Chap. 2 Plastic Hinge

1. Introduction
2. Moment-curvature relationship and Plastic Hinge Length
3. Full Plastic Moment
4. Design of a cross Section
5. Effect of Axial Load
6. Effect of Shear Force
7. Effect of Combined Axial and Shear Force
8. Compactness
9. Connections

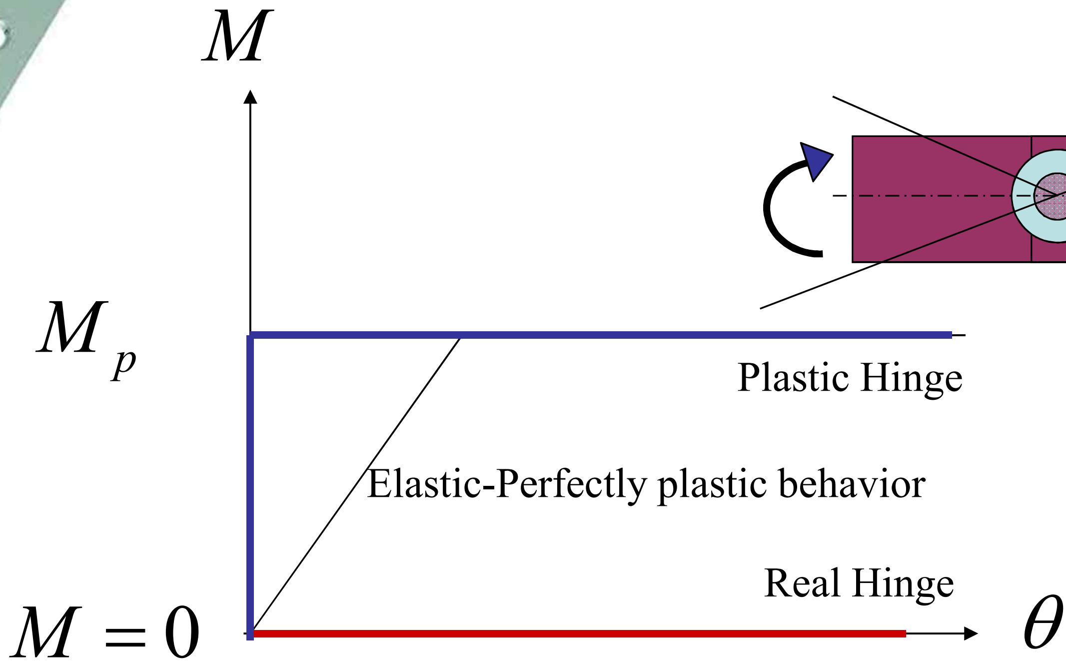
2.1 Introduction

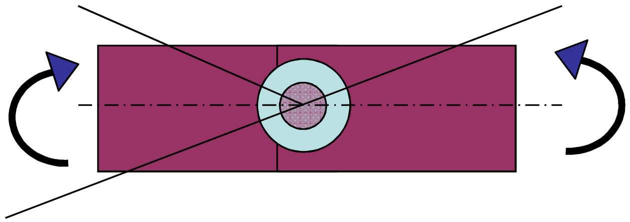
- To elaborate the concept of the plastic hinge and plastic moment

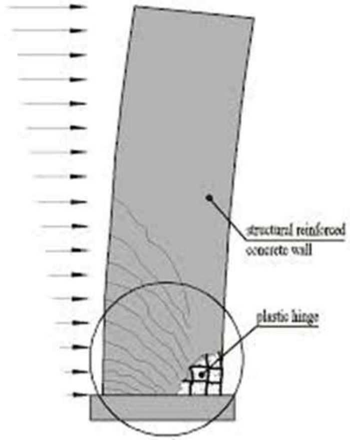
  we extend

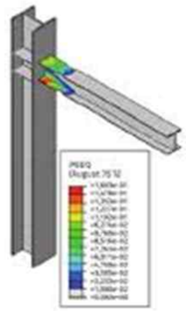
- Effect of axial force and shear
- Compactness (local buckling in plastic hinge zone)

Concept of Plastic Hinge

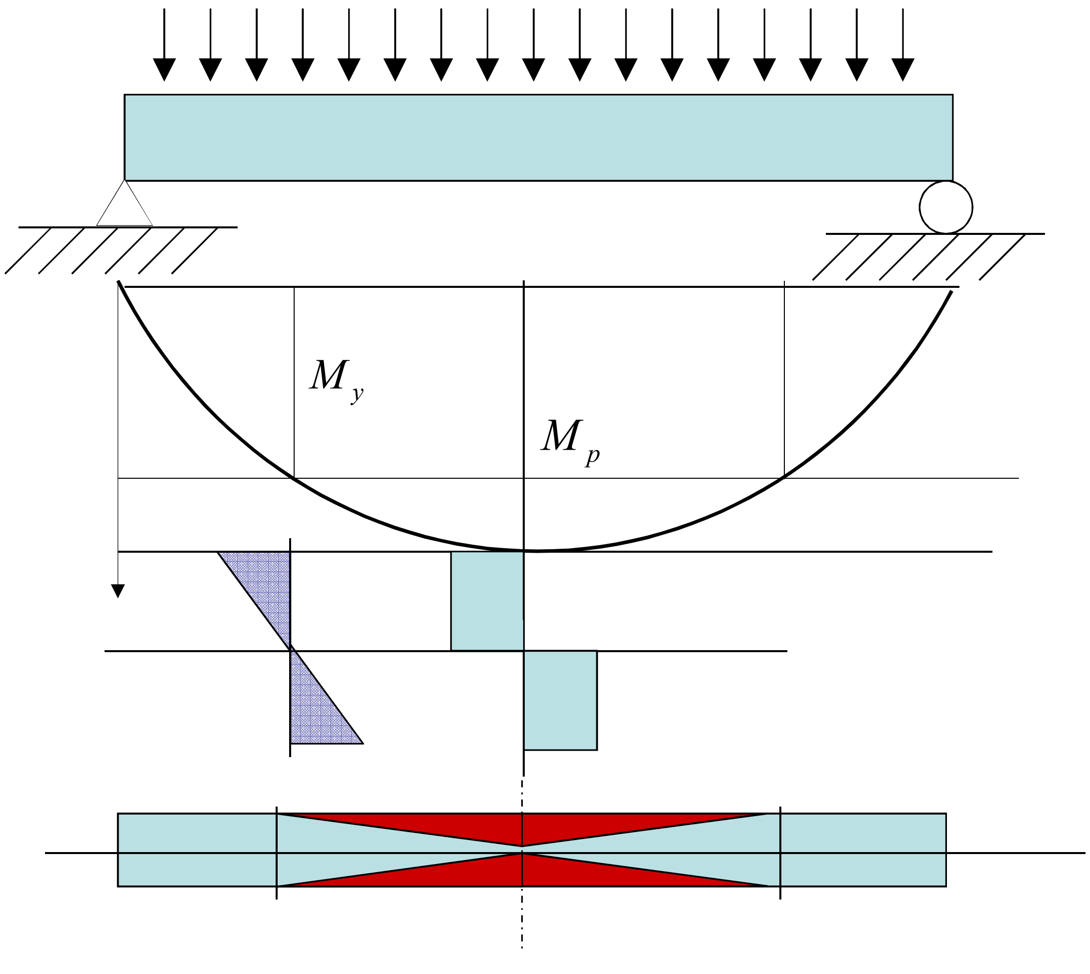




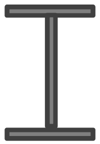




Plastic Hinge Length (Zone)



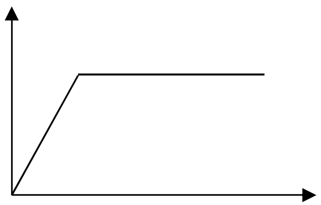
Moment-Curvature Relationship and Plastic Hinge Length

