CH. 8
DEFLECTIONS DUE TO BENDING

8.1 Introduction

i) We consider the deflections of slender members which transmit bending moments.

ii) We shall treat statically indeterminate beams which require simultaneous consideration of all three of the steps (2.1)

iii) We study mechanisms of plastic collapse for statically indeterminate beams.

iv) The calculation of the deflections is very important way to analyze statically indeterminate beams and confirm whether the deflections exceed the maximum allowance or not.

8.2 The Moment – Curvature Relation

▶ From Ch.7

→ When a symmetrical, linearly elastic beam element is subjected to pure bending, as shown in Fig. 8.1, the curvature of the neutral axis is related to the applied bending moment by the equation.

$$\frac{1}{\rho} = \lim_{\Delta s \to 0} \frac{\Delta\phi}{\Delta s} = \frac{d\phi}{ds} = \frac{M_b}{EI_{zz}} = \frac{M_b}{EI} \tag{8.1}$$

cf. For simplification, $I_{zz} \to I$

▶ Simplification

i) When M_b is not a constant, the effect on the overall deflection by the shear force can be ignored.

ii) Assume that although M_b is not a constant the expressions defined from pure bending can be applied.

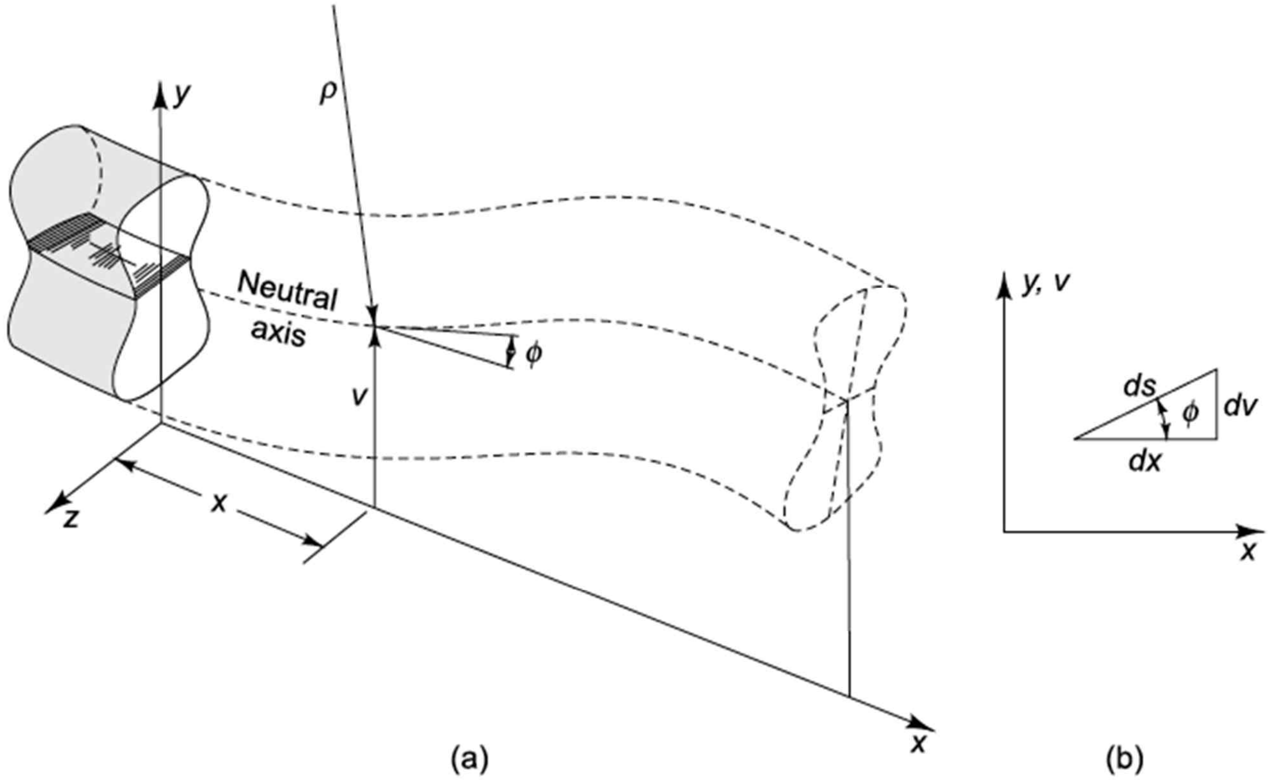


Fig. 8.2 *Geometry of the neutral axis of a beam bent in the xy plane*

▶ **Differential equations between the curvature and the deflection**

1 ▷ The case of the large deflection

The *slope* of the neutral axis in Fig. 8.2 (a) is

$$\frac{dv}{dx} = \tan\phi$$

Next, differentiation with respect to arc length s gives

$$\frac{d}{ds}\left(\frac{dv}{dx}\right) = \frac{d}{ds}(\tan\phi)$$

$$\therefore \quad \frac{d^2v}{dx^2}\frac{dx}{ds} = \sec^2\phi\,\frac{d\phi}{ds}$$

$$\rightarrow \frac{d\phi}{ds} = \frac{d^2v}{dx^2}\frac{dx}{ds}\cos^2\phi \qquad \text{(a)}$$

From Fig. 8.2 (b)

$$(ds)^2 = (dx)^2 + (v)^2$$

$$\rightarrow \left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dv}{dx}\right)^2$$

$$\rightarrow \left(\frac{dx}{ds}\right)^2 = \frac{1}{1+(dv/dx)^2} \tag{b}$$

$$\& \ \cos\phi = \frac{dx}{ds} = \frac{1}{[1+(dv/dx)^2]^{1/2}} \tag{c}$$

If substitutng (b) and (c) into the (a),

$$\frac{d\phi}{ds} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}} = \frac{v''}{[1+(v')^2]^{3/2}} = \kappa \tag{8.2}$$

$$\therefore \quad \kappa = \frac{v''}{[1+(v')^2]^{3/2}} = \frac{M_b}{EI}$$

cf. When the slope angle ϕ shown in Fig. 8.2 is small, then dv/dx is small compared to unity. If we neglect $(dv/dx)^2$ in the denominator of the right-hand term of (8.2), we obtain a simple approximation for the curvature

$$\frac{d\phi}{ds} \approx \frac{d^2v}{dx^2} \approx \frac{M_b}{EI} \qquad (8.3)(8.4)$$

→ There is less than a 1% error involved in the approximation (8.3) to the exact curvature expression (8.2) when $\phi < 4.7°$.

