Chapter 6

Fundamentals of Positioning

Table 6.1 Overview of positioning methods

Positioning method	Observable	Measured by
Proximity sensing	Cell-Id, coordinates	Sensing for pilot signals
Lateration	Range or	Traveling time of pilot signals
		Path loss of pilot signals
	Range difference	Traveling time difference of pilot signals
		Path-loss difference of pilot signals
Angulation	Angle	Antenna arrays
Dead reckoning	Position and	Any other positioning method
	Direction of motion and	Gyroscope
	Velocity and	Accelerometer
	Distance	Odometer
Pattern matching	Visual images or	Camera
	Fingerprint	Received signal strength

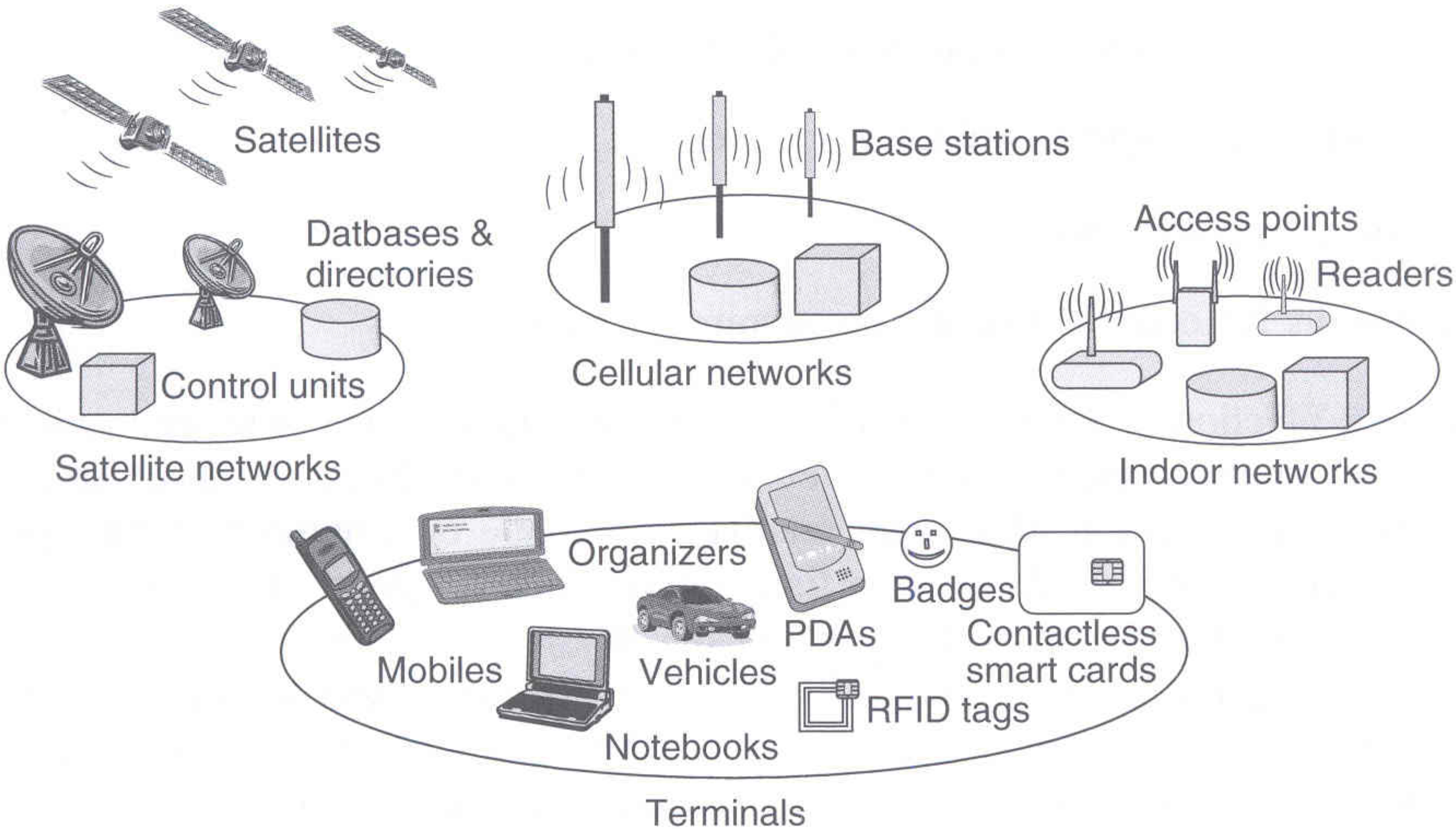


Figure 6.1 Positioning infrastructures.

- **Accuracy and precision**
- **Yield and consistency**
- **Overhead**
- **Power consumption**
- **Latency**
- **Roll-out and operating costs**

6.1 Classification of Positioning Infrastructures

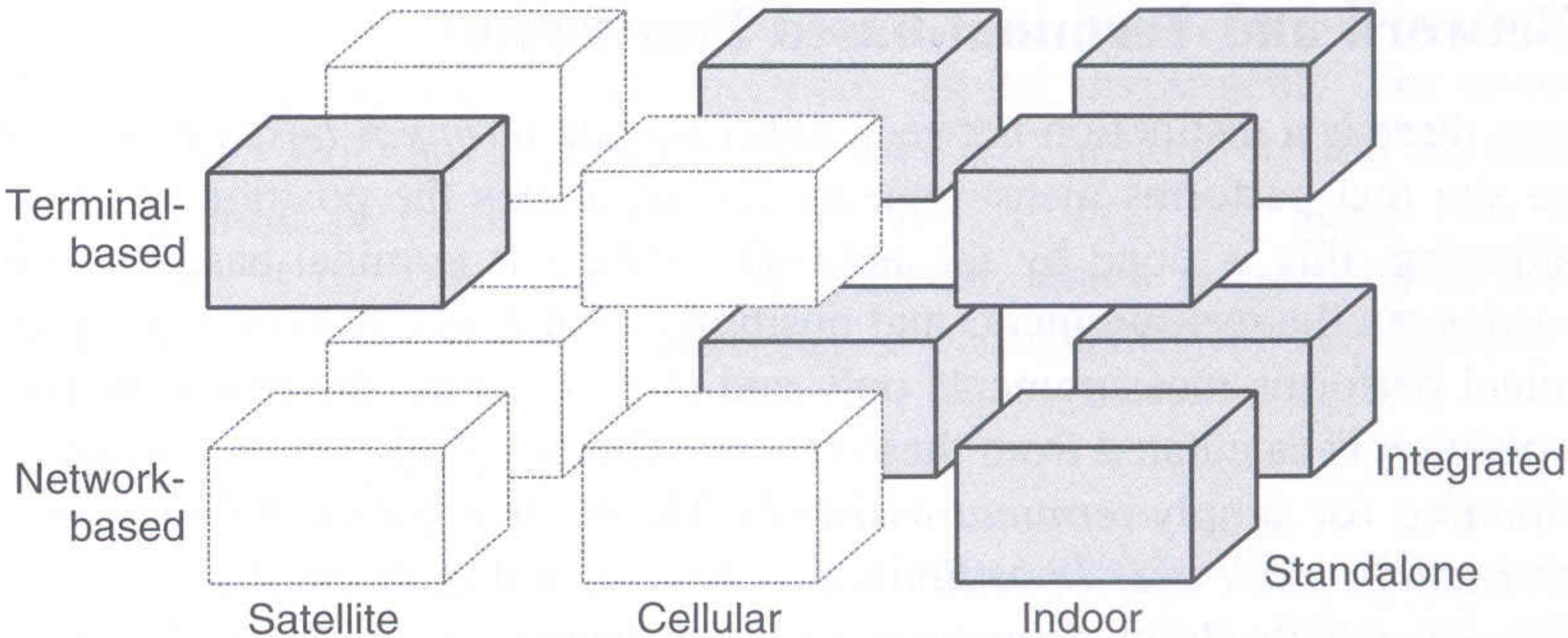


Figure 6.2 Classification scheme for positioning infrastructures.

