

Chapter 7

Satellite Positioning

7.1 Historical Background

7.2 Orbital Motion of Satellite Systems

7.2.1 Satellite Orbits

- **Orbit plane**
- **Orbit shape**
- **Orbit altitude**

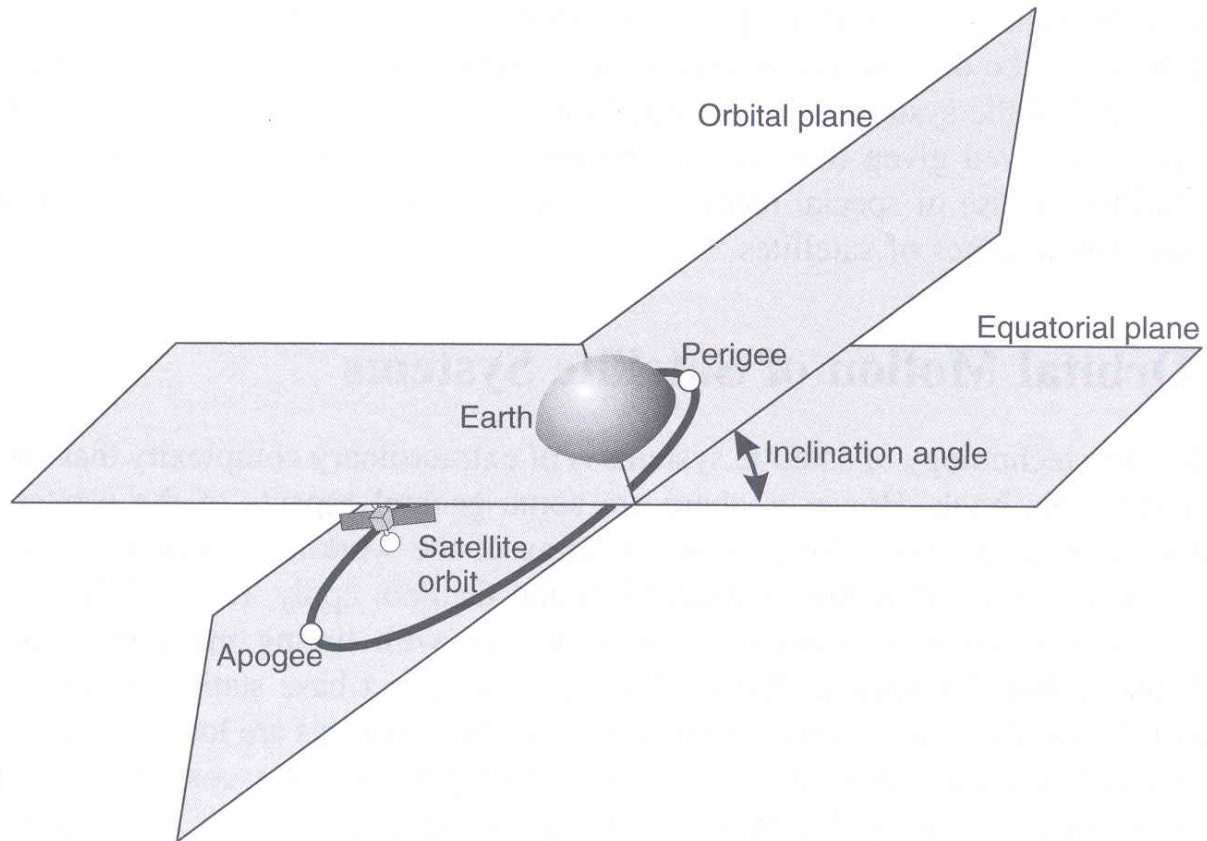


Figure 7.1 Satellite orbits.

$$F_{\text{centripetal}} = m \frac{v^2}{r} = ma \quad (7.1)$$

$$F_{\text{gravity}} = m \frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} = mg_{\text{Earth}} \quad (7.2)$$

$$F_{\text{gravity}} = m \frac{GM_{\text{Earth}}}{R_c^2} = m \cdot g_c = m \frac{v_c^2}{R_c} = F_{\text{centripetal}} \quad (7.3)$$

$$R_c = \frac{GM_{\text{Earth}}}{v_c^2} \quad (7.4)$$

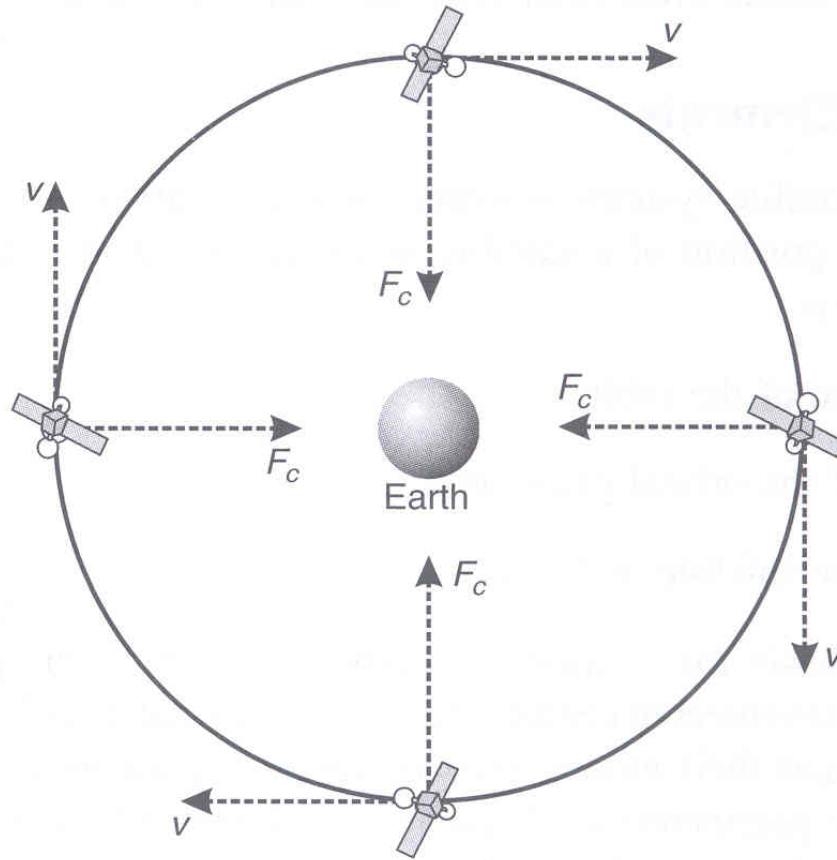


Figure 7.2 Inertia and centripetal force.