2 ▷ The case of the small deflection

The slope of the neutral axis in Fig. 8.2 (a) is

$$\frac{dv}{dx} = \tan\phi$$

When the deflection is very small,

$$ds \approx dx, \ \tan\phi \approx \phi$$

$$\rightarrow \kappa = \frac{d\phi}{ds} \approx \frac{d\phi}{dx} \approx \frac{d}{dx}\left(\frac{dv}{dx}\right) = \frac{d^2v}{dx^2} \tag{8.3}$$

$$\therefore \quad \kappa = \frac{1}{\rho} = \frac{d\phi}{ds} = \frac{d^2v}{dx^2} = \frac{M_b}{EI} \tag{8.4}$$

▶ Comment on Eq.(8.4)

$$\kappa = \frac{d\phi}{ds} \approx \frac{d^2v}{dx^2} \approx \frac{M_b}{EI} \tag{8.4}$$

i) This relation is linear like $M_b = EI\frac{d^2v}{dx^2}$

ii) *EI* is the flexural rigidity or the bending modulus.

iii) The sign convention of the M_b and the curvature are same as what it has been.

▶ The solution of the deflection-curvature

i) Integration of the moment-curvature relation

ii) Method of the singularity functions

iii) Moment-area method

iv) Superposition technique

v) Load-deflection differential equation

vi) Elastic energy method

8.3 Integration of the moment-curvature relation

▶ Differential equation of deflection-curvature in case of the linear elastic materials and very small deflection.

$$EI\frac{d^2v}{dx^2} = M_b\ , \quad EI\frac{d^3v}{dx^3} = -V\ , \quad EI\frac{d^4v}{dx^4} = q$$

$$\frac{dv}{dx} = v' \rightarrow \quad EIv'' = M_b\ , \quad EIv''' = -V\ , \quad EIv'''' = q$$

▶ Example 8.1

Determine the deflection curvature of the deformed neutral axis in simple beam like in Fig. 8.3

Sol)

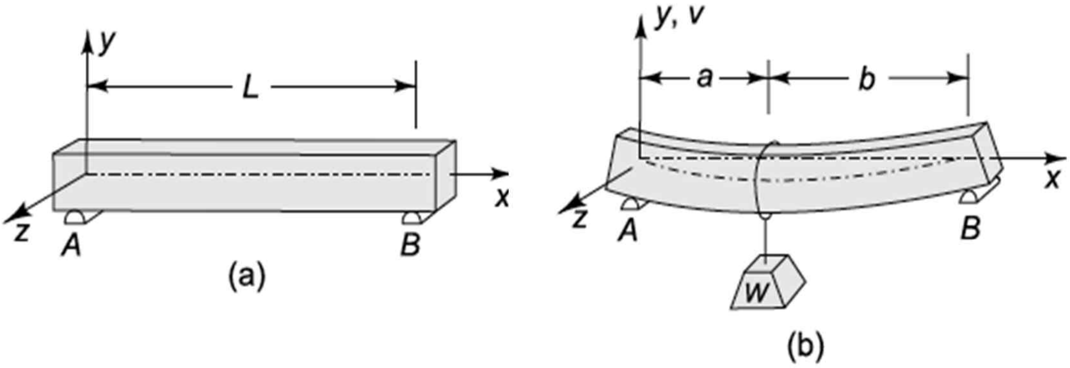


Fig. 8.3 *Example 8.1. Simply supported beam (a) before and (b) after application of a concentrated load W*

$$M_b = \frac{Wb}{L}x - W\langle x-a \rangle^1$$

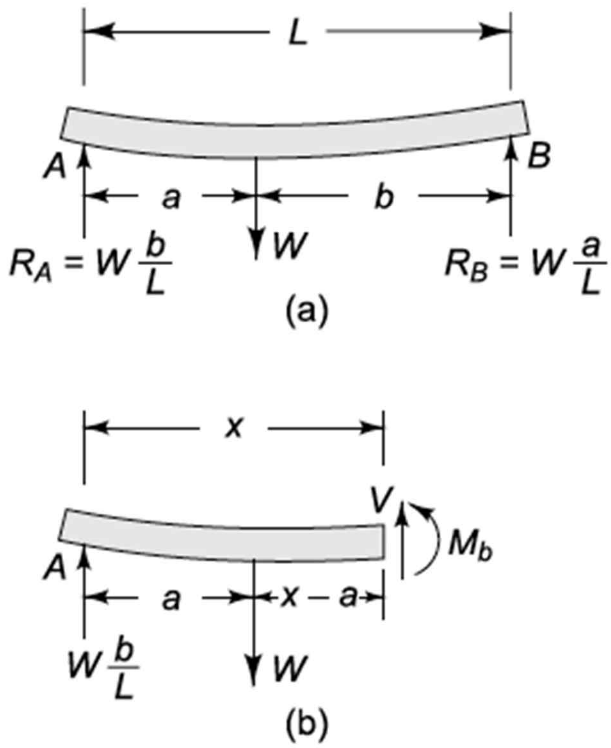


Fig. 8.4 *Example 8.1. Free-body diagram of beam and segment of beam*

→ Using the singularity functions and bracket notation introduced m Sec 3 6, we can write a single expression for the bending moment *M*, directly from the free body of Fig. 8 4b.

$$EIv'' = M_b = \frac{W\,b}{L}x - W\langle x-a\rangle^1 \qquad \text{(a)}$$

$$EIv' = \frac{W\,b}{L}\frac{x^2}{2} - W\frac{\langle x-a\rangle^2}{2} + c_1 \qquad \text{(b)}$$

$$EIv = \frac{W\,b}{L}\frac{x^3}{6} - W\frac{\langle x-a\rangle^2}{2} + c_1 x + c_2 \qquad \text{(c)}$$

