Ch. 1
Fundamental Principles of Mechanics

1.2 Generalized Procedure

▶ General steps to solve problems in applied mechanics

i) Select system of interest

ii) Postulate characteristics of system. This usually involves idealization and simplification of the real situation

iii) Apply principles of mechanics to the idealized model. Deduce the consequences

iv) Compare these predictions with the behavior of the actual system. This usually involves recourse to tests and experiments

v) If satisfactory agreement is not achieved, the foregoing steps must be reconsidered. Very often progress is made by altering the assumptions regarding characteristics of the system, i.e., by constructing a different idealized model of system

→ In this book, steps i), ii), iii) are mainly treated

cf. In chapter 1, we apply the principles of mechanics to rigid body

1.3 Fundamental Principles of Mechanics

▶ Two different types of movement which are important in the mechanics of solids

i) Gross overall changes in position with time = motion

ii) Local distortions of shapes = deformation

→ In solid mechanics, we shall consider situations in which there is deformation

▶ Analysis of a mechanical system

i) Study of forces applied to a system

→ Focused on the equilibrium of a system

ii) Study of motions and deformation in a system

→ Focused on the deformation of local and global deformation

iii) Application of laws relating the forces to the motion and deformation

1.4 Concept of Force

▶ Concept

i) Newton, in his third law, postulated equal and opposite effectiveness of force on the two interacting systems

Force interactions have two principal effects
1) Alter the motion of the systems
2) Deform or distort the shape of the systems

ii) A very important property of force is that the superposition of forces satisfies the laws of vector addition

iii) Force is a vector interaction which can be characterized by a pair of equal and opposite vectors having the same line of action

iv) The magnitude of a force can be established in terms of a standardized experiment.

v) When two or more forces act simultaneously, at one point, the effect is the same as if a single force equal to the vector sum of the individual forces were acting.

▶ Units of forces

i) SI-unit ~ 1 Newton = 1 kgf·m/s^2

ii) United States unit ~ 1 lb·ft/s^2 1 slug= 32.174 lbm = $\frac{1\ \text{lbf}}{1\ \text{ft/s}^2}$

Prefixes for SI system

Factor by which unit is multiplied	*Prefix* *Name*	*Symbol*
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

Conversion factors

Length:	1 in. = 25.40 mm	1 m = 39.37 in.
	1 ft = 0.3048 m	1 m = 3.281 ft
Force:	1 lbf =4.448 N	1 N = 0.2248 lb
Pressure:	1 psi = 6.895 kN/m^2	1 MN/m^2 = 145.0 psi
	1 psf = 47.88 N/m^2	1 kN/m^2 = 20.88 psf
Distributed load:	1 lb/ft = 14.59 N/m	1 kN/m = 68.53 lb/ft
Moment of force:	1 ft-lb = 1.356 N·m	1 N·m = 0.7376 ft-lb

1.5 Moment of a force

▶ Definition

→ The moment or torque of F about the point O is defined as the vector cross product

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

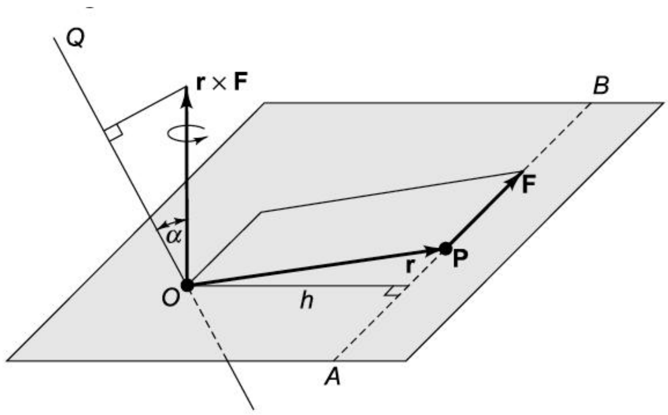


Fig. 1.5 *The moment of a force* **F** *about a* **point** *O is* **r** × **F**

▶ <u>Magnitude of the 'M'</u>

$$|\mathbf{M}| = |\mathbf{r} \times \mathbf{F}| = F\ r sin\phi$$
$$= h|\mathbf{F}| = hF$$

