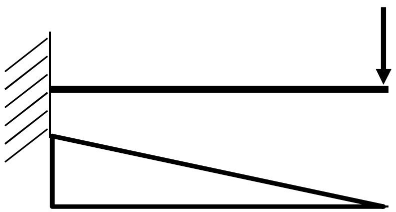
Chap. 4 Equilibrium Method

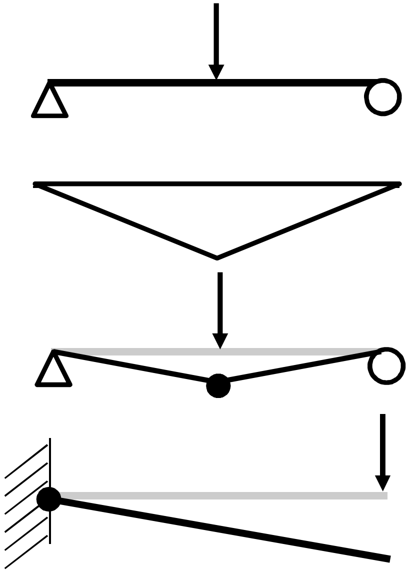
4.1 **Introduction**

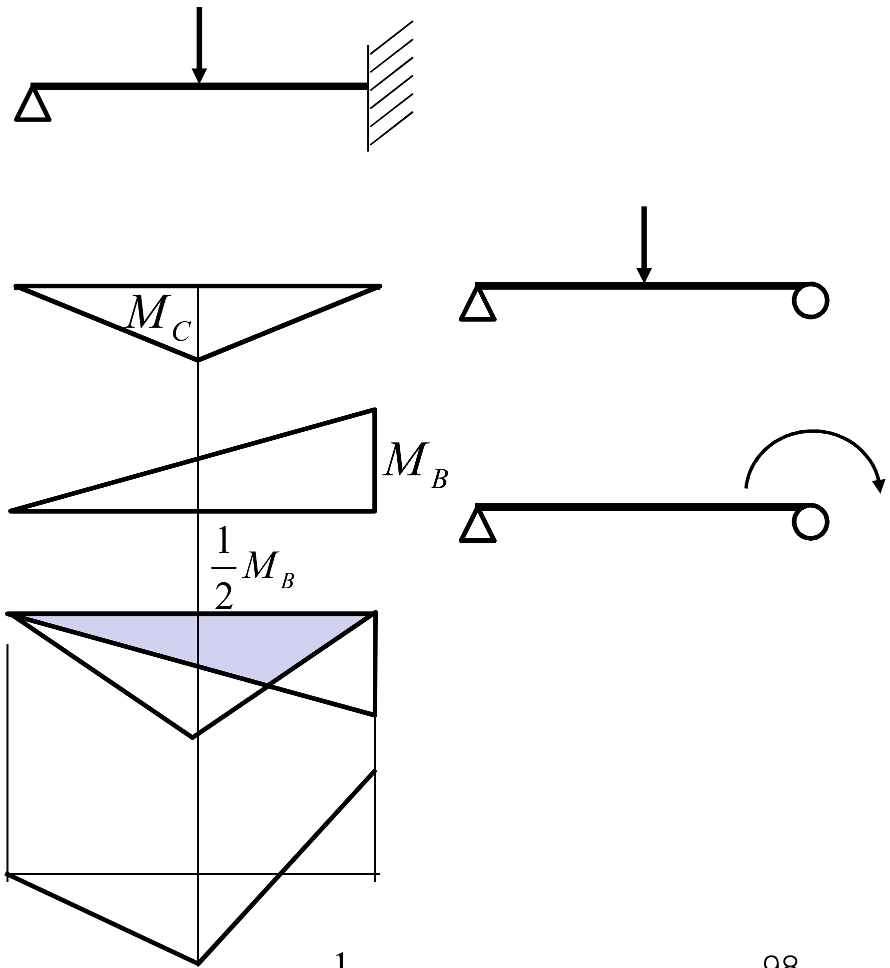
- Based on lower bound
- Relationship between the strength of structure and
  the the applied load is found by adjusting
  the unknown redundants
- The moment condition is not violated
- The mechanism condition may or may not be satisfied

4.2 **Basis of the method**

- Determinate structures
- Indeterminate structures







$$M_P = M_C - \frac{1}{2}M_B$$

4.3 Moment equilibrium equations

a) Select redundants to make determinate structures

b) Draw BMD for determinate structure for applied load

c) Draw BMD for redundants

d) Superpose them

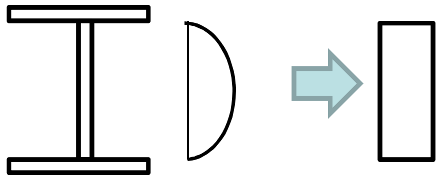
e) Write moment equation at critical section

4.5 Strength of beams

a) Shear force effect

b) Lateral torsional buckling

4.5.1 Shear force effect

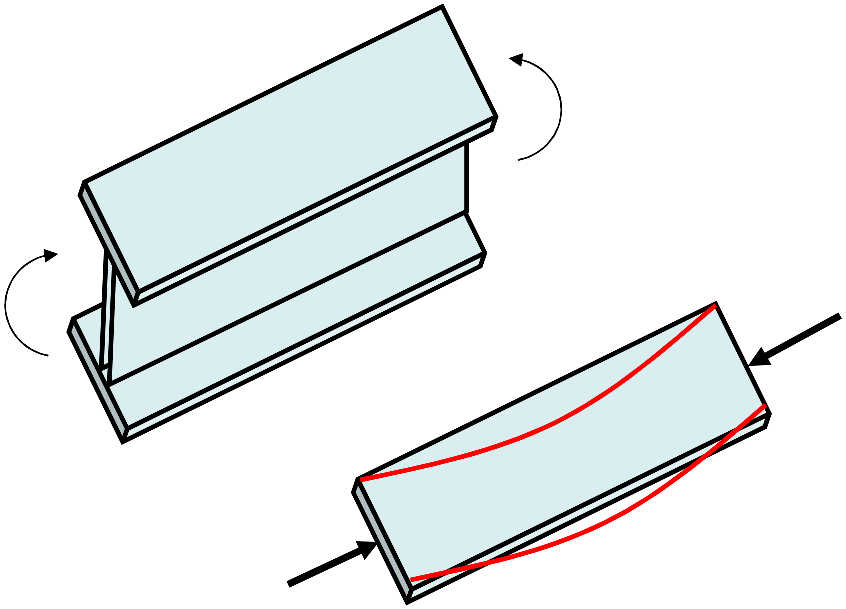


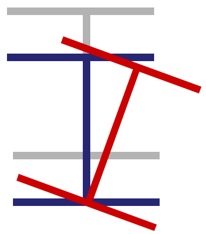
$$V_P = \tau_p t_w \left(d - 2t_f\right)$$

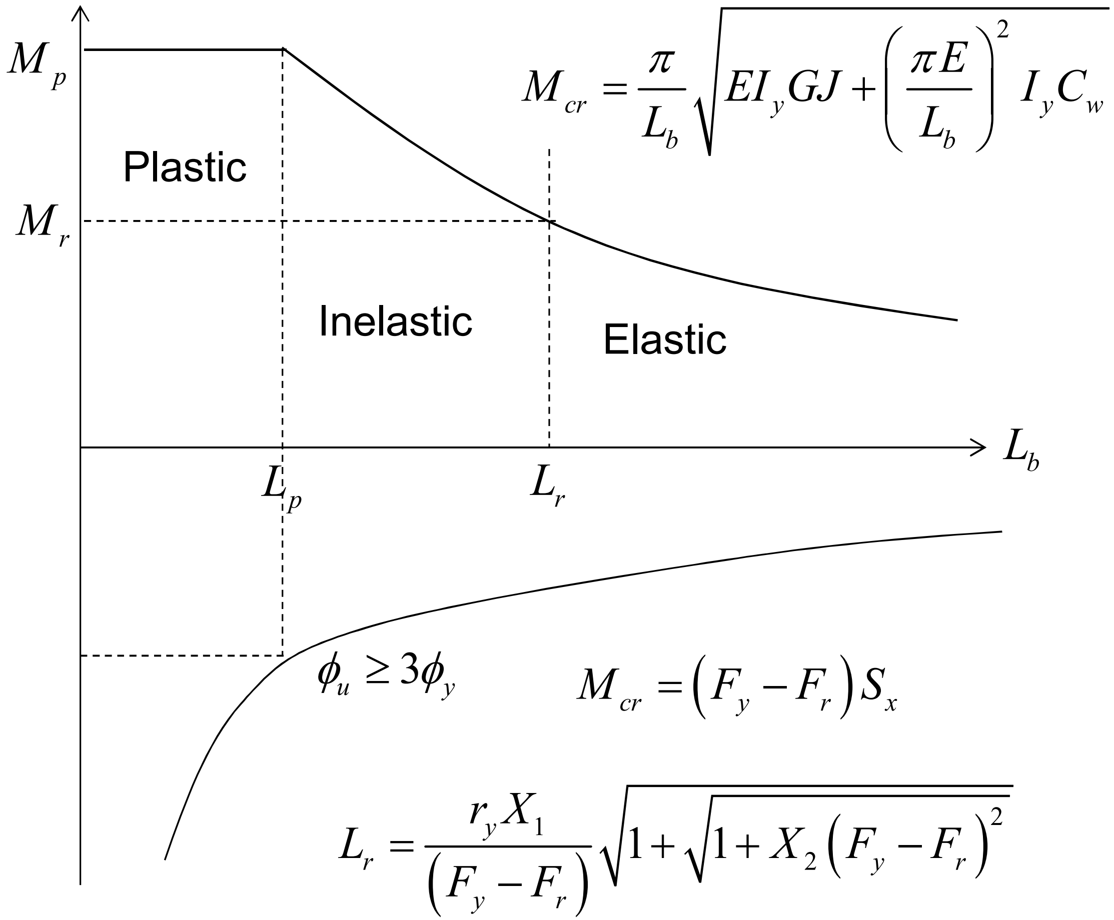
$$= \frac{F_y}{\sqrt{3}} t_w d \left(\frac{d - 2t_f}{d}\right)$$