B.C.) $v(0) = v(L) = 0$

$eq.(c)$ is ; $\quad 0 = c_2$

$eq.(c)$ is ; $\quad 0 = \frac{W\,b}{6L}L^3 - \frac{Wb^3}{6} + c_1 L$

$\rightarrow c_1 = \frac{W\,b}{6L}(b^2 - L^2)\,, \quad c_2 = 0$

∴ From eq.(c)

$$v = -\frac{W}{6EI}\left[\frac{bx}{L}(L^2 - b^2 - x^2) + \langle x-a\rangle^3\right]$$

cf. $(\phi)_{x=0} = (v')_{x=0} = -\frac{WL^2}{16EI}$

To give some idea of order of magnitudes, let us consider the following particular case:

$L = 3.70\,m\,, \quad a = b = 1.85\,m$

$W = 1.8\,kN\,, \;\; E = 11\,GN/m^2\,, \;\; I = 3.33(10)^7\,mm^4$

Then, the maximum deflection and slope is

$$(v)_{max} = (v)_{x=L/2} = -\frac{WL^3}{48EI} = -\frac{1800\,(3.70)^3}{48(11\times10^9)(3.33\times10^{-5})}$$

$$= -5.19(10)^{-3}\,m = -5.19\,mm$$

$$\phi_{max} = v'(0) = v'(L) = -\frac{WL^2}{16EI} = -0.00420\,rad = -0.2409°$$

▶ Example 8.2

Determine the deflection δ and the slope angle ϕ of the beam in Fig.8.5

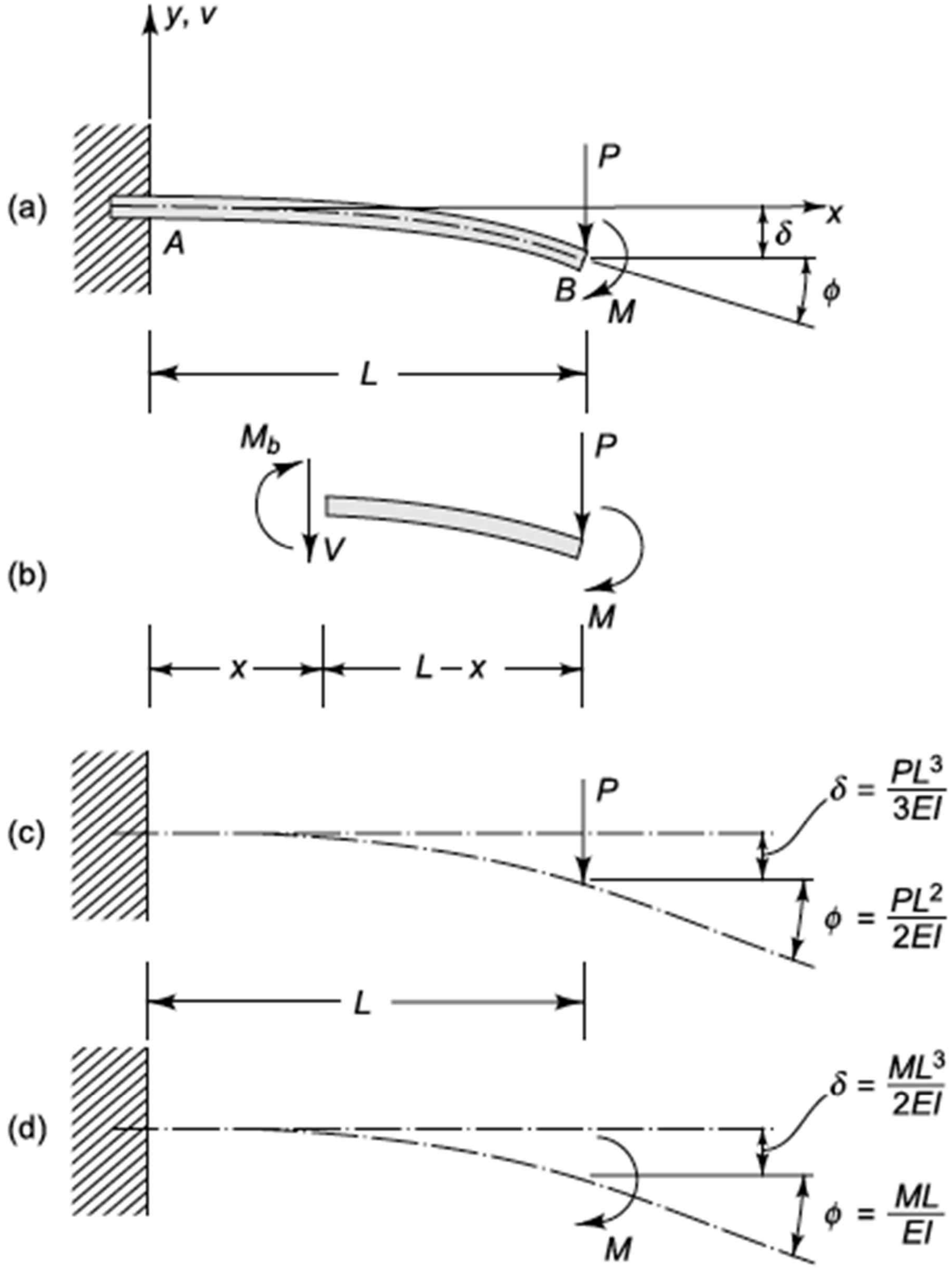


Fig. 8.5 *Example 8.2. Cantilever beam with force and moment load*

In order to obtain the bending moment in the interior of the beam, we isolate the segment of length L - x shown m Fig. 8.5 (b). From this free body we obtain the bending moment

$$M_b = P(L - x) - M \qquad \text{(a)}$$

Inserting (a) into the moment-curvature relation (8.4), we find the differential equation for the beam displacement $v(x)$.

$$\therefore \quad EI\frac{d^2v}{dx^2} = -PL + Px - M \qquad \text{(b)}$$

$$EIv' = -PLx + P\frac{x^2}{2} - Mx + c_1 \qquad \text{(d)}$$

$$EIv = -PL\frac{x^2}{2} + P\frac{x^3}{6} - M\frac{x^2}{2} + c_1x + c_2 \qquad \text{(e)}$$

B.C.) $v'(0) = 0$, $v(0) = 0$

$$\therefore \quad c_1 = c_2 = 0$$

$$\therefore \quad v = -\frac{1}{EI}\left[P\frac{x^2}{6}(3L - x) + M\frac{x^2}{2}\right] \qquad \text{(f)}$$

$$\therefore \quad \delta = -v(L) = \frac{PL^3}{3EI} + \frac{ML^2}{2EI}$$

$$\phi = -v'(L) = \frac{PL^2}{2EI} + \frac{M\,L}{EI}$$

cf. Figure 8.5 (c) shows the case where the moment $M = 0$.

cf. Figure 8.5 (d) shows the case where the moment $P = 0$.