2.2.1 M−ϕ relationship of 

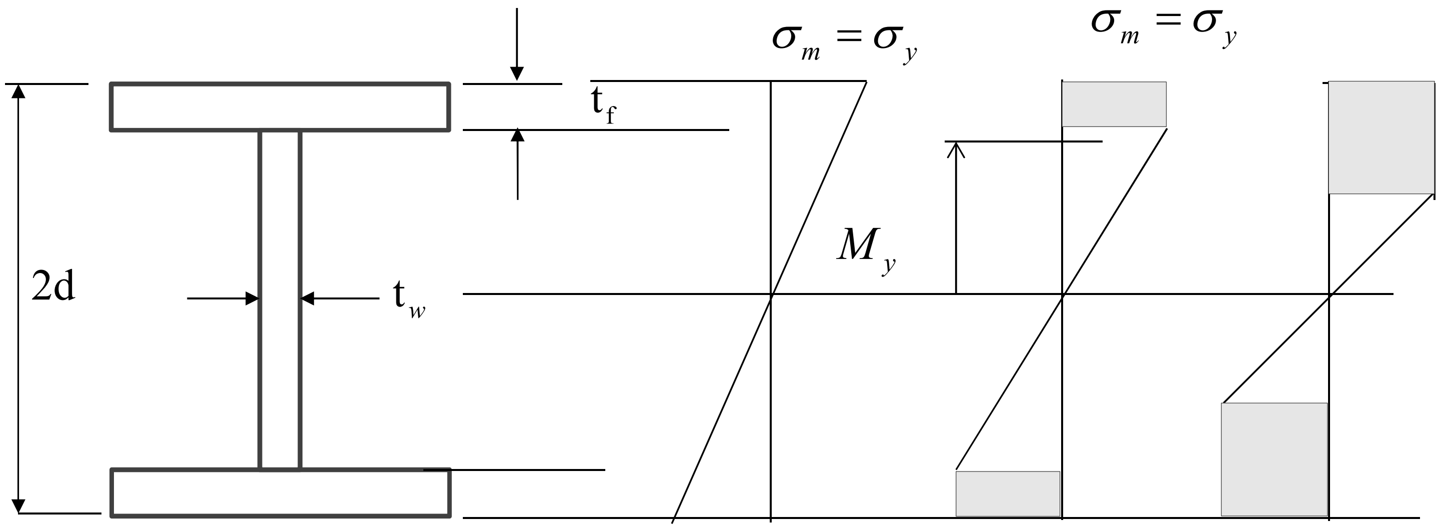
Assumptions

- Plane sections remain plane after bending

- Material model



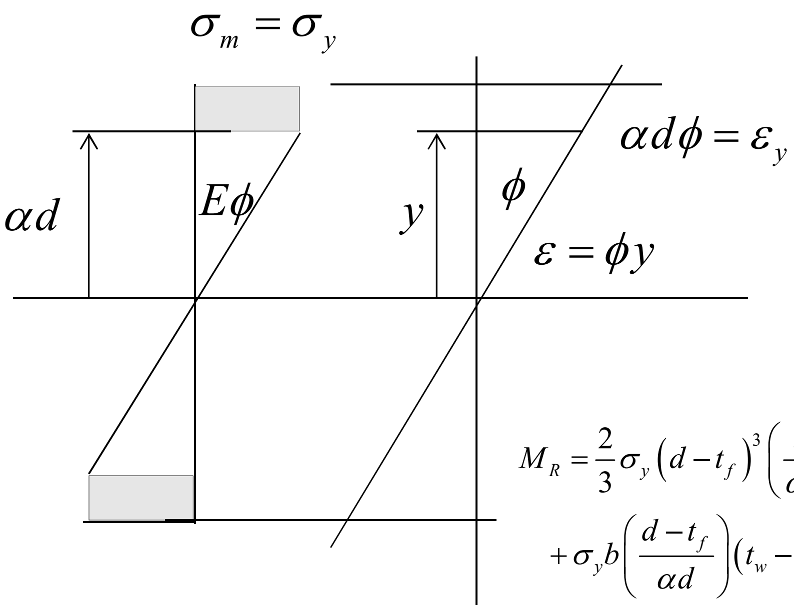
- Equilibrium



1. Elastic regime

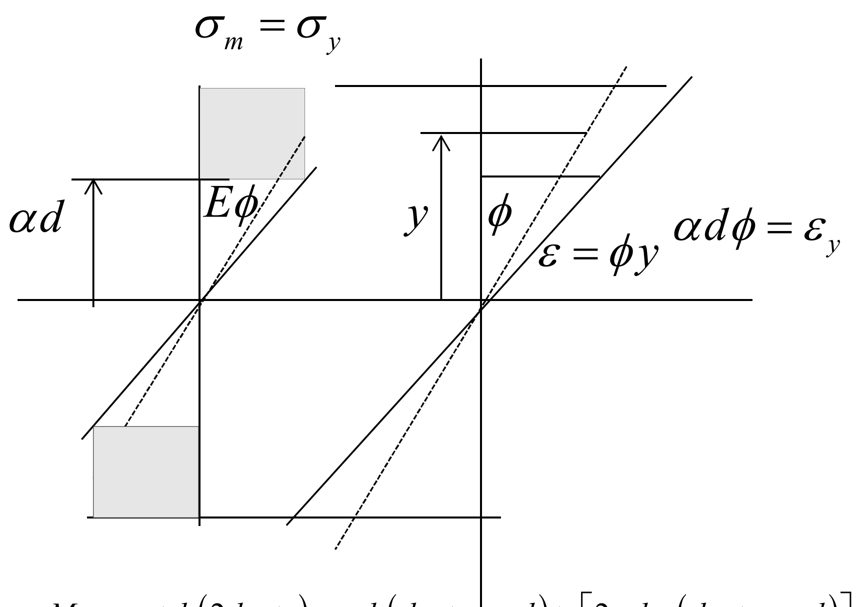
$$\phi_y = \frac{\varepsilon_y}{d} = \frac{\sigma_y}{E}\frac{1}{d}$$

2. Regime II: flange is partially yielded



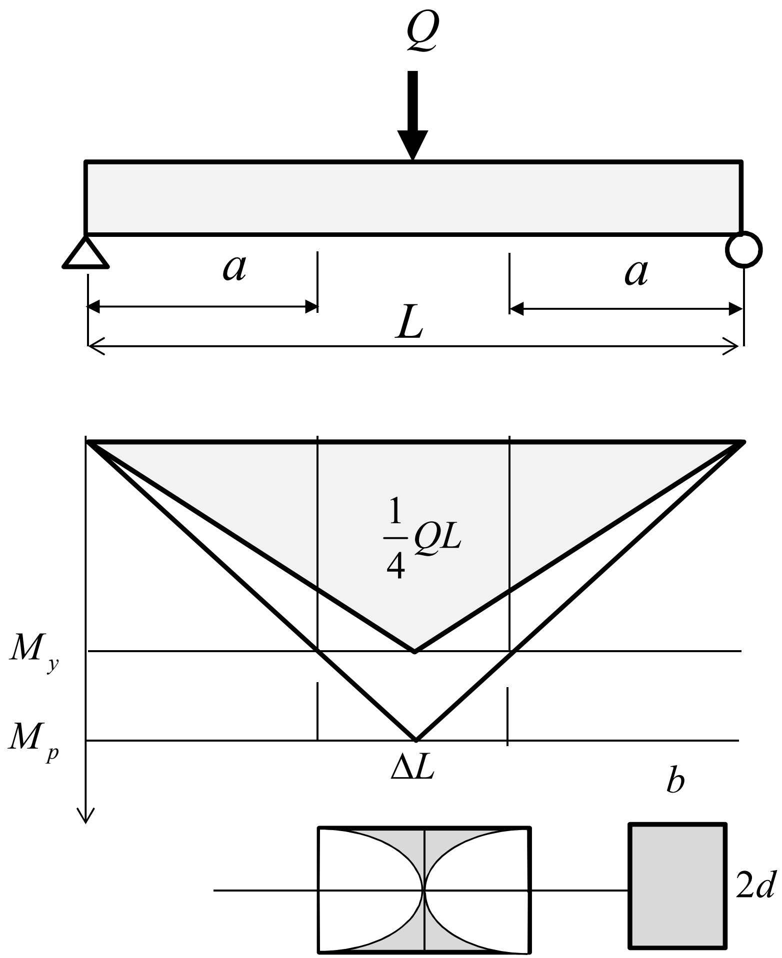
$$M_R = \frac{2}{3}\sigma_y\left(d - t_f\right)^3\left(\frac{t_w}{\alpha d}\right) + \sigma_y b\left(t_f - d + \alpha d\right)^2\left(\frac{2}{3}\alpha d + \frac{1}{3}d - \frac{1}{3}t_f\right)\left(\frac{1}{\alpha d}\right)$$
$$+\sigma_y b\left(\frac{d - t_f}{\alpha d}\right)\left(t_w - d + \alpha d\right)\left(t_f - d + \alpha d\right) + \sigma_y b d^2\left(1 - \alpha^2\right)$$

3. Regime III: Web is partially yielded



$$M_R = \sigma_y t_f b\left(2d - t_f\right) + \sigma_y b\left(d - t_f - \alpha d\right)t_w\left[2\alpha d - \left(d - t_f - \alpha d\right)\right]$$
$$+\frac{1}{2}\sigma_y\left(\alpha d\right)t_w\left(\frac{4}{3}\alpha d\right)$$

2.2.2 Plastic Hinge Length



At the yield $\frac{1}{4}Q_y L = M_y$

At the ultimate $\frac{1}{4}Q_p L = M_p$ $\quad Q_p = \frac{4M_p}{L}$

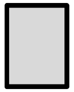
$$M_c = \frac{Q_p}{2} a = M_y \qquad a = \frac{2M_y}{Q_p}$$

$$a = \frac{L}{2\frac{M_p}{M_y}} = \frac{L}{2f}$$

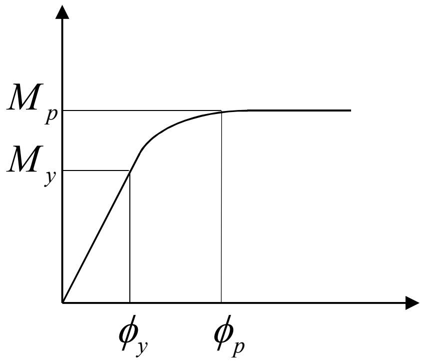
The plastic hinge length

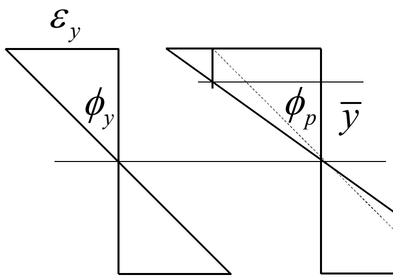
$$\Delta L = L - 2a = L\left(1 - 1/f\right)$$

The length of the plastic hinge length depends on the
The boundary and loading patterns

The distribution of the yield zone for 

$$M = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{\phi_y}{\phi} \right)^2 \right]$$





$$\phi = \frac{\varepsilon_y}{\bar{y}} \qquad \phi_y = \frac{\varepsilon_y}{d}$$

Section

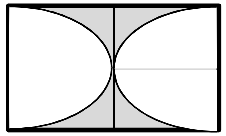
$$M(\bar{y}) = M_p \left[1 - \frac{1}{3} \left(\frac{\bar{y}}{d} \right)^2 \right] \quad \Rightarrow \quad A$$

Member length

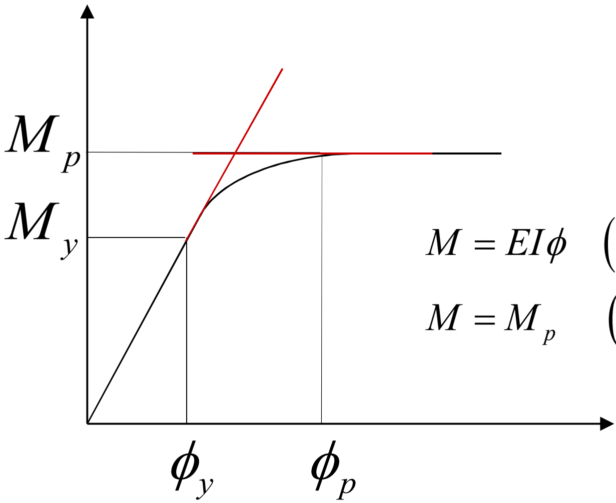
$$M(x) = M_p \frac{L - 2x}{L} \quad \Rightarrow \quad B$$

A=B

$$\bar{y} = d\sqrt{\frac{6x}{L}}$$



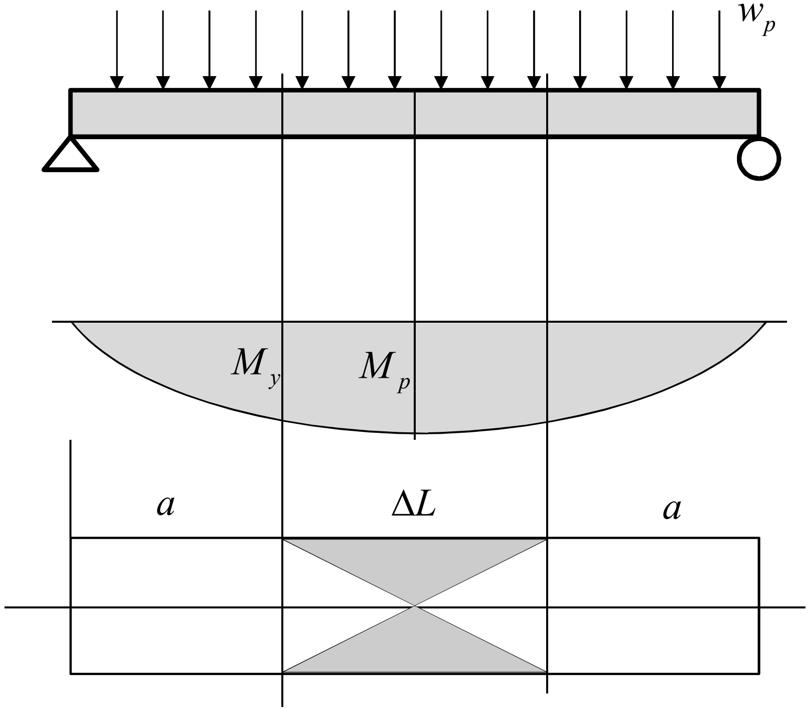
2.2.3 Plastic Hinge Idealization



$$M = EI\phi \quad (0 \le \phi \le \phi_p)$$

$$M = M_p \quad (\phi_p \le \phi)$$

2.2.4 Another example of Plastic Hinge Length



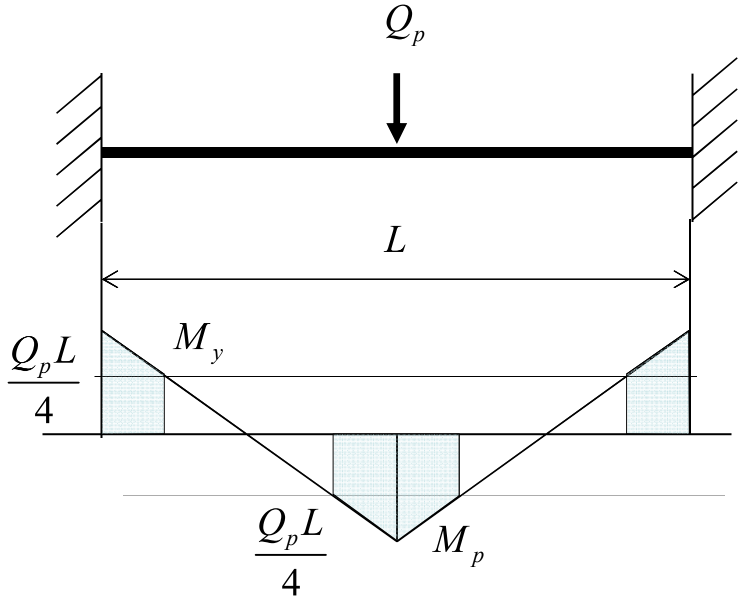
$$w_p = \frac{8M_p}{L^2}$$

$$\frac{w_p L}{2} a - w_p \frac{a^2}{2} = M_y$$

$$a = \frac{L}{2}\left[1 - \sqrt{1 - 1/f}\right]$$

$$\Delta L = L - 2a = a = L\sqrt{1 - 1/f}$$

Example)