7.2.2 Keplerian Elements

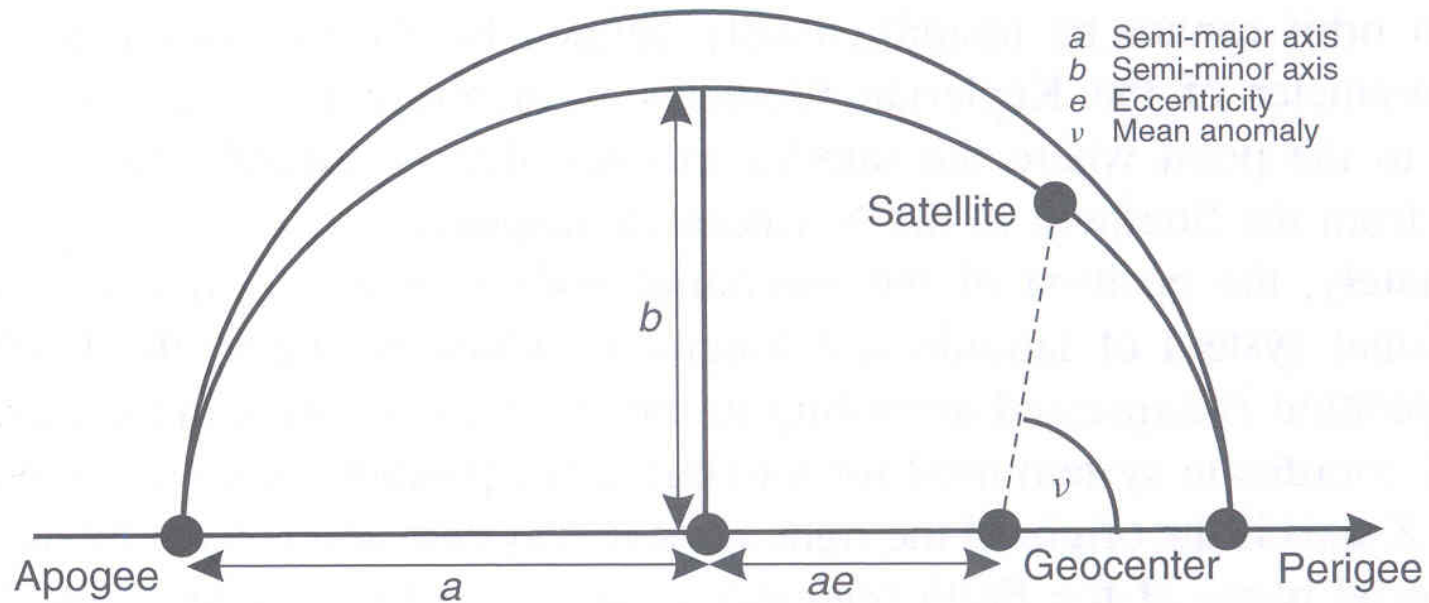


Figure 7.3 Shape and size of orbit.

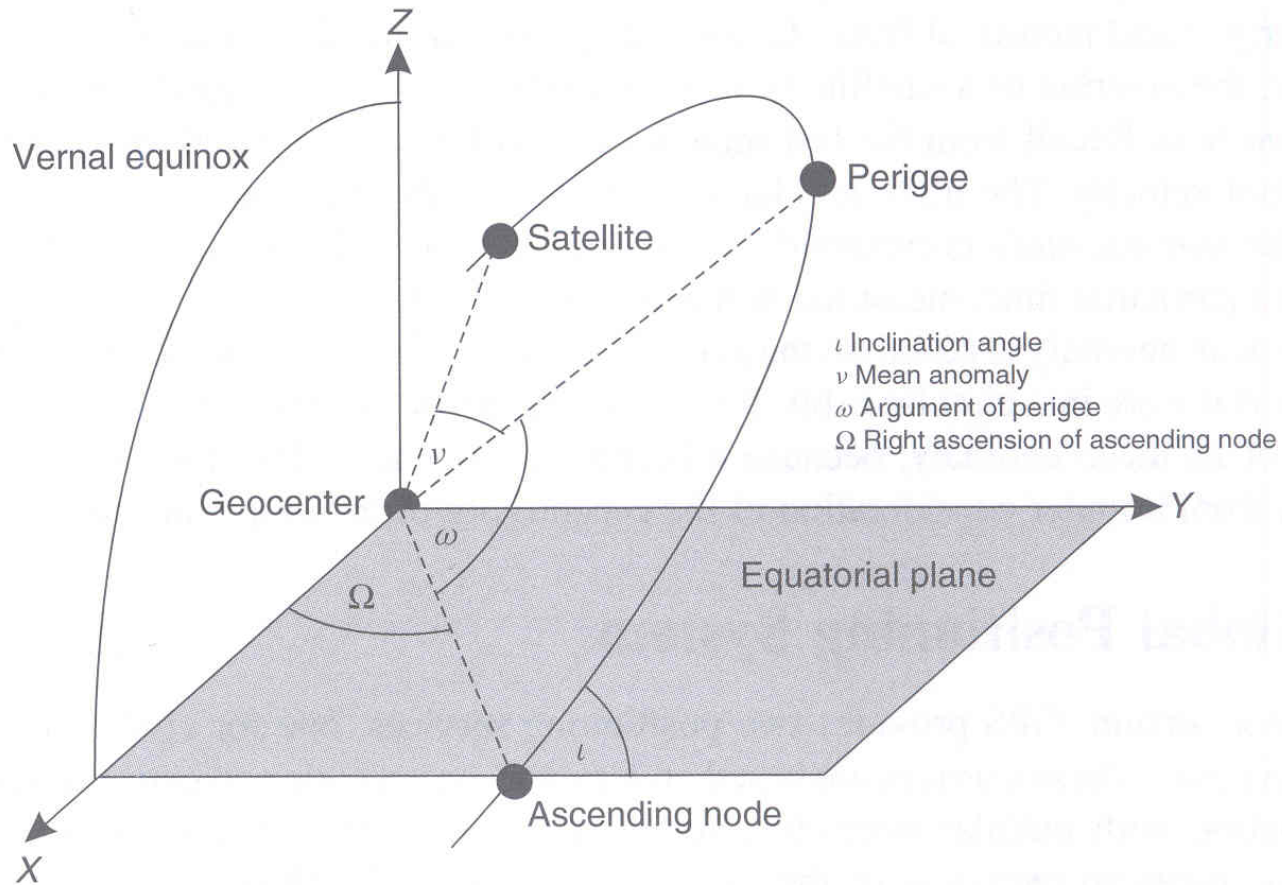


Figure 7.4 Orientation of orbital plane and satellite position.