6.1.1 Integrated and Stand-alone Infrastructures

6.1.2 Network and Terminal-based Positioning

6.1.3 Satellites, Cellular, and Indoor Infrastructures

6.1.3.1 Satellite Infrastructures

6.1.3.2 Cellular Infrastructures

6.1.3.3 Indoor Infrastructures

6.2 Basic Positioning Methods

6.2.1 Proximity Sensing

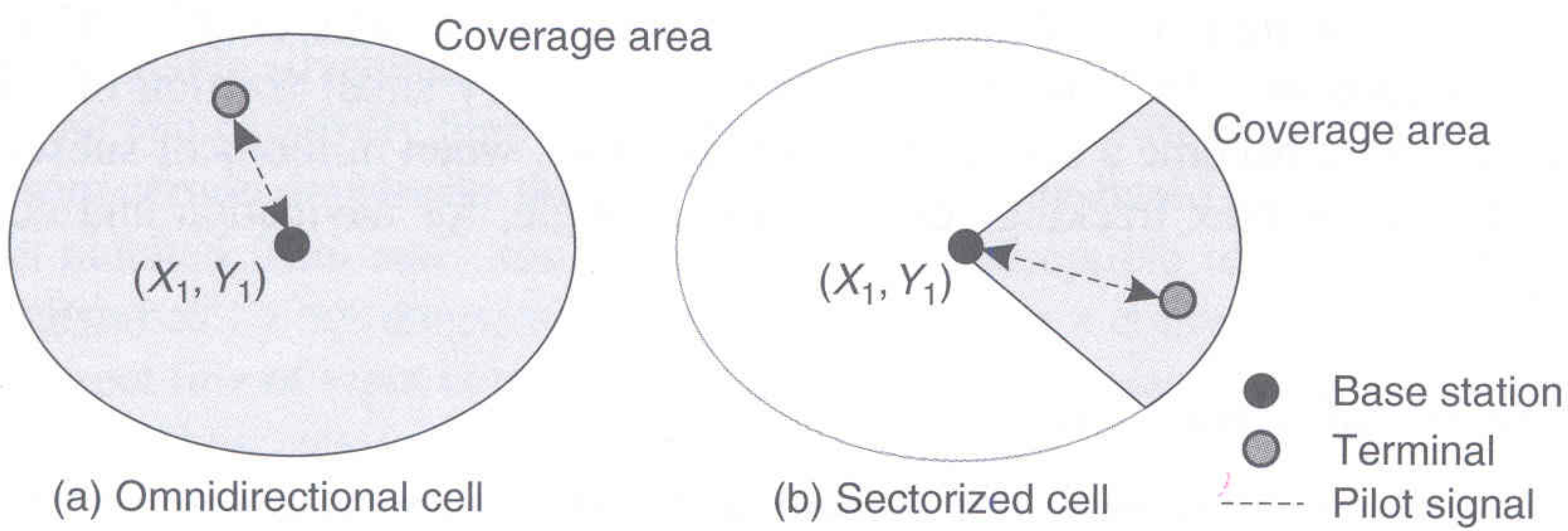


Figure 6.3 Proximity sensing.

6.2.2 Lateration

6.2.2.1 Circular Lateration

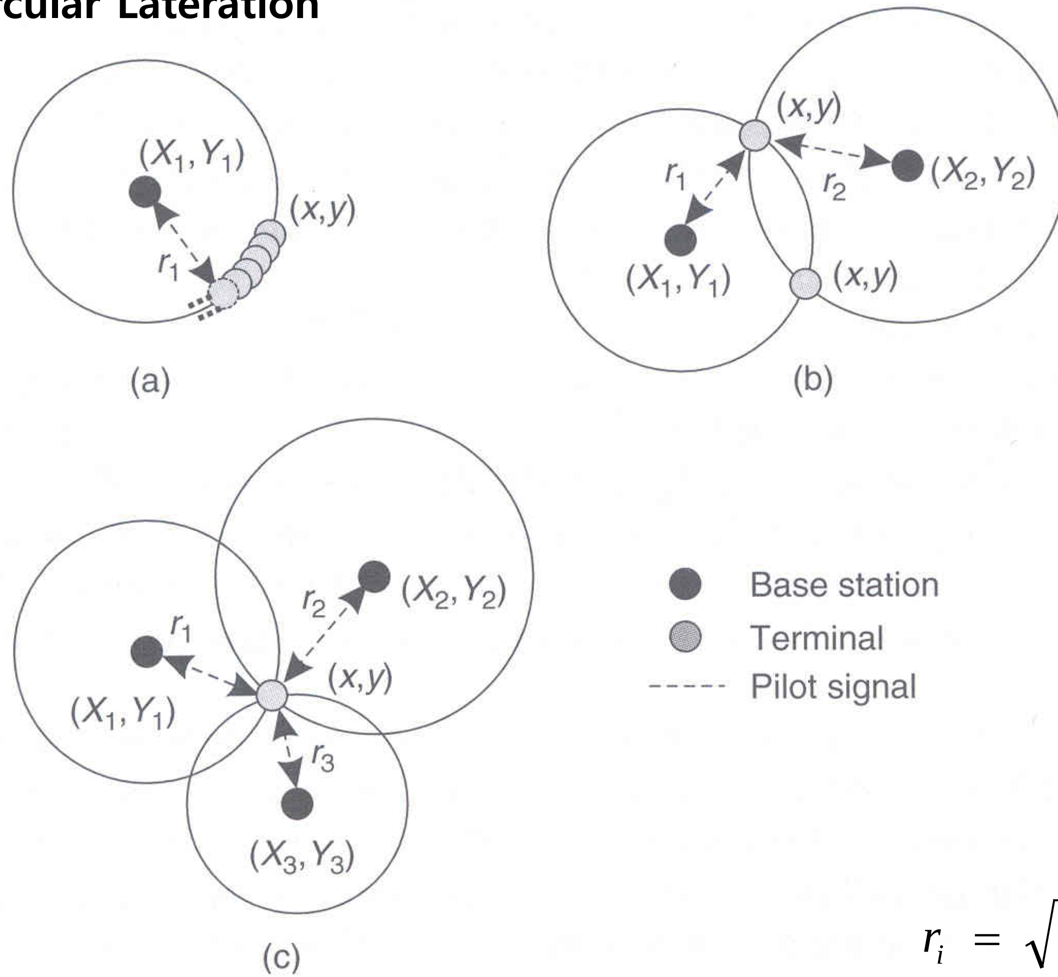


Figure 6.4 Circular lateration in 2D.

$$r_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2} \tag{6.1}$$

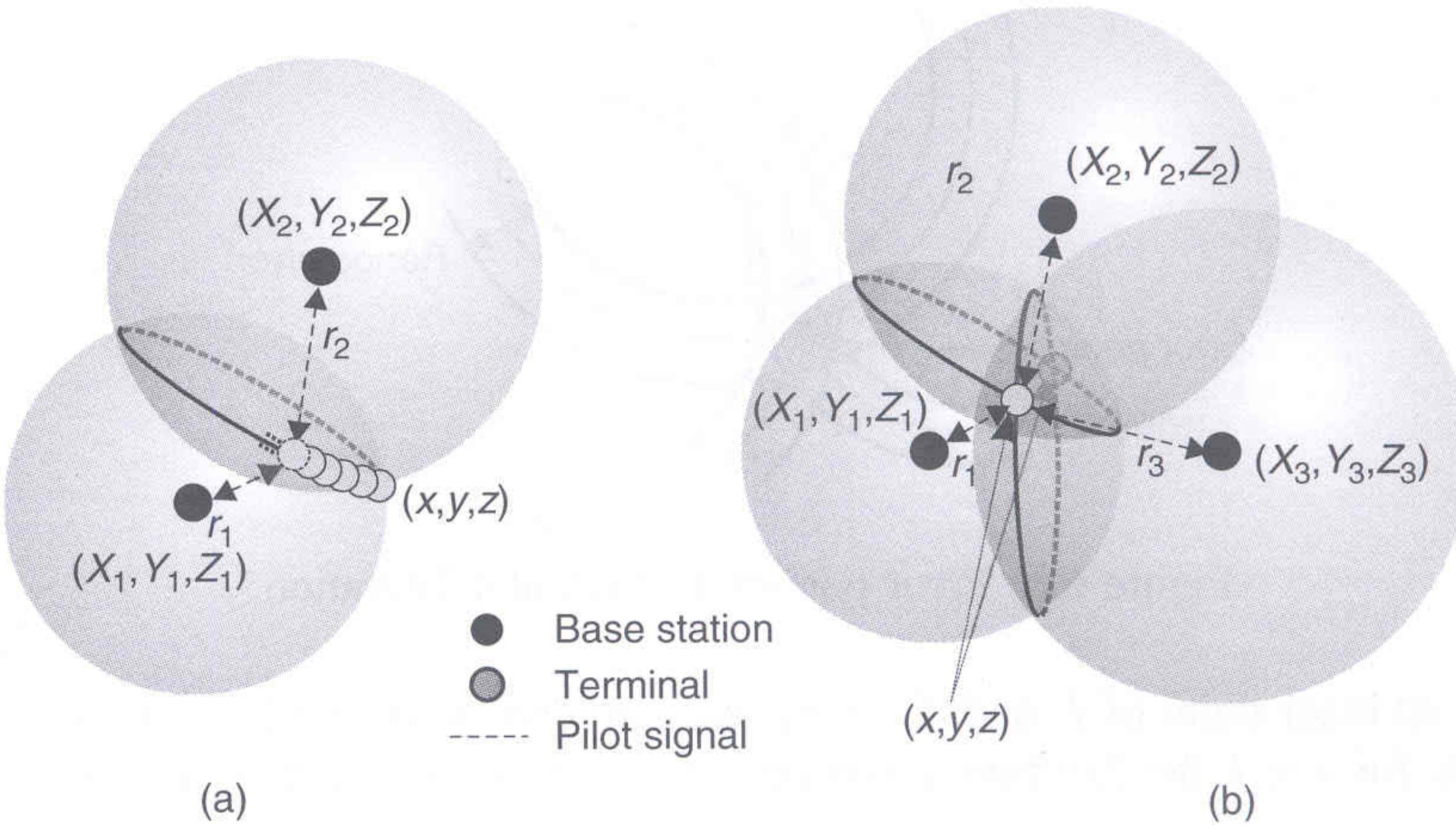


Figure 6.5 Circular lateration in 3D.

