine (SVM)
   - Relevance Vector Machine (RVM)
   - Fuzzy Logic

Homework 1: *Sources of uncertainty in a vibration problem*
Let us consider an undamped system with a lumped mass and spring. The motion behavior of the system can be ideally modeled using a second-order ordinary differential equation as

$$my''(t) + ky(t) = 0; \quad y(0) = 15, \quad y'(0) = 0$$

where m and k are the mass and spring coefficient of the system, respectively. According to the manufacturer of the system, the mass and spring coefficient are believed to be 10 kg and 1000 N/m, respectively. At time t = 1 second, ten experimental tests show a set of y data as (4.4456, 4.2094, 4.3348, 4.2441, 4.1768, 4.1756, 4.4057, 4.2448, 4.2303, 4.0952). Answer the following questions:

(1) Identify all possible sources of uncertainties involved in this problem.
(2) Please explain why experimentally measured y values are random.
(3) Also, provide possible reasons for what causes the difference between experimental and analytical y values.

Homework 2: *Probability Distribution & Statistical Moments*

Let us recall the example of fatigue tests. The sample data can be obtained about the physical quantities in the damage model below.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$$

Let us consider a 30 data set (Table 5) for the fatigue ductility coefficient (ε_f') and exponent (c) used in the strain-life formula shown above. Answer the following questions:

(1) Construct the covariance matrix and find out the coefficient of correlation using the data set given in Table 5.

(2) Use normal, weibull, and lognormal distributions. Find out the most suitable parameters of three distributions for the fatigue ductility coefficient (ε_f') and exponent (c) using the MLE method.

(3) Find out the most suitable distributions for the data set (ε_f', c) using the chi-square GOF.

(4) Verify the results with the graphical methods (histogram and probability plots).

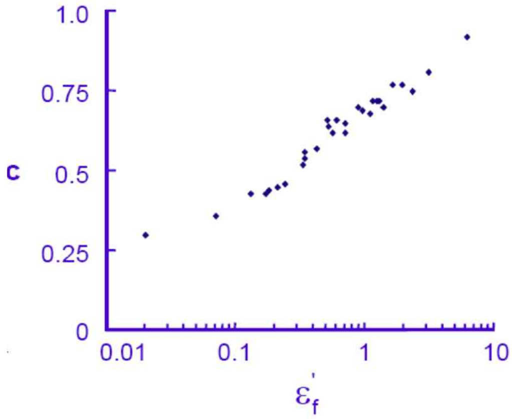


Figure 8: Statistical Correlation

Table 5: Data for the fatigue ductility coefficient and exponent

(ε_f', c)		(ε_f', c)		(ε_f', c)		(ε_f', c)		(ε_f', c)	
0.022	0.289	0.253	0.466	0.539	0.630	0.989	0.694	1.611	0.702
0.071	0.370	0.342	0.531	0.590	0.621	1.201	0.690	1.845	0.760
0.146	0.450	0.353	0.553	0.622	0.653	1.304	0.715	1.995	0.759
0.185	0.448	0.354	0.580	0.727	0.635	1.388	0.717	2.342	0.748
0.196	0.452	0.431	0.587	0.729	0.645	1.392	0.716	3.288	0.821
0.215	0.460	0.519	0.655	0.906	0.703	1.426	0.703	6.241	0.894

Homework 3: *Reliability Function*

Supposed it is desired to estimate the failure rate of an electronic component. A test can be performed to estimate its failure rate. A target life is set to 2000 minutes. $R(t) = P(T > 2000 \text{ minutes})$ Answer the following questions:

(1) Construct a histogram of TTF.

(2) Find out a probability distribution model and its parameters, $f_T(t)$, for the TTF data.

(3) Construct a reliability function.

(4) Determine MTTF, standard deviation of TTF, and hazard function.

(5) Compare the reliabilities from n_f/N from the TTF data and the reliability function when t = 2000 where n_f is the number of failed components and N (= 100) is the total components.