7.3 Global Positioning System

7.3.1 GPS Segments

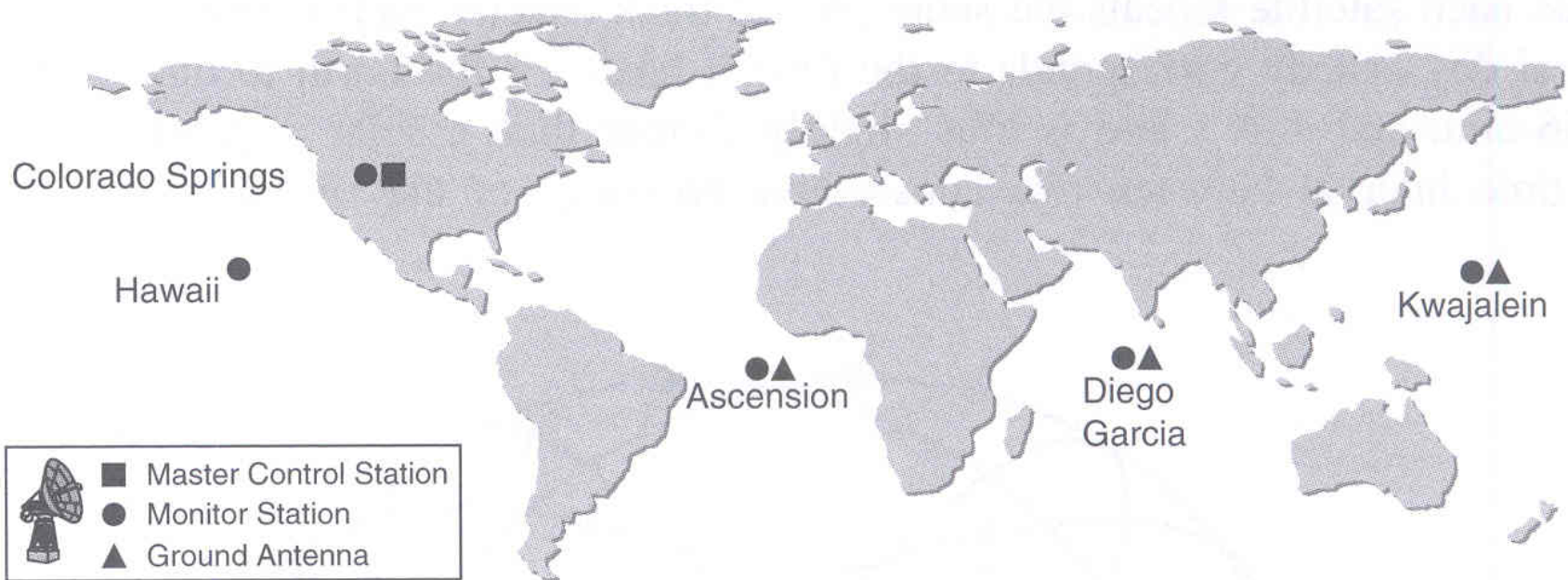


Figure 7.5 GPS control stations.

7.3.2 Satellite Constellation

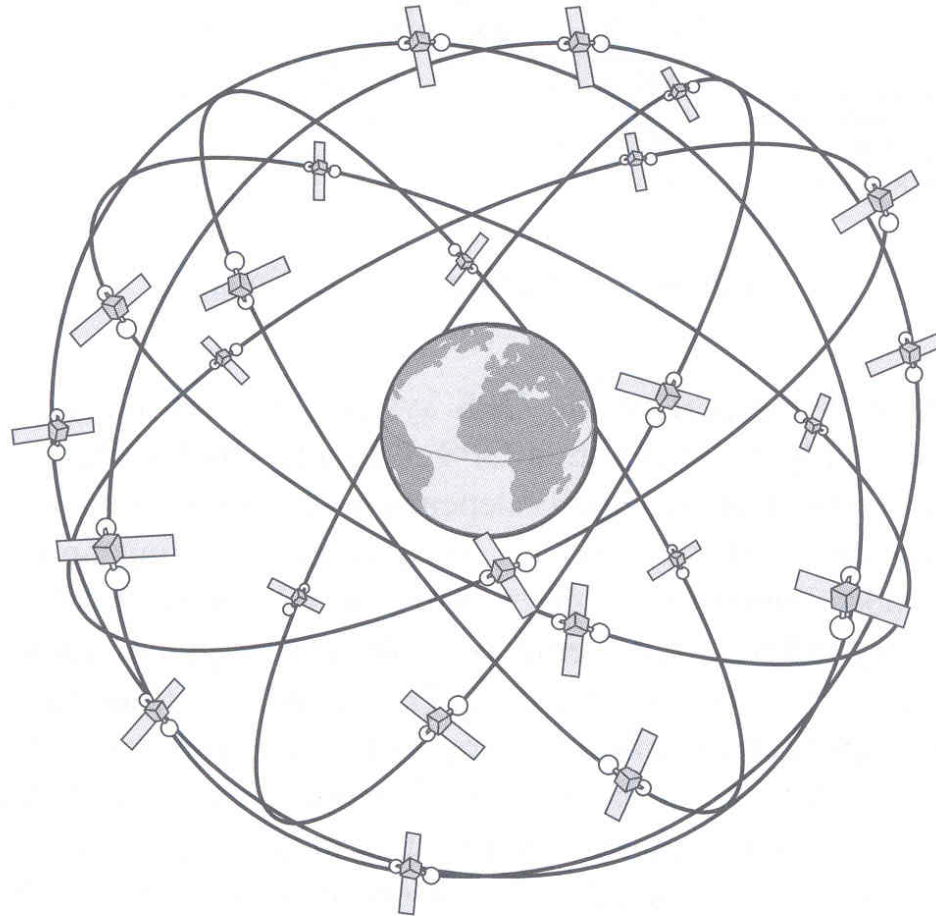


Figure 7.6 GPS constellation.

- **Block I**
- **Block II**
- **Block IIA**
- **Block IIR**
- **Block IIF and Block III**