$$r_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} \qquad (6.2)$$

$$r_i = p_i + \varepsilon \tag{6.3}$$

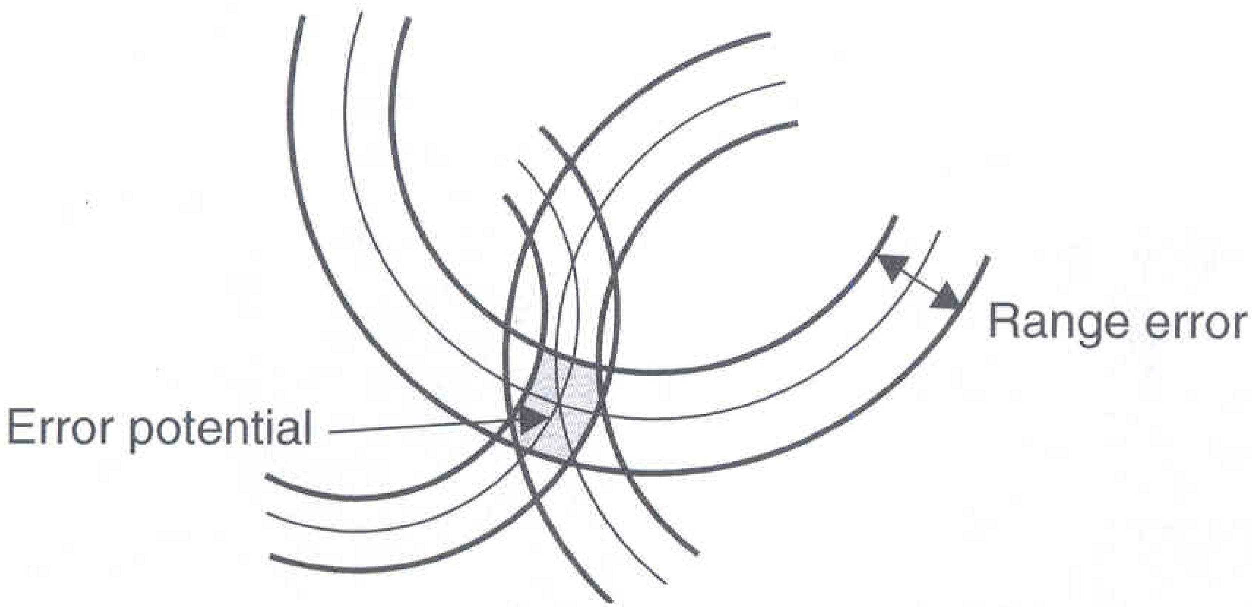


Figure 6.6 Error potential of circular lateration.

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i + R_n(x,a) \tag{6.4}$$

$$\begin{aligned} p_i(x,y,z) &= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} \\ &= p_i(\tilde{x} + \Delta x,\ \tilde{y} + \Delta y,\ \tilde{z} + \Delta z) \end{aligned} \tag{6.5}$$

$$\begin{aligned} & p_i(\tilde{x} + \Delta x,\ \tilde{y} + \Delta y,\ \tilde{z} + \Delta z) \\ & \quad = p_i(\tilde{x}, \tilde{y}, \tilde{z}) + \frac{\partial p_i}{\partial \tilde{x}} \Delta x + \frac{\partial p_i}{\partial \tilde{y}} \Delta y + \frac{\partial p_i}{\partial \tilde{z}} \Delta z \end{aligned} \tag{6.6}$$

$$\begin{aligned} \frac{\partial p_i}{\partial \tilde{x}} &= \frac{-X_i + \tilde{x}}{\sqrt{(X_i - \tilde{x})^2 + (Y_i - \tilde{y})^2 + (Z_i - \tilde{z})^2}} = a_i \\ \frac{\partial p_i}{\partial \tilde{y}} &= \frac{-Y_i + \tilde{y}}{\sqrt{(X_i - \tilde{x})^2 + (Y_i - \tilde{y})^2 + (Z_i - \tilde{z})^2}} = b_i \\ \frac{\partial p_i}{\partial \tilde{z}} &= \frac{-Z_i + \tilde{z}}{\sqrt{(X_i - \tilde{x})^2 + (Y_i - \tilde{y})^2 + (Z_i - \tilde{z})^2}} = c_i \end{aligned} \tag{6.7}$$

$$p_i(\tilde{x}+\Delta x, \tilde{y}+\Delta y, \tilde{z}+\Delta z) \\ = \; p_i(\tilde{x}, \tilde{y}, \tilde{z}) + a_i \Delta x + b_i \Delta y + c_i \Delta z \tag{6.8}$$

$$\Delta p_i \; = \; a_i \Delta x + b_i \Delta y + c_i \Delta z \tag{6.9}$$

$$\mathbf{b} \; = \; \mathbf{A}\mathbf{x} \tag{6.10}$$

$$\mathbf{b} \; = \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \cdots \\ \Delta p_n \end{bmatrix}, \; \mathbf{A} \; = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \cdots & \cdots & \cdots \\ a_n & b_n & c_n \end{bmatrix}, \; \mathbf{x} \; = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \tag{6.11}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \tag{6.12}$$

$$\|r\|_2^2 = r^T r = (\mathbf{b} - \mathbf{A}\tilde{\mathbf{x}})^T(\mathbf{b} - \mathbf{A}\tilde{\mathbf{x}}) \tag{6.13}$$
$$= \mathbf{b}^T\mathbf{b} - 2\tilde{\mathbf{x}}^T\mathbf{A}^T\mathbf{b} + \tilde{\mathbf{x}}^T\mathbf{A}^T\mathbf{A}\tilde{\mathbf{x}}$$

$$\min\|\mathbf{b} - \mathbf{A}\tilde{\mathbf{x}}\|_2^2 \tag{6.14}$$

$$-2\mathbf{A}^T\mathbf{b} + 2\mathbf{A}^T\mathbf{A}\tilde{\mathbf{x}} = 0 \tag{6.15}$$

$$\begin{aligned} \mathbf{A}^T\mathbf{A}\tilde{\mathbf{x}} &= \mathbf{A}^T\mathbf{b} \\ \Leftrightarrow \quad \tilde{\mathbf{x}} &= (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \end{aligned} \tag{6.16}$$