Table 5: Data for 100 Electronics Time-To-Failure (TTF) [minute]

1703.2	1071.4	2225.8	1826.5	1131	2068.9	1573.5	1522.1	1490.7	2226.6
1481.1	2065.1	1880.9	2290.9	1786.4	1867.2	1859.1	1907.5	1791.8	1871
1990.4	2024.1	1688.6	1962.7	2191.7	1841	1814.1	1918.1	2237.5	1396.8
1692.8	707.2	2101.3	2165.4	1975.2	1961.6	2116.7	1373	1798.8	2248.4
1872.3	1597.8	1865.1	742.8	1436.7	1380.8	2258.2	1960	2182.8	1772.7
2003.6	1589.4	1988.3	1874.9	1859	2051.9	1763	1854.6	1974.7	2289.9
1945.7	1774.8	1579.6	1430.5	1855	1757.9	1029.3	1707.2	1864.7	1964.8
1719.4	1565.2	1736.8	1759.4	1939.4	2065.7	2258.5	2292.8	1452.5	1692.2
2120.7	1934.8	999.4	1919.9	2162.4	2094.9	2158.2	1884.2	1748.7	2260.3
1040.8	1535	1283.4	2267.7	2100.3	2007.9	2499.8	1902.9	1599.6	1567.5

(6) Attempt to update the TTF mean value (θ) with aggregation of 100 TTF data using Bayesian inference. Assume that the TTF follows a normal distribution with the standard deviation (σ = 315.16) and the prior distribution of θ be $P(\theta) = N(u = 1750.0,\ \tau^2 = 500^2)$.

Homework 4: *Reliability Analysis*

Consider the following simply supported beam subject to a uniform load, as illustrated in Fig. 19. Suppose L = 5 m and w=10 kN/m.

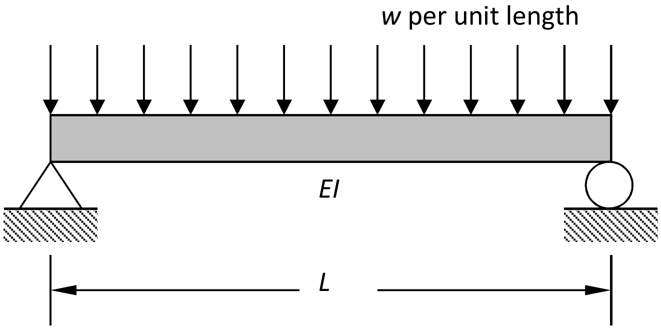


Figure 19: Simply Supported Beam

Random Vector:

$$EI = X_1 \sim N(\mu_{X_1} = 3\times10^7, \sigma_{X_1} = 10^5)$$
$$w = X_2 \sim N(\mu_{X_2} = 10^4, \sigma_{X_2} = 10^3)$$

The maximum deflection of the beam is shown as

$$Y = g(X_1, X_2) = -\frac{5X_2L^4}{384X_1}$$

Using Monte Carlo simulation, first-order expansion method, MPP-based method (HL-RF) and Eigenvector Dimension Reduction (EDR) method, determine the PDF (or CDF) of the maximum deflection and estimate its reliability when the failure is defined as $Y < y_c = -3\times10^{-3}$m. Make your own discussion and conclusion.

Homework 5: *Bayesian Reliability Analysis*

Consider the following simply supported beam subject to a uniform load, as illustrated in Fig. 19. Suppose L = 5 m and w=10 kN/m.

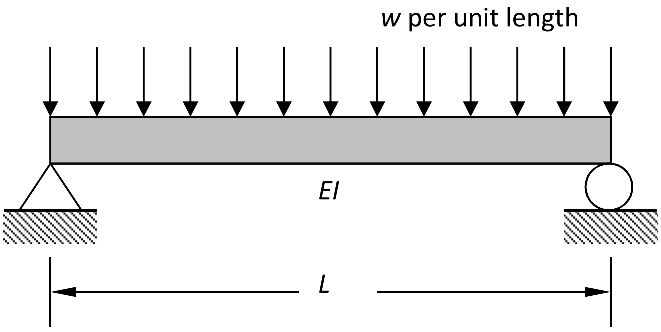


Figure 19: Simply Supported Beam

Random Vector:

$$EI = X_1 \sim N(\mu_{X_1} = 3\times 10^7, \sigma_{X_1} = 10^5)$$
$$w = X_2 \sim \text{epitemic}$$