▶ Example 8.3

Determine the deflection δ of the beam in Fig.8.6

Sol)

$$EIv' = M_b = wbx - wb\left(a + \frac{b}{2}\right) - \frac{w\langle x-a\rangle^2}{2} \qquad \text{(a)}$$

$$EIv' = wb\frac{x^2}{2} - wb\left(a + \frac{b}{2}\right)x - \frac{w\langle x-a\rangle^3}{6} + c_1 \qquad \text{(b)}$$

$$EIv = wb\frac{x^3}{6} - wb\left(a + \frac{b}{2}\right)\frac{x^2}{2} - \frac{w\langle x-a\rangle^4}{24} + c_1x + c_2 \qquad \text{(c)}$$

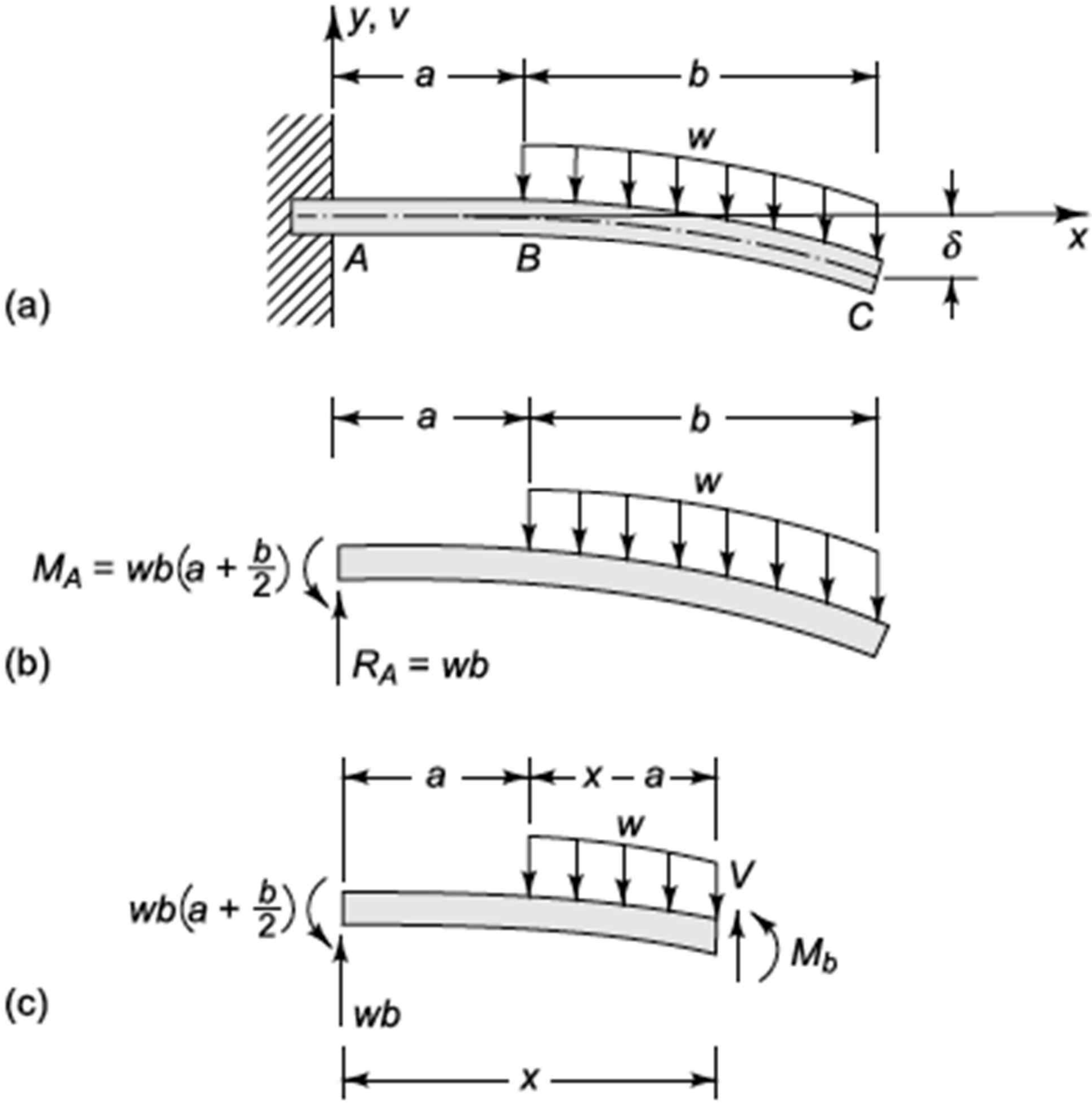


Fig. 8.6 *Example 8.3*

B.C.) $v(0)' = 0$, $v(0) = 0$

$\therefore \;\; c_1 = c_2 = 0$

$\therefore \;\; \delta = -v(a+b) = \frac{w\,b}{EI}\left(\frac{a^3}{3} + \frac{3a^2b}{4} + \frac{ab^2}{2} + \frac{b^3}{8}\right)$

cf. When $a = 0$;