$$V_P = \frac{F_y}{\sqrt{3}} t_w d \frac{1}{1.07} \cong 0.55 F_y t_w d$$

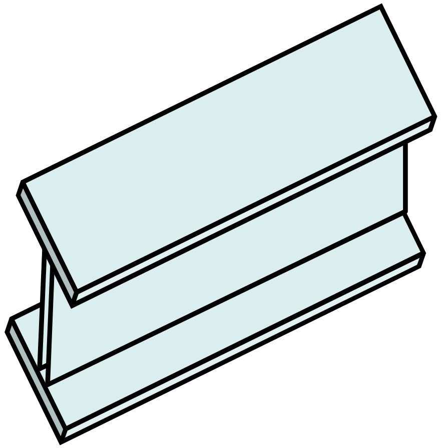
4.5.2 Lateral Torsional Buckling

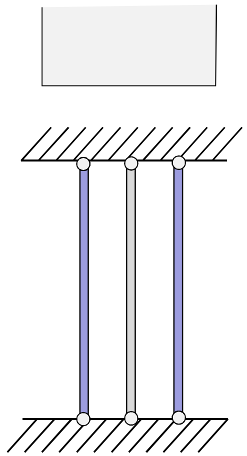




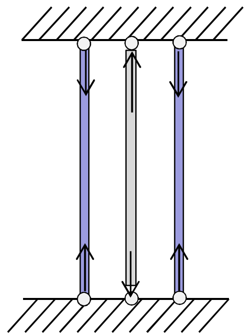


Residual stress in rolled section



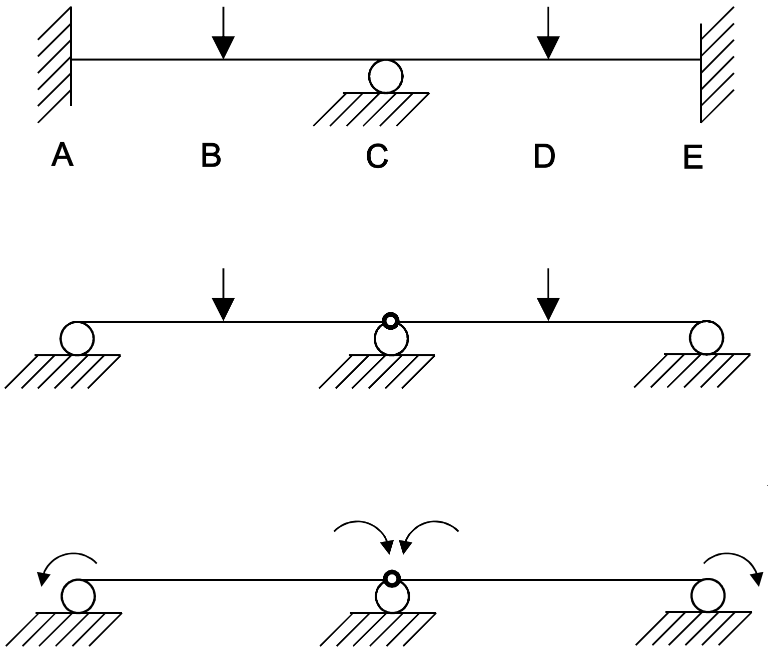


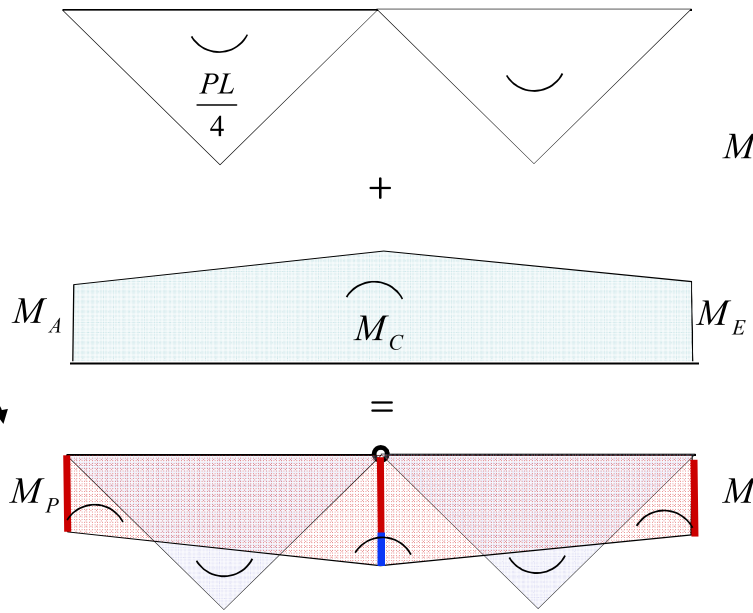
Both flange sides get cool
Then fixed

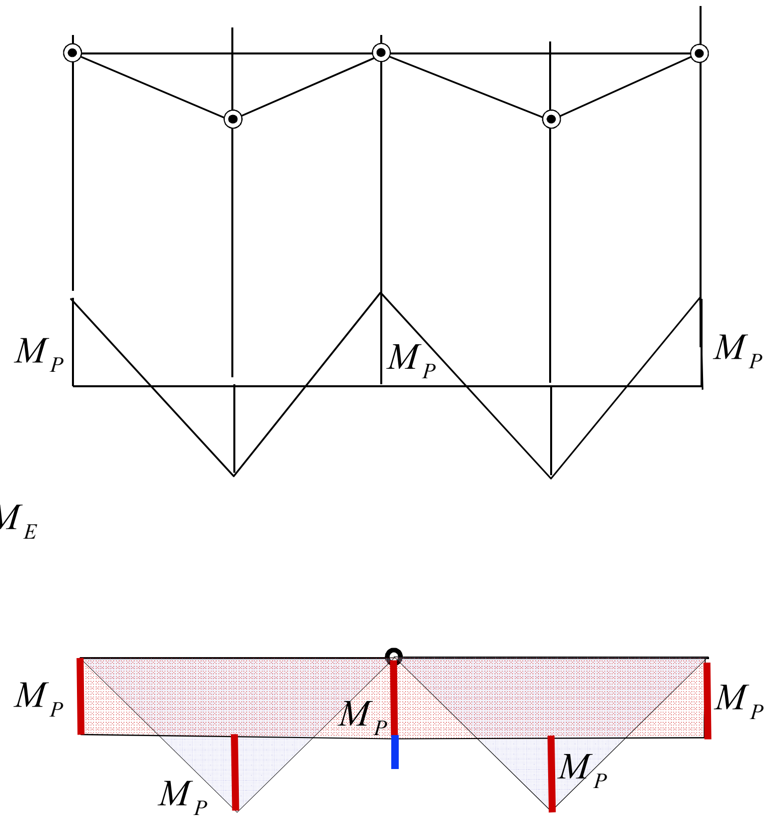


Tension

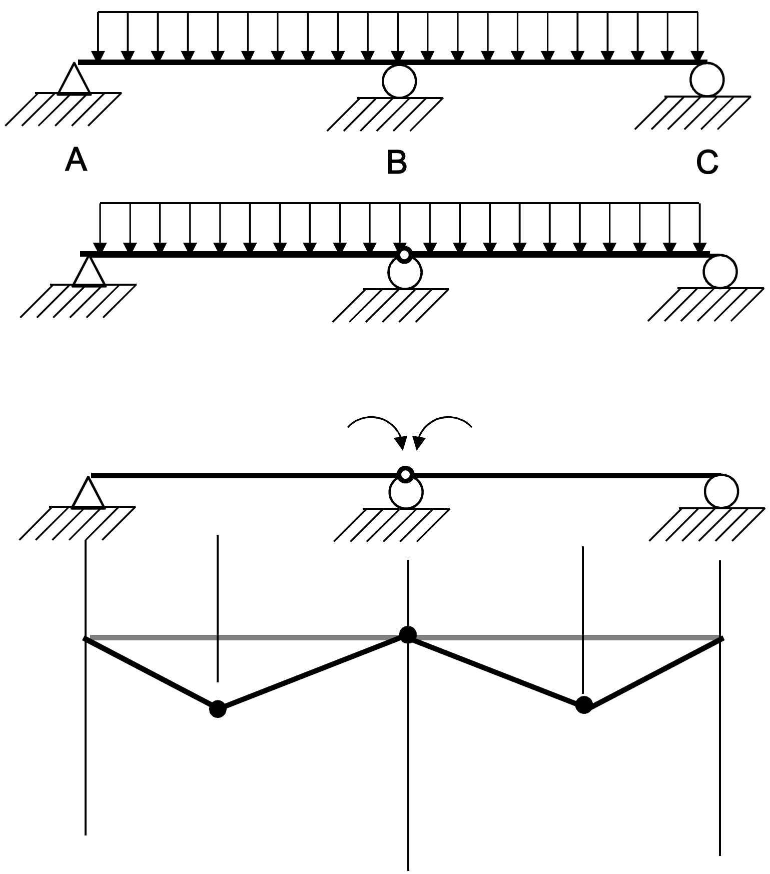
Equilibrium method example



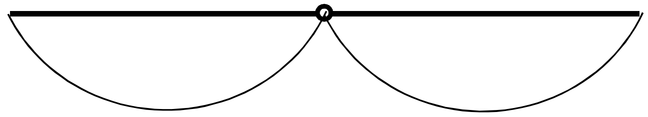




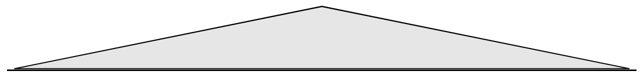
Equilibrium method example

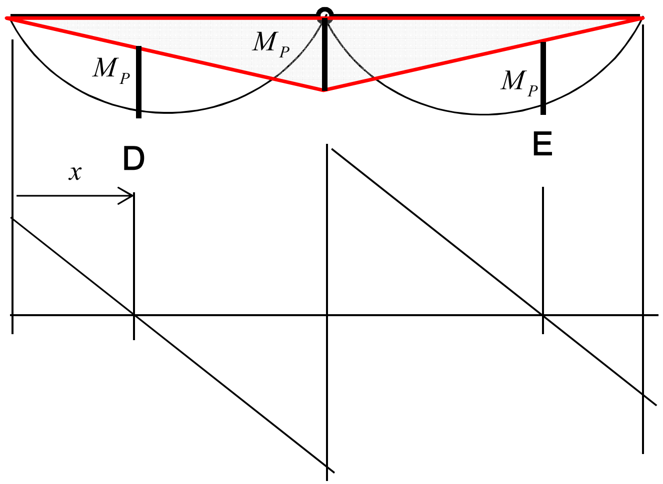


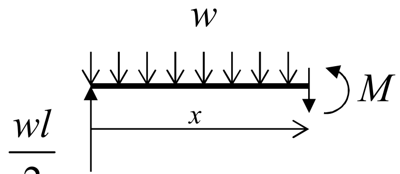
Primary structure and Primary moment



redundant structure and redundant moment



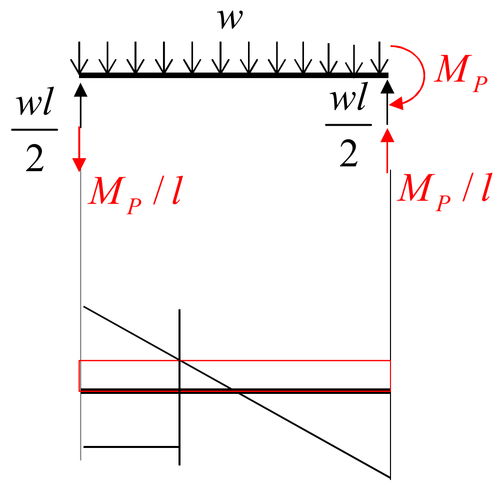


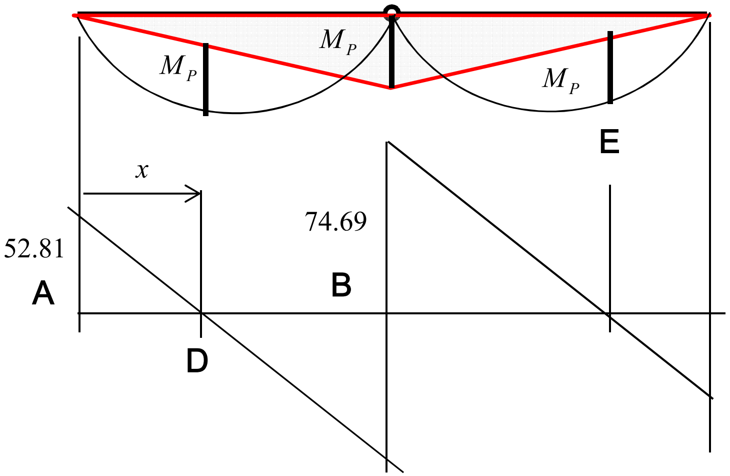


$$M = \frac{wl}{2}x - \frac{w}{2}x^2 - M_P\frac{x}{l}$$

$$\frac{dM}{dx} = 0 \quad \Rightarrow \quad x_{\max} = \frac{l}{2} - \frac{M_P}{wl}$$

$$M_P = \frac{wl^2}{2}\left(3 - \sqrt{8}\right)$$





$$M_P = \frac{wl^2}{2}\left(3-\sqrt{8}\right)$$

$$\begin{cases} w = 5 \text{ kips/ft} \\ l = 15 \text{ ft} \end{cases}$$

$$Z = \frac{M_P}{F_y} = 54.69 \text{ in}^3$$

Max. Shear @ B

$$A_w = \frac{V_B}{\tau_y} = \frac{74.69}{36/\sqrt{3}} = 3.59 \text{ in}^2$$

Try WF16 x 36

$$\begin{cases} Z = 64 \text{ in}^3 \\ A_w = 4.42 \text{ in}^2 \end{cases}$$

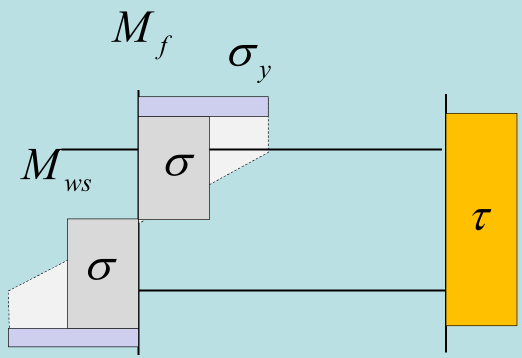
$$\tau_w = \frac{74.69}{4.42} = 16.89 \text{ ksi}$$

$$Z_{ps} = Z - Z_w\left(1-\frac{\sigma}{\sigma_y}\right)$$

$$= Z - Z_w\left(1-\frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y}\right)$$

$$Z_{ps} = 57.07 > 54.69$$

Use WF16 x 36



$$M_{ps} = M_f + M_{ws}$$

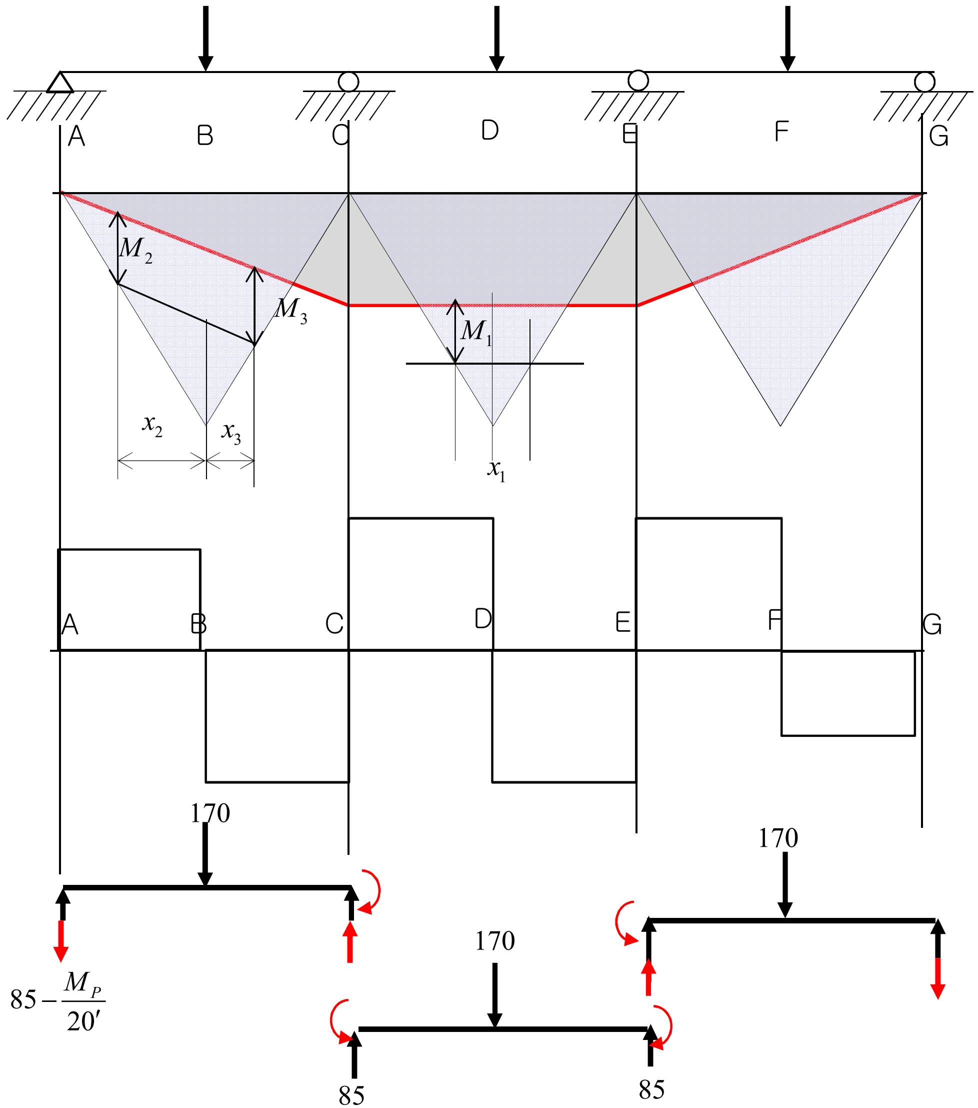
$$M_{ws} = M_w \frac{\sigma}{\sigma_y}$$

$$M_{ps} = M_f + M_{ws} - M_{ws} + M_{ws}$$

$$= M_p - M_w\left(1-\frac{\sigma}{\sigma_y}\right)$$

$$Z_{ps} = Z_p - Z_w\left(1-\frac{\sigma}{\sigma_y}\right)$$

Equilibrium method example



M_P by WF18x50(A36)