6.2.2.2 Hyperbolic Lateration

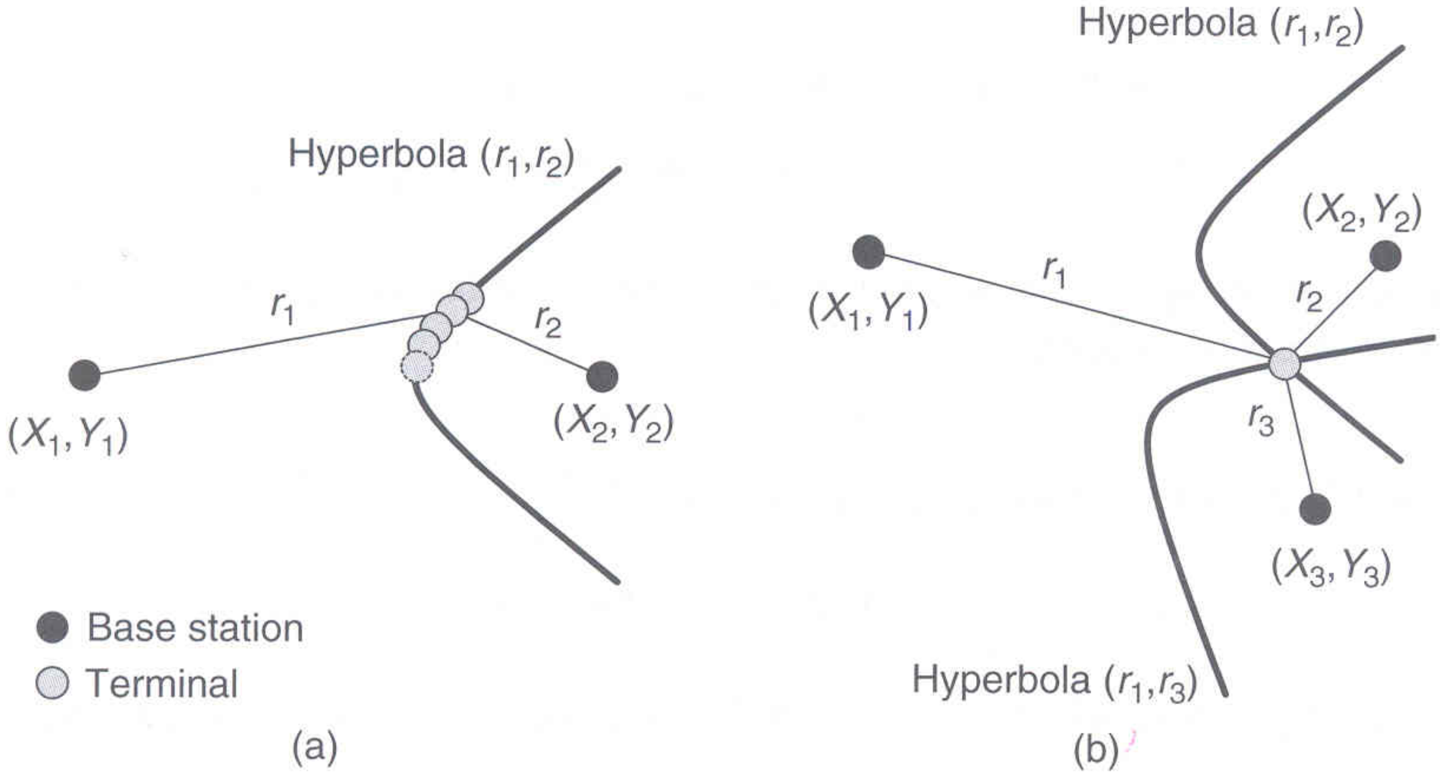


Figure 6.7 Hyperbolic lateration.

$$d_{ij} = r_i - r_j \tag{6.17}$$

$$= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} - \sqrt{(X_j - x)^2 + (Y_j - y)^2 + (Z_j - z)^2}$$

$$\frac{\partial d_{1j}}{\partial \tilde{x}} = \frac{-X_1 + \tilde{x}}{\tilde{r}_1} - \frac{-X_j + \tilde{x}}{\tilde{r}_j} = a_i$$

$$\frac{\partial d_{1j}}{\partial \tilde{y}} = \frac{-Y_1 + \tilde{y}}{\tilde{r}_1} - \frac{-Y_j + \tilde{y}}{\tilde{r}_j} = b_i \tag{6.18}$$

$$\frac{\partial d_{1j}}{\partial \tilde{z}} = \frac{-Z_1 + \tilde{z}}{\tilde{r}_1} - \frac{-Z_j + \tilde{z}}{\tilde{r}_j} = c_i$$

$$\tilde{r}_i = \sqrt{(X_i - \tilde{x})^2 + (Y_i - \tilde{y})^2 + (Z_i - \tilde{z})^2} \tag{6.19}$$

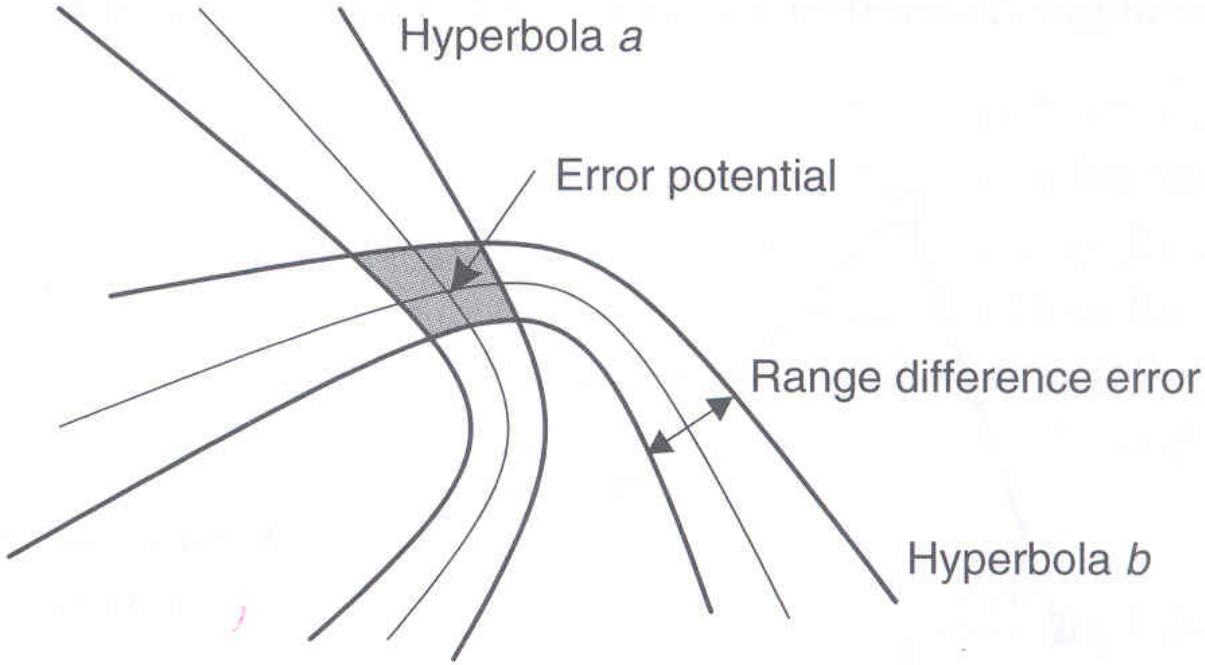


Figure 6.8 Error potential of hyperbolic lateration.

$$\tilde{\mathbf{x}} = (\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{b} \tag{6.20}$$

6.2.3 Angulation

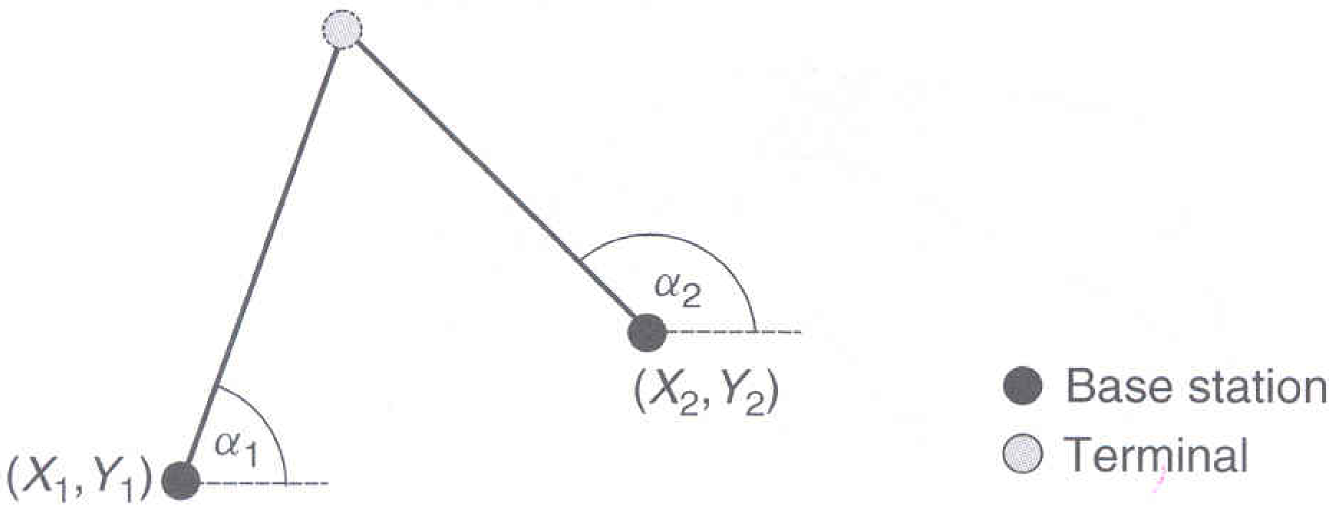


Figure 6.9 Angulation.