$\delta = \frac{w\,b^4}{8EI}$

▶ Example 8.4

Draw the B.M.D in Fig.8.7, statically indeterminate beam

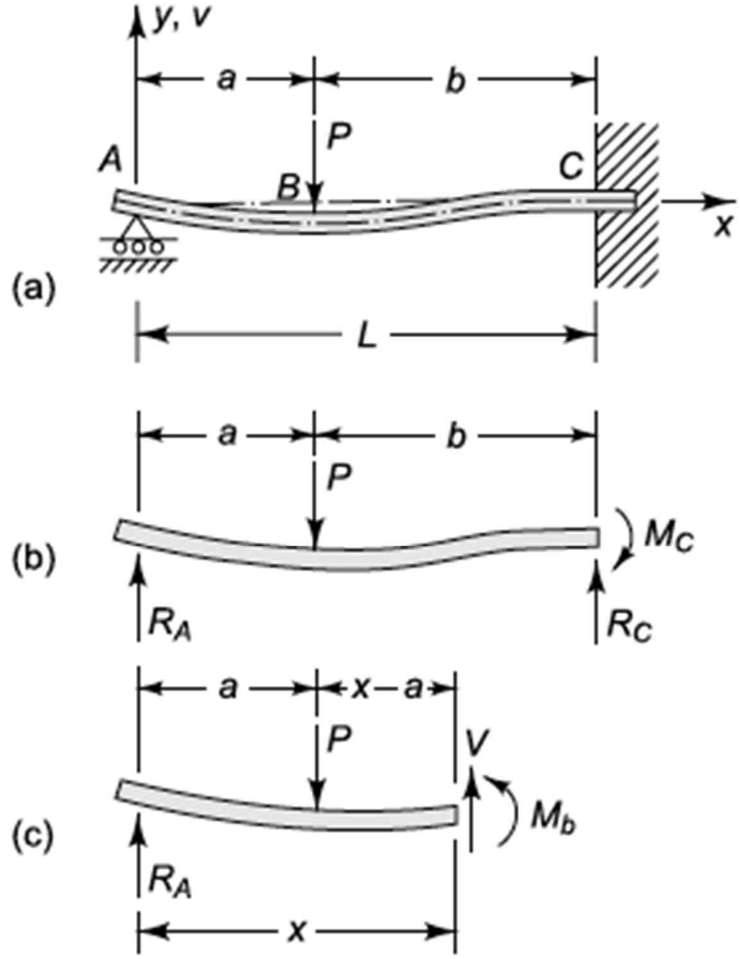


Fig. 8.7 Example 8.4

$$EIv'' = M_b = R_A x - P\langle x-a\rangle^1 \quad \text{(a)}$$

$$EIv' = R_A\frac{x^2}{2} - P\frac{\langle x-a\rangle^2}{2} + c_1 \quad \text{(b)}$$

$$EIv = R_A\frac{x^3}{6} - P\frac{\langle x-a\rangle^3}{6} + c_1 x + c_2 \quad \text{(c)}$$

B.C.) $v(0) = v(L) = v'(L) = 0$

eq (b) is ;

$$0 = R_A\frac{L^2}{2} - P\frac{(L-a)^2}{2} + c_1$$

$$\rightarrow \; c_1 = \frac{Pb^2}{2} - \frac{R_A L^2}{2}$$

eq (c) is

$$0 = c_2$$

$$\therefore 0 = R_A \frac{L^3}{6} - P\frac{b^3}{6} + P\frac{b^2 L}{2} - \frac{R_A L^3}{2}$$

$$\therefore R_A = \frac{Pb^2}{2L^3}(3L - b)$$

cf. By equating the magnitudes given in Fig. 8.8, we find that when $a = (\sqrt{2}-1)L = 0.414\,L$ the bending moments at *B* and at *C* have equal magnitude.

$$\begin{cases} a < 0.414\,L & \rightarrow \quad (M_b)_{max}\, at\ B \\ a > 0.414\,L & \rightarrow \quad (M_b)_{max}\, at\ C \end{cases}$$

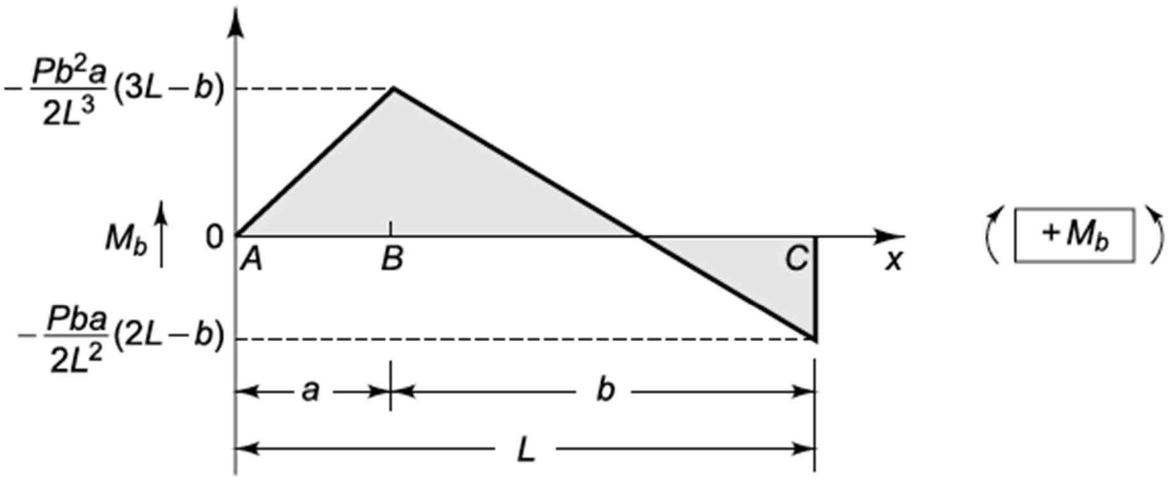
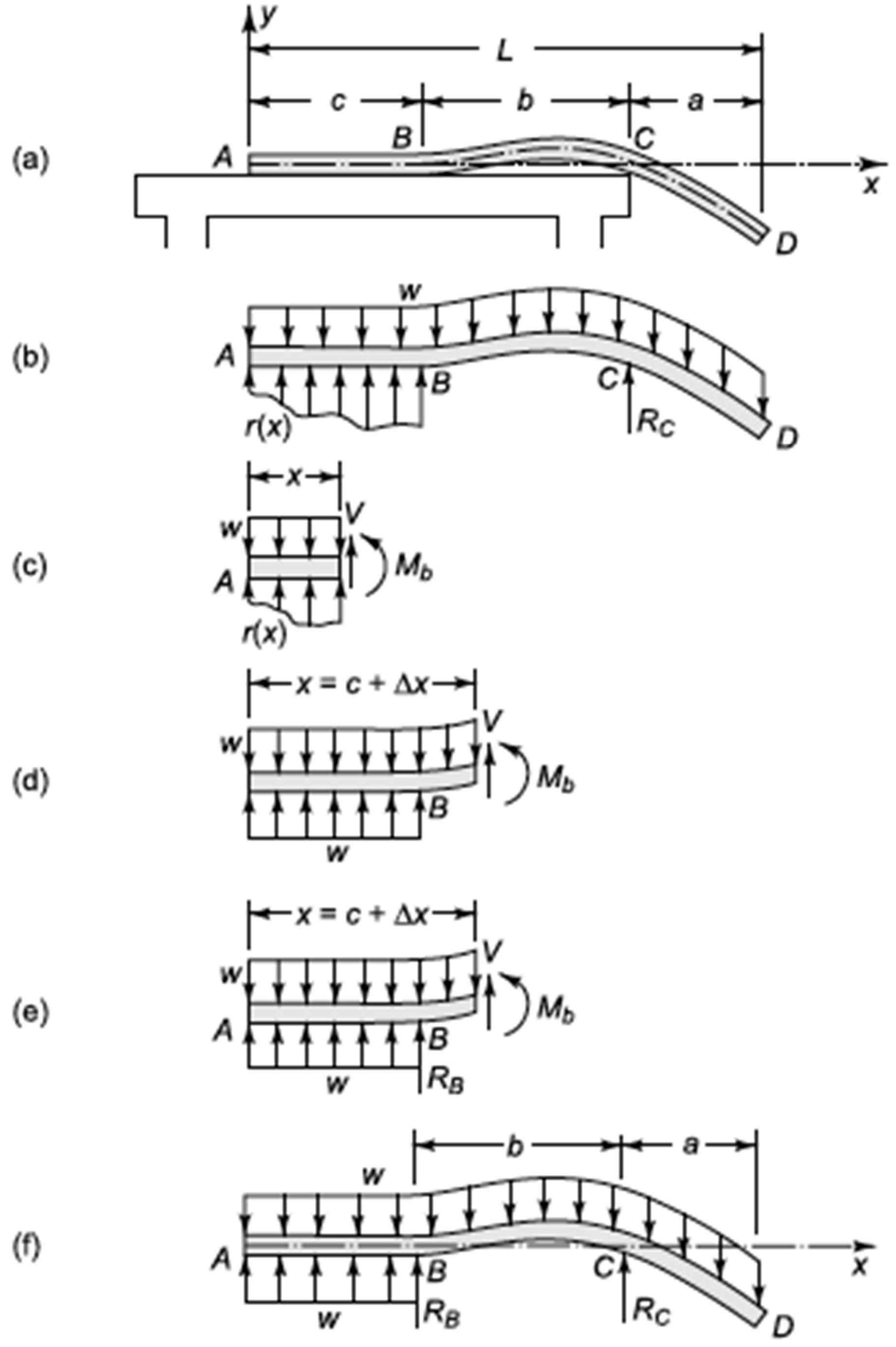