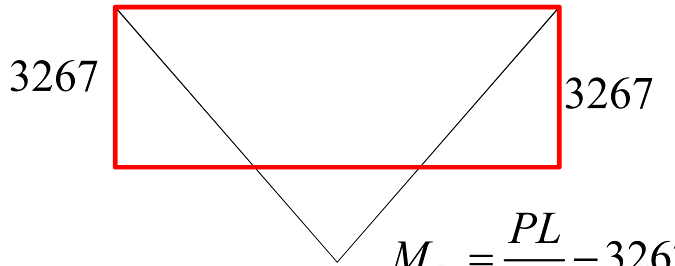
Mc and ME are not determined

M_P by WF18x50(A36)

$$(M_{PS})_{@C,E} = M_P - \sigma_y Z_W \left[1 - \frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y}\right]$$

$$(M_{PS})_{@C,E} = 3267$$

1) Cover plate for the mid-span



$$M_D = \frac{PL}{4} - 3267 = 6934$$

Shear stress in the web @D

$$\tau = \frac{85}{d_w t_w} = 14.21$$

Shear stress in the web @D

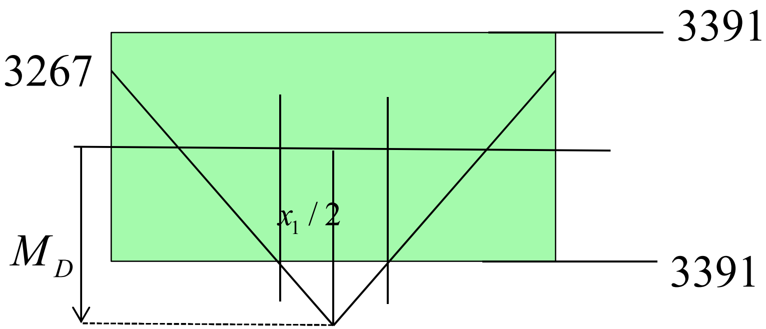
$$\tau = \frac{85}{d_w t_w} = 14.21$$

$$M_{PS} = M_P - \sigma_y Z_W \left[1 - \frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y} \right] = 3391$$

required Z_{PL}

$$Z_{PL} = \frac{6934 - 3391}{36} \text{ in}^3$$

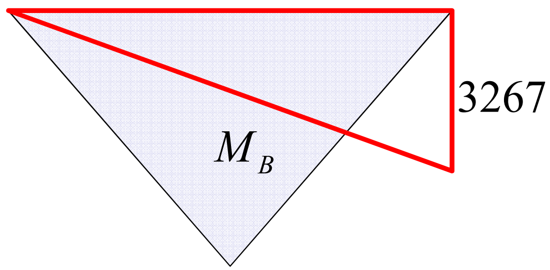
$$Z_{PL} = b_{PL} t_{PL} \left[17.99 + t_{PL} \right] = 99.17 \text{ in}^3$$



$$M(x) = \frac{10200}{10} x - 3267 \Rightarrow 3391$$

$$x = 10 - \frac{x_1}{2} \Rightarrow x_1 = 6.95'$$

2) Cover plate for the end-spans



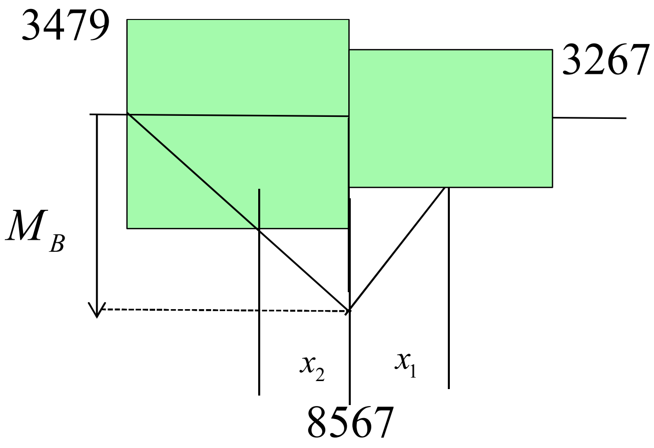
$$M_B = \frac{170 \times 20 \times 12}{4} - \frac{3267}{2} = 8567$$

required Z_{PL}

$$Z_{PL} = \frac{8567 - 3267}{36} 147.23 \text{ in}^3$$

Try $t_{PL} = 0.75''$ $b_{PL} = 10.54''$

$$M_{PS} = M_P - \sigma_y Z_W \left[1 - \frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y} \right] = 3479$$



$$M_2 = \frac{8567}{10} x \Rightarrow 3479$$

$$x = 10 - x_2 \Rightarrow x_2 = 5.94'$$

$$M_3 = \frac{8567 + 3267}{10} x - 3267 \Rightarrow 3267$$

$$x = 10 - x_3 \Rightarrow x_3 = 4.48'$$

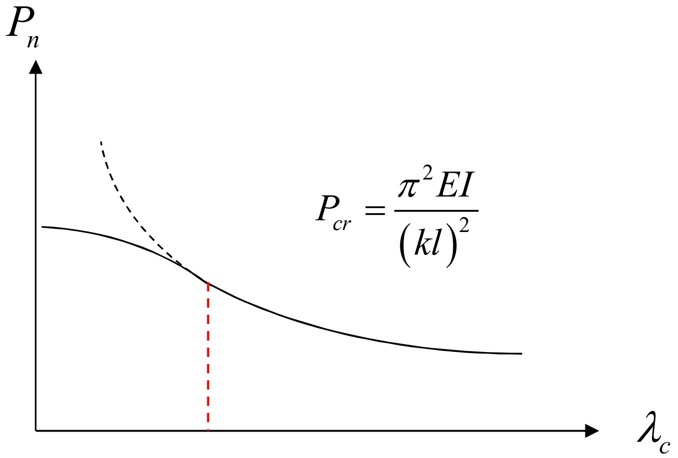
Lateral support spacing

$$L_{pd} = \frac{3600 + 200 \frac{M_1}{M_P}}{F_y} r_y$$

For W 16 x 36 $\quad r_y = 1.52''$

$$L_{pd} = 12.67''$$

Design of columns



$$\lambda_c = \frac{1}{\pi}\frac{kl}{r}\sqrt{\frac{F_y}{E}}$$

$$I = Ar^2 \Rightarrow r = \sqrt{\frac{I}{A}}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{kl}{r}\right)^2} \Rightarrow F_{cr}$$

$$\frac{P_{cr}}{F_y A} = \frac{P_{cr}}{P_y} = \frac{\pi^2 E}{F_y}\frac{1}{\left(\frac{kl}{r}\right)^2} = \frac{1}{\lambda_c^2}$$

λ_c : slenderness parameter

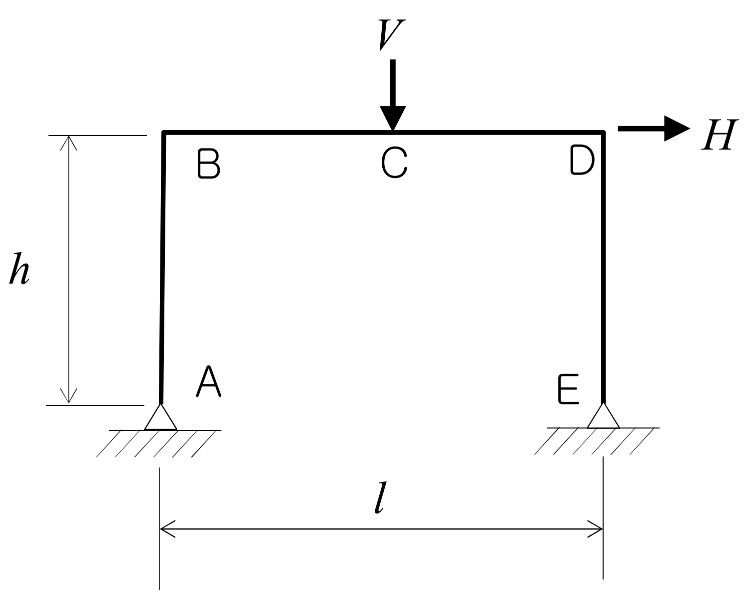
$$\frac{P_{cr}}{P_y} = \frac{1}{\lambda_c^2}$$

$$\frac{79}{90} = 0.877$$

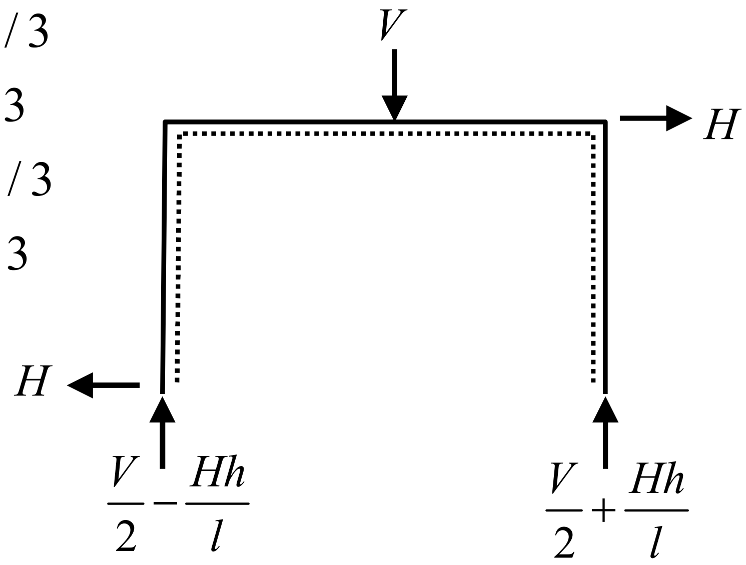
For Design of columns

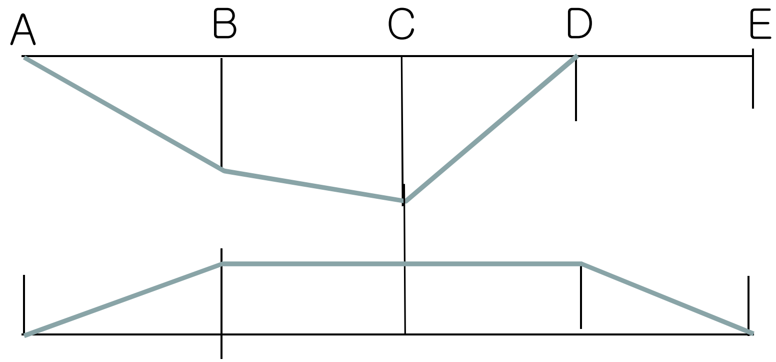
$$\lambda_c \geq 1.5 \quad P_n = \frac{0.877}{\lambda_c^2} P_y$$

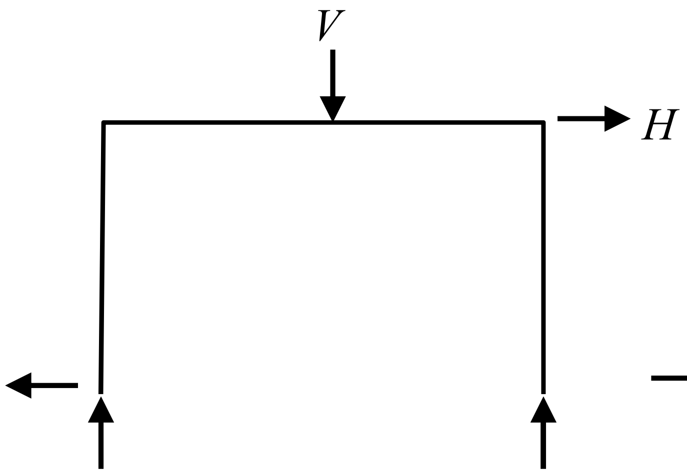
$$\lambda_c \leq 1.5 \quad P_n = 0.658^{\lambda_c^2} P_y$$



$$H\begin{cases} l/h=1 \quad \text{and} \quad V/H=1/3 \\ l/h=1 \quad \text{and} \quad V/H=3 \\ l/h=3 \quad \text{and} \quad V/H=1/3 \\ l/h=3 \quad \text{and} \quad V/H=3 \end{cases}$$

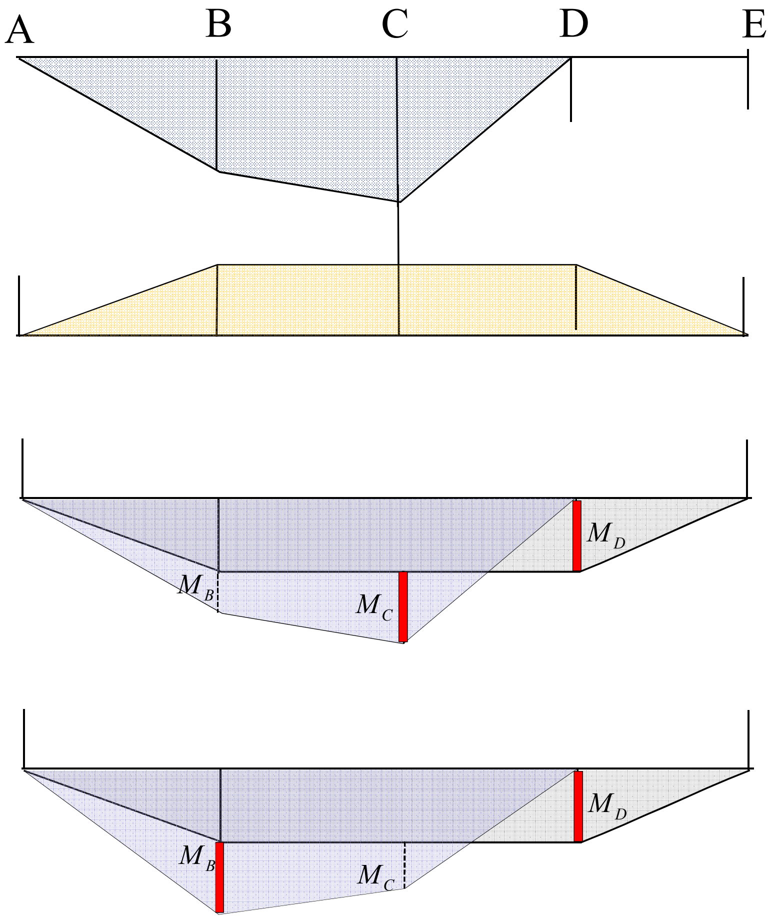






free

redundant



$$\begin{cases} M_D = M_P = Sh \\ M_B = Hh - M_P \\ M_C = \dfrac{Hh}{2} + \dfrac{Vl}{4} - M_P \end{cases}$$