$$Q_p = \frac{8M_p}{L}$$

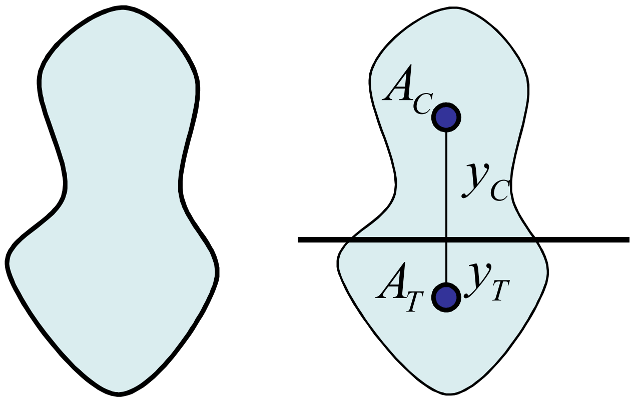
$$M_x = \frac{Q_p}{2} x - M_p$$

$$-M_y = \frac{Q_p}{2} \Delta L_1 - M_p$$

$$\Delta L_1 = \frac{L}{4}\left(1 - 1/f\right)$$

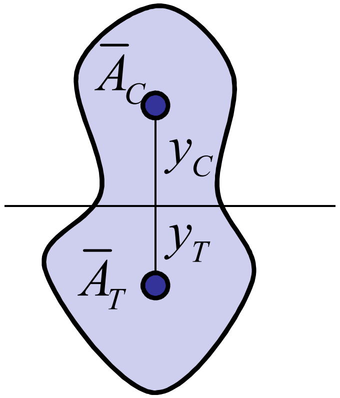
$$\Delta L_2 = 2\frac{L}{4}\left(1 - 1/f\right)$$