→ |M|is independent of the position of P along AB;

▶ <u>Other expression of the moment (consider two-dimensional structure shown in Fig. 1.7)</u>

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = h|\mathbf{F}|\mathbf{k}$$
$$= (x\ \mathbf{i} + y\ \mathbf{j}) \times (F_x\ \mathbf{i} + F_y\ \mathbf{j}) = (xF_y - yF_x)\ \mathbf{k}$$

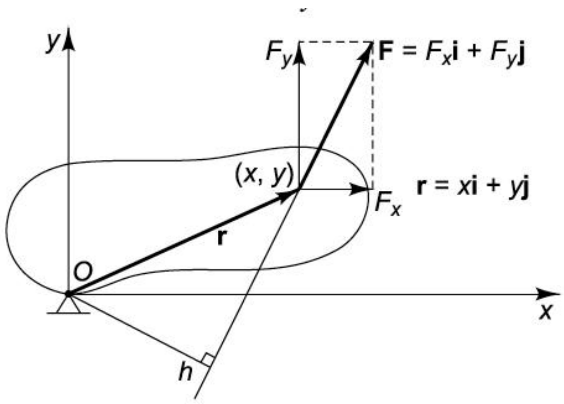


Fig. 1.7 *Moment about O*

▶ <u>The magnitude of the moment of 'F' about OQ-axis</u>

$$(\mathbf{r}\ \times \mathbf{F}) \cdot \mathbf{e} = |\mathbf{r} \times \mathbf{F}| \cos\alpha = h\ |\mathbf{F}|\ cos\ \alpha$$

▶ Units of the moment

→ [m·N] or [ft·lbf]

▶ **Examples 1.1** Determine the moment M about the shaft axis OO' due to the force P applied

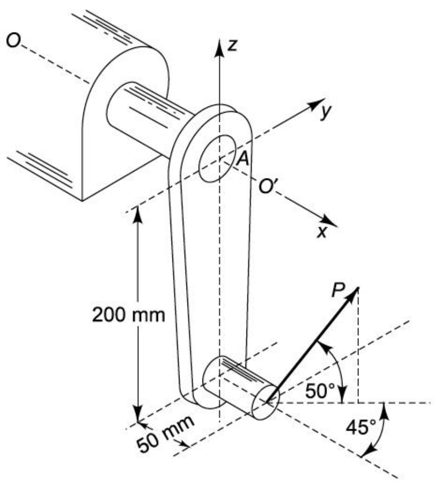


Fig. 1.8 *Example 1.1*

Sol)

$\mathbf{r} = 50\,\mathbf{i} - 200\,\mathbf{k}$

$$\mathbf{P} = P(cos50°\,cos45°\,\mathbf{i} + cos50°\,sin45°\,\mathbf{j} + sin50°\,\mathbf{k})$$

∴ $M_{OO'} = \mathbf{i}\cdot[\mathbf{r}\times\mathbf{P}] = 200Pcos50°sin45°$

▶ Couple

→ Two equal and parallel forces which have opposite sense

$$\begin{aligned}\mathbf{M} &= \mathbf{r_1}\times\mathbf{F_1} + \mathbf{r_2}\times\mathbf{F_2}\\ &= (\mathbf{r_2}+\mathbf{a})\times\mathbf{F_1} + \mathbf{r_2}\times\mathbf{F_2}\\ &= \mathbf{r_2}\times(\mathbf{F_1}+\mathbf{F_2}) + \mathbf{a}\times\mathbf{F_1}\\ &= \mathbf{a}\times\mathbf{F_1}\end{aligned}$$