7.3.3 Pilot Signals and Spreading Codes

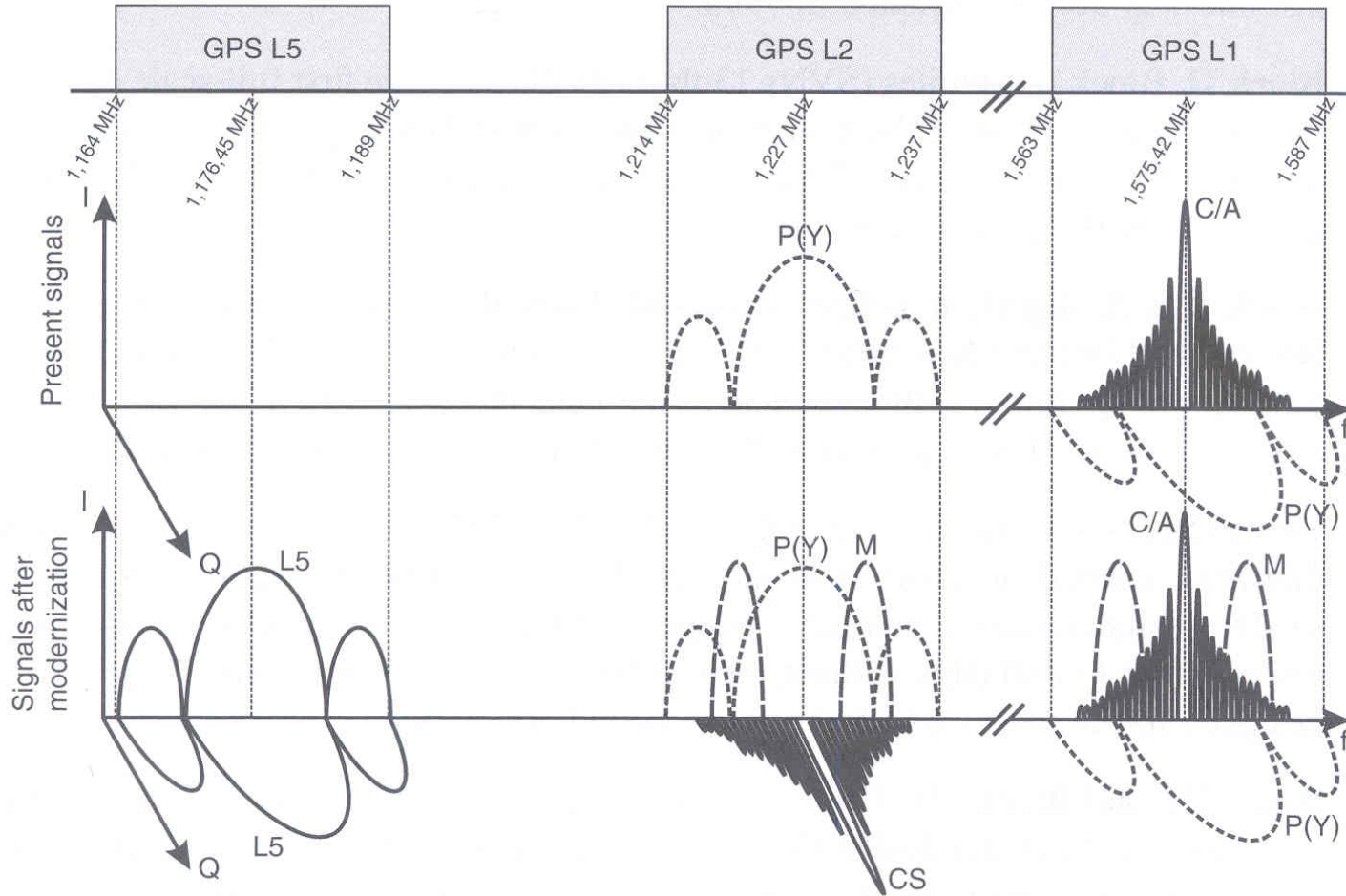


Figure 7.7 GPS frequency spectrum.

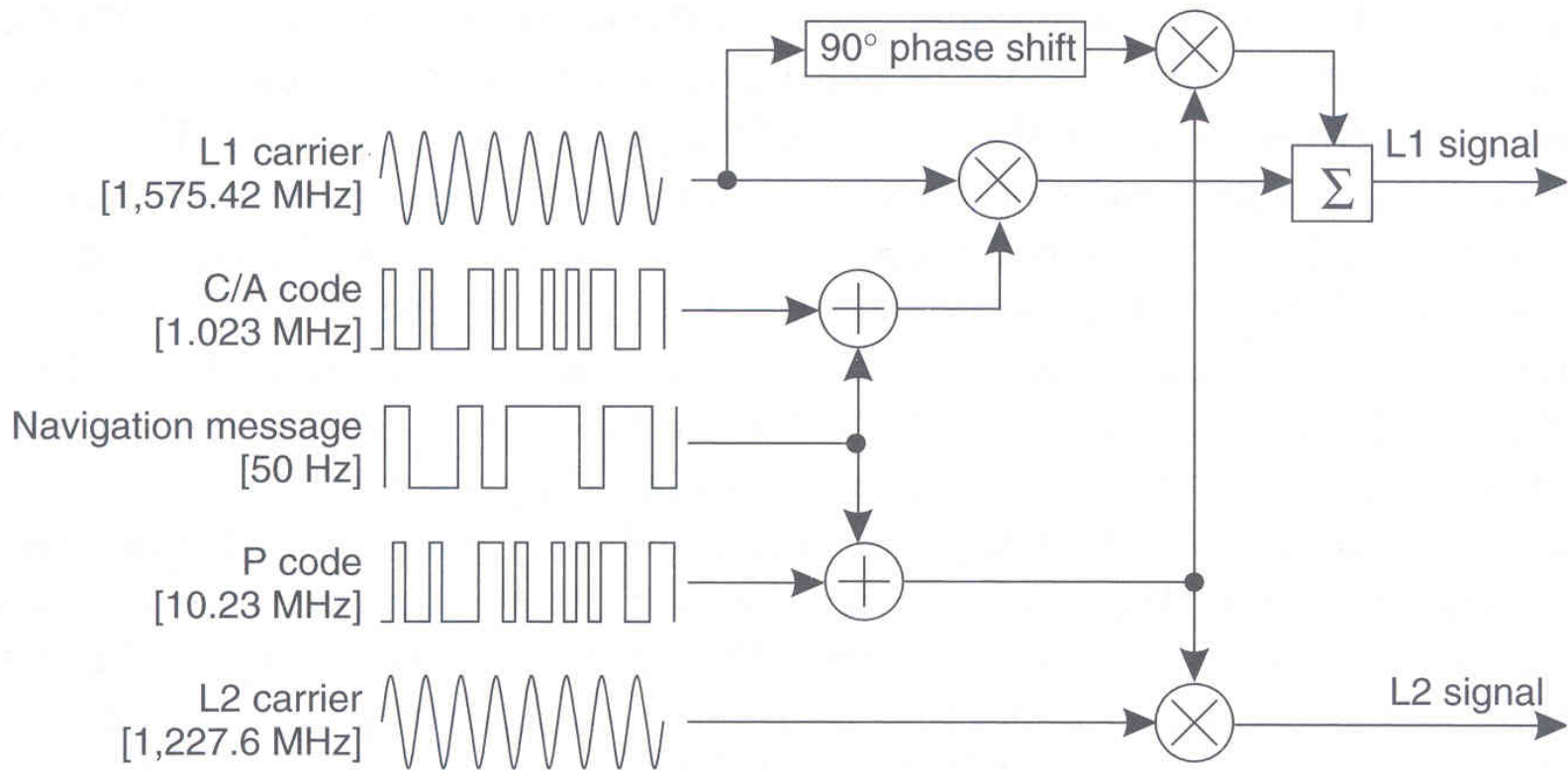


Figure 7.8 Pilot signals and codes.