The maximum deflection of the beam is shown as

$$Y = g(X_1, X_2) = -\frac{5X_2L^4}{384X_1}$$

The X_2 is an epistemic uncertainty. For X_2, it is assumed that 10 data sets are gradually obtained at different times. Using MPP-based method (HL-RF) and Eigenvector Dimension Reduction (EDR) method, determine the reliability of the maximum deflection constraint, $P(Y(X_1) \geq y_c = -3\times 10^{-3}\text{m})$, at all individual X_2 points in the table. Predict reliability in a Bayesian sense using the first 10 data set and gradually update the reliability using the second and third data sets. Make your own discussion and conclusion, and attach your code used for Bayesian reliability analysis.

Table 9 Three sets of 10 data for X_2 ($\times 10^4$)

Set1	1.0000	0.8126	1.0731	1.0677	0.9623	0.9766	1.1444	1.0799	1.0212	0.9258
Set2	0.9682	1.0428	1.0578	1.0569	0.9704	1.0118	0.9649	1.0941	1.0238	1.1082
Set3	1.1095	1.0896	1.0040	0.9744	0.8525	1.0315	1.0623	0.9008	0.8992	0.9869

Homework 6: *Design Optimization of Crashworthiness Problem*

A vehicle side impact problem is considered for design optimization. All the design variables are shown in Table 11. In this example, the abdomen load is treated as an objective function with nine constraints defined in Table 12.

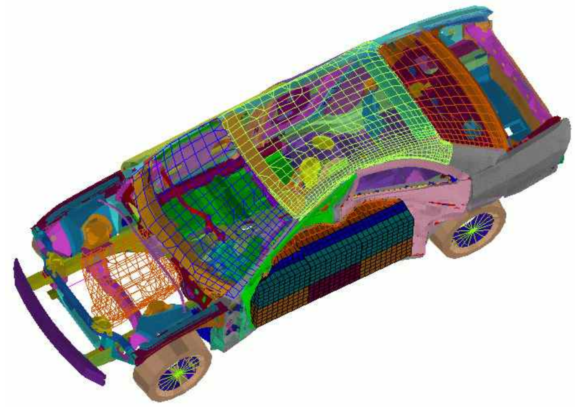


Table 11: Properties of design variables (X_{10} and X_{11} have "0" value)

Design Variables	d^L	d	d^U
X_1	0.500	1.000	1.500
X_2	0.500	1.000	1.500
X_3	0.500	1.000	1.500
X_4	0.500	1.000	1.500
X_5	0.500	1.000	1.500
X_6	0.500	1.000	1.500
X_7	0.500	1.000	1.500
X_8	0.192	0.300	0.345
X_9	0.192	0.300	0.345
X_{10} X_{11}		0.000 (not design variable)	

Table 12: Design variables and their bounds

Components		Safety criteria
Objective: Abdomen load (kN)		≤1
G_1-G_3: Rib Deflection	Upper Middle Lower	≤32
G_4-G_6: VC (m/s)	Upper Middle Lower	≤0.32
G_7: Pubic symphysis force (kN)		≤4
G_8: Velocity of B-pillar		≤9.9
G_9: Velocity of front door at B-pillar		≤15.7