Case I) $l/h = 1$ and $V/H = 1/3$

Then $M_B = Hh - M_P$

$$M_C = \frac{Hh}{2} + \frac{Hh}{12} - M_P = \frac{7Hh}{12} - M_P$$

Since $|M_B| > |M_C|$

the second PH occurs @B

$$\therefore\ H = \frac{2M_P}{h}$$

Case II) $l/h = 1$ and $V/H = 3$

Then $M_B = Hh - M_P$

$$M_C = \frac{Hh}{2} + \frac{3Hh}{4} - M_P = \frac{5Hh}{4} - M_P$$

Since $|M_C| > |M_B|$

the second PH occurs @C

$$\therefore\ H = \frac{1.6M_P}{h}$$

Case III) $l/h = 3$ and $V/H = 1/3$

Then $M_B = Hh - M_P$

$$M_C = \frac{Hh}{2} + \frac{Hh}{4} - M_P = \frac{3Hh}{4} - M_P$$

Since $|M_B| > |M_C|$

the second PH occurs @B

$$\therefore\ H = \frac{2M_P}{h}$$

Case IV) $l/h = 3$ and $V/H = 3$

Then $M_B = Hh - M_P$

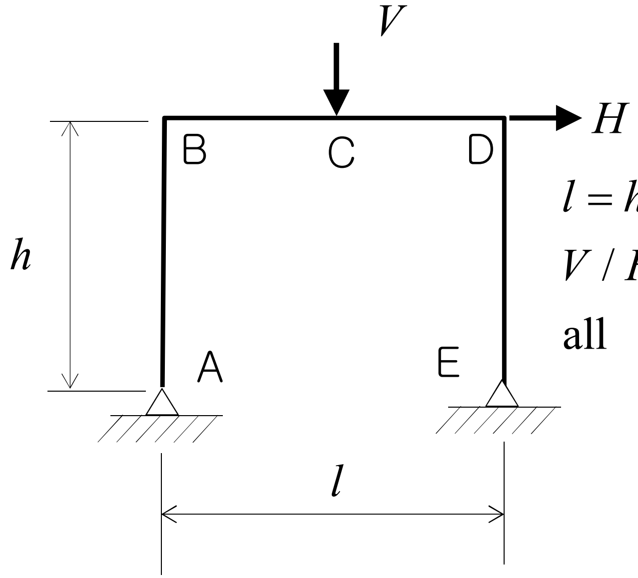
$$M_C = \frac{Hh}{2} + \frac{3H \times 3h}{4} - M_P = 2.75Hh - M_P$$

Since $|M_C| > |M_B|$

the second PH occurs @C

$$\therefore\ H = \frac{0.727M_P}{h}$$

Ex. 4.6.2



$l = h = 20'$

$V / H = 3$

all WF16×45

Determine the limit values V and H

a) w/o considering the effect of axial force

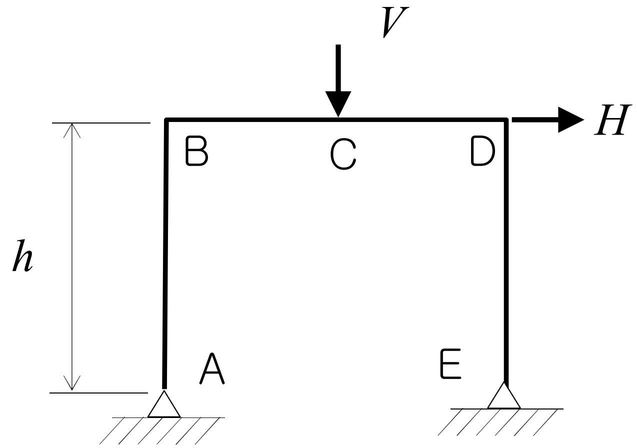
From the previous solution

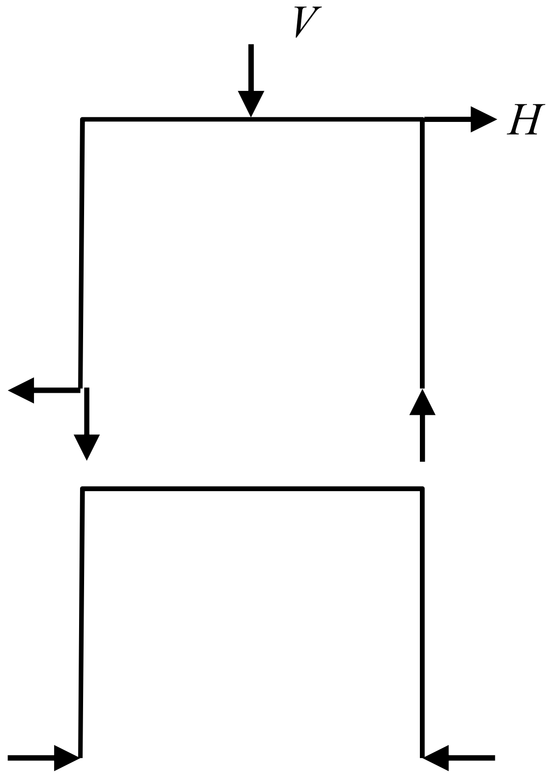
$$H = 1.6\frac{M_P}{h}$$

From AISC $M_{Px} = Z_x F_y = 2963$ kip-in

$$H = 1.6\frac{M_P}{(20)(12)} = 19.75 \text{ kip-in}$$

$$V = 3H = 59.25 \text{ kips}$$





b) w/ the effect of axial force

For Member BD

$$T = H - S = 0.6\frac{M_P}{h} = 7.41 \text{ kips}$$

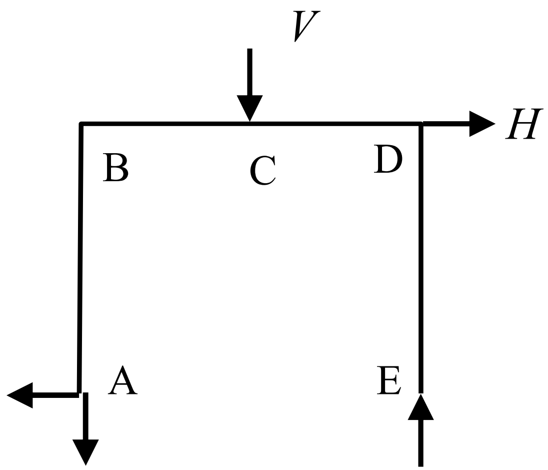
$$\text{WF16} \times 45 \Rightarrow P_y = 13.3 \times 36 = 478.8 \text{ kips}$$

$$\frac{P}{\phi_t P_y} = 0.018 < 0.2$$

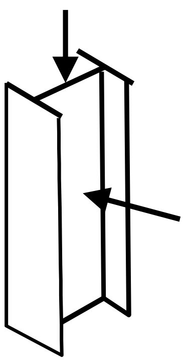
Reduced Mpc $\quad \dfrac{P}{\phi_t P_y} + \dfrac{M_{pc}}{\phi_b M_p} \le 1.0 \Rightarrow$