Fig. 8.8 *Example 8.4. Bending-moment diagram for the beam of Fig. 8.7*

▶ Example 8.5

A long uniform rod of length *L*, weight *w* per unit length, and bending modulus *EI* is placed on a rigid horizontal table. Determine the length *b* in Fig.8.9



(a)
y
L
c
b
a
A
B
C
x
D
(b)
w
A
B
C
r(x)
R_C
D
(c)
x
w
V
A
M_b
r(x)
(d)
$x = c + \Delta x$
w
V
M_b
B
w
(e)
$x = c + \Delta x$
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V
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(g)

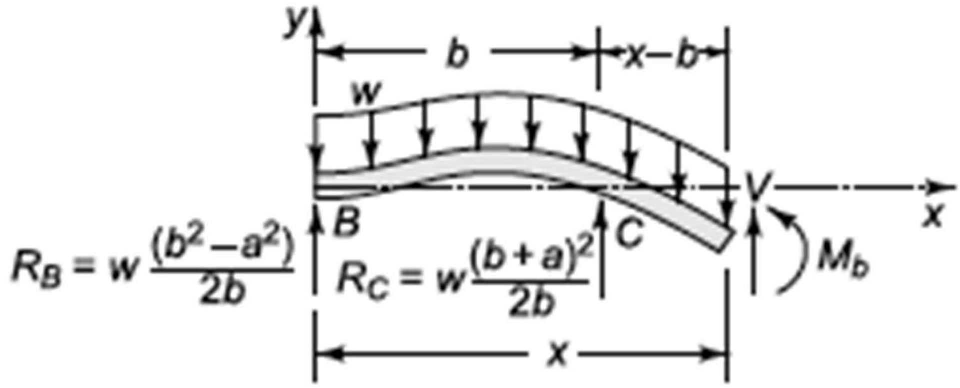


Fig. 8.9 *Example 8.5. Beam overhanging edge of the table causes segment BC to lift up from the table*

▶ Verbal analysis

i) The curvature of the beam is 0 in the region AB. → ∴ Between A and B, $M_b = 0$

ii) In section AB, V=0. → ∴Net load intensity is 0

iii) From Fig (d), M_b is positive. ∵ The positive curvature is needed in order that the beam is detached from the table.

iv) From Fig (e), the reaction force R_B should exist which offsets the bending moment in order to satisfy the equilibrium.

v) The deflection and the slope angle are 0 at the point B since the deformation should be continuous.

vi) The bending moment at the point B is 0 but the shear force appears suddenly.

▶ Formulated analysis

$$EIv'' = M_b = w\frac{(b^2-a^2)}{2b}x + w\frac{(b+a)^2}{2b}\langle x-b\rangle^1 - w\frac{x^2}{2} \quad \text{(a)}$$

$$EIv' = \frac{w(b^2-a^2)}{4b}x^2 + \frac{w(b+a)^2}{4b}\langle x-b\rangle^2 - \frac{wx^3}{6} + c_1 \quad \text{(b)}$$

$$EIv = \frac{w(b^2-a^2)}{12b}x^3 + \frac{w(b+a)^2}{12b}\langle x-b\rangle^3 - \frac{wx^4}{24} + c_1x + c_2 \quad \text{(c)}$$

B.C.) $v(0) = 0$, $v(b) = 0$, $v'(0) = 0$

$\therefore\ c_1 = c_2 = 0 \quad \rightarrow \quad v(b) = 0$

$$\therefore\ 0 = \frac{w(b^2 - a^2)}{12b} b^3 + 0 - \frac{wb^4}{24}$$

$\rightarrow\ b = \sqrt{2}a$

8.4 Superposition

→ The method based on linear relation between the load and the deflection.