7.3.4 Navigation Message

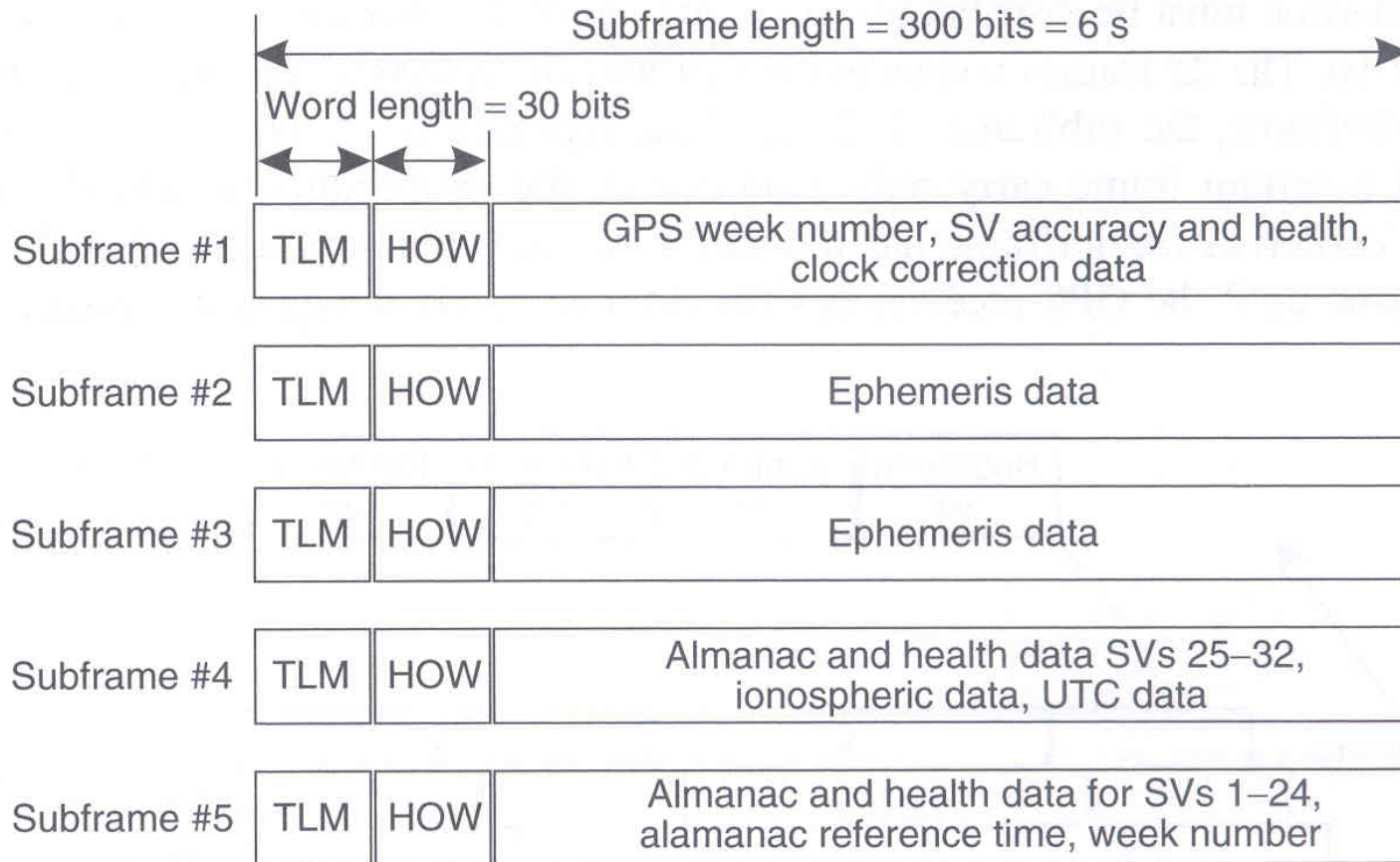


Figure 7.9 GPS navigation message: frame structure.

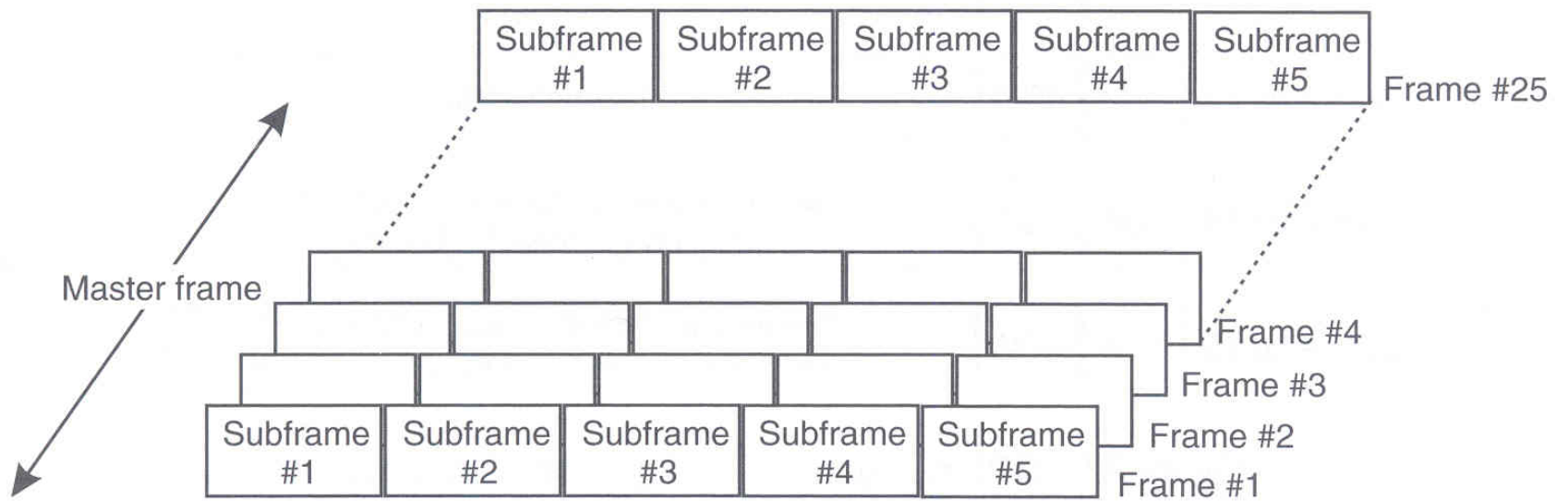


Figure 7.10 GPS navigation message: masterframe structure.