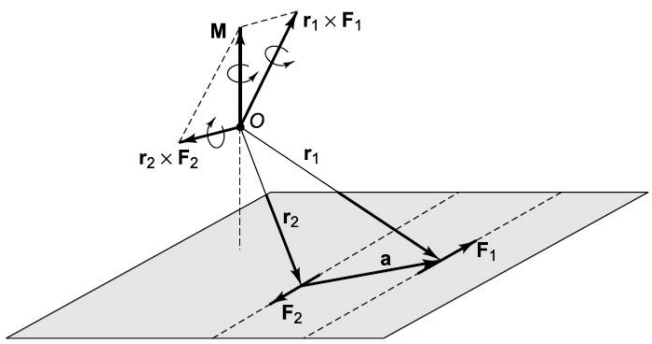


Fig. 1.9 *The moment of a couple about the point O*

∴ It is independent of the location of O

→ The moment of a couple is the same about all points in space

▷ Magnitude of the couple

$$|\mathbf{M}| = h|\mathbf{F}|$$

1.6 Conditions for equilibrium

▶ Equilibrium of a particle

→ According to Newton's law of motion, a particle has no acceleration if the resultant force acting on it is zero. We say that such a particle is in equilibrium.

Necessary and sufficient condition for the equilibrium

$$\mathbf{F_1} + \mathbf{F_2} + \cdots + \mathbf{F_n} = \sum_{\mathbf{j}} \mathbf{F_j} = \mathbf{0}$$

▶ Equilibrium of particles

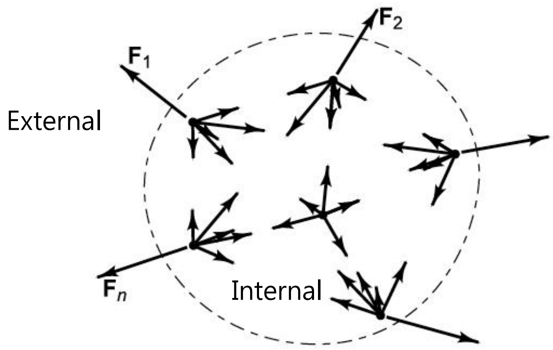


Fig. 1.12 *An isolated system of particles showing external and internal forces*

1) For rigid body ~ A necessary and sufficient condition for a perfectly rigid body to be in equilibrium is that the vector sum of all the external forces should be zero and that the sum of the moments of all the external forces about an arbitrary point (together with any external applied moments) should be zero.

$$\mathbf{F_1} + \mathbf{F_2} + \cdots + \mathbf{F_n} = \textstyle\sum_{\mathbf{j}} \mathbf{F_j} = \mathbf{0} \tag{1.5}$$

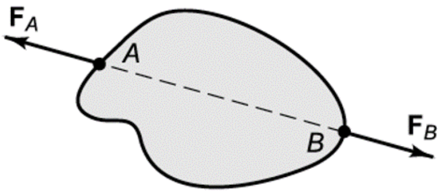
$$\mathbf{r_1} \times \mathbf{F_1} + \mathbf{r_2} \times \mathbf{F_2} + \cdots + \mathbf{r_n} \times \mathbf{F_n} = \textstyle\sum_{\mathbf{j}} \mathbf{r_j} \times \mathbf{F_j} = \mathbf{0} \tag{1.6}$$

2) For deformable system ~ A necessary and sufficient condition for the equilibrium of a deformable system is that the sets of external forces which act on the system and on every possible subsystem isolated out of the original system should all be sets of forces which satisfy both (1.5) and (1.6).

▷ Two-force member

→ The line of action of $\mathbf{F_B}$ must pass through **A** and the line of action of $\mathbf{F_A}$ must pass through **B.**

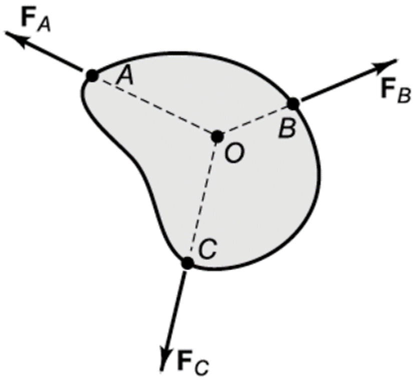
→ $\mathbf{F_A} = -\mathbf{F_B}$



▷ Three-force member

→ The three forces must all lie in the plane ABC and must all intersect in a common point O