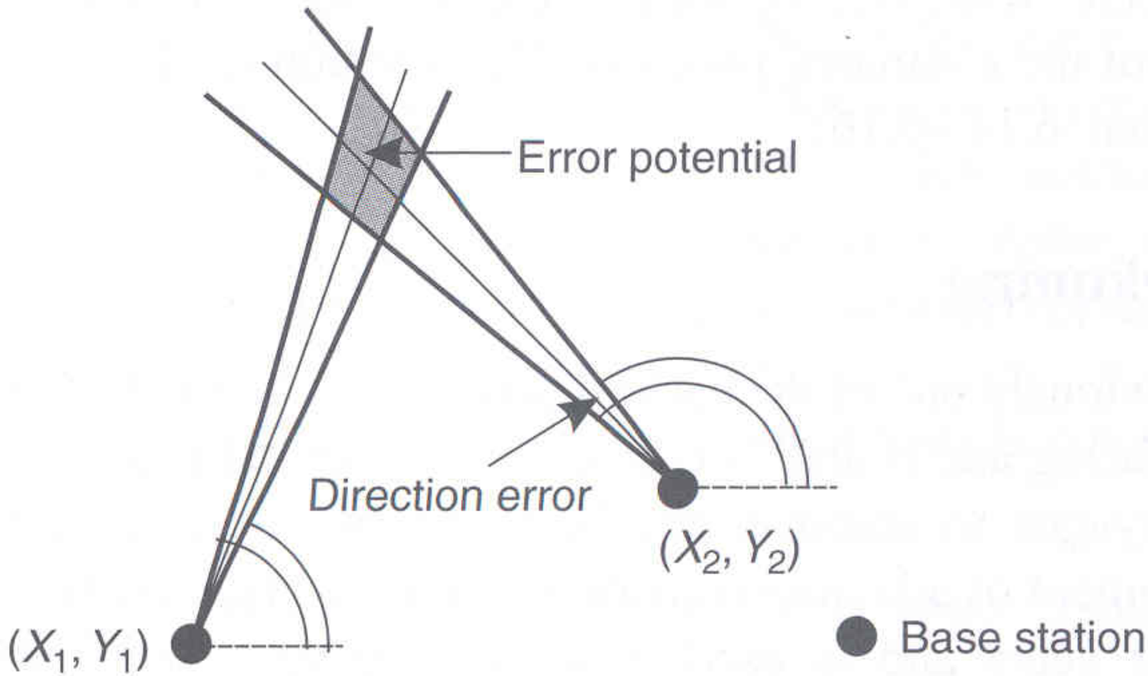


Figure 6.10 Error potential of angulation.

$$\alpha = \phi + \varepsilon \tag{6.21}$$

$$\alpha_i = \arctan\left(\frac{Y_i - y}{X_i - x}\right) \tag{6.22}$$

$$\begin{aligned} \phi_i(x, y, z) &= \arctan\left(\frac{Y_i - y}{X_i - x}\right) \\ &= \phi_i(\tilde{x} + \Delta x, \tilde{y} + \Delta y) \end{aligned} \tag{6.23}$$

$$\begin{aligned} \frac{\partial \phi_i}{\tilde{x}} &= \frac{\sin \phi_i}{r_i} &= a_i \\ \frac{\partial \phi_i}{\tilde{y}} &= -\frac{\cos \phi_i}{r_i} &= b_i \end{aligned} \tag{6.24}$$

6.2.4 Dead Reckoning

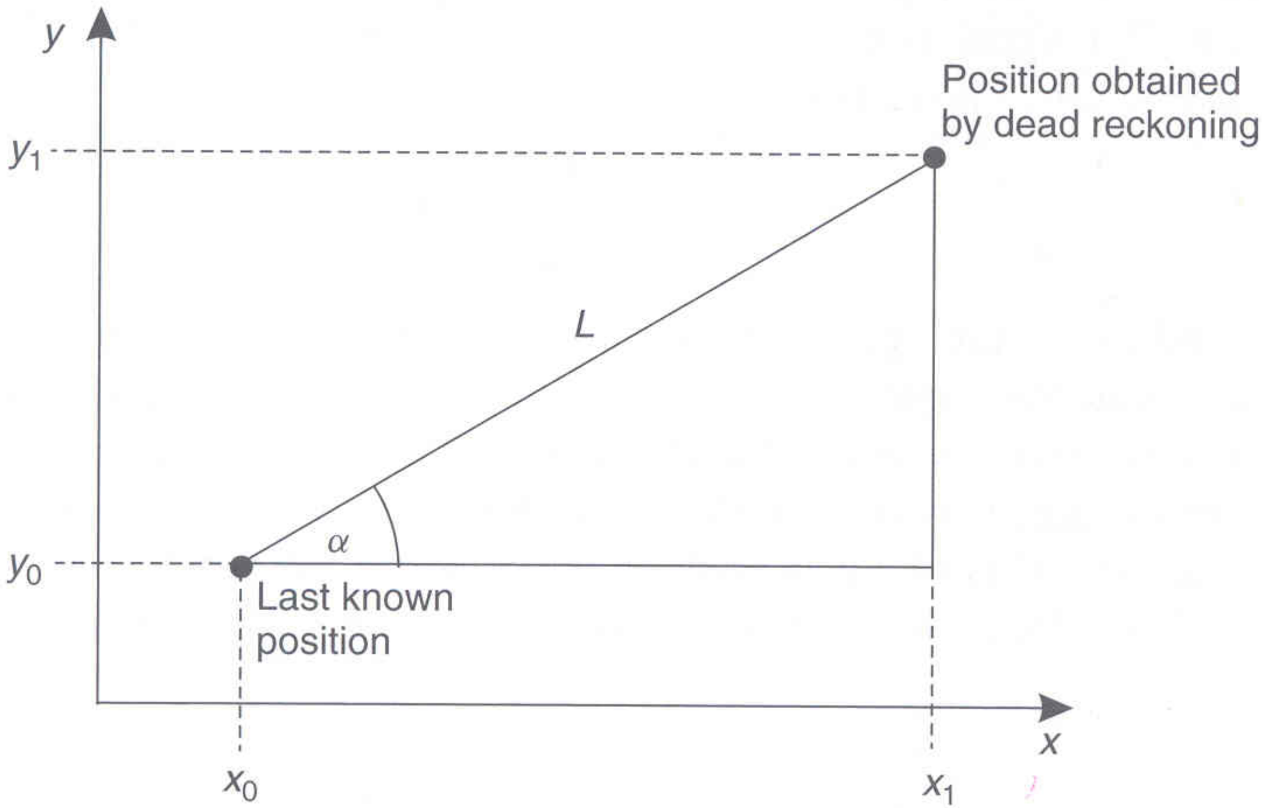


Figure 6.11 Dead reckoning.

$$x_1 = x_0 + L\cos\alpha, \quad y_1 = y_0 + L\sin\alpha \tag{6.25}$$

$$L = v\Delta t \tag{6.26}$$

6.2.5 Pattern Matching

6.2.6 Hybrid Approaches

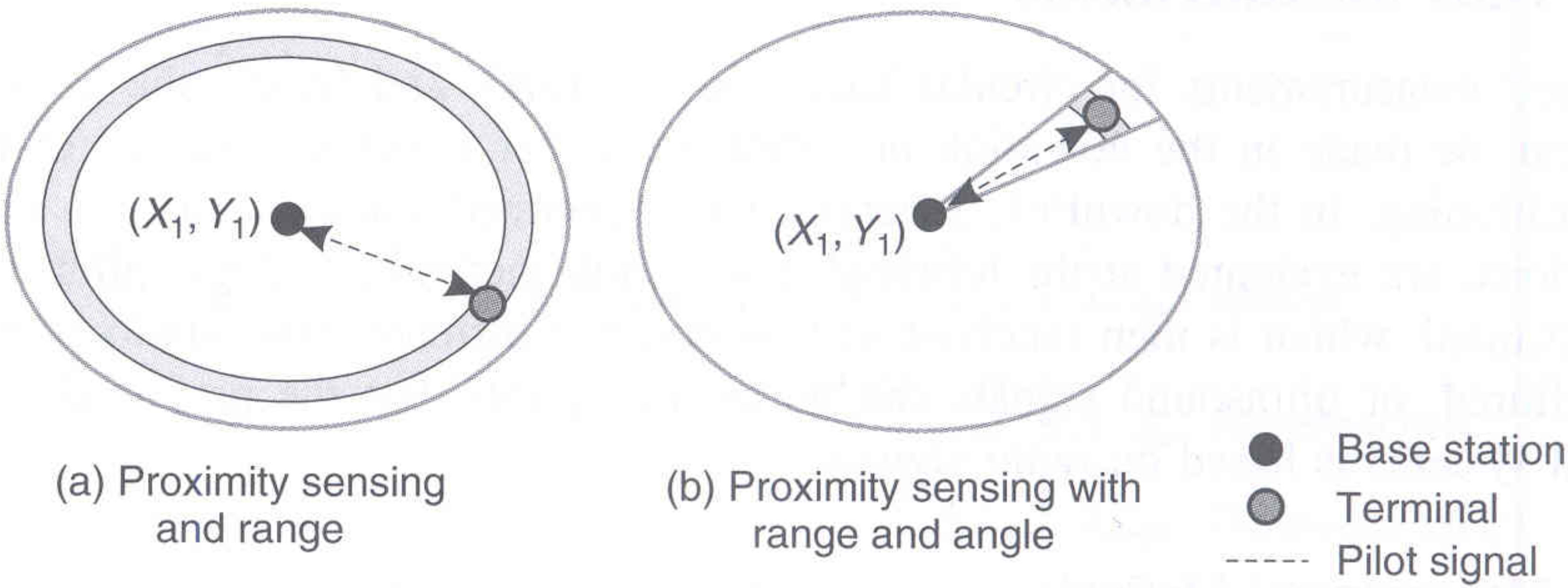


Figure 6.12 Combination of proximity sensing with range and angle.