▶ The linearity is based on the following.

i) Linearity between the bending moment and the curvature. $\frac{d\phi}{ds} = \frac{M_b}{EI}$

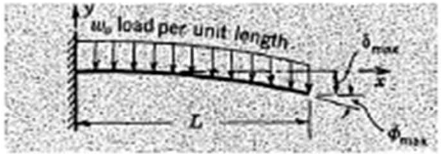
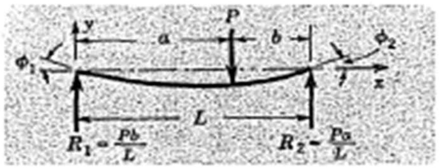
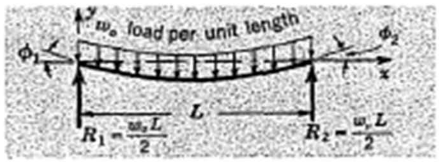
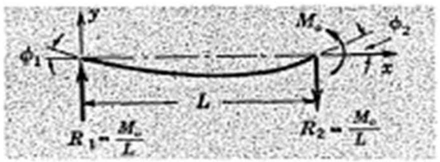
ii) Linearity between the curvature and the deflection. $\frac{d\phi}{ds} \approx \frac{dv^2}{dx^2}$

→ This expression can be applied only when the load-deflection is linear and the deformation is infinitesimal.

▶ Eq. (2.1) in superposition is satisfied with the equilibrium, the geometry and the force-deformation relation.

Table 8.1 Deflection formulas for uniform beams

δ is positive downward

1.		$\delta = \dfrac{P}{6EI}(\langle x-a\rangle^3 - x^3 + 3x^2a)$	$\delta_{max} = \dfrac{Pa^2(3L-a)}{6EI}$	$\phi_{max} = \dfrac{Pa^2}{2EI}$
2.		$\delta = \dfrac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$	$\delta_{max} = \dfrac{w_o L^4}{8EI}$	$\phi_{max} = \dfrac{w_o L^3}{6EI}$
3.		$\delta = \dfrac{M_o x^2}{2EI}$	$\delta_{max} = \dfrac{M_o L^2}{2EI}$	$\phi_{max} = \dfrac{M_o L}{EI}$
4.		$\delta = \dfrac{Pb}{6LEI}\left[\dfrac{L}{b}\langle x-a\rangle^3 - x^3 + (L^2-b^2)x\right]$	$\delta_{max} = \dfrac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}\,LEI}$ at $x = \sqrt{\dfrac{L^2-b^2}{3}}$	$\phi_1 = \dfrac{Pab(2L-a)}{6LEI}$ $\phi_2 = \dfrac{Pab(2L-b)}{6LEI}$
5.		$\delta = \dfrac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$	$\delta_{max} = \dfrac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \dfrac{w_o L^3}{24EI}$
6.		$\delta = \dfrac{M_o Lx}{6EI}\left(1 - \dfrac{x^2}{L^2}\right)$	$\delta_{max} = \dfrac{M_o L^2}{9\sqrt{3}\,EI}$ at $x = \dfrac{L}{\sqrt{3}}$	$\phi_1 = \dfrac{M_o L}{6EI}$ $\phi_2 = \dfrac{M_o L}{3EI}$

▶ Example 8.7

Draw the B.M.D.