$$M_{PC} = \left(1 - \frac{0.018}{2}\right)\phi_b M_p = 0.991\phi_b M_p$$

b) w/ the effect of axial force
For Member DE



$$P=\frac{V}{2}+\frac{Hh}{l}=49.38$$



$$\lambda_{cy}=\frac{1}{\pi}\frac{(kL)_y}{r_y}\sqrt{\frac{F_y}{E}}=0.843\vee$$

$$\lambda_{cx}=\frac{1}{\pi}\frac{(kL)_x}{r_x}\sqrt{\frac{F_y}{E}}=0.398$$

$$P_n=0.658^{\lambda_c^2}P_y=355.6\text{ kips}$$

The ratio $\frac{P}{\phi_c P_n}=\frac{49.38}{0.85\times 355.6}=0.163$

The reduced M_{pc}

$$\frac{P}{2\phi_c P_n}+\frac{M_{pc}}{\phi_b M_{nx}}\le 1$$

$$\frac{0.163}{2}+\frac{M_{pc}}{0.9M_{nx}}=1.0$$

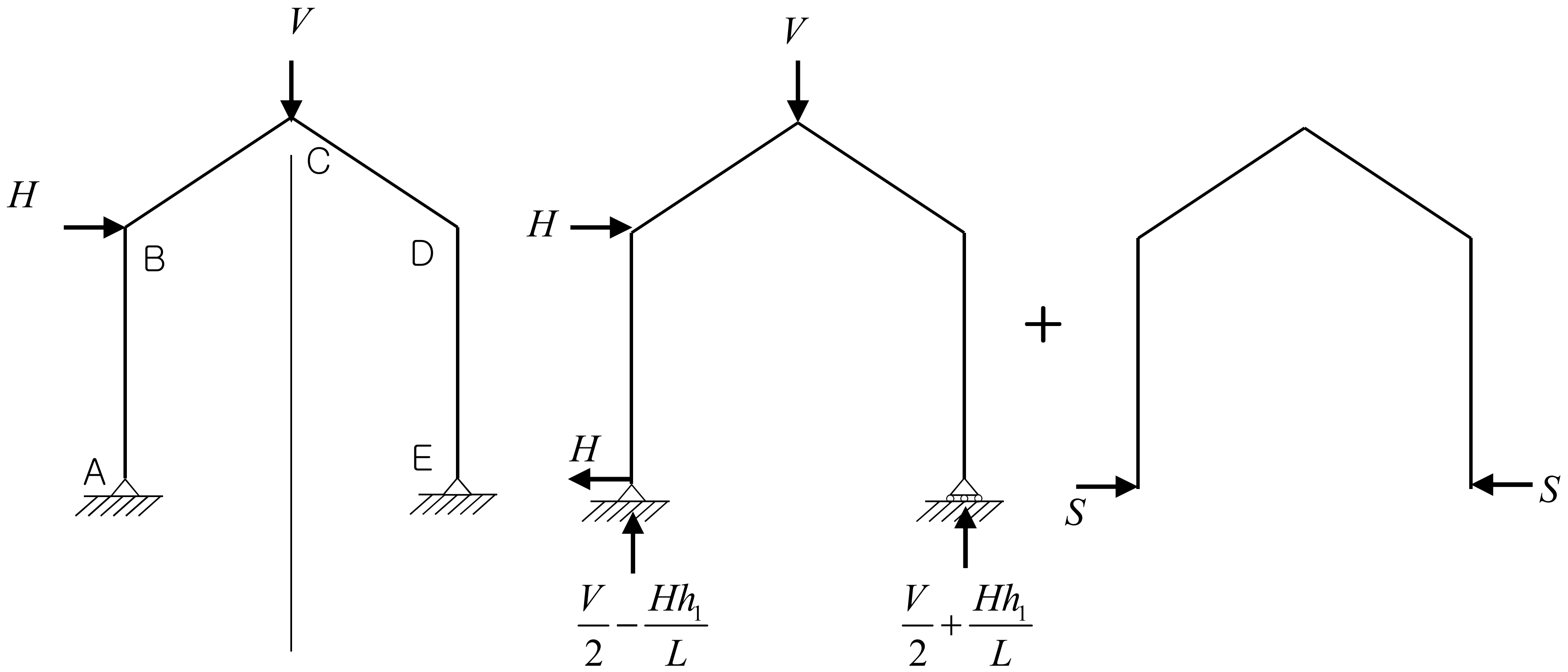
$$M_{pc}=0.827M_p$$

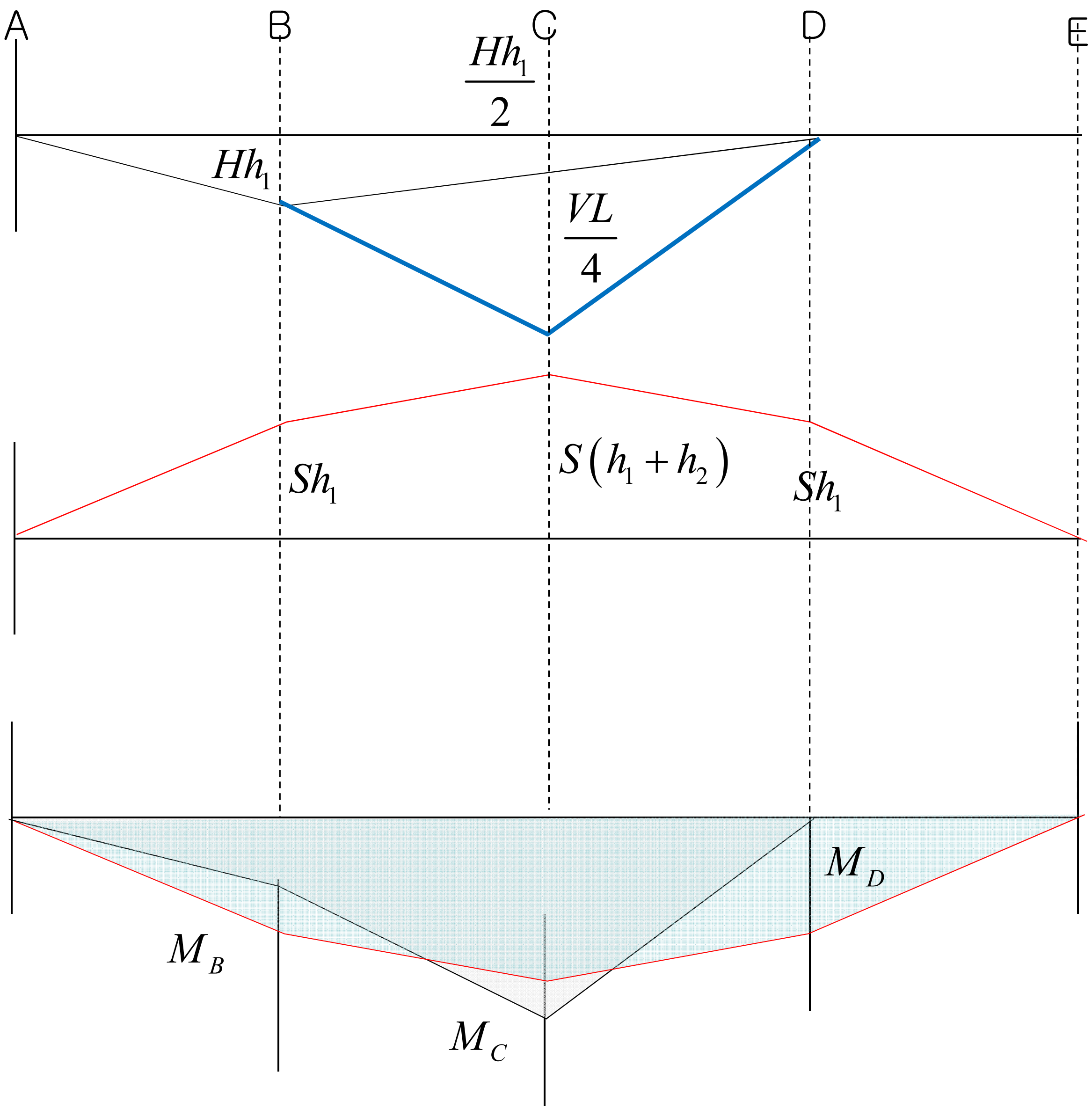
Now

$$H=1.6\frac{M_{pc}}{h}=16.34\text{kips}$$

$$V=3H=49\text{kips}$$

Ex. 4.6.3





$$Sh_1 = M_p \Rightarrow S = \frac{M_p}{h_1} = 12.35 \text{ kips}$$

$$M_B = Hh_1 - Sh_1 = 20H - 247$$

$$M_C = \frac{Hh_1}{2} + \frac{VL}{4} - S(h_1 + h_2)$$

$$M_C = 55H - 370.5$$

assume $M_B = M_p$

$$H = 24.7 \text{ kips}$$

$$M_C = 55 \times 24.7 - 370.5 = 988$$

$$|M_C| > M_p$$

So the second PH occurs @C

$$M_C = M_p = \frac{2962.8}{12} = 55H - 370.5$$

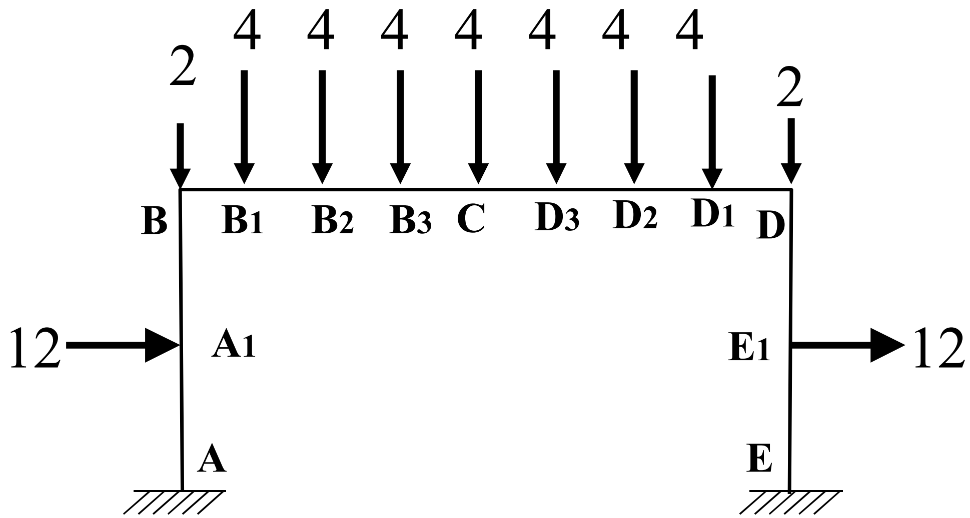
$$H = 24.7 \text{ kips}$$

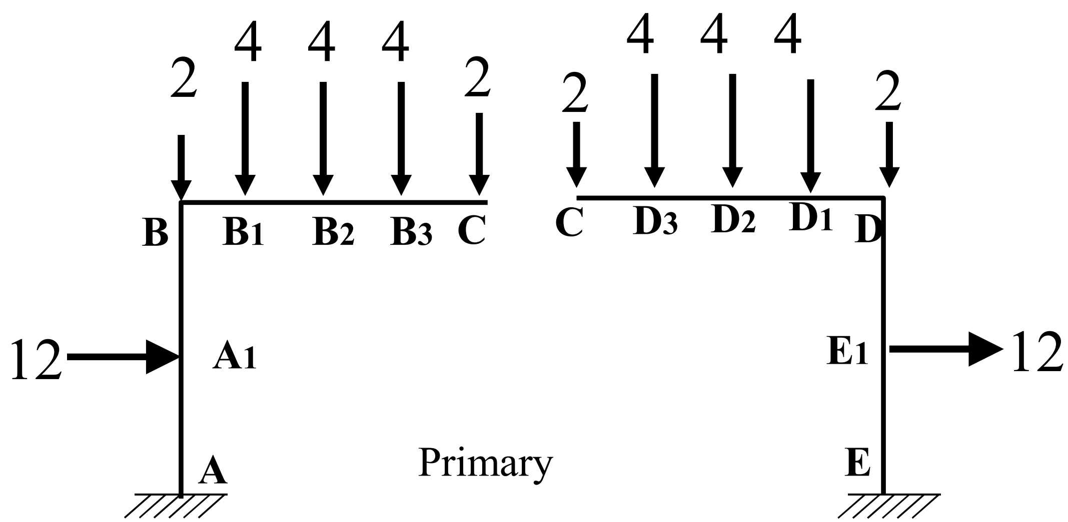
$$V = 3H = 33.69 \text{ kips}$$

4.8 Practical procedure for large structures

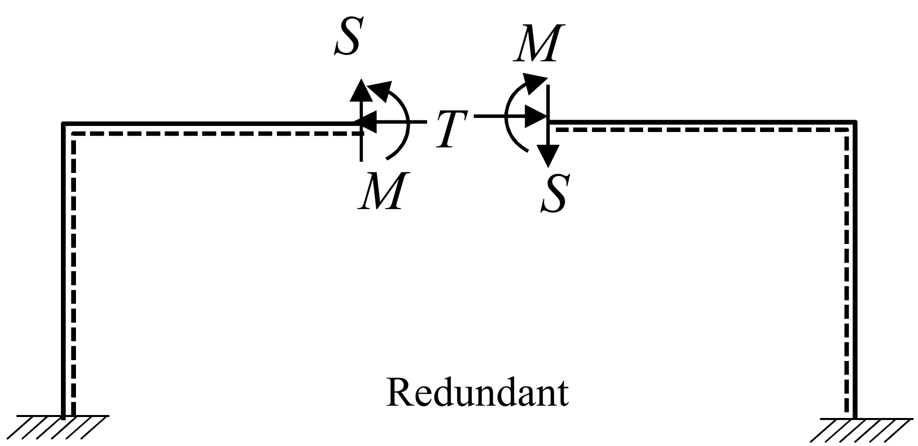
a) Select the redundants
b) Obtain the primary moments
c) Obtain BMD for redundants
d) Assume a failure mechanism
e) Combine b)+c) and equate the sum of moment to the Mp
f) Solve the equations to determine the redundants and the limit
g) Check the yield conditions
h) If the solution is exact, If not proceed further to determine the upper and lower solutions

$$P_{t0} = P_{up} \frac{M_p}{M_{\max}}$$





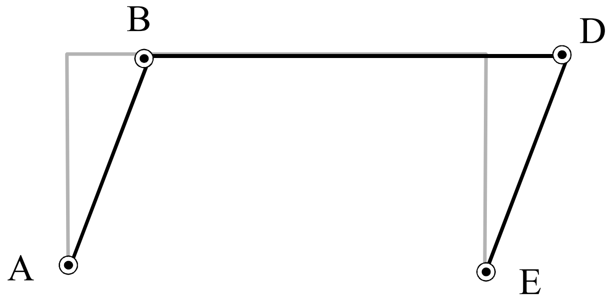
Primary



Redundant

joint	**A**	**A1**	**A2**	**B**	**B1**	**B2**	**B3**	**C**	**D3**	**D2**	**D1**	**D**	**E1**	**E**
[redacted]														
T	16T	8T	0	0	0	0	0	0	0	0	0	0	8T	16T
[redacted]														
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
[redacted]														
Total 2	-128	-16	0	68	112	132	128	100	48	-28	-128	-128	-48	128
[redacted]														

Mechanism 1

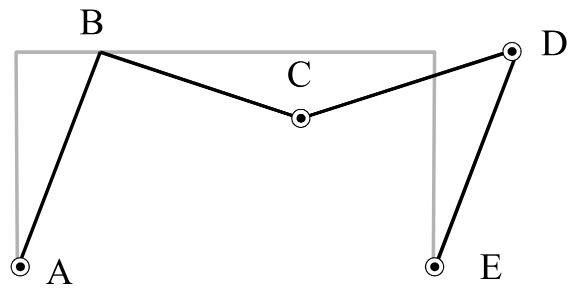


$$\begin{cases} M_A = -384 + 16T + 24S + M = -M_p \\ M_B = -192 + \cancel{16T} + 24S + M = M_p \\ M_D = -192 + \cancel{16T} + 24S + M = -M_p \\ M_E = \cancel{-384} + 16T + 24S + M = -M_p \end{cases}$$

$$\begin{cases} M_p = 96 \text{ kip-ft} \\ M = 192 \text{ kip-ft} \\ S = 4 \text{ kips} \\ T = 0 \end{cases}$$

$$M_{\max} = 204$$
$$96 \le M_p \le 204$$

Mechanism 2

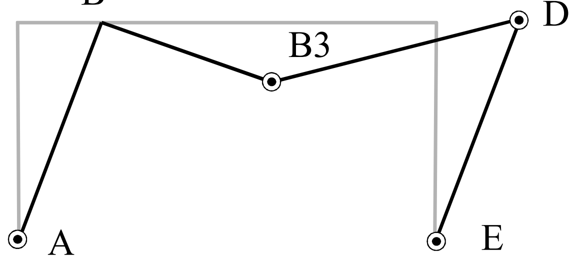


$$\begin{cases} M_A = -384 + 16T + 24S + M = -M_p \\ M_B = \cancel{-192} + \cancel{16T} + \cancel{24S} + M = M_p \\ M_D = -192 + \cancel{16T} - 24S + M = -M_p \\ M_E = \cancel{-384} + 16T - 24S + M = M_p \end{cases}$$

$$\begin{cases} M_p = 128 \text{ kip-ft} \\ M = 128 \text{ kip-ft} \\ S = 2.67 \text{ kips} \\ T = 4 \text{ kips} \end{cases}$$

$$M_{\max} = 132$$
$$128 \le M_p \le 132$$

Mechanism 3

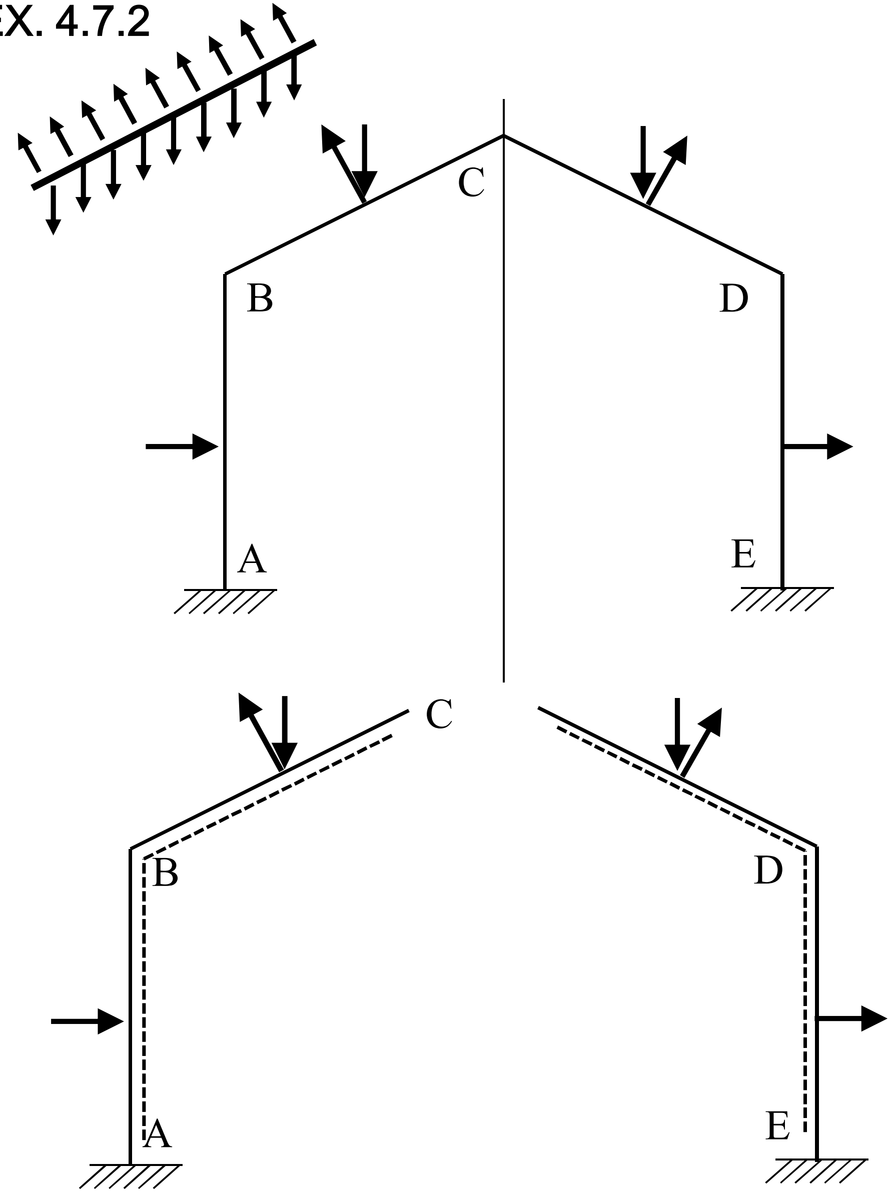


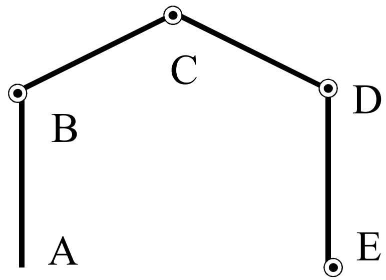
$$\begin{cases} M_A = -384 + 16T + 24S + M = -M_p \\ M_B = -12 + \cancel{16T} + 6S + M = M_p \\ M_D = -192 + \cancel{16T} - 24S + M = -M_p \\ M_E = \cancel{-384} + 16T - 24S + M = M_p \end{cases}$$

$$\begin{cases} M_p = 129.2 \text{ kip-ft} \\ M = 125.6 \text{ kip-ft} \\ S = 2.615 \text{ kips} \\ T = 4.15 \text{ kips} \end{cases}$$