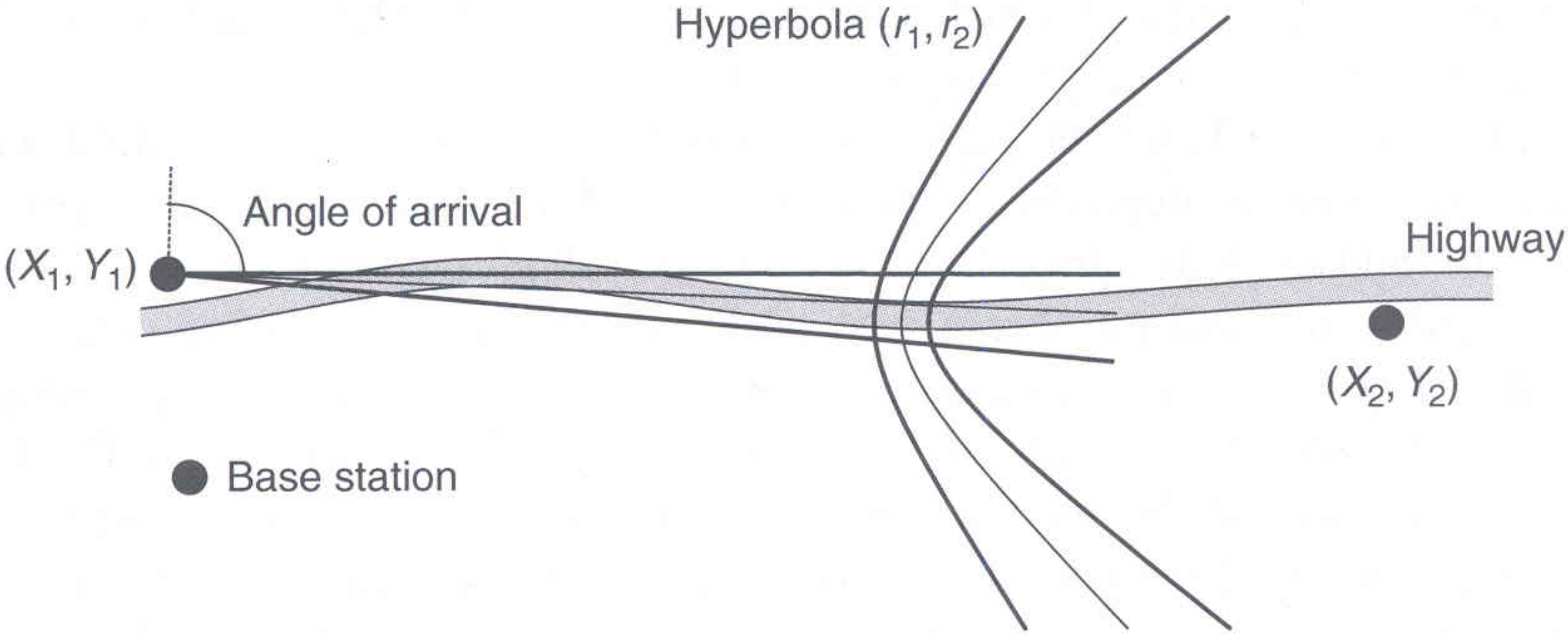


Figure 6.13 Combination of angulation and hyperbolic lateration.

6.3 Range Measurements

6.3.1 Time Measurements

6.3.1.1 Measurement Methods

- Pulse ranging
- Carrier phase ranging
- Code phase ranging

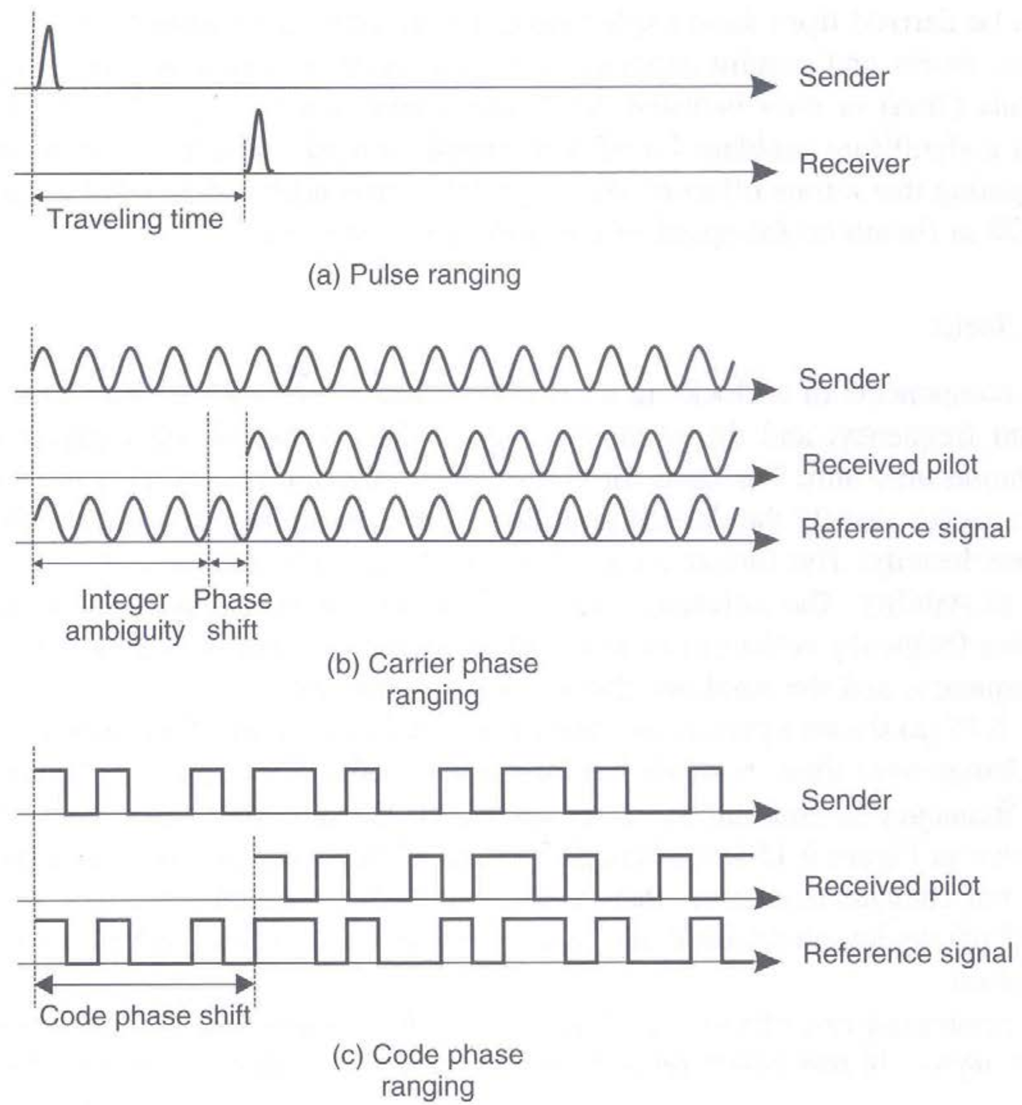


Figure 6.14 Time-based ranging methods.