∴ The total moment about the intersection of any two of the lines of action should be zero



▷ General two-dimensional equilibrium

$$\sum_j F_{jx} = \mathbf{0}$$
$$\sum_j F_{jy} = \mathbf{0} \quad \textbf{(1.7)}$$

$$\sum_j \mathbf{r}_j \times \mathbf{F}_j = \mathbf{0} \quad \textbf{(1.8)}$$

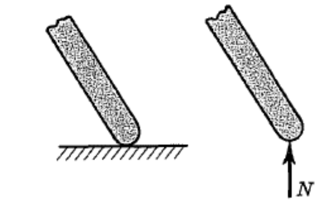
1.7 Engineering Applications (Free-body diagram; Principles of mechanics; Statically (in)determinate system)

▶ The general method of analysis that is followed throughout this book

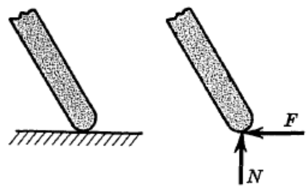
i) Selection of system

ii) Idealization of system characteristics

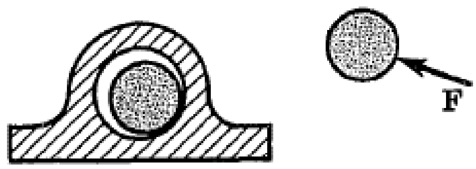
Table 1.2 Force-transmitting properties of some idealized mechanical elements



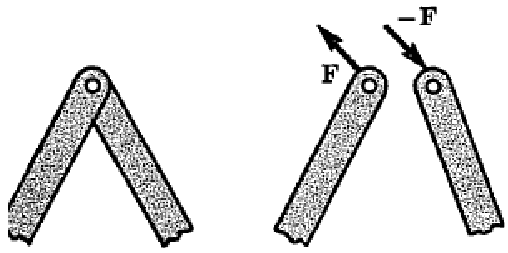
→ **A frictionless surface**



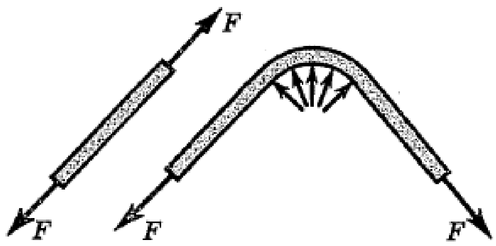
→ **A frictional surface**



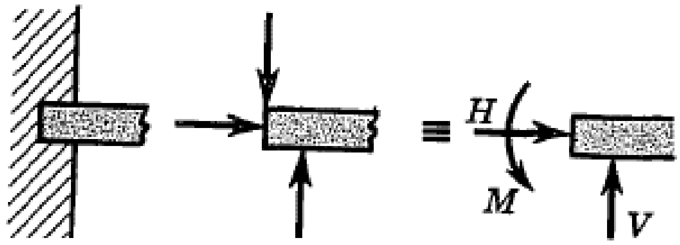
→ **A frictionless bearing**



→ **A frictionless pinned joint**



→ **The direction of the force is altered but its magnitude remains constant**



→ **An Ideal clamped support**

iii) These are followed by an analysis based on the principles of mechanics including the following steps:

iii-1) Study of forces and equilibrium requirements → Static conditions (Ch. 3)

iii-2) Study of deformation and conditions of geometric fit

iii-3) Applications of force-deformation relations

cf. Statically determinate system: Possible to determine all the forces involved by using only the equilibrium requirements without regard to the deformations ↔ Statically Indeterminate system

1.8 Friction

▶ Given pair of surfaces the forces F_s, F_k are proportional to the normal force N

$$F_s = f_s N$$
$$F_k = f_k N$$

where f_s: static coefficient of friction f_k: kinetic coefficient of friction.