2.3 Full Plastic Moment



Plastic Neutral Axis

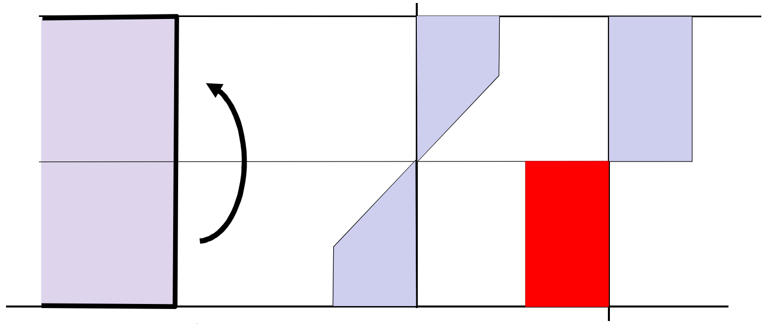
$$A_C = A_T$$

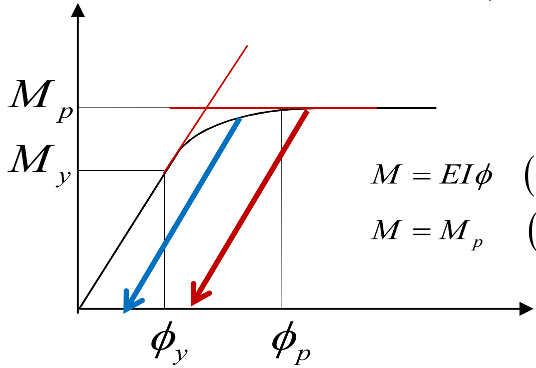


Neutral Axis

$$\bar{A}_C \bar{y}_C = \bar{A}_T \bar{y}_T$$

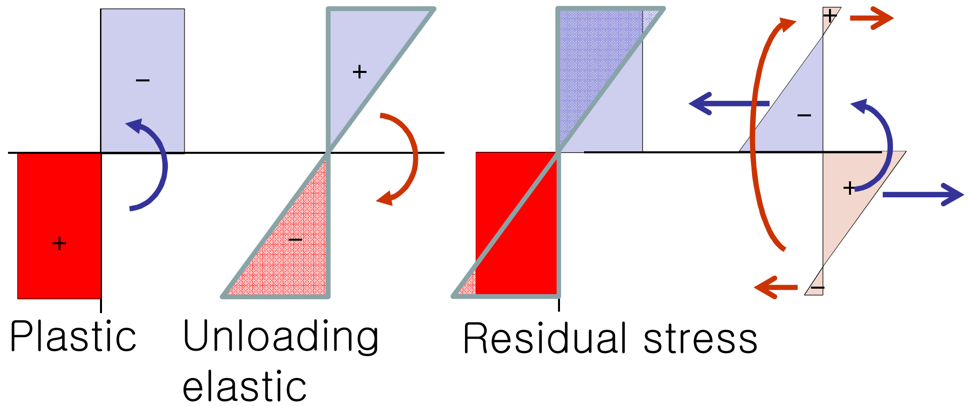
Unloading after Full Plastic Moment



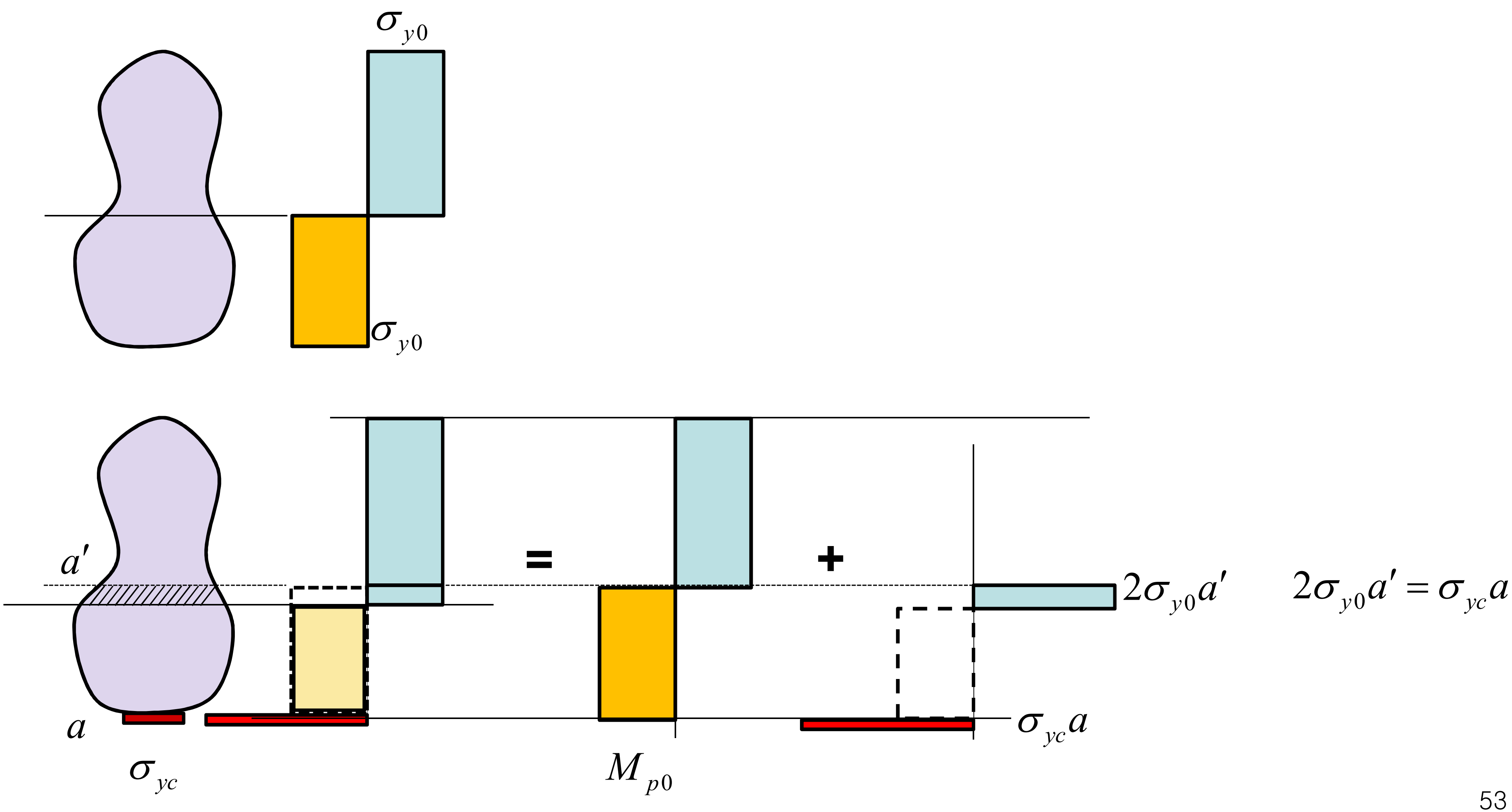


$$M = EI\phi \quad (0 \le \phi \le \phi_p)$$

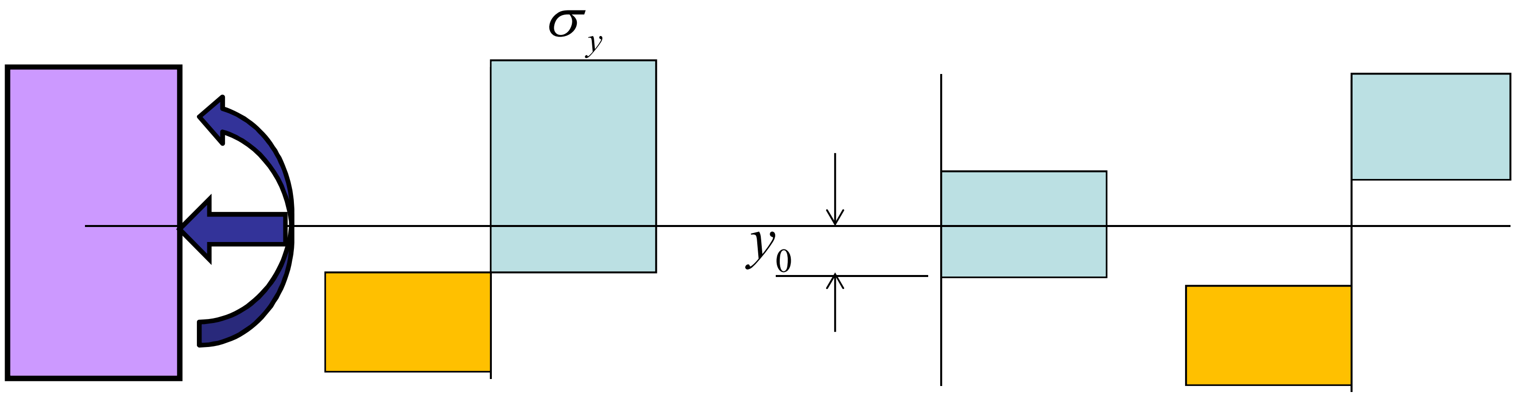
$$M = M_p \quad (\phi_p \le \phi)$$



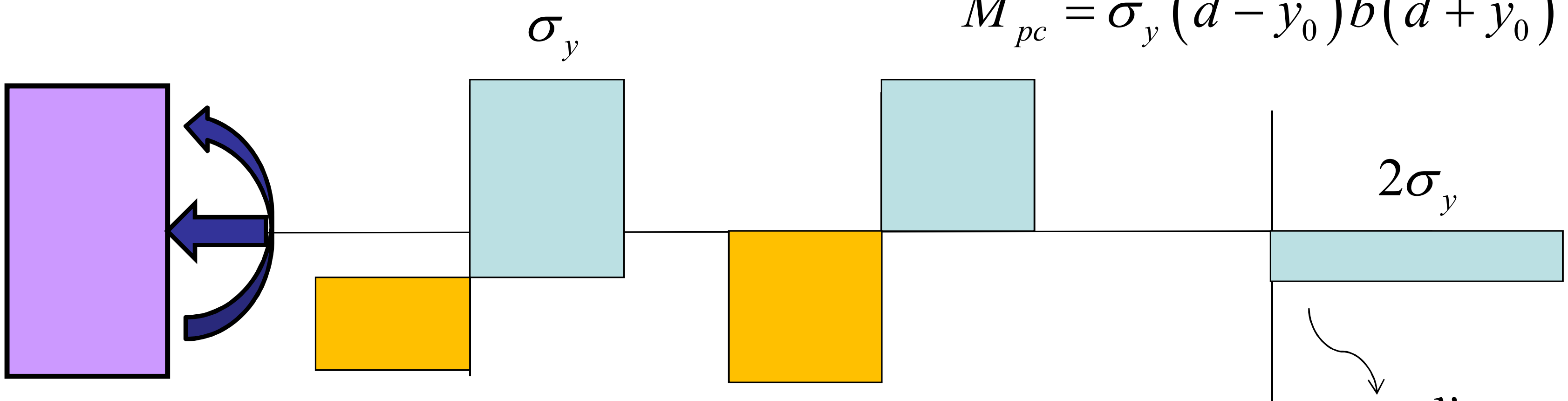
2.4 Design of Cross Section



Axial Force Effect:
two approaches



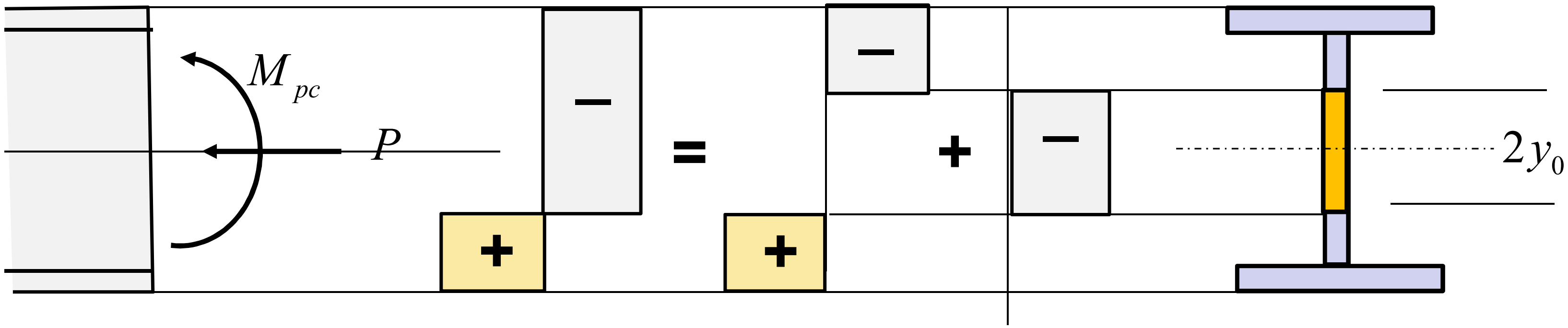
$$M_{pc} = \sigma_y (d - y_0) b (d + y_0)$$



$$M_{pc} = M_p - 2\sigma_y y_0 b \frac{y_0}{2}$$

2.5.3 WF bending about strong axis

PNA in web

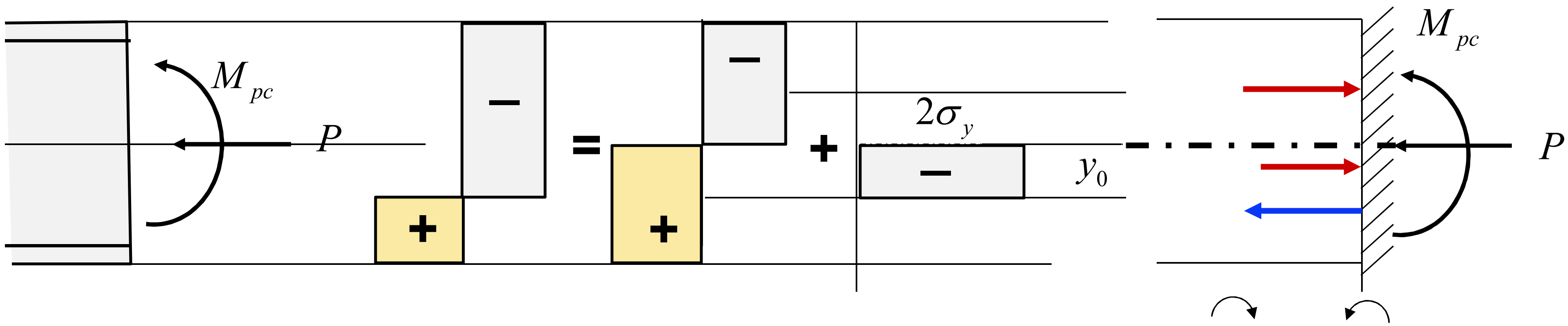


$$P = \sigma_y \times t_w \times 2y_0 \quad \left(P_y = \sigma_y \left(2bt_f + t_w d_w\right)\right)$$

$$\frac{P}{\sigma_y t_w} \le d_w$$

$$\frac{P}{P_y} \le \frac{d_w t_w}{\left(2bt_f + t_w d_w\right)} = \frac{1}{1 + \frac{2bt_f}{t_w d_w}}$$

PNA in web



$$M_{pc} = M_p - 2\sigma_y t_w y_0 \frac{y_0}{2}$$

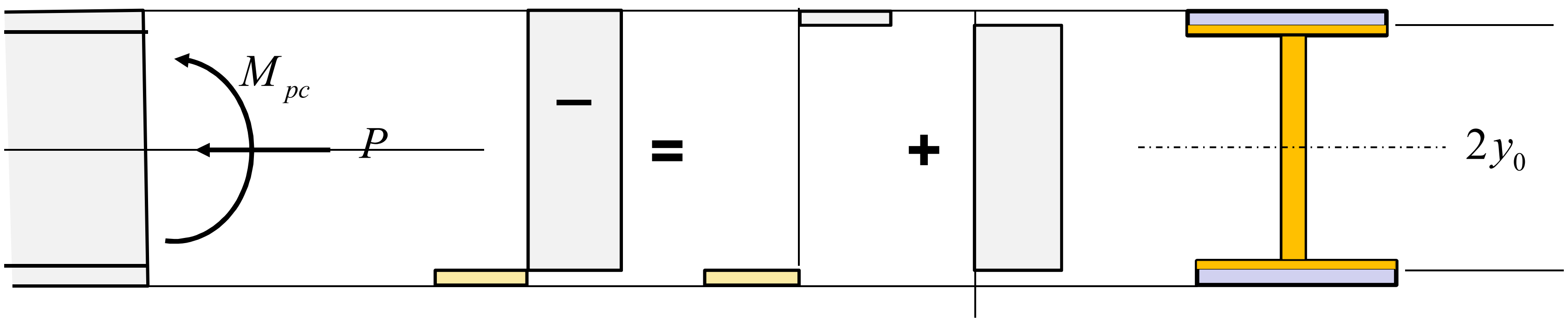
$$= \sigma_y Z - \sigma_y t_w y_0^2$$

since $y_0 = \dfrac{P}{2\sigma_y t_w}$

$$M_{pc} = \sigma_y \left[Z - \frac{P^2}{4\sigma_y^2 t_w} \right]$$

$$\frac{M_{pc}}{M_p} = 1 - \frac{A^2}{4t_w Z}\left(\frac{P}{P_y}\right)^2$$

PNA in flange



$$P=\sigma_y\left[A-b\left(d-2y_0\right)\right]$$

$$M_{pc}=\sigma_y b\frac{d-2y_0}{2}\frac{d+2y_0}{2}$$

$$y_0=\frac{P}{2b\sigma_y}-\frac{A}{2b}+\frac{d}{2}$$

$$M_{pc}=\sigma_y b\left[\frac{1}{2}\frac{PA}{b^2\sigma_y}+\frac{Ad}{2b}-\frac{P^2}{4b^2\sigma_y^2}-\frac{A^2}{4b^2}-\frac{Pd}{2b\sigma_y}\right]$$

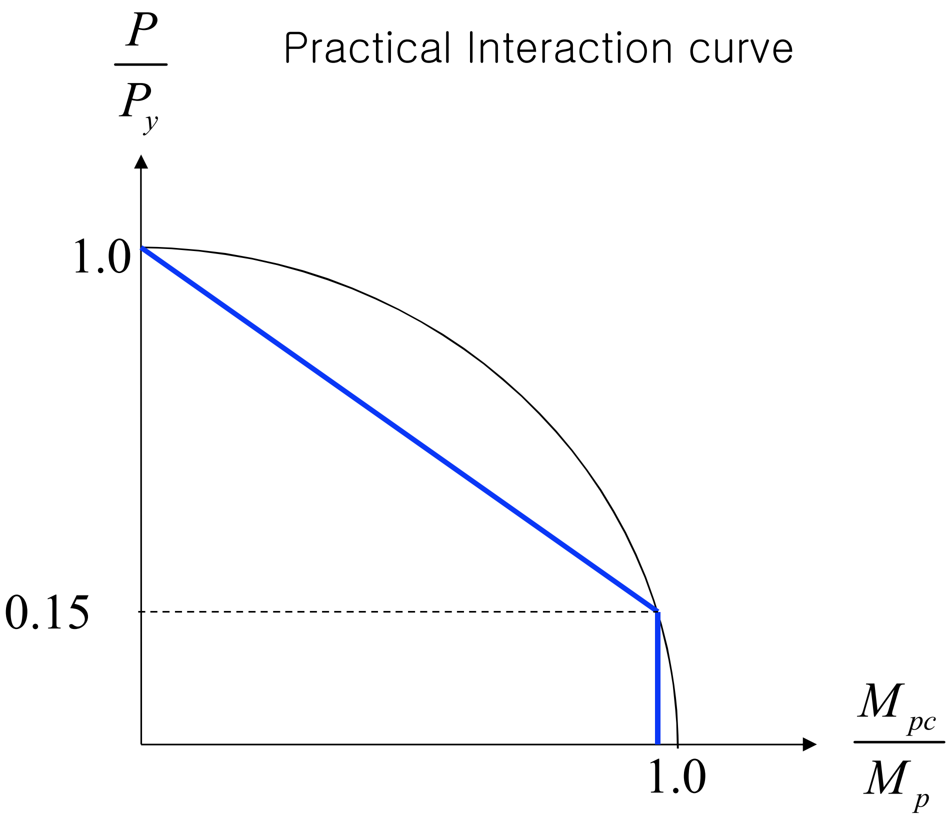
$$M_{pc}=\frac{\sigma_y}{2}\left[d\left(A-\frac{P}{\sigma_y}\right)-\frac{1}{2b}\left(A-\frac{P}{\sigma_y}\right)^2\right]$$