7.3.5 GPS Services

Table 7.1 Positioning and timing accuracy standard

Accuracy standard	95% Vertical error	95% Horizontal error
Global average	≤ 22 m	≤ 13 m
Worst site	≤ 77 m	≤ 36 m

7.3.6 GPS Positioning

Identification of satellites

Range measurements

Position calculation

$$p_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + c\Delta t \quad (7.5)$$

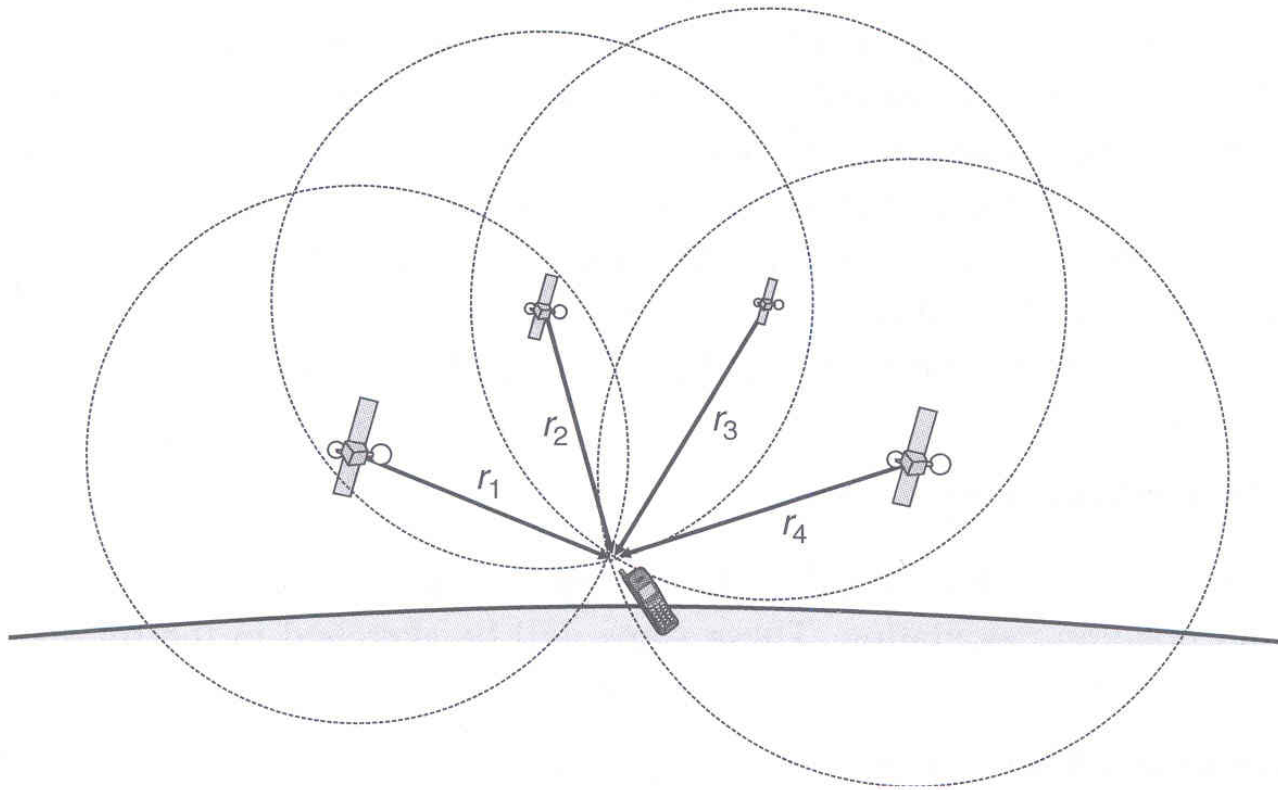


Figure 7.11 Positioning in GPS.

7.3.7 GPS Error Budget

Table 7.2 Example of observed GPS error budget (National Research Council 1995)

Error source	Range error magnitude [m, 1 σ]		
	SPS (SA on)	SPS (SA off)	PPS
Selective availability	24.0	0.0	0.0
Ionospheric error	7.0	7.0	0.01
Tropospheric error	0.7	0.7	0.7
Clock and ephemeris error	3.6	3.6	3.6
Receiver noise	1.5	1.5	0.6
Multipath	1.2	1.2	1.8
Total user equivalent range error	25.3	8.1	4.1
Typical horizontal dilution of precision	2.0	2.0	2.0
Total horizontal accuracy (2 σ)	101.2	32.5	16.4

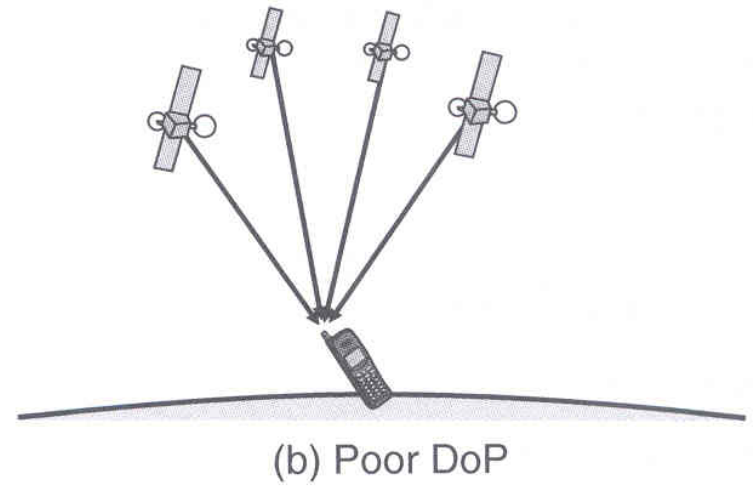
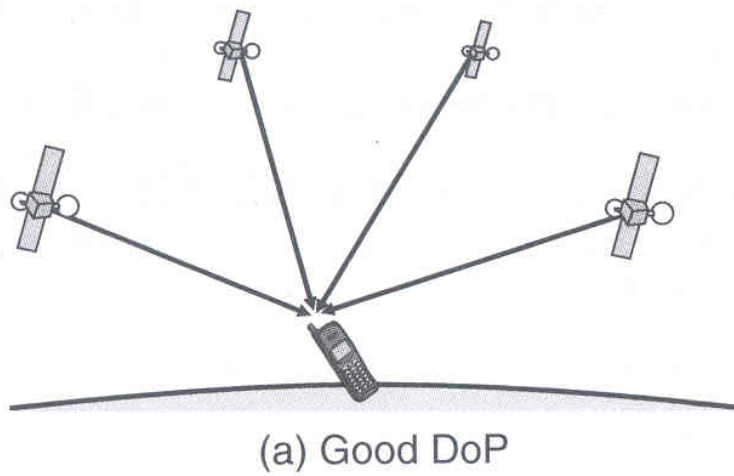


Figure 7.12 Good and poor DoP.