→ These coefficients are intrinsic properties of the interface between the materials A and B, being determined by the materials A and B and by the state of lubrication or contamination at the interface

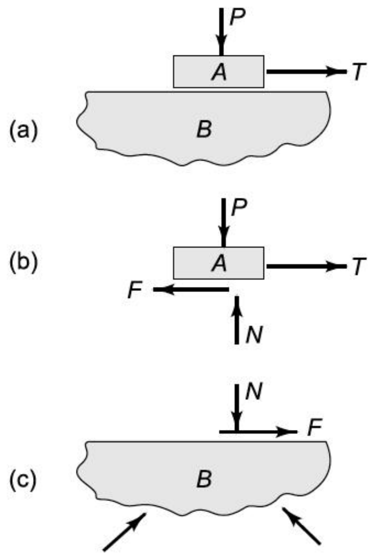


Fig. 1.18 *(a) Body A pressed against B; (b) free-body diagram of body A; (c) free-body diagram of body B*

▶ Properties of the coefficients of friction

i) Both coefficients of friction are nearly independent of the area of the interface. In particular, if body A in Fig. 1.18 were tipped up so that only an edge or a corner was in contact with B, we should still find approximately the same coefficients of friction. Note that under these circumstances the tangential and normal directions are determined only by the surface of B.

ii) Both coefficients are nearly independent of the roughness of the two surfaces, although this is a conclusion which many people find hard to accept.

iii) The static coefficient f_s is nearly independent of the time of contact of the surfaces at rest. Similarly, the kinetic coefficient f_k is nearly independent of the relative velocity of the two surfaces. Figure 1.19 shows a schematic representation of typical static-friction-time and kinetic-friction-velocity plots.

1.10 Hooke's Joint (Reading homework)

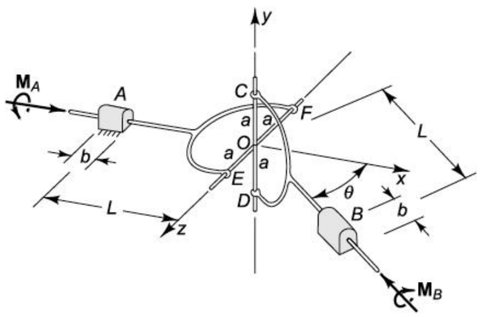


Fig. 1.32 Hooke's joint mechanism

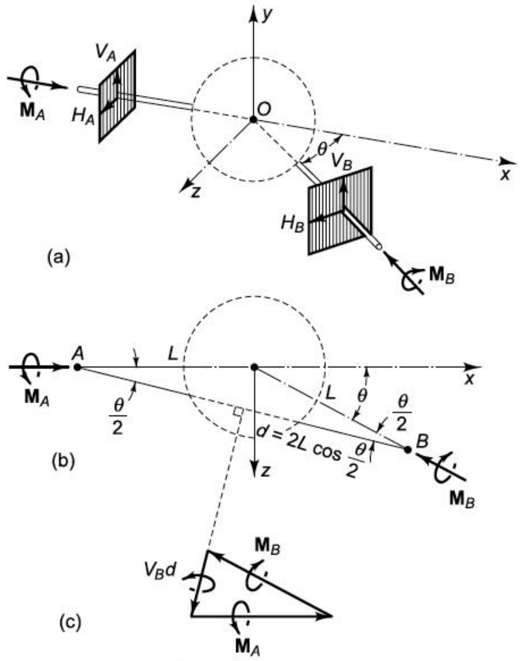


Fig. 1.33 Isolation of entire mechanism

A Comparison an incorrect conclusion of the over-idealized model with a correct solution.

▶ Incorrect solution.