Since $P_y=\sigma_y A$

$M_p=\sigma_y Z$

$$\frac{M_{pc}}{M_p}=\frac{Ad}{2Z}\left[\left(1-\frac{P}{P_y}\right)-\frac{A}{2bd}\left(1-\frac{P}{P_y}\right)^2\right]$$

Practical Interaction curve

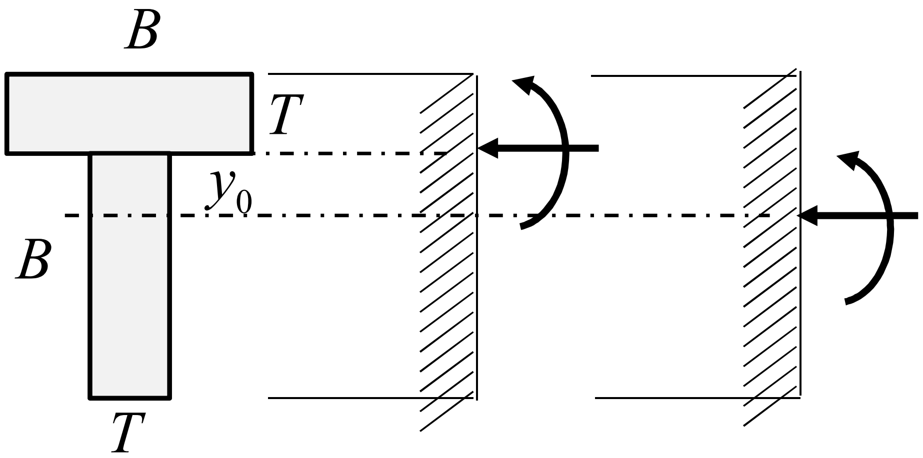


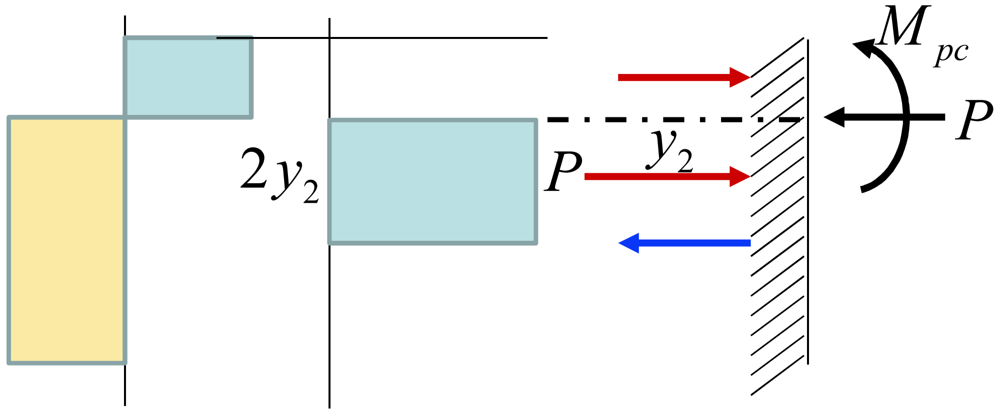
$$M_{pc} = M_p \quad \text{for} \quad 0 \le P \le 0.15P_y$$

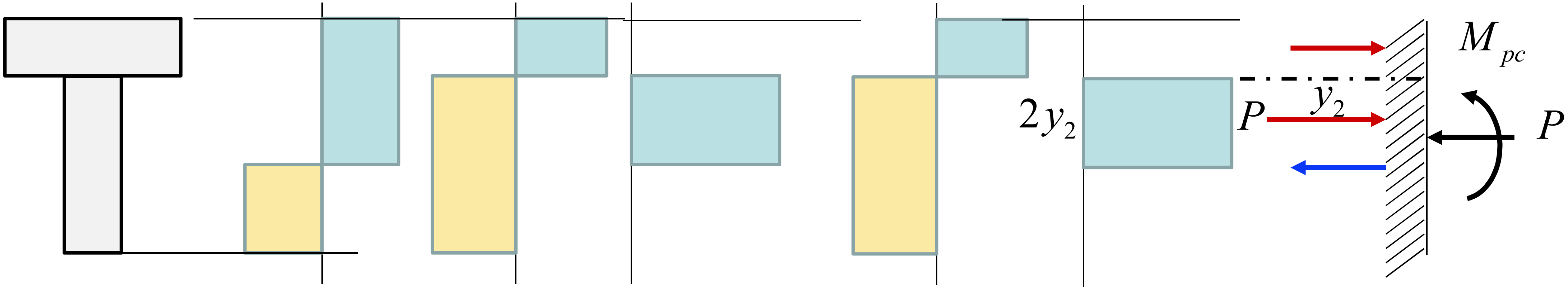
$$M_{pc} = 1.18\left(1 - \frac{P}{P_y}\right)M_p \quad \text{for} \quad 0.15P_y \le P \le P_y$$

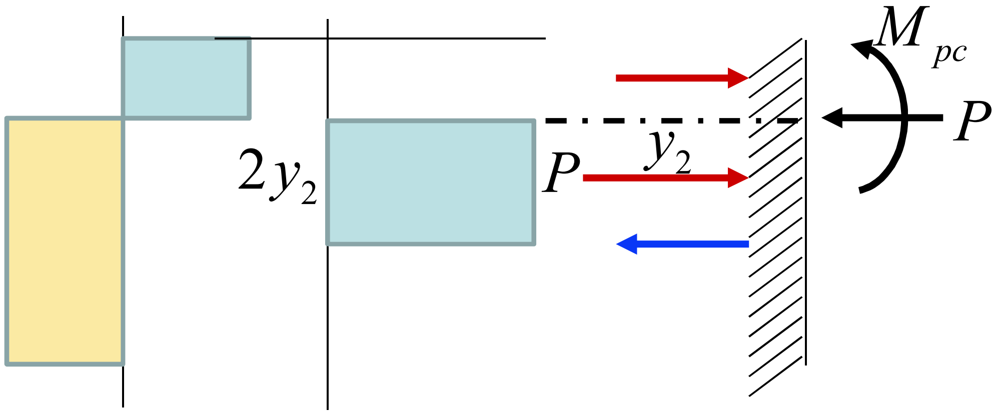
2.5.5 T-section

1) P thru PNA
2) P thru centroid







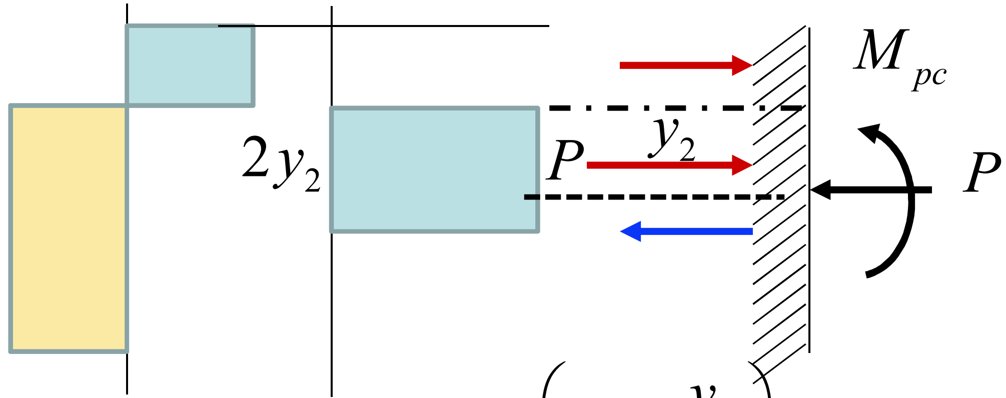


$$M_{pc} = M_p - P\frac{y_2}{2}$$

$$= M_p - P\frac{P}{4\sigma_y T}$$

$$= M_p - \frac{M_p}{P_y^2}\frac{P_y^2}{M_p}\frac{P^2}{4\sigma_y T}$$

$$= M_p\left[1 - \left(\frac{P}{P_y}\right)^2 \frac{2B}{B+T}\right]$$



$$M_{pc} = M_p + P\left(y_0 - \frac{y_2}{2}\right)$$

$$= M_p - P\frac{y_2}{2} + Py_0$$

for $\dfrac{B-T}{4} \geq \dfrac{P}{4\sigma_y T}$

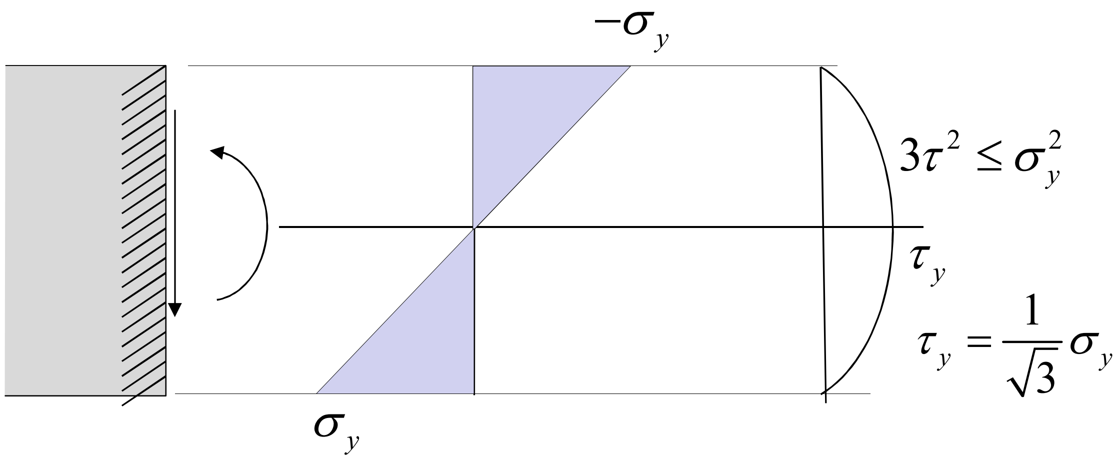
$$T(B-T)\sigma_y \geq P$$

Shear Force Effect

- Rectangular Section
- Wide Flange Section
- Effects of Combined Axial Force and Shear Force