$$\mathbf{Q}_{\text{ECEF}} = (\mathbf{A}^T \mathbf{A})^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \end{bmatrix} \quad (7.6)$$

$$\mathbf{Q}_{\text{geo}} = \begin{bmatrix} \sigma_n^2 & \sigma_{ne} & \sigma_{nu} \\ \sigma_{en} & \sigma_e^2 & \sigma_{eu} \\ \sigma_{un} & \sigma_{ue} & \sigma_u^2 \end{bmatrix} \quad (7.7)$$

$$\sigma = \sigma_R DoP \quad (7.8)$$

Table 7.3 Dilution of precision

DoP measure	Defined by
VDoP	$\sqrt{\sigma_u^2}$
HDoP	$\sqrt{\sigma_n^2 + \sigma_e^2}$
PDoP	$\sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_u^2}$
TDoP	$\sqrt{\sigma_t^2}$
GDoP	$\sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_u^2 + (c\sigma_t^2)}$

7.4 Differential GPS

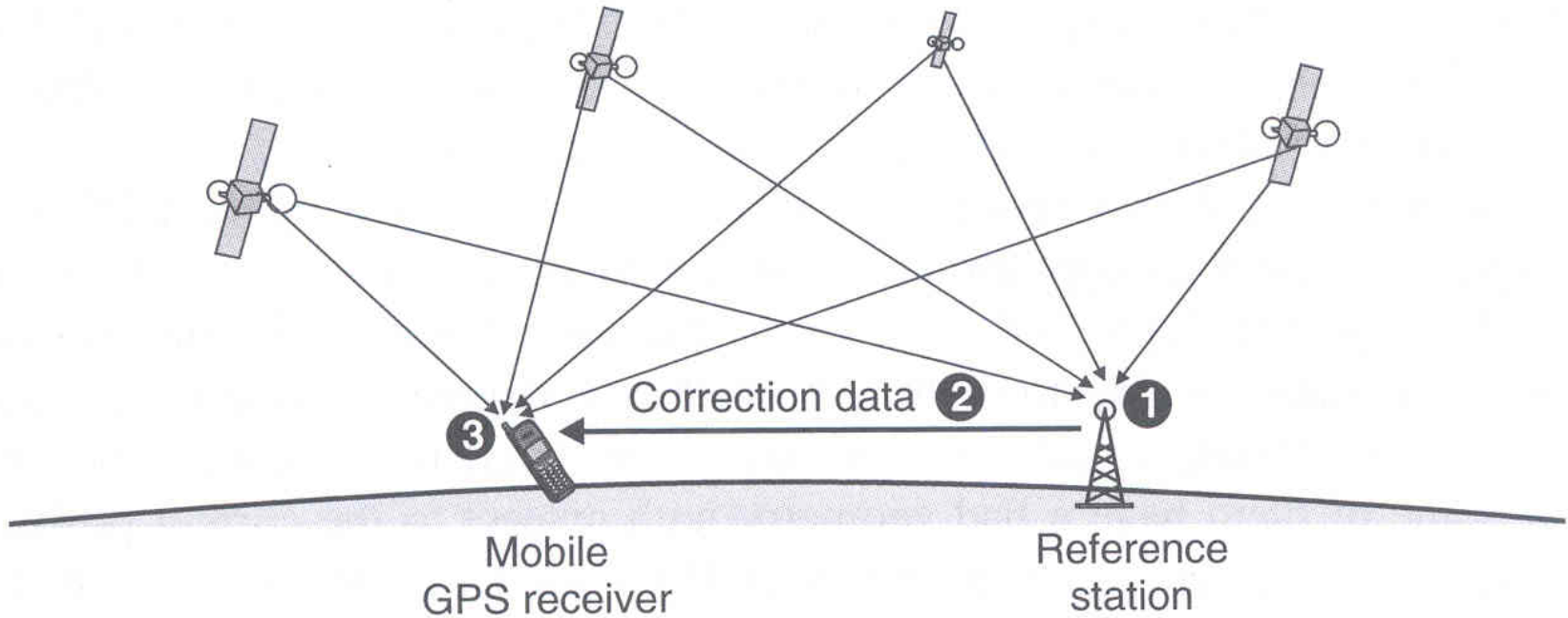


Figure 7.13 Differential GPS.

$$r_{i,rs} = \sqrt{(X_i - x_{rs})^2 + (Y_i - y_{rs})^2 + (Z_i - z_{rs})^2} \quad (7.9)$$

$$p_{i,rs} = r_{i,rs} + c \cdot \Delta t_{i,rs} + \varepsilon_{i,rs} \quad (7.10)$$

$$\Delta p_i = p_{i,rs} - r_{i,rs} = c \cdot \Delta t_{i,rs} \quad (7.11)$$

$$p_{i,mr} - \Delta p_i = r_{i,mr} + c \cdot \Delta t_{i,mr} + \varepsilon_{i,mr} - (c \cdot \Delta t_{i,rs} + \varepsilon_{i,rs}) \quad (7.12)$$

$$p_i^* \approx r_{i,mr} + c \cdot (\Delta t_{i,mr} - \Delta t_{i,rs}) = r_{i,mr} + \Delta t_{\Sigma} \quad (7.13)$$