Considering the conditions of force balance from Fig.1.33a

$$H_A = H_B = 0$$
$$-V_A = V_B$$

From Fig. 1.33c

$$M_A = M_B \tag{a}$$

Alternatively, if we proceed directly with moment equilibrium about the point O, we have

$$M_A \mathbf{i} - M_B(\mathbf{i}\cos\theta + \mathbf{k}\sin\theta) - L\mathbf{i} \times V_A \mathbf{j} - L(\mathbf{i}\cos\theta + \mathbf{k}\sin\theta) \times V_A \mathbf{j} = 0$$

Working out the cross products,

$$V_A L(1+\cos\theta) = -M_A \sin\theta$$
$$M_A + V_A L \sin\theta = M_B \cos\theta$$

The result as above is

$$V_A L = \frac{-M_A \sin\theta}{1+\cos\theta} \quad * \text{Incorrect solution.}$$

Suppose we were to continue our analysis further by considering the shaft B separately as in Fig. 1.34.

The system of Fig. 1.34. can't possibly be in equilibrium because there is nothing to balance V_B in the vertical direction.

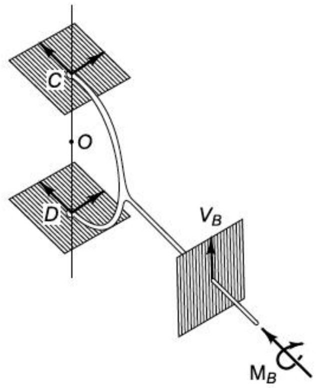


Fig. 1.34 *Isolated free-body diagram of the shaft B*

▶ <u>Correct solution.</u>

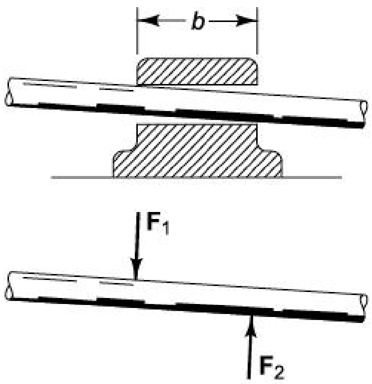


Fig. 1.35 *When a shaft cocks slightly in a long bearing, there is double contact with two reactions F_1 and F_2*

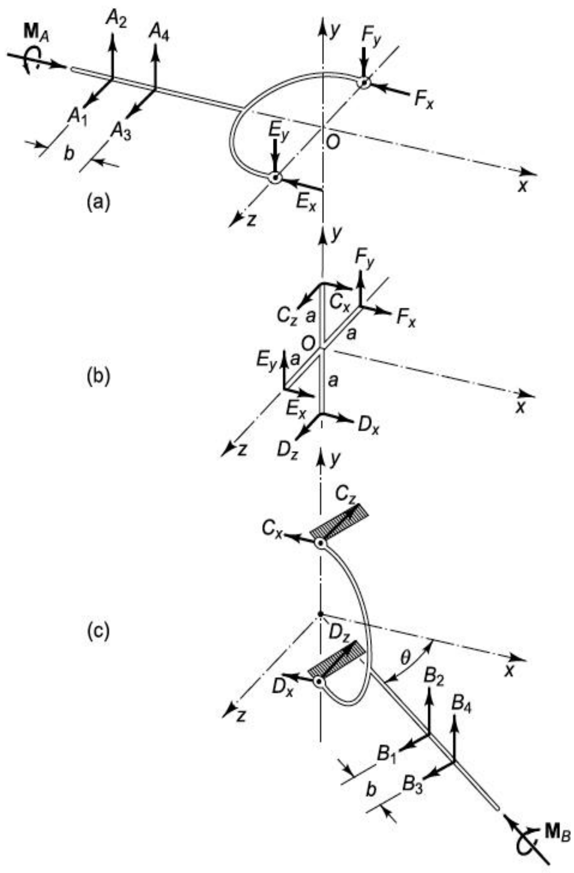


Fig. 1.36 *Free-body diagrams of the three parts of the Hooke's joint*

To provide for the double-contact type of reaction shown in Fig. 1.35, we have four components A_1, A_2, A_3, and A_4 at bearing A and the four components B_1, B_2, B_3, and B_4 at bearing B.