Rectangular Sections

Von Mises yield criteria $\sigma^2 + 3\tau^2 \le \sigma_y^2$



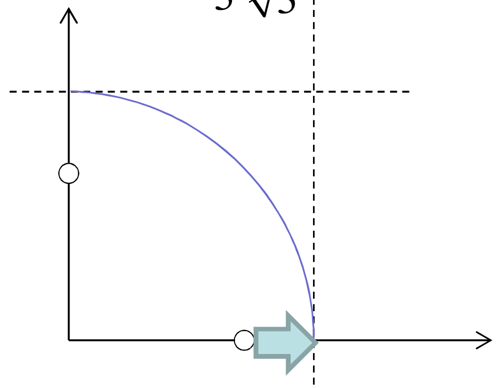
Lower bound 1

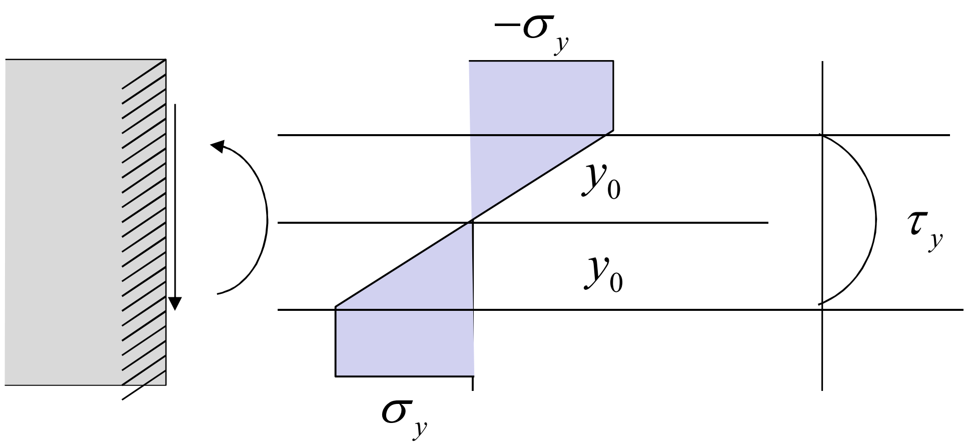
$$\sigma = \sigma_y \frac{y}{d/2}$$

$$\tau = \frac{\sigma_y}{\sqrt{3}}\left[1 - \left(\frac{y}{d/2}\right)^2\right]^{1/2}$$

$$M_{ps} = \int \sigma y dA = \frac{1}{6}\sigma_y bd^2 = \frac{2}{3}M_p$$

$$V = \int \tau dA = \frac{2}{3}\frac{\sigma_y}{\sqrt{3}}bd$$





Lower bound 2

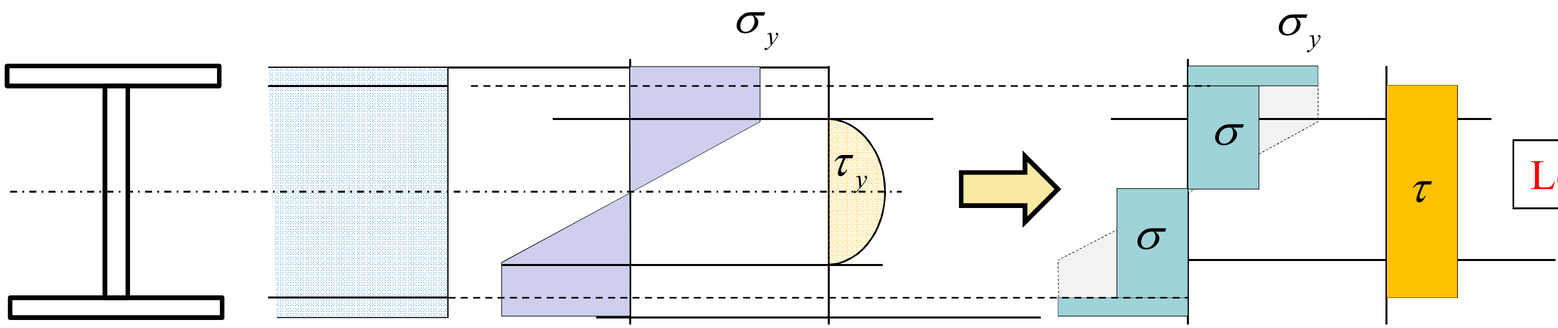
$$M_{ps} = M_p\left[1 - \frac{1}{3}\left(\frac{2y_0}{d}\right)^2\right]$$

$$V = \frac{4}{3}\tau_y b y_0$$

$$\frac{M_{ps}}{M_p} = 1 - \frac{3}{4}\left(\frac{V}{V_p}\right)^2$$

where $V_p = \frac{\sigma_y}{\sqrt{3}}bd$

2.6.2 Wide-flange section



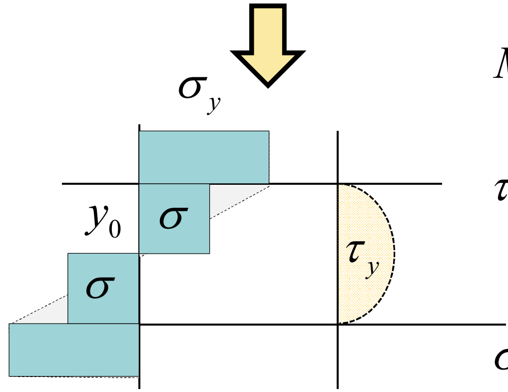
Lower bound 1

$$M_{ps} = \sigma_y b t_f + \frac{1}{4}\sigma t_w d_w^2$$

$$\tau = \frac{V}{t_w d_w}$$

$$\sigma^2 + 3\tau^2 = \sigma_y^2 \Rightarrow \frac{\sigma}{\sigma_y} = \sqrt{1-\left(\frac{3\tau}{\sigma_y}\right)^2} = \sqrt{1-\left(\frac{V}{V_p}\right)^2}$$

$$\frac{M_{ps}}{M_p} = \frac{1+\frac{t_w d_w^2}{4t_f d_f^2}\sqrt{1-\left(\frac{V}{V_p}\right)^2}}{1+\frac{t_w d_w^2}{4t_f d_f^2}}$$

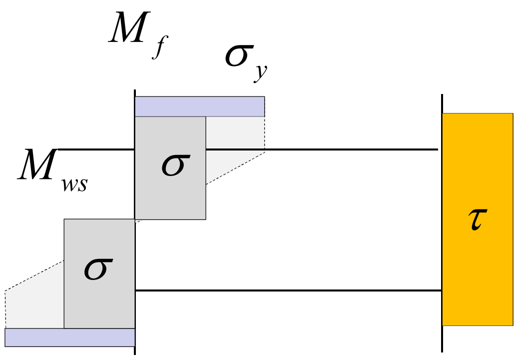


Lower bound 2

$$M_{ps} = M_p - \frac{1}{3}\sigma_y y_0^2 t_w$$

$$V = \frac{4}{3}\frac{\sigma_y}{\sqrt{3}} t_w y_0$$

$$\frac{M_{ps}}{M_p} = 1 - \frac{\frac{3}{4}\left(\frac{V}{V_p}\right)}{1+\frac{4bt_f d_f}{t_w d_w^2}}$$



Ex 2.6.1)

$$M_{ps} = M_f + M_{ws}$$

$$M_{ws} = M_w \frac{\sigma}{\sigma_y}$$

$$M_{ps} = M_f + M_{ws} - M_{ws} + M_{ws}$$

$$= M_p - M_w \left(1 - \frac{\sigma}{\sigma_y}\right)$$

$$Z_{ps} = Z_p - Z_w \left(1 - \frac{\sigma}{\sigma_y}\right)$$

For W14×82

$$V_u = 100\text{kips}$$

$$Z = 139\ \text{in}^3$$

$$d_w = d - 2t_f = 12.6\ \text{in}$$

$$t_w = 0.51\ \text{in}$$

$$Z_w = t_w \frac{d_w^2}{4}$$

to find σ

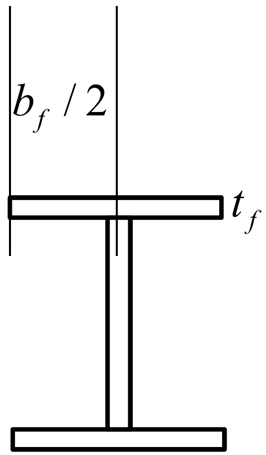
$$\tau = \frac{V}{d_w t_w} = \frac{100}{12.6 \times 0.51} = 15.56\ \text{ksi}$$

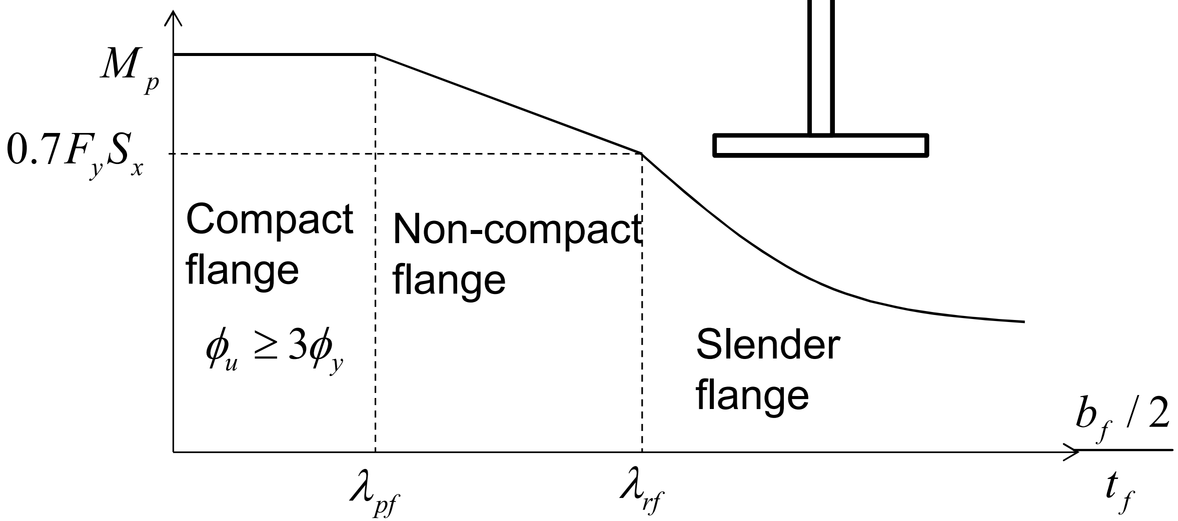
$$\sigma = \sqrt{\sigma_y^2 - 3\tau^2} = 23.81\ \text{ksi}$$