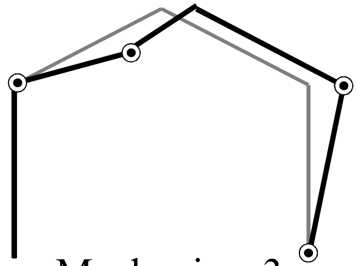
$$M_{\max} = 129.2$$
$$129.2 = M_p$$

EX. 4.7.2

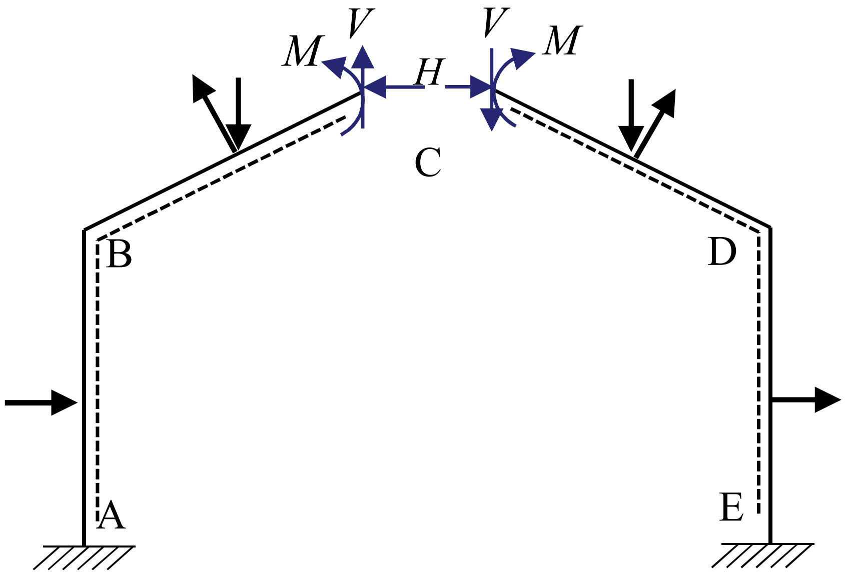




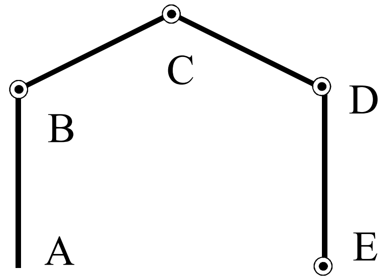
Mechanism 1



Mechanism 3



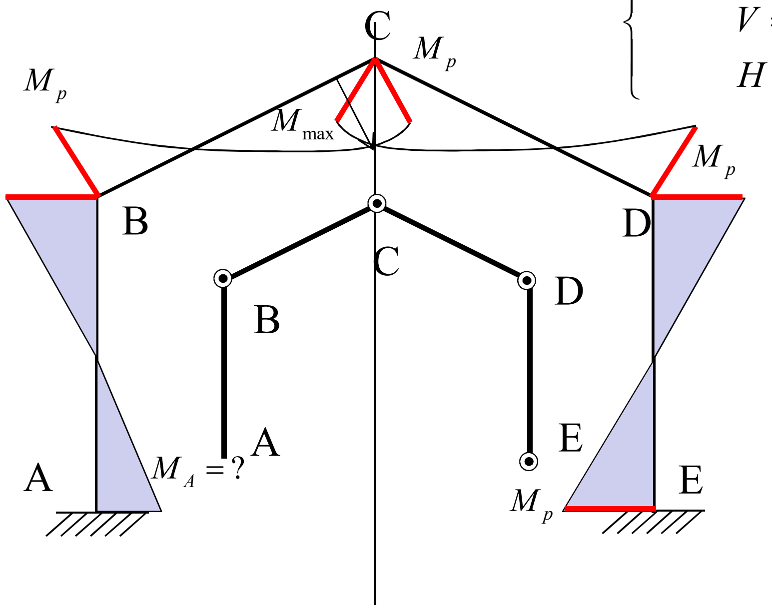
joint	A	B	C	D	E
primary	-274.66	-261.03	0	-222.07	-153.33
M	M	M	M	M	M
V	-12V	-12V	0	12V	12V
H	12.97H	4.97H	0	4.97H	12.97H



Mechanism 1

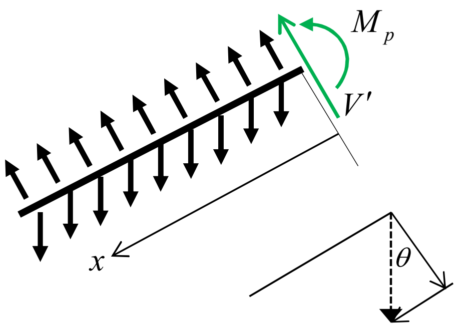
$$\begin{cases} M_B = -261.03 + M - 12V + 4.97H = -M_p \\ M_C = M = M_p \\ M_D = -222.07 + M + 12V + 4.97H = -M_p \\ M_E = -153.33 + M + 12V + 12.97H = -M_p \end{cases}$$

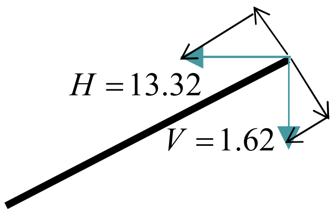
$$\begin{cases} M_C = M = M_p = 87.68 \\ V = -1.62 \\ H = 13.32 \end{cases}$$



$$\begin{cases} M_C = M = M_p = 87.68 \\ V = -1.62 \\ H = 13.32 \end{cases}$$

$$M_A = -274.66 + M_p - 12V + 12.97H = 5.22 < M_p$$





$$V' = 13.32\sin\theta + 1.62\cos\theta$$

$$M_x = M_p + V'x - \frac{wx^2}{2}$$

where

$$\begin{cases} V' = 6.6 \text{ kips} \\ w = \dfrac{50\cos 22.5^\circ - 6}{12.98} = 3.1 \text{ kips/ft} \end{cases}$$

$$M_x = 87.68 + 6.6x - \frac{3.1x^2}{2}$$

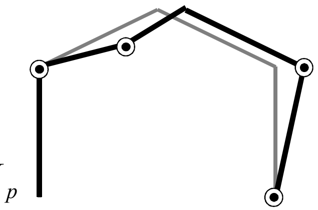
$$\frac{dM_x}{dx} = 6.6 - 3.1x = 0 \quad \Rightarrow x = 2.13'$$

$$M_{C1} = 87.68 + 6.6 \times 2.13 - 3.1\frac{(2.13)^2}{2} = 94.71 \text{ kips-ft}$$

$$87.68 \le M_p \le 94.71$$

Try exact mechanism

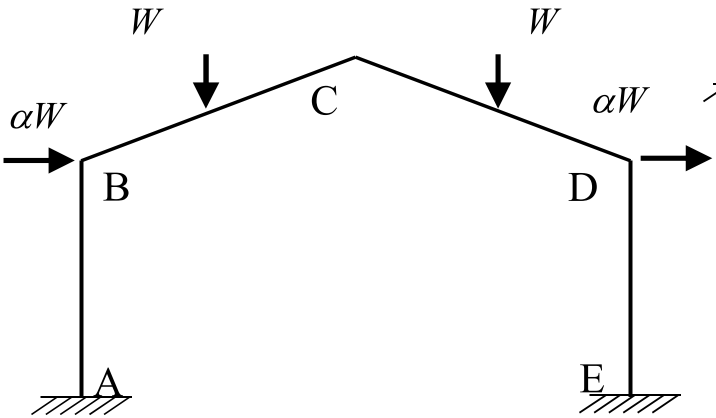
$$\begin{cases} M_B = -261.03 + M - 12V + 4.97H = -M_p \\ M_{C1} = -7.03 + M - 1.97V + 0.82H = M_p \\ M_D = -222.07 + M + 12V + 4.97H = -M_p \\ M_E = -153.33 + M + 12V + 12.97H = M_p \end{cases}$$



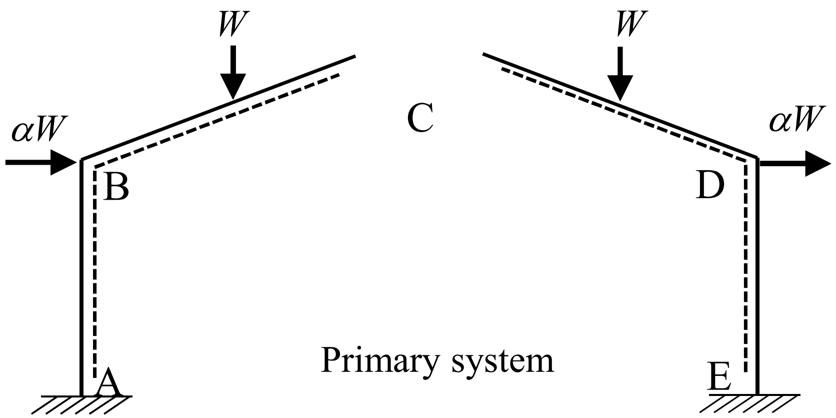
Mechanism 3

$$\Rightarrow \begin{cases} M = 82.48 \\ H = 13.9 \\ V = -1.624 \\ M_p = 89.96 \end{cases}$$

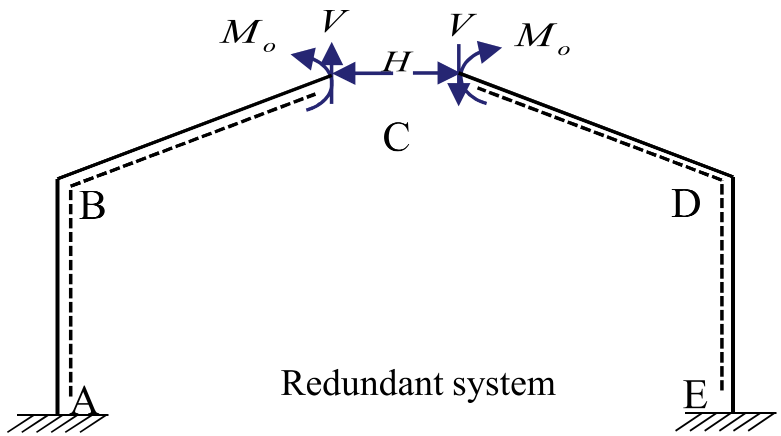
Ex. 4.7.3



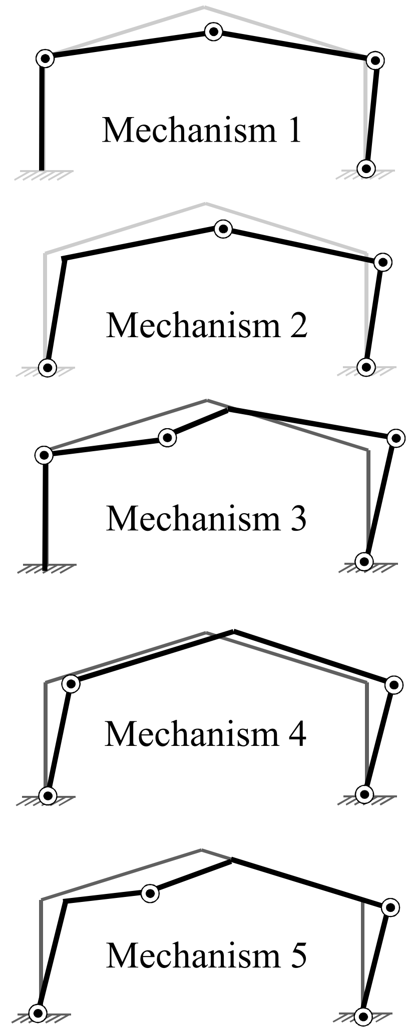
W
W
αW
αW
B
C
D
A
E



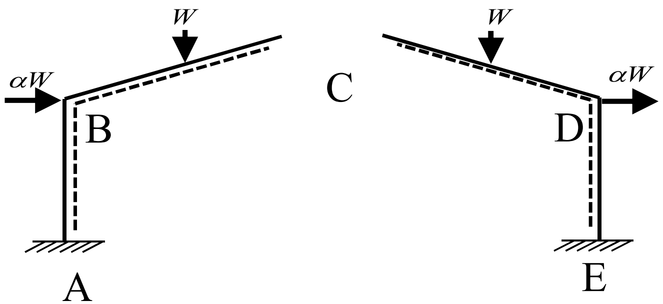
W
W
αW
αW
C
B
D
Primary system
A
E

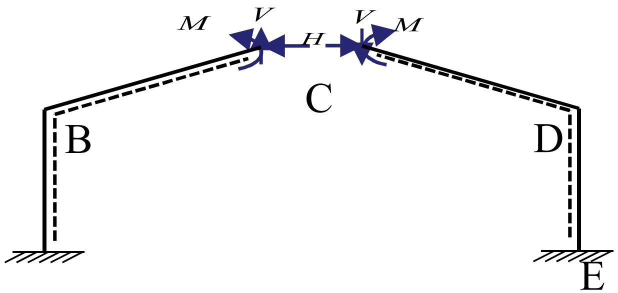


M_o
V
H
V
M_o
C
B
D
Redundant system
A
E

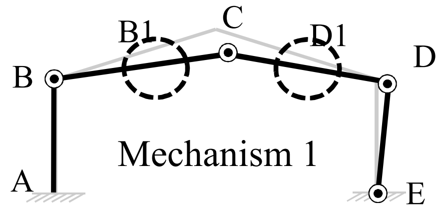


Mechanism 1
Mechanism 2
Mechanism 3
Mechanism 4
Mechanism 5





joint	A	B	C	D	E
primary	-WL(1/4+2α/5)	-WL/4	0	-WL/4	-WL(1/4+2α/5)
Mo	Mo	Mo	Mo	Mo	Mo
V	1/2VL	1/2VL	0	-1/2VL	-1/2VL
H	3/5HL	1/5HL	0	1/5HL	3/5HL



$$\begin{cases} M_B = -M_p \\ M_C = M_p \\ M_D = -M_p \\ M_E = M_p \end{cases} \qquad \begin{aligned} M_0 = M_p &= \frac{WL}{12} + \frac{\alpha WL}{15} \\ V &= 0 \\ H &= \frac{5W}{4} - 10\frac{M_p}{L} \end{aligned}$$

Now we must ascertain that

$$|M| \le M_p \quad @ \quad B_1 \& D_1, \quad A$$

$$M_{B1} = 0 + M_0 + \frac{1}{4}VL + \frac{1}{10}HL \le M_p$$

$$M_{D1} = 0 + M_0 - \frac{1}{4}VL + \frac{1}{10}HL \le M_p$$

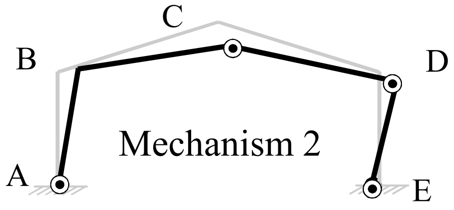
$$\Rightarrow \alpha \ge 0.625$$

At A we have

$$M_A = -WL\left(\frac{1}{4} + \frac{2}{5}\alpha\right) + M_0 + \frac{1}{2}VL + \frac{3}{5}HL \ge -M_p$$

$$\Rightarrow \alpha \le 0.25$$

Not possible mechanism



$$\begin{cases} M_A = -M_p \\ M_C = M_p \\ M_D = -M_p \\ M_E = M_p \end{cases}$$

$$M_0 = M_p = \frac{WL}{16} + WL\left(\frac{3\alpha}{20}\right)$$

$$V = -\frac{W}{8} + \frac{\alpha W}{2}$$

$$H = \frac{5W}{16} - \frac{\alpha W}{4}$$

Now we must ascertain that

$$|M| \le M_p \quad @ \quad B_1 \& D_1, \quad B$$

$$M_{B1} = M_0 + \frac{1}{4}VL + \frac{1}{10}HL \le M_p \Rightarrow \alpha \le 0$$