- From fig. 1-36(b)~

The statements of force and moment balance for the cross (Fig. 1.36b) are

$$\sum \mathbf{F} = (C_x + D_x + E_x + F_x)\mathbf{i} + (E_y + F_y)\mathbf{j} + (C_z + D_z)\mathbf{k} = 0$$
$$\sum \mathbf{M_0} = (aC_z - aD_z - aE_y + aF_y)\mathbf{i} + (aE_x - aF_x)\mathbf{j} + (-aC_x + aD_x)\mathbf{k} = 0 \tag{b}$$

Setting each component separately equal to zero,

$$C_x = D_x = -E_x = -F_x$$
$$C_z = -D_z = E_y = -F_y \tag{c}$$

- From fig. 1-36 (a) ~

In Fig. 1.36a, equilibrium of forces parallel to x yields $F_x = 0$ and equilibrium of moments about the x axis yields $2aF_y = M_A$.

$$C_x = D_x = -E_x = -F_x = 0$$
$$C_z = -D_z = E_y = -F_y = -\frac{M_A}{2a} \tag{d}$$

For completeness, considering the other equilibrium conditions in Fig 1.36a

$$A_1 = A_2 = A_3 = A_4 = 0$$

- From fig. 1-36 (c)~

Using values (d) in Fig. 1.36c

$$M_B - aD_z\cos\theta + aC_z\cos\theta = 0$$
$$M_B = M_A\cos\theta \tag{e}$$

For completeness, considering the other equilibrium conditions in Fig 1.36c

$$B_1 = B_3 = 0$$
$$B_4 = -B_2 = \frac{M_A}{b}\sin\theta$$

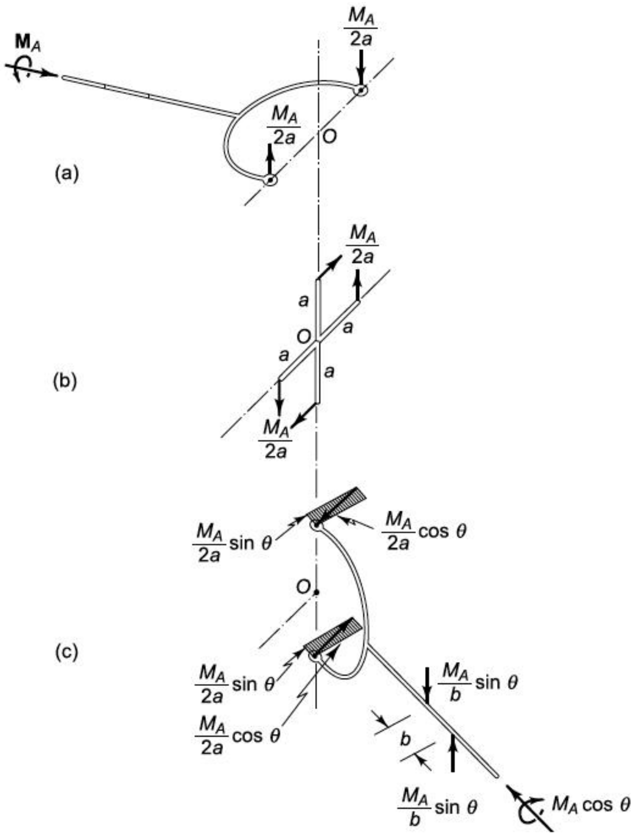


Fig. 1.37 *Equilibrium solution of the free-body diagrams of Fig. 1.36*

To aid visualization the complete solution is shown in Fig. 1.37.

It is important to emphasize that our solution (e) is for the configuration shown if Fig. 1.32.

An exact solution for arbitrary angle of orientation ϕ of shaft A measured from EF in the direction of twist of M_A (Fig. 1.32) can be found. The result is

$$M_B = \frac{\sin^2\phi + \cos^2\theta\cos^2\phi}{\cos\theta} M_A \tag{f}$$

When ϕ=0, the result (f) reduces to (e).