7.5 Galileo

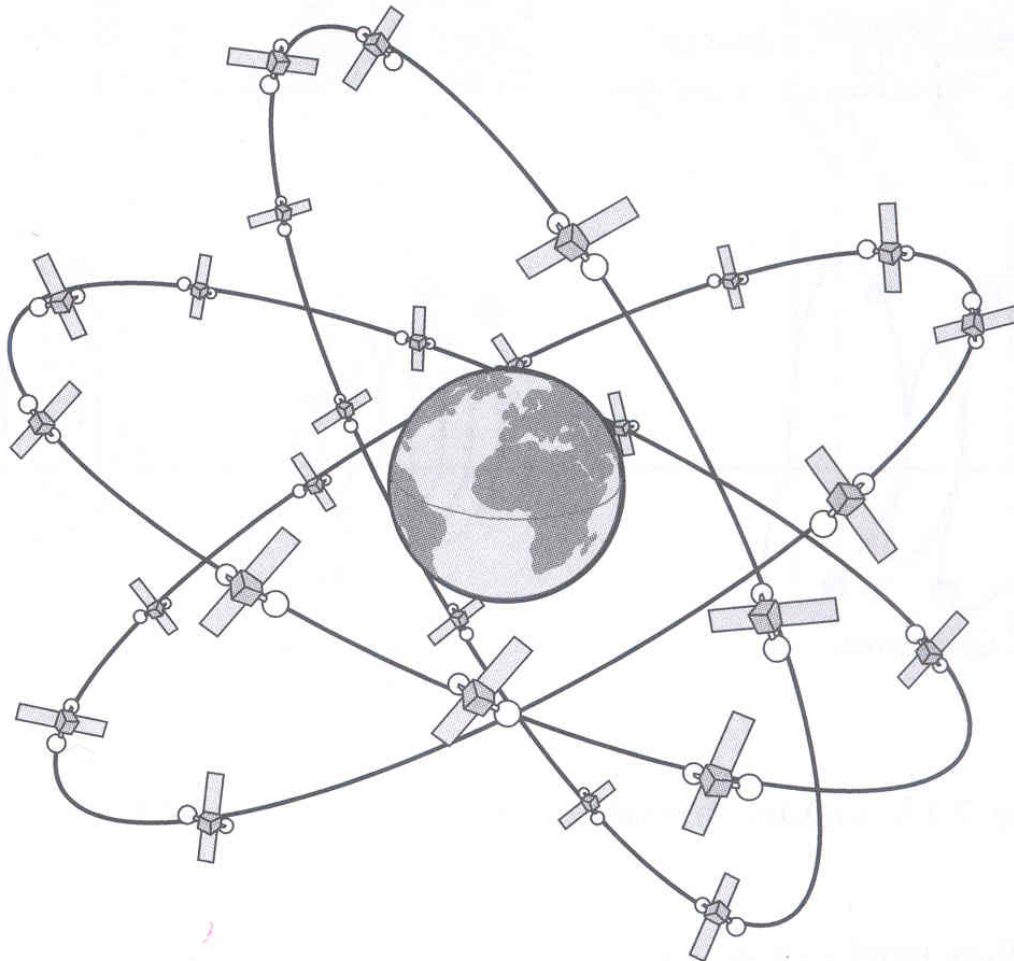


Figure 7.14 Galileo constellation.

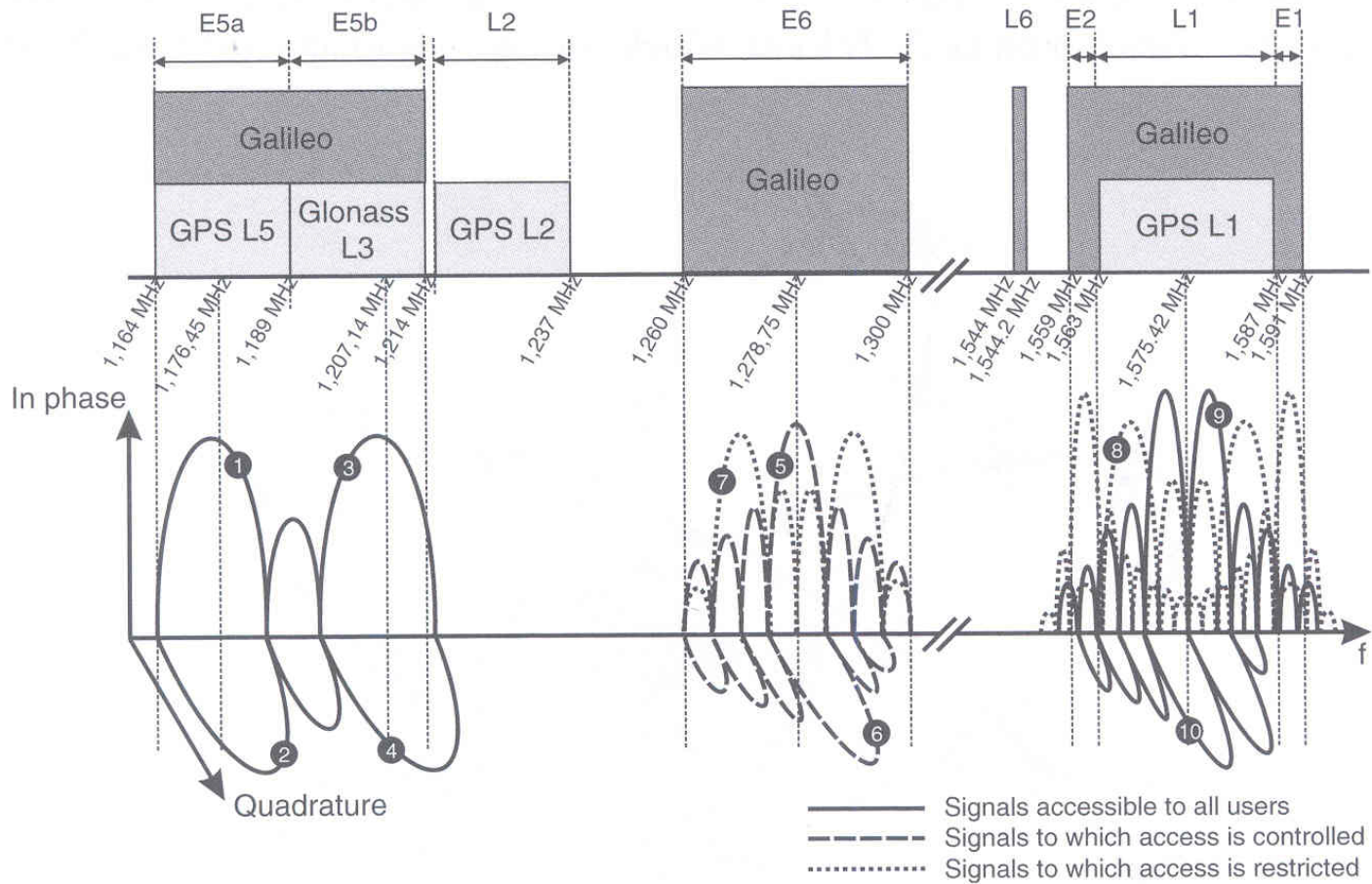


Figure 7.15 Galileo frequency spectrum (Hein and Pany 2002).

Table 7.4 Quality parameters of Galileo services

Level	Open service	Commercial service	Public reg. service	Safety of life service	
				Critical	Noncritical
Accuracy					
– horizontal(h)	h = 4 m	<1 m	h = 6.5 m	h = 4 m	h = 220 m
– vertical(v)	v = 8 m (dual frequency) h = 15 m v = 35 m (single frequency)	(dual frequency)	v = 12 m (dual frequency)	v = 8 m (dual frequency)	
Availability	99.8%	99.8%	99–99.9%	99.8%	
Integrity		optional			
– Alarm limit	–	–	h = 20 m v = 35 m	h = 12 m v = 20 m	h = 556 m
– Time-to-alarm	–	–	10 s	6 s	6 s
– Integrity risk	–	–	$3.5 \times 10^{-7} / 150$ s	$3.5 \times 10^{-7} / 150$ s	$10^{-7} / h$

Table 7.5 Mapping of Galileo services onto signals

	1 + 2	3 + 4	5	6 + 7	8	9 + 10
Open service (single frequency)						×
Open service (dual frequency)	×					×
Open service (improved accuracy)	×	×				×
Safety-of-life service	×	×				×
Commercial service				×		×
Commercial service (multiple carrier)	×	×		×		×
Public regulated service			×		×	

7.6 Conclusion