6.3.1.2 Clocks

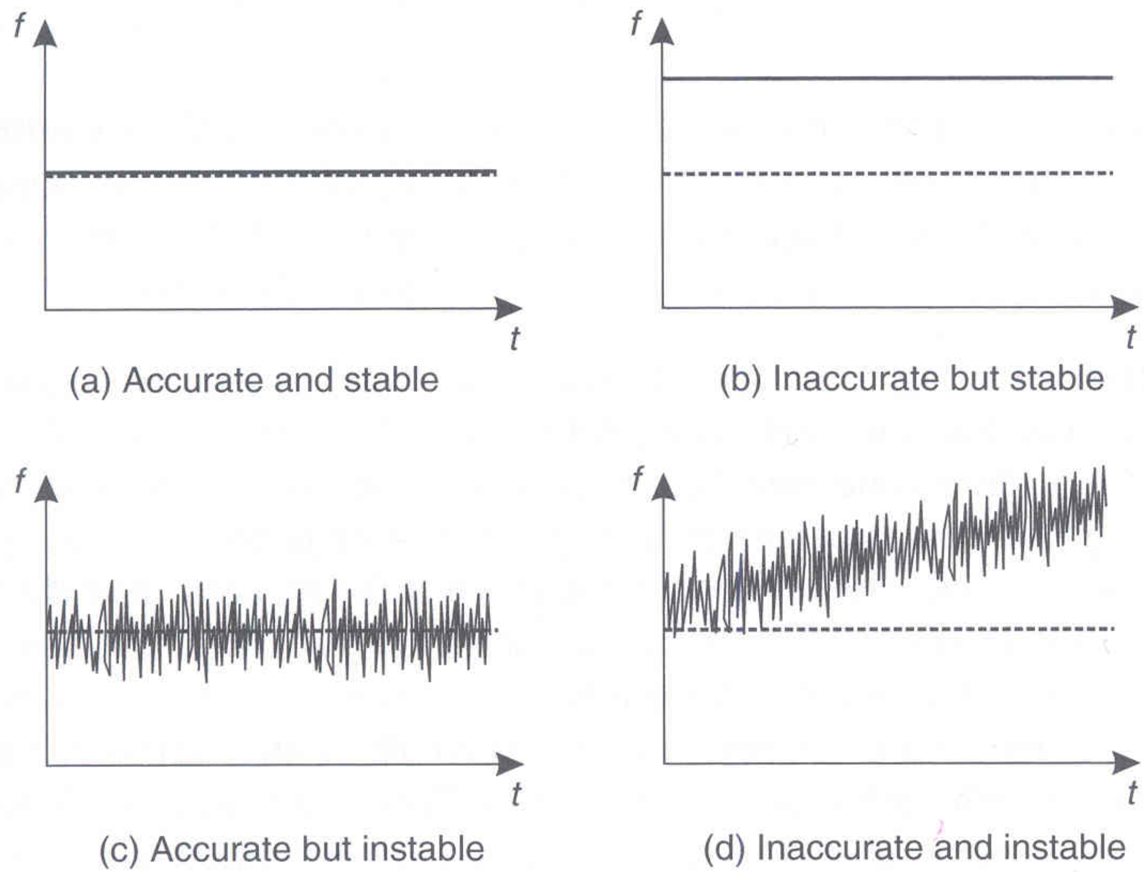


Figure 6.15 Accuracy and stability of clocks.

Table 6.2 Overview of oscillators used in positioning equipment

Oscillator type	Used in	Frequency offset
OCXO	GSM and UMTS base stations	10^{-8} to 10^{-10}
TCXO	Mobile devices	10^{-6}
Caesium	GPS satellites	5×10^{-12} to 1×10^{-14}
Rubidium	GPS satellites	5×10^{-10} to 5×10^{-12}

6.3.2 Received Signal Strength

6.4 Accuracy and Precision

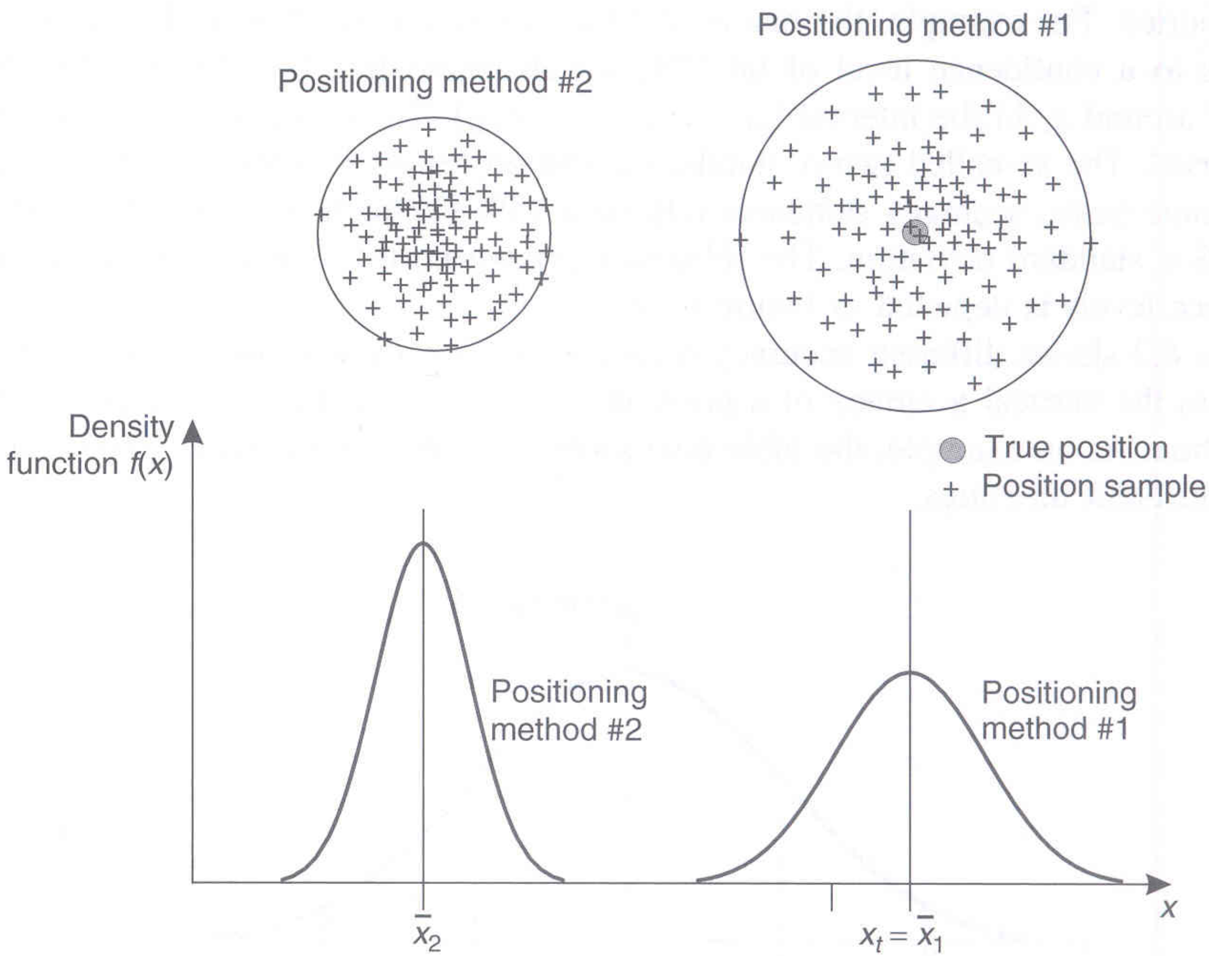


Figure 6.16 Accuracy and precision (modified from (Leick 2004)).

$$\sigma_{1D} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x_t)^2}{n}} \tag{6.27}$$