Responses:


```
OBJ= (1.16-0.3717*x(2)*x(4)-0.00931*x(2)*x(10)-
0.484*x(3)*x(9)+0.01343*x(6)*x(10));

G1 = (28.98+3.818*x(3)-4.2*x(1)*x(2)+0.0207*x(5)*x(10)+6.63*x(6)*x(9)-
7.7*x(7)*x(8)+0.32*x(9)*x(10))-32;

 G2= (33.86+2.95*x(3)+0.1792*x(10)-5.057*x(1)*x(2)-11*x(2)*x(8)-
0.0215*x(5)*x(10)-9.98*x(7)*x(8)+22*x(8)*x(9))-32;

 G3 = (46.36-9.9*x(2)-12.9*x(1)*x(8)+0.1107*x(3)*x(10))-32;

G4 = (0.261-0.0159*x(1)*x(2)-0.188*x(1)*x(8)-
0.019*x(2)*x(7)+0.0144*x(3)*x(5)+0.0008757*x(5)*x(10)+0.08045*x(6)*x(9)+0.00
139*x(8)*x(11)+0.00001575*x(10)*x(11))-0.32;

G5 = (0.214+0.00817*x(5)-0.131*x(1)*x(8)-0.0704*x(1)*x(9)+ 0.03099*x(2)*x(6)-
0.018*x(2)*x(7)+0.0208*x(3)*x(8)+ 0.121*x(3)*x(9)-
0.00364*x(5)*x(6)+0.0007715*x(5)*x(10)-
0.0005354*x(6)*x(10)+0.00121*x(8)*x(11)+0.00184*x(9)*x(10)- 0.018*x(2).^2)-
0.32;

G6 = (0.74-0.61*x(2)-0.163*x(3)*x(8)+0.001232*x(3)*x(10)-
0.166*x(7)*x(9)+0.227*x(2).^2)-0.32;

G7 = (4.72-0.5*x(4)-0.19*x(2)*x(3)-
0.0122*x(4)*x(10)+0.009325*x(6)*x(10)+0.000191*x(11).^2)-4;

G8 = (10.58-0.674*x(1)*x(2)-1.95*x(2)*x(8)+0.02054*x(3)*x(10)-
0.0198*x(4)*x(10)+0.028*x(6)*x(10))-9.9;

G9 = (16.45-0.489*x(3)*x(7)-0.843*x(5)*x(6)+0.0432*x(9)*x(10)-
0.0556*x(9)*x(11)-0.000786*x(11).^2)-15.7;
```

The Design Optimization is formulated as

$$\begin{aligned} &\text{Minimize} \quad f(\mathbf{x}) \\ &\text{Subject to} \quad g_j(\mathbf{x}) \le 0, \quad j = 1, \cdots, 9 \\ &\qquad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U, \quad \mathbf{x} \in R^9 \end{aligned}$$

Solve this optimization problem using the sequential quadratic programming (use the matlab function, ‘fmincon’, in Matlab). Make your own discussion and conclusion.

Homework 7: *RBDO of a Crashworthiness Problem*

A vehicle side impact problem is considered for design optimization. All the design variables are shown in Table A. In this example, the abdomen load is treated as an objective function with nine constraints defined in Table B.

Table A: Properties of random and design variables (X_{10} and X_{11} have "0" value)

Random Variables	Distr. Type	Std Dev.	d^L	d	d^U
X_1	Normal	0.050	0.500	1.000	1.500
X_2	Normal	0.050	0.500	1.000	1.500
X_3	Normal	0.050	0.500	1.000	1.500
X_4	Normal	0.050	0.500	1.000	1.500
X_5	Normal	0.050	0.500	1.000	1.500
X_6	Normal	0.050	0.500	1.000	1.500
X_7	Normal	0.050	0.500	1.000	1.500
X_8	Lognorm	0.006	0.192	0.300	0.345
X_9	Lognorm	0.006	0.192	0.300	0.345
X_{10}	Normal	10.0	X_{10} and X_{11} are not design variables		
X_{11}	Normal	10.0			

Table B: Design variables and their bounds

Components		Safety criteria
Objective: Abdomen load (kN)		≤1
G_1-G_3: Rib Deflection	Upper Middle Lower	≤32
G_4-G_6: VC (m/s)	Upper Middle Lower	≤0.32
G_7: Pubic symphysis force (kN)		≤4
G_8: Velocity of B-pillar		≤9.9
G_9: Velocity of front door at B-pillar		≤15.7

The Design Optimization is formulated as

$$\text{Minimize} \quad f(\mathbf{x})$$

$$\text{Subject to} \quad P\left(G_j(\mathbf{x}) \le 0\right) \ge 99\%, \quad j = 1, \cdots, 9$$

$$\mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U, \quad \mathbf{x} \in R^9$$

Solve the RBDO optimization problem using the matlab function, 'fmincon',

Mode 1:
17.73 Hz

in Matlab) starting at the initial design (d_1 to d_7 = 1.000, d_8 = d_9 = 0.300) and deterministic optimum design (obtained in the previous homework). Make your own discussion and conclusion.