$$Z_{ps} = 139 - 20.24\left(1 - \frac{23.87}{36}\right) = 132.18\ \text{in}^2$$

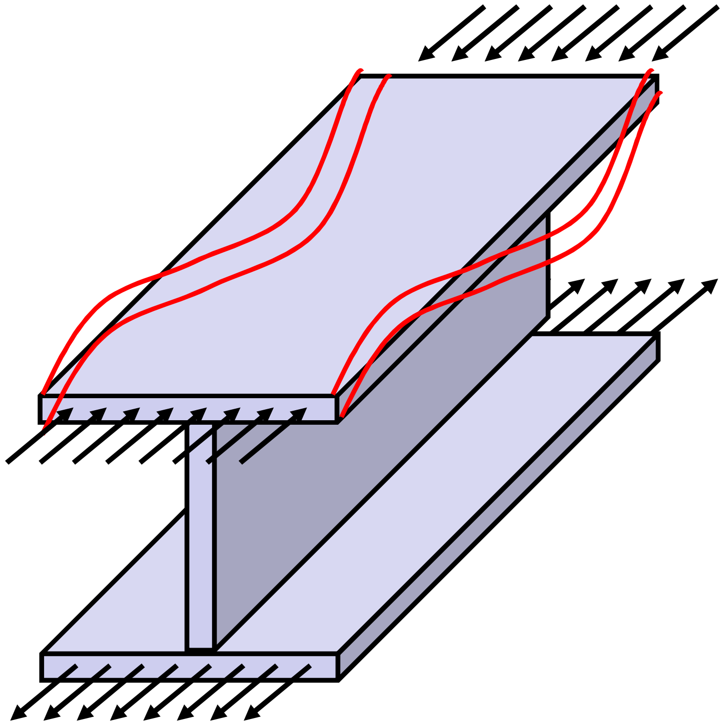
2.8 Compactness

2.8.1 LRFD



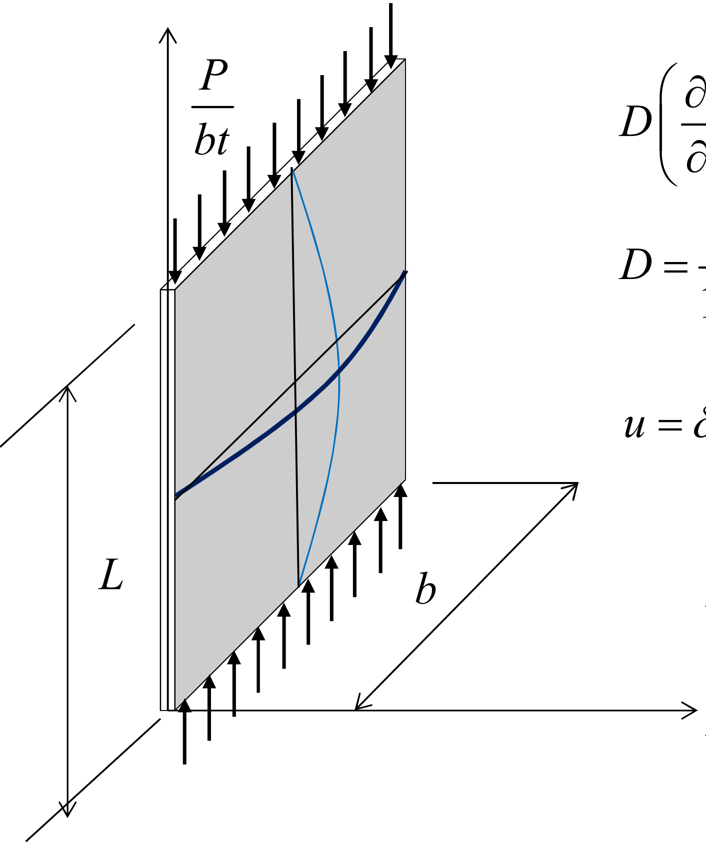


$$0.38\sqrt{\frac{E}{F_y}} = \frac{65}{\sqrt{F_y \text{ ksi}}} \qquad 1.0\sqrt{\frac{E}{F_y}} = \frac{170}{\sqrt{F_y \text{ ksi}}}$$



$$\frac{b}{t} \leq \alpha\sqrt{\frac{k}{F_y}}$$

2.8.1 Plate Buckling



$$D\left(\frac{\partial^4 u}{\partial z^4}+2\frac{\partial^4 u}{\partial^2 z \partial^2 y^2}+\frac{\partial^4 u}{\partial y^4}\right)=-\frac{P}{b}\frac{\partial^2 u}{\partial z^2}$$

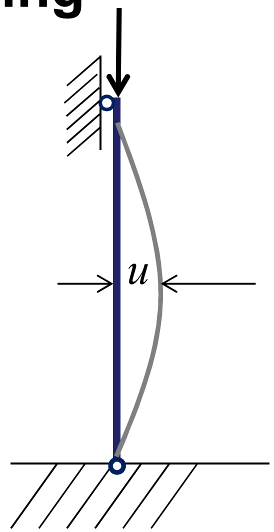
$$D=\frac{Et^3}{12\left(1-\nu^2\right)}$$

$$u=\delta \sin\frac{m\pi y}{b}\sin\frac{n\pi z}{L}$$

$$F_{01}=\frac{P_{01}}{bt}$$

$$F_{01}=\frac{\pi^2 E}{12\left(1-\nu^2\right)}\frac{k}{\left(b/t\right)^2}$$

Column Buckling



$$u=\delta \sin\frac{\pi z}{L}$$

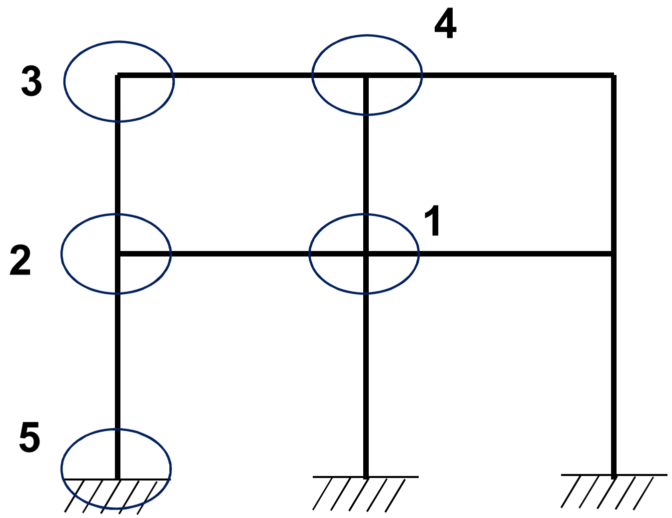
$$P_{cr}=\frac{\pi^2 EI}{L^2}$$

$$P_{cr}=\frac{\pi^2 EA}{\left(L/r\right)^2} \quad r=\sqrt{\frac{I}{A}}$$

Plate Buckling

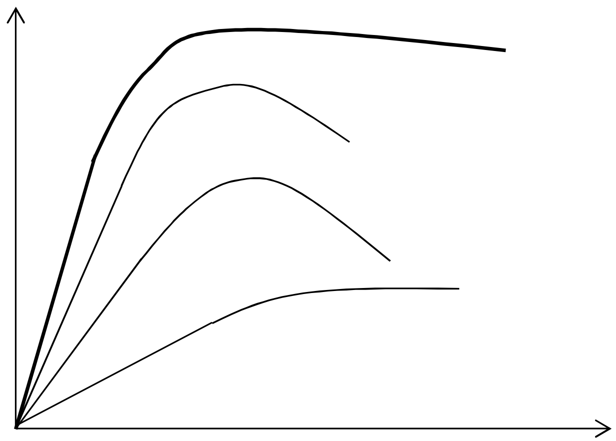
$$F_{cr}=\frac{\pi^2 E}{12\left(1-\nu^2\right)}\frac{k}{\left(b/t\right)^2}$$

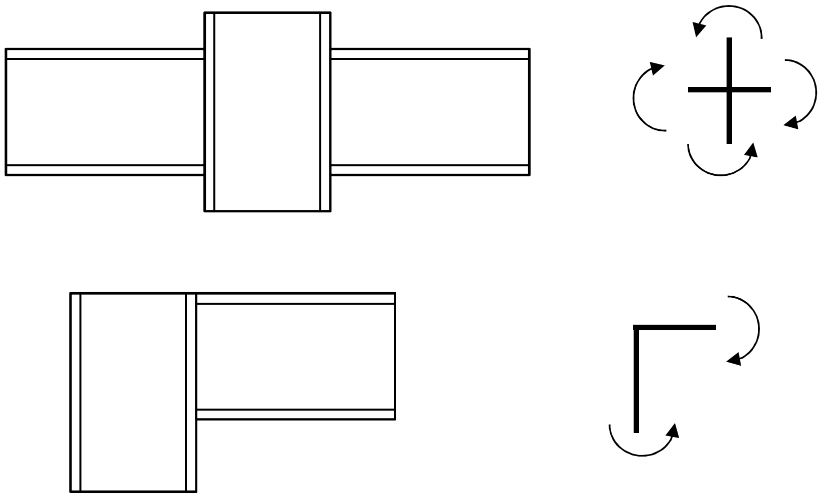
2.9 Connections



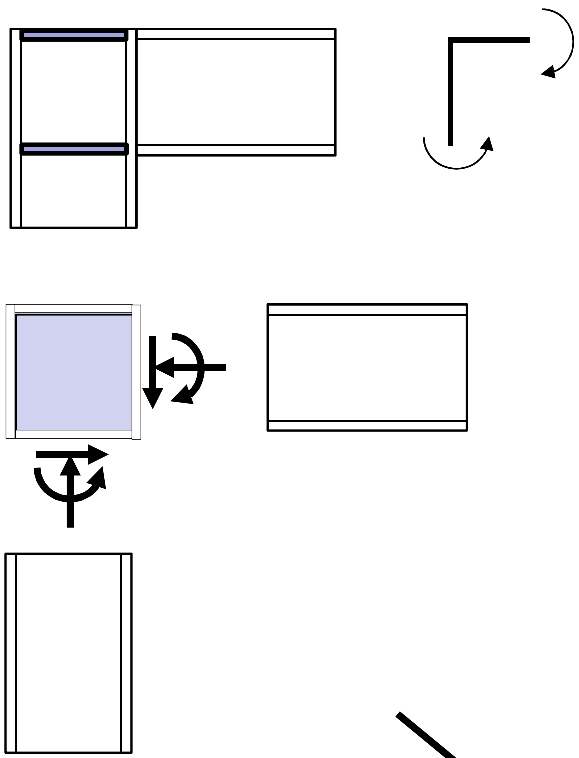
Requirement for connections

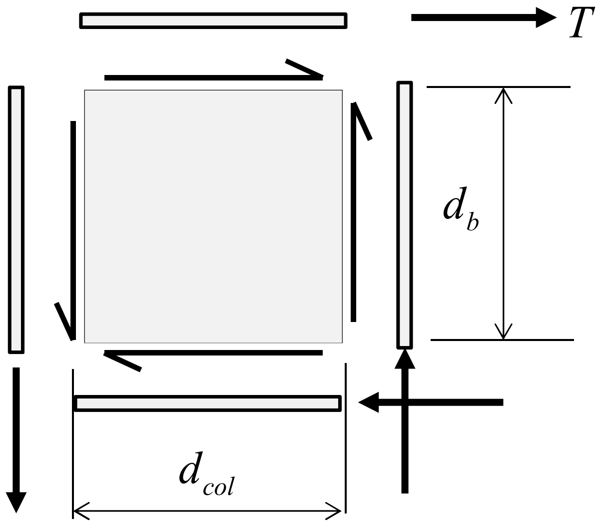
1. **Strength**
2. **Rotation capacity**
3. **Adequate stiffness**
4. **Constructability**