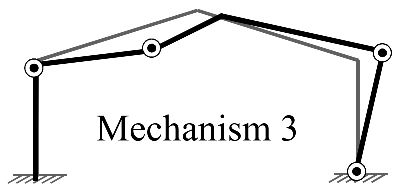
For moment @D1 &B

$$M_{D1} = M_0 - \frac{VL}{4} + \frac{1}{10}HL \le M_p$$

$$M_B = -\frac{WL}{4} + M_0 + \frac{VL}{2} + \frac{1}{5}HL \le M_p$$

$$\Rightarrow \alpha \ge 0.417, \quad \alpha \le \frac{5}{4}$$

Not possible mechanism



$$M_0 = \frac{WL}{20} + 0.12\alpha WL$$

$$V = 0$$

$$H = \frac{W}{2} - \frac{4}{5}\alpha W$$

$$M_p = \frac{WL}{10} + \frac{1}{25}\alpha WL$$

$$\begin{cases} M_B = -M_p \\ M_{D1} = M_p \\ M_D = -M_p \\ M_E = M_p \end{cases}$$

Now we must ascertain that

$$|M| \le M_p \quad @ \quad A\&C, \quad D_1$$

$$M_A = -WL\left(\frac{1}{4} + \frac{2}{5}\alpha\right) + M_0 + \frac{1}{2}VL + \frac{3}{5}HL \le M_p$$

$$\Rightarrow \alpha \ge 0$$

$$M_A \ge -M_p \Rightarrow \alpha \le 0.278$$

For moment @C

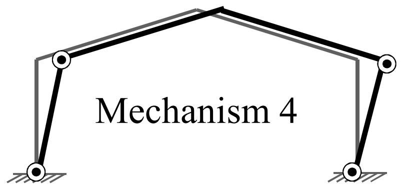
$$M_C = M_0 = \frac{1}{20}WL + 0.12\alpha WL \le M_p$$

$$\Rightarrow \alpha \le 0.625$$

$$M_C = M_0 \ge -M_p$$

$$0 \le \alpha \le 0.278$$

$$W = \frac{M_p}{L(0.1 + 0.04\alpha)}$$



$$M_0 = \frac{WL}{4}$$

$$\begin{cases} M_A = -M_p \\ M_B = M_p \\ M_D = -M_p \\ M_E = M_p \end{cases}$$

$$V = \frac{2}{5}\alpha W$$

$$H = 0$$

$$M_p = \frac{1}{5}\alpha WL$$

Now we must ascertain that

$$|M| \le M_p \quad @ \quad C\&\mathrm{B}_1,\quad \mathrm{D}_1$$

$$M_C = \frac{1}{4}WL \le M_p = \frac{1}{5}\alpha WL \Rightarrow \alpha \ge 1.25$$

$$M_C \ge -M_p$$

$$\frac{1}{4}WL \ge -M_p = -\frac{1}{5}\alpha WL \Rightarrow \alpha \ge 1.25$$

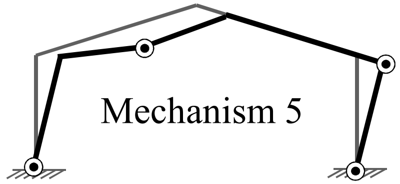
$$M_{B1} \le M_p \Rightarrow \alpha \ge 2.5$$

$$M_{B1} \ge -M_p \Rightarrow \alpha \ge -0.833$$

$$M_{D1} \le M_p \Rightarrow \alpha \ge 0.833$$

$$M_{D1} \ge -M_p \Rightarrow \alpha \ge -2.5$$

$$\therefore \quad \alpha \ge 2.5 \Rightarrow W = \frac{5M_p}{\alpha L}$$



$$M_0 = 0.0625WL$$

$$V = -\frac{1}{8}W + \frac{9}{20}\alpha W$$

$$H = 0.3125W - 0.125\alpha W$$

$$M_p = \frac{1}{16}WL + \frac{7}{40}\alpha WL$$

$$\begin{cases} M_A = -M_p \\ M_{B1} = M_p \\ M_D = -M_p \\ M_E = M_p \end{cases}$$

Now we must ascertain that

$$|M| \le M_p \quad @ \quad B\&C$$

$$M_B \le M_p \Rightarrow \alpha \ge 2.5$$

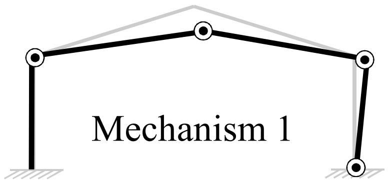
$$M_B \ge -M_p \Rightarrow \alpha \ge 0.278$$

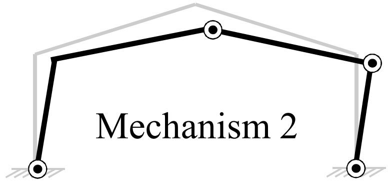
$$M_C \le M_p \Rightarrow \alpha \ge 0$$

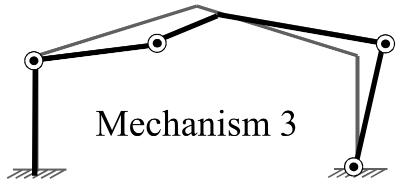
$$M_C \ge -M_p \Rightarrow \alpha \ge -0.5$$

So $\quad 0.278 \le \alpha \le 2.5$

$$W = \frac{M_p}{L\left(1/16 + 7\alpha/40\right)}$$

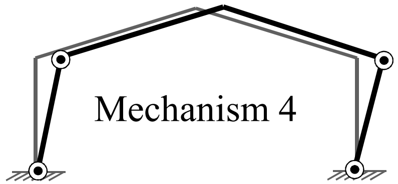






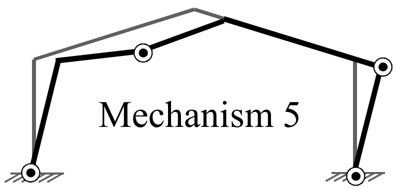
$0 \le \alpha \le 0.278$

$$W = \frac{M_p}{L\left(0.1 + 0.04\alpha\right)}$$



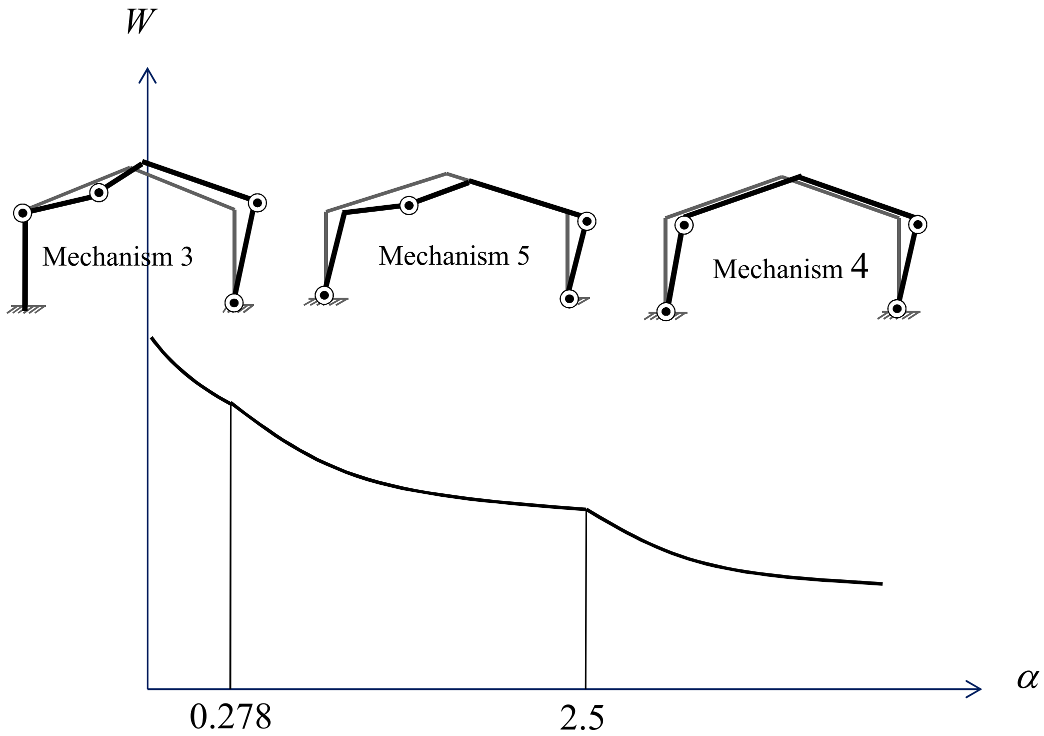
$\alpha \ge 2.5$

$$W = \frac{5M_p}{\alpha L}$$

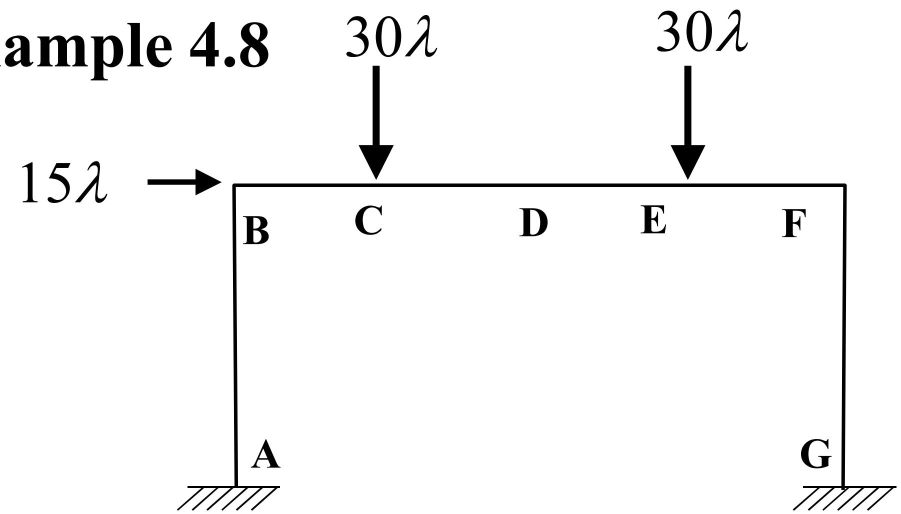


$0.278 \le \alpha \le 2.5$

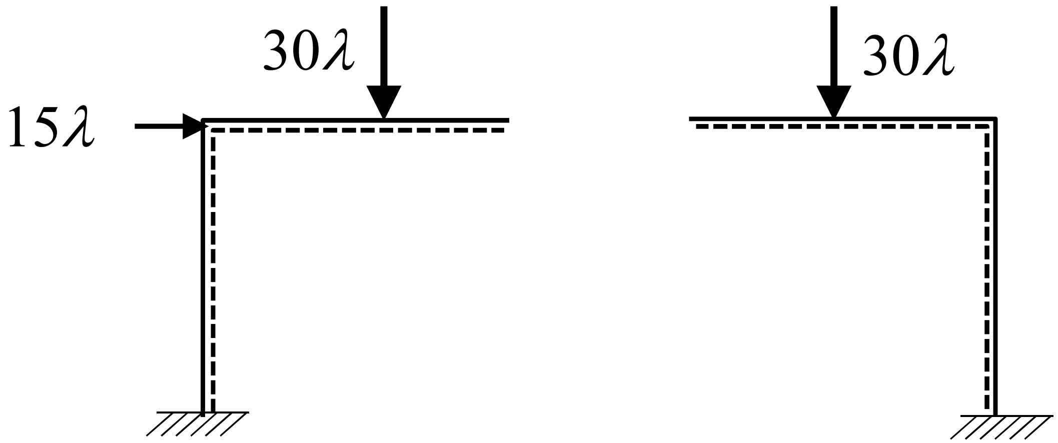
$$W = \frac{M_p}{L\left(1/16 + 7\alpha/40\right)}$$

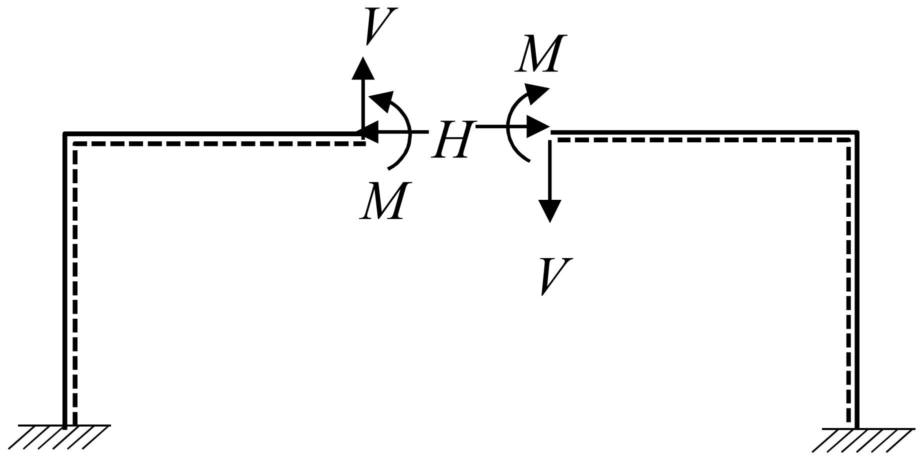


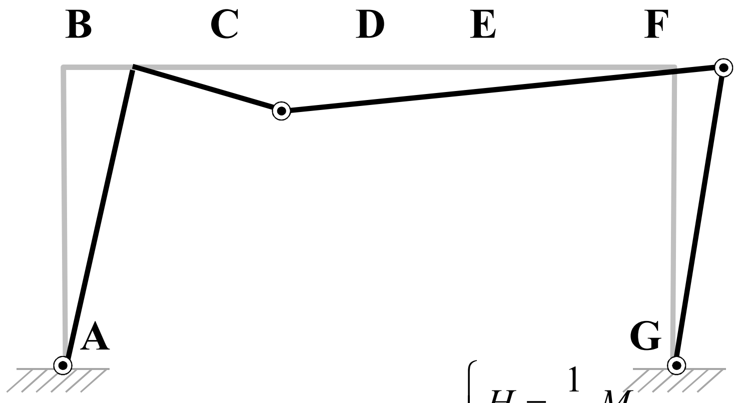
Example 4.8



joint	A	B	C	D	E	F	G
primary	-525λ	-525λ	0	0	0	-525λ	-525λ
M	M	M	M	M	M	M	M
V	15V	15V	7.5V	0	-7.5V	-15V	-15V
H	20H	0	0	0	0	0	20H