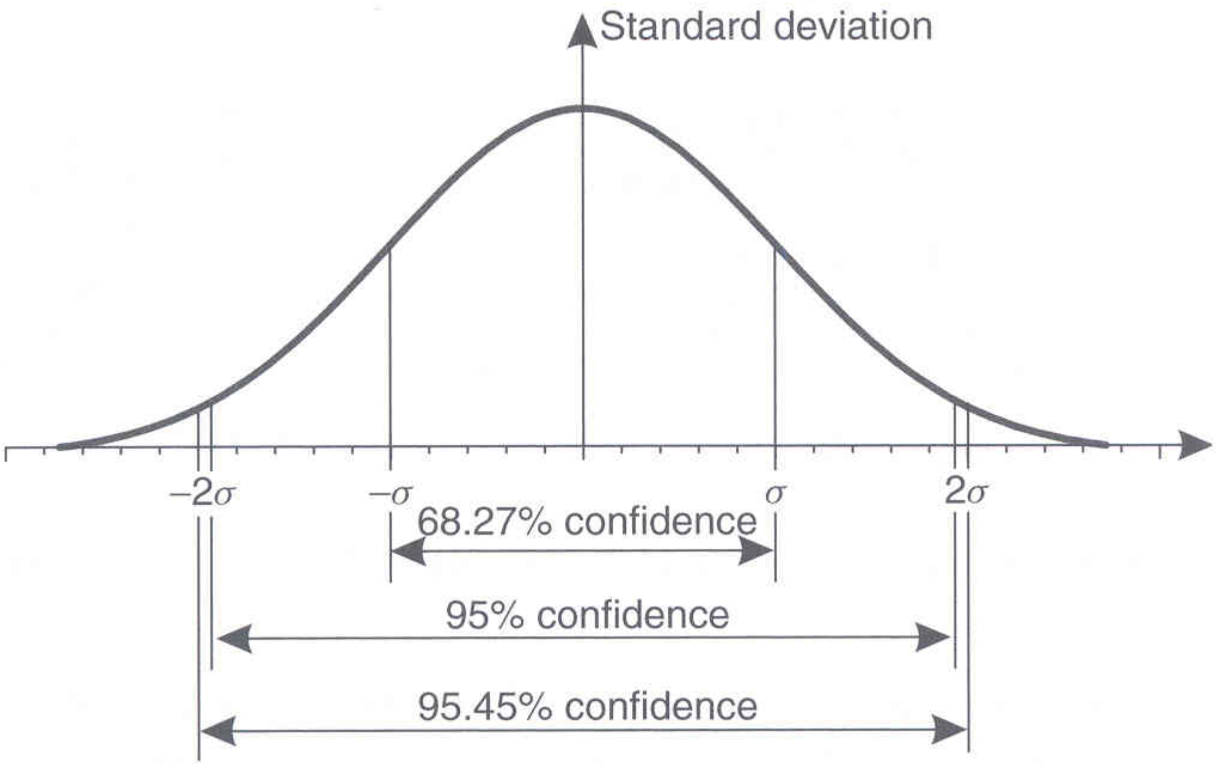
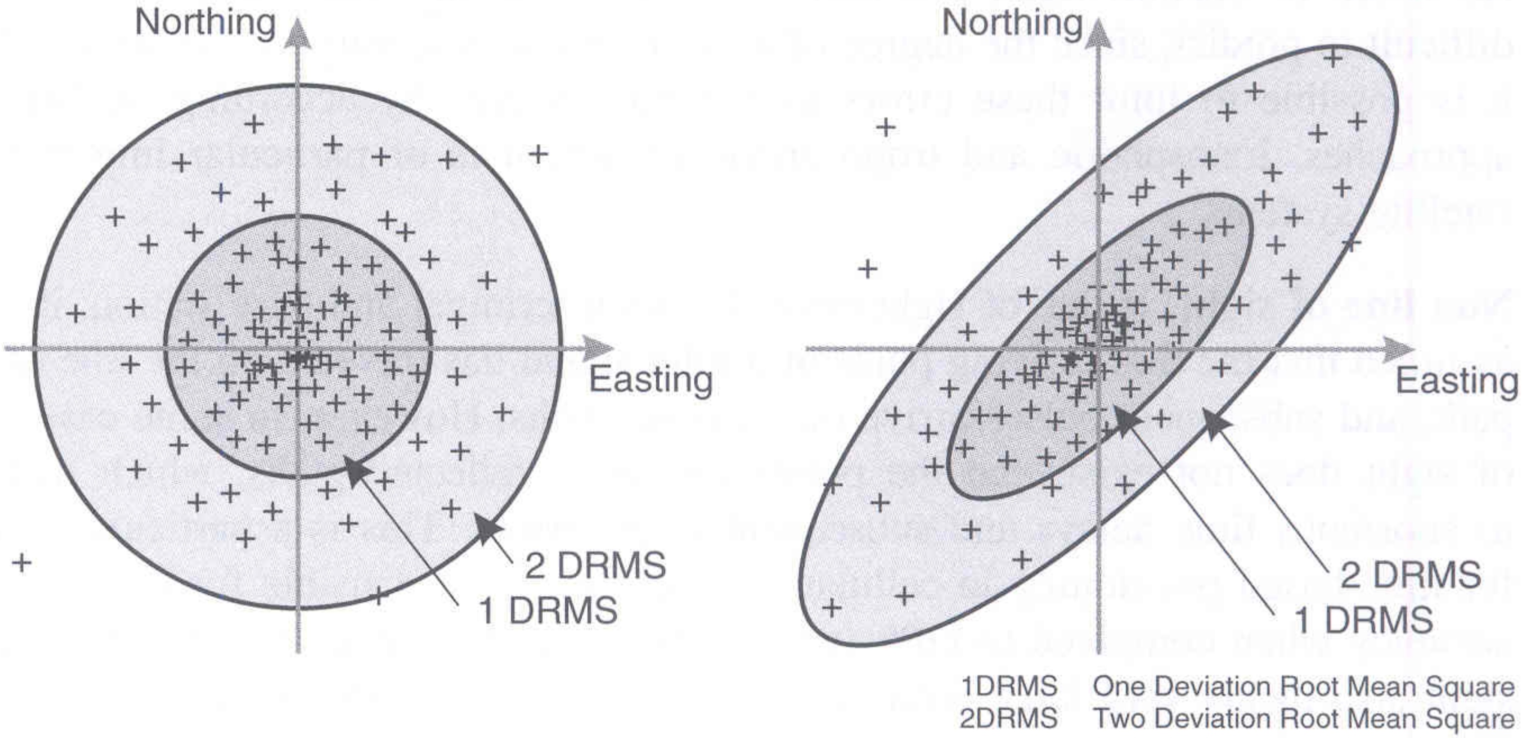


Figure 6.17 Confidence levels for one dimension.

Table 6.3 Accuracy measures

	Confidence level [%]	Relative distance [σ]	GPS accuracy [m]
One dimension			**Vertical**
One-σ standard deviation	68.27	$1\sigma_{1D}$	± 13.8 m
95% probability/confidence	95	$1.96\sigma_{1D}$	± 27 m
Two-σ standard deviation	95.45	$2\sigma_{1D}$	± 27.7 m
Three-σ standard deviation	99.73	$3\sigma_{1D}$	± 42 m
Two dimensions			**Horizontal**
One-σ standard error circle	39	$1\sigma_{2D}$	± 6 m
One deviation RMS	63	$1.414\sigma_{2D}$	± 9 m
95% 2-D positional confidence circle	95	$2.447\sigma_{2D}$	± 15 m
Two deviation RMS	98	$2.83\sigma_{2D}$	± 17.8 m
Three deviation RMS	99.9	$4.24\sigma_{2D}$	± 27 m



(a) Confidence circles

(b) Confidence ellipses

Figure 6.18 Confidence circles and ellipses.

$$\sigma_{D2} = \sqrt{2\sigma_N^2} = \sqrt{2\sigma_E^2} \qquad (6.28)$$
$$= 1.4142\sigma_N = 1.4142\sigma_E$$

$$\sigma_{D2} = 0.5(\sigma_N + \sigma_E) \qquad (6.29)$$

6.5 Error Sources

- Clocks
- Ionospheric and tropospheric refraction
- Non line of sight
- Multipath propagation
- Medium access
- Base station coordinates
- Bad geometry

6.6 Conclusion

Table 6.4 Overview of satellite, cellular, and indoor positioning

Name	Basic method	Mode			Type of signal	Measurement	Type of network
		ta	tb	nb			
Satellite positioning							
GPS	Circ. lat.		×		Radio	Time	
D-GPS	Circ. lat.		×		Radio	Time	
Galileo	Circ. lat.		×		Radio	Time	
Cellular positioning							
Cell-Id	Prox. sens.			×	Radio	Cell-Id (+RTT)	GSM
E-OTD	Hyp. lat.	×	×		Radio	Time	GSM
U-TDoA	Hyp. lat.			×	Radio	Time	GSM
Cell-Id	Prox. sens. (+angul.)			×	Radio	Cell-Id (+RTT + AoA)	UMTS
OTDoA	Hyp. lat.	×	×		Radio	Time	UMTS
E-FLT	Hyp. lat.			×	Radio	Time	cdmaOne/2000
A-FLT	Hyp. lat.	×	×		Radio	Time	cdmaOne/2000
A-GPS	Circ. lat.	×	×		Radio	Time	all
Indoor positioning							
RADAR	Fingerprint.			×	Radio	RSS	WLAN
Ekahau	Fingerprint.	×			Radio	RSS	WLAN
Indoor GPS	Circ. lat.		×		Radio	Time	
RFID	Prox. sens.	×	×	×	Radio	ID	
ActiveBadge	Prox. sens.			×	Infrared	ID	
WIPS	Prox. sens.	×			Infrared	ID	WLAN
ActiveBat	Circ. lat.			×	Ultrasound	Time	418 MHz radio
Cricket	Prox. sens.		×		Ultrasound	ID + time	