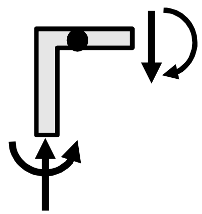




2.9.2 Corner Connections







Strong column and weak beam

$$T = \frac{M_p}{d_b} = \tau_{yb} t_w d_{col}$$

$$\tau_{yb} = \frac{\sigma_{yb}}{\sqrt{3}} \Rightarrow 0.6\sigma_y$$

$$t_w = \frac{M_p}{\phi_v \tau_{yb} d_b d_{col}}$$

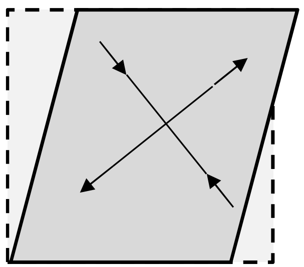
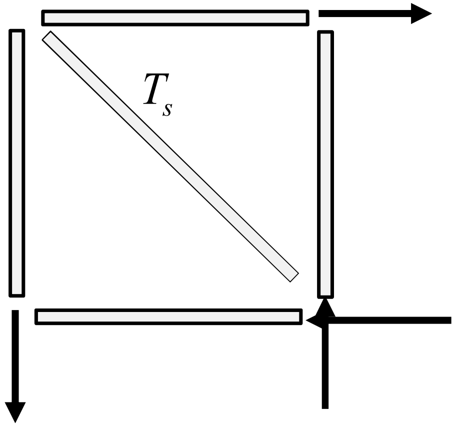


Plate buckling by shear

$$T = \frac{M_p}{d_b}$$

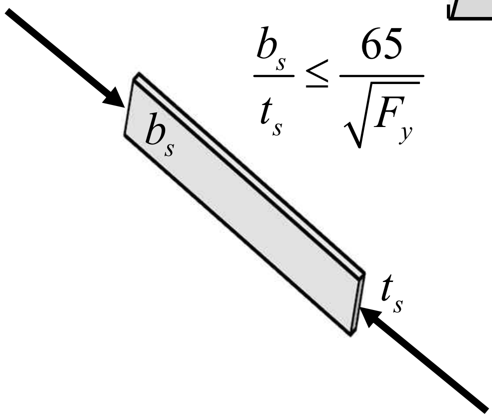


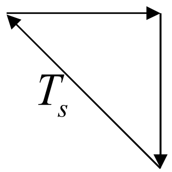
$$T_{req} \le T_{PL} + T_s$$

$$\frac{M_p}{d_b} \le \phi_v \tau_{yp} t_w d_{col} + T_s \cos\theta$$

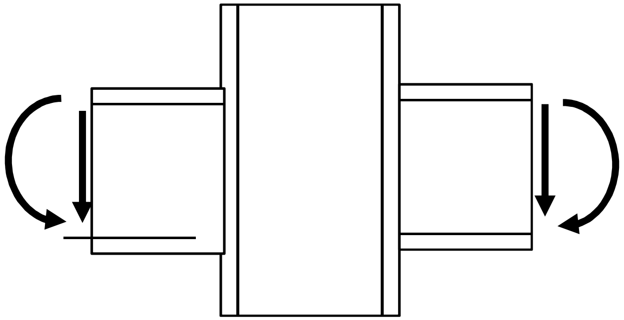
$$T_s \ge \frac{1}{\cos\theta}\left[\frac{M_p}{d_b} - \frac{\phi_v \sigma_{yp} t_w d_{col}}{\sqrt{3}}\right]$$

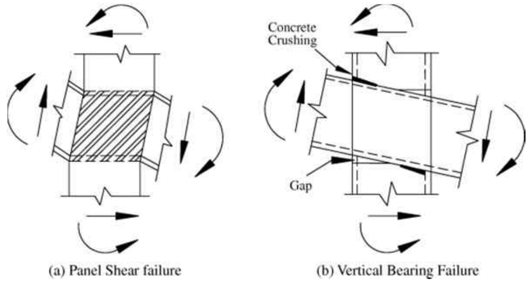
$$\frac{b_s}{t_s} \le \frac{65}{\sqrt{F_y}}$$

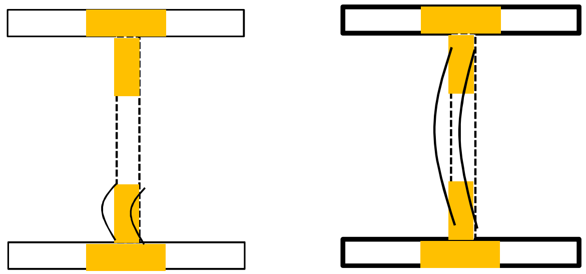




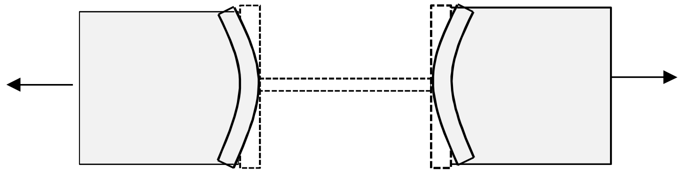
2.9.3 interior Connections



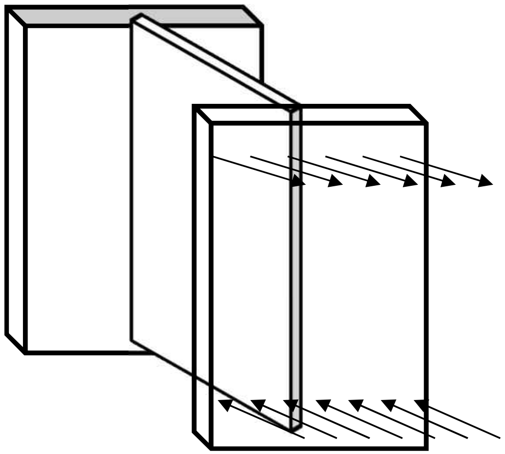


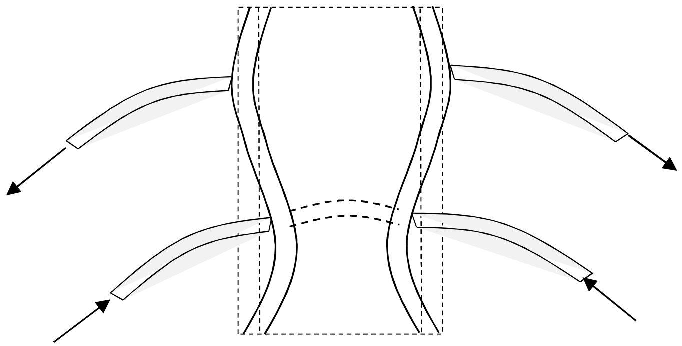


Column web yielding Column web buckling



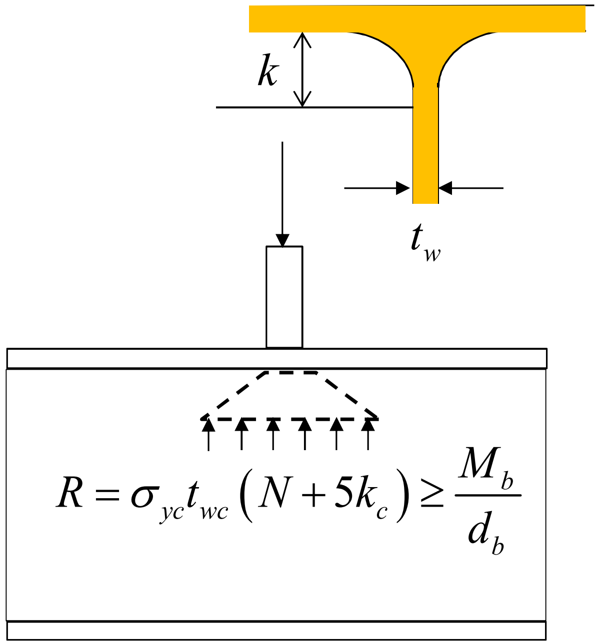
Column flange bending in tension





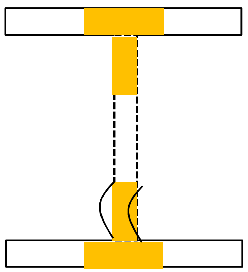
Column web yielding

Column web buckling

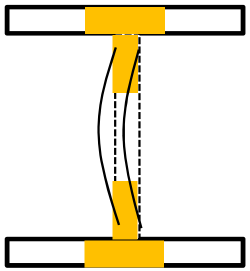


$$R = \sigma_{yc} t_{wc} \left(N + 5k_c \right) \geq \frac{M_b}{d_b}$$

2.9.3 Unbalanced interior Connections

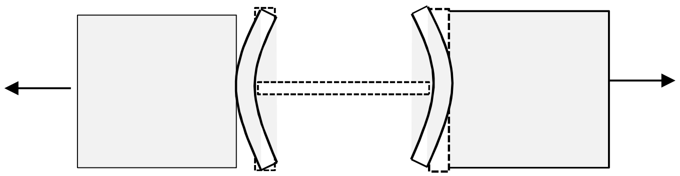


Column web yielding



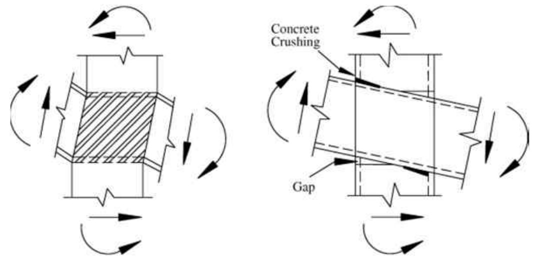
Column web buckling

$$R = \frac{4100\phi t_w^3 \sqrt{\sigma_{yc}}}{d_c}$$

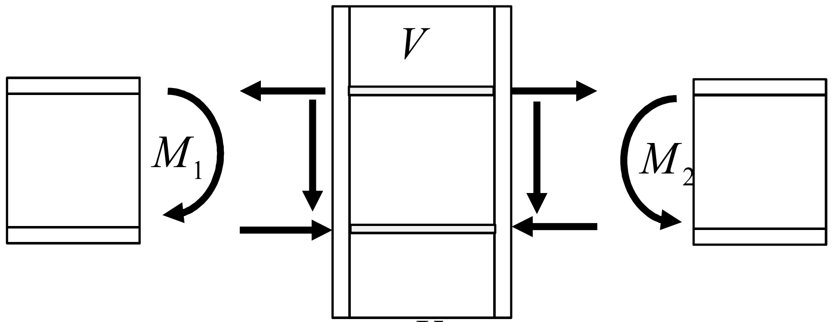


Column flange bending

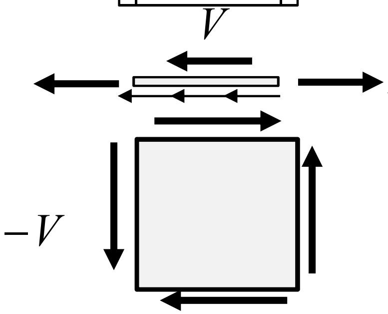
$$\phi R = \phi 6.25 t_{fc}^3 \sigma_{yc}$$



(a) Panel Shear failure (b) Vertical Bearing Failure



$$T_1 = \frac{M_1}{d_b}$$



$$T_2 = \frac{M_2}{d_b}$$

$$\phi_v \tau t_{wc} d_{col} = \frac{M_2}{d_{b2}} - \frac{M_1}{d_{b1}} - V$$

$$t_{wc} = \frac{1}{\phi_v \tau d_{col}} \left[\frac{M_2}{d_{b2}} - \frac{M_1}{d_{b1}} - V \right]$$