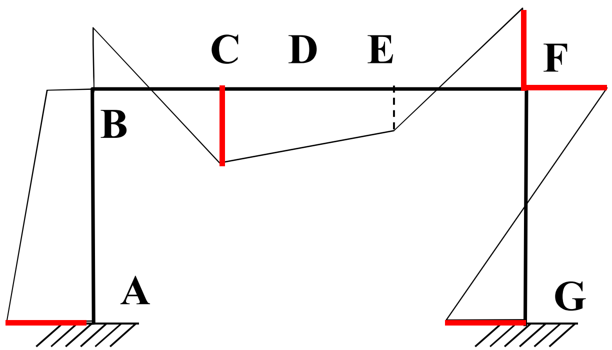




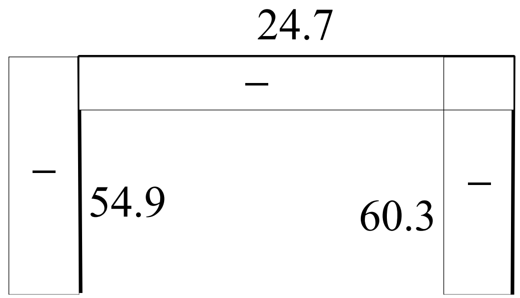


$$\begin{cases} M_A = -525\lambda + M + 12V + 20H = -M_p \\ M_C = M + 7.5V = M_p \\ M_D = -225\lambda + M - 15V = -M_p \\ M_E = -225\lambda + M - 15V + 20H = M_p \end{cases}$$

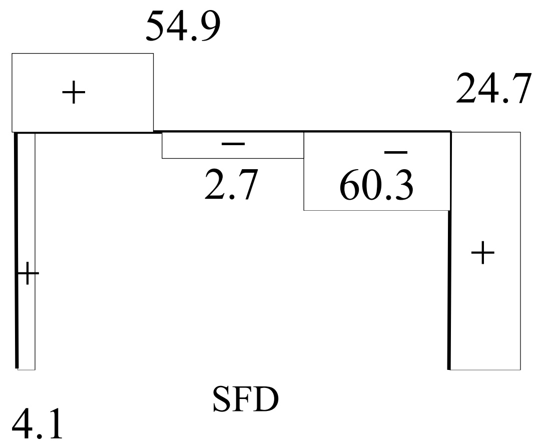
$$\begin{cases} H = \dfrac{1}{10} M_p \\ \lambda = \dfrac{7}{900} M_p \\ V = \dfrac{1}{90} M_p \\ M = \dfrac{11}{12} M_p \end{cases}$$



$$\begin{cases} M_B = -225\lambda + M + 15V = -\dfrac{2}{3} M_p \\ M_D = M = \dfrac{11}{12} M_p \\ M_E = M - 7.5V = \dfrac{10}{12} M_p \end{cases}$$



AFD



SFD

Try W16X45

$$M_p = M_{pc} = F_y Z_x = 247\text{kip-ft}$$

Member FG is critical

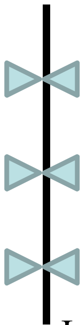
$$P_{FG} = 60.3 \text{ kips}$$

$$P_y = 478.8$$

$$\lambda_{cx} = \frac{1}{\pi}\frac{kL}{r_x}\sqrt{\frac{F_y}{E}} = 0.398$$

$$\lambda_{cy} = 1.686 \quad \vee \text{ control}$$

$$P_n = \left[\frac{0.877}{\lambda_c^2}\right] P_y = 147.7$$



$$\lambda'_{cy} = \frac{1.686}{4} < 1.5$$

$$P_n = 0.658^{\lambda_c^2} P_y = 444.4$$

Since $\frac{P}{\phi_c P_n} = \frac{60.6}{0.85 \times 444.4} = 0.16 < 0.2$

Interaction Eq. $\frac{P}{2\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} \le 1.0$

$$\frac{M_x}{0.9 \times M_{nx}} \le \left(1 - \frac{0.16}{2}\right)$$

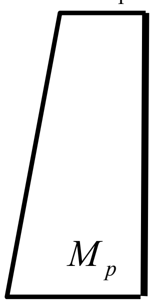
$$M_{pc} = 0.92 \times 0.9 M_p = 0.828 M_p$$

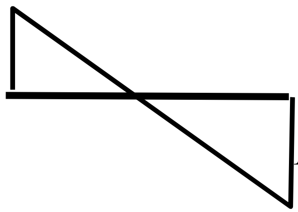
We reduce the load factor

$$\lambda = \frac{7M_{pc}}{900} = 1.59$$

Lateral torsional buckling

$$L_{pd} = \frac{\left(3600 + 2200\frac{M_1}{M_p}\right)}{F_y} r_y$$





$\frac{M_1}{M_p} \Rightarrow$ negative $\quad L_{pd} = \frac{\left(3600 - 2200\frac{2}{3}\right)}{36} 1.57 = 93''$

$\frac{M_1}{M_p} \Rightarrow$ postive $\quad L_{pd} = \frac{\left(3600 + 2200\frac{2}{3}\right)}{36} 1.57 = 221''$

Portion CE

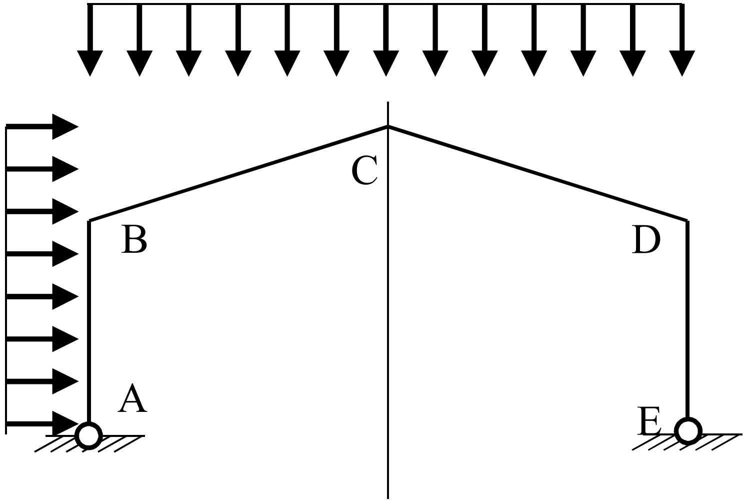
$$L_{pd} = \frac{\left(3600 - 2200\frac{10}{12}\right)}{36} 1.57 = 77'' < 180''$$

Portion EF
Portion FG ⟹ OK

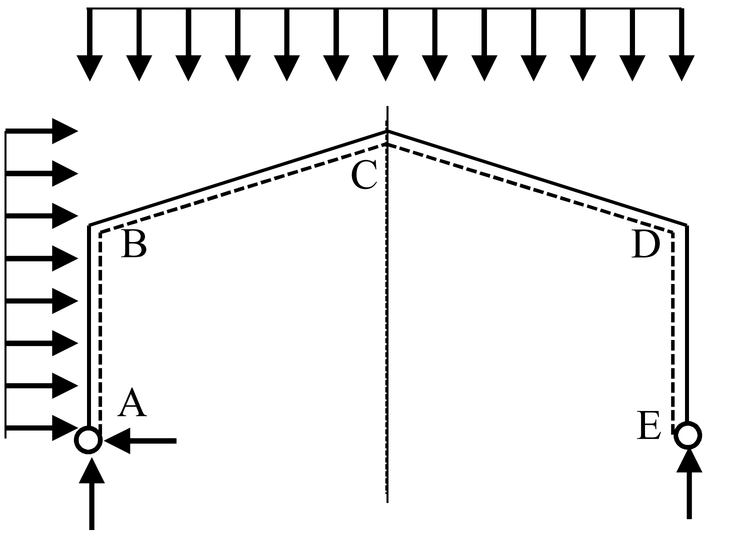
Shear force

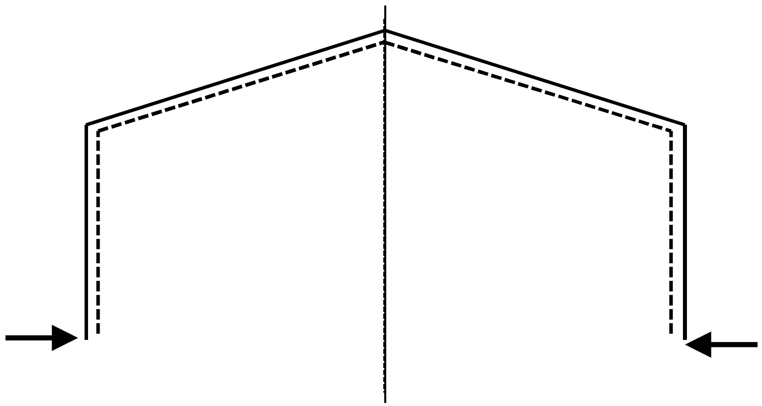
$$V_p = 0.55 F_y t_w d = 110 \text{ kips}$$

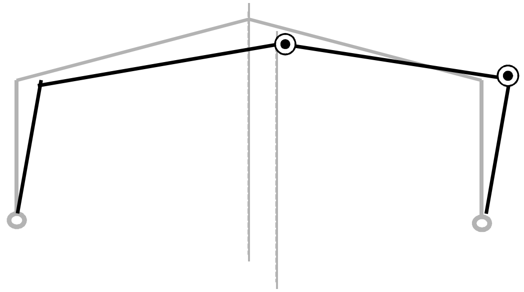
4.8.2



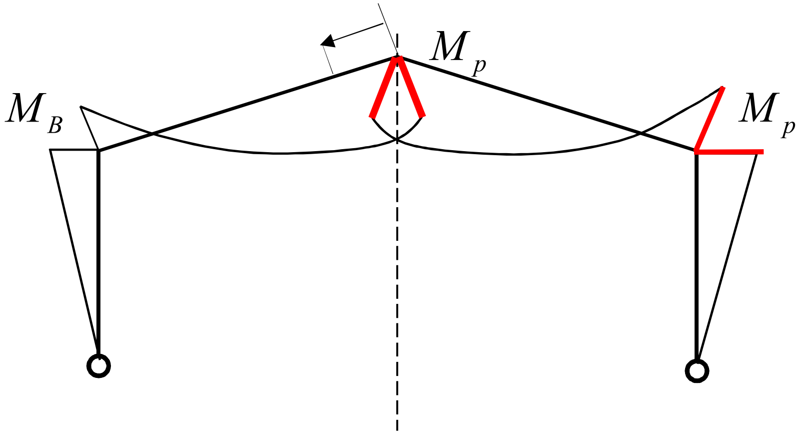
joint	A	B	C	D	E
primary	0	400	2495	0	0
S	0	-20S	-35S	-20S	0





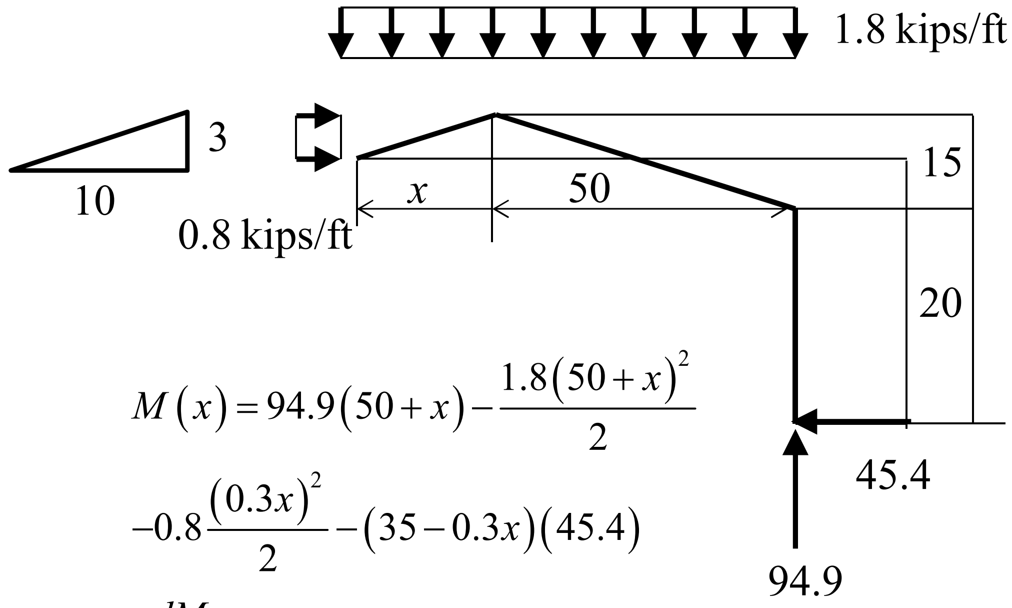


$$\begin{cases} M_C = -35S + 2495 = M_p \\ M_D = -20S = -M_p \end{cases}$$



$$M_p = 907$$

$$M_B = 400 - 20S = \left|-505\right| \le M_p$$



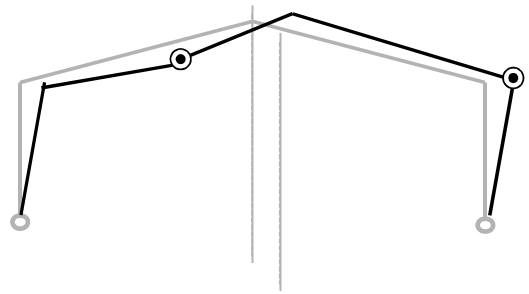
$$M(x) = 94.9(50+x) - \frac{1.8(50+x)^2}{2}$$

$$-0.8\frac{(0.3x)^2}{2} - (35-0.3x)(45.4)$$

$$\frac{dM}{dx} = 0 \Rightarrow x = 9.9 \text{ ft}$$

$$M_{\max} = 997$$

$$907 \le M_p \le 997$$



$$M_D = -20S = -M_p$$

$$M_{C1} = 94.9(59.9) - \frac{1.8(59.9)^2}{2} - 0.8(2.97)^2 - 32S = M_p$$

$$M_{C1} = 2448 - 32S = M_p$$

$$\begin{cases} M_p = 942 \text{ kip-ft} \\ S = 47.1 \text{ kips} \end{cases}$$

Try W30X116

Axial force: Member DE

$$P_{DE} = V_E = 94.9$$

$$P_y = 34.2 \times 36 = 1231$$

$$\lambda_{cx} = 0.221$$

$$\lambda_{cy} = 0.302$$

$$P_n = 0.658^{\lambda_c^2} P_y = 1185$$

$$\frac{P}{\phi_c P_n} = 0.094 \leq 0.2$$

$$\frac{P}{2\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} = 1.0$$