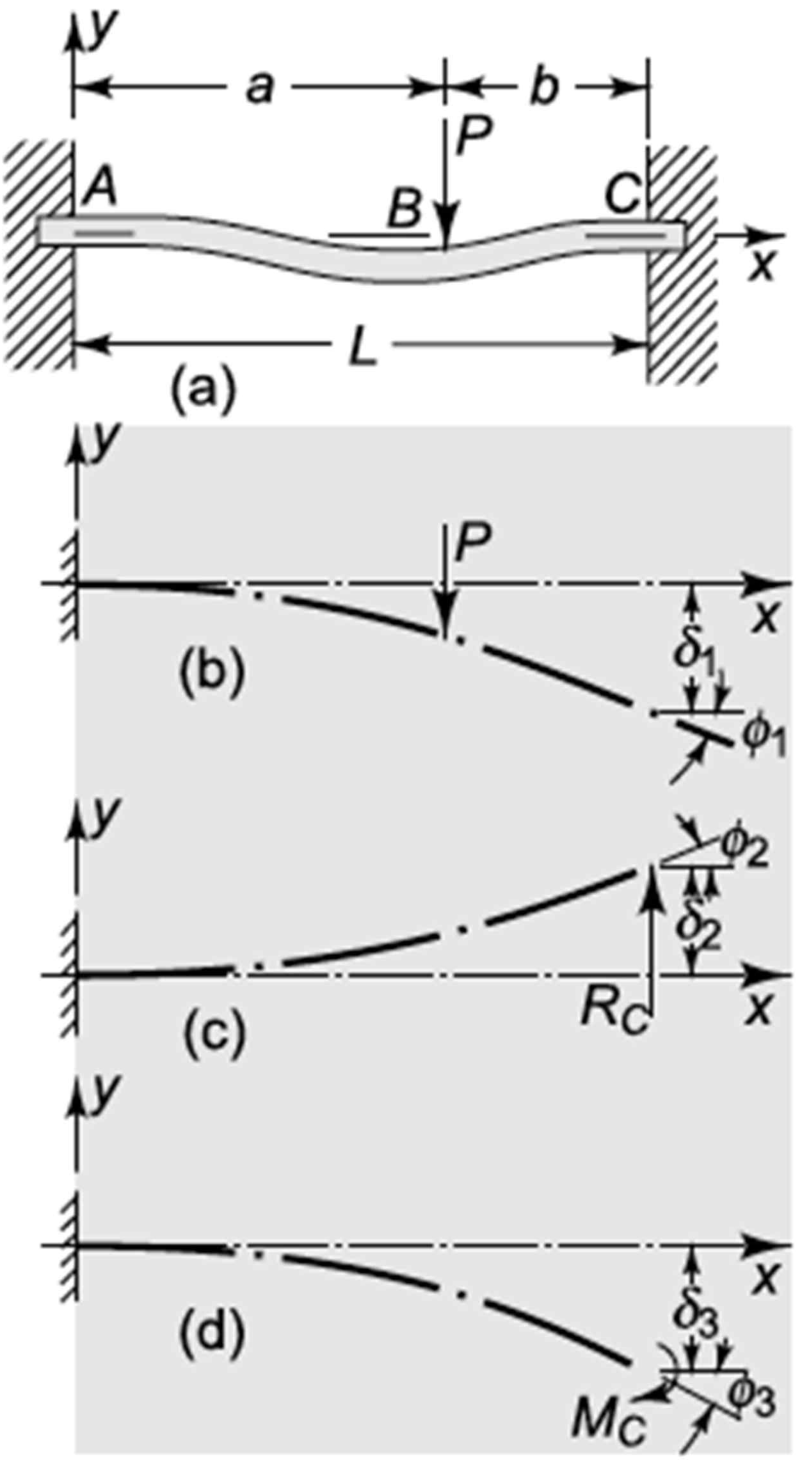


Fig. 8.12 *Example 8.7*

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▶ Assumptions

i) There is no stress in the beam when P=0.

ii) Although the shear force P is applied on the beam, the deflection is infinitesimal sufficiently. Thus the effect of the axial stress on the bending can be ignored.

cf. This beam is statically indeterminate since there are four unknowns, R_A R_c M_A M_c, but two equilibrium equation, $\sum F_y = 0, \sum M = 0$

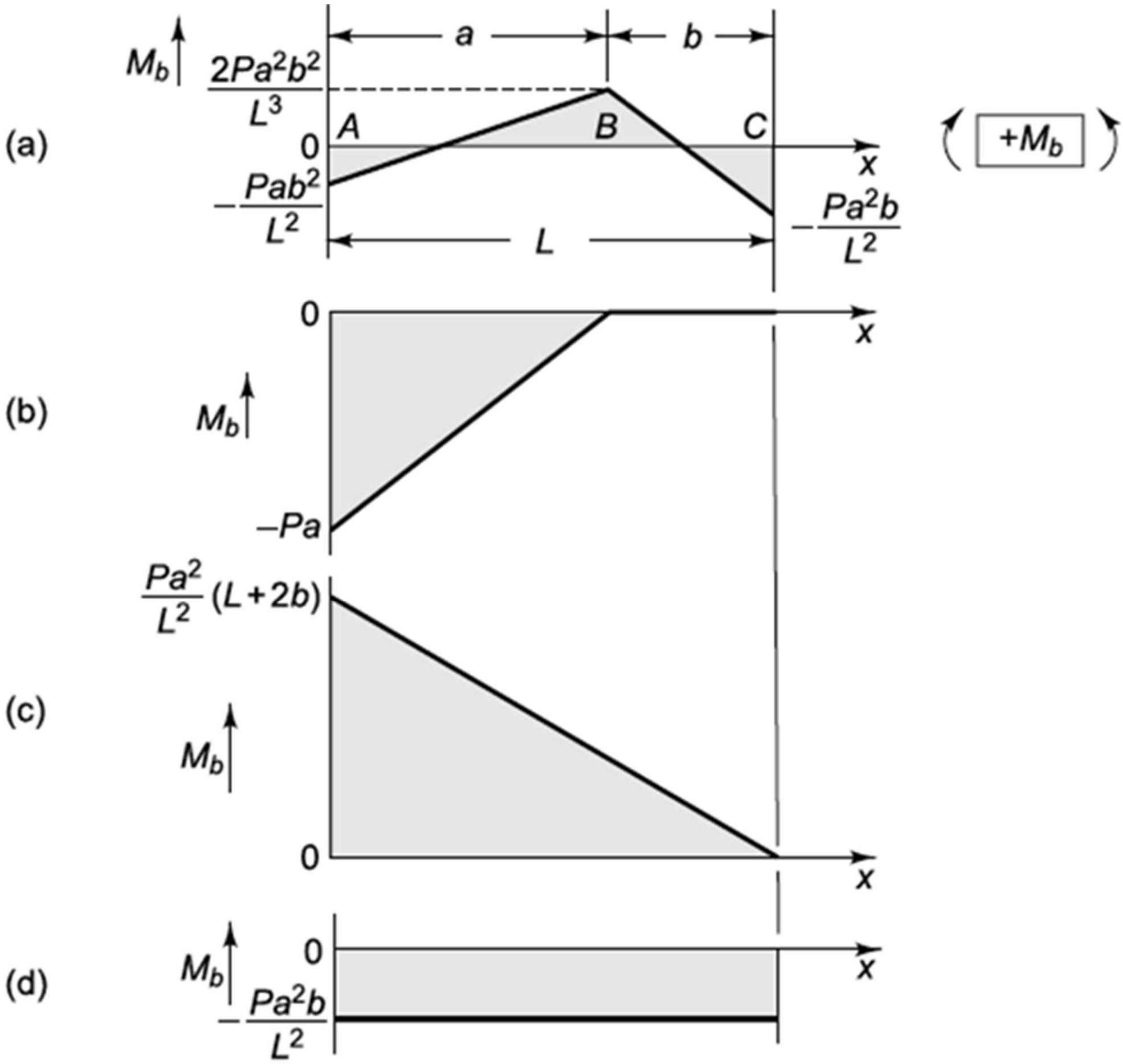

Fig. 8.13 *Example 8.7. Superposition of bending-moment diagrams*

▶ Analysis

$$\begin{cases} \delta_1 - \delta_2 + \delta_3 = 0 \\ \phi_1 - \phi_2 + \phi_3 = 0 \end{cases} \qquad \text{(a)}$$

$$\begin{cases} \delta_1 = \dfrac{Pa^2(3L-a)}{6EI}, & \phi_1 = \dfrac{Pa^2}{2EI} \\ \delta_2 = \dfrac{R_c L^3}{3EI}, & \phi_2 = \dfrac{R_c L^2}{2EI} \\ \delta_3 = \dfrac{M_c L^2}{2EI}, & \phi_3 = \dfrac{M_c L}{EI} \end{cases}$$

$$R_c = \frac{Pa^2(3L-2a)}{L^3}$$

$$M_c = \frac{Pa^2(L-a)}{L^2}$$

$$M_A = \frac{Pab^2}{L^2}$$

▶ **Example 8.8**

Determine the δ_v *and* δ_H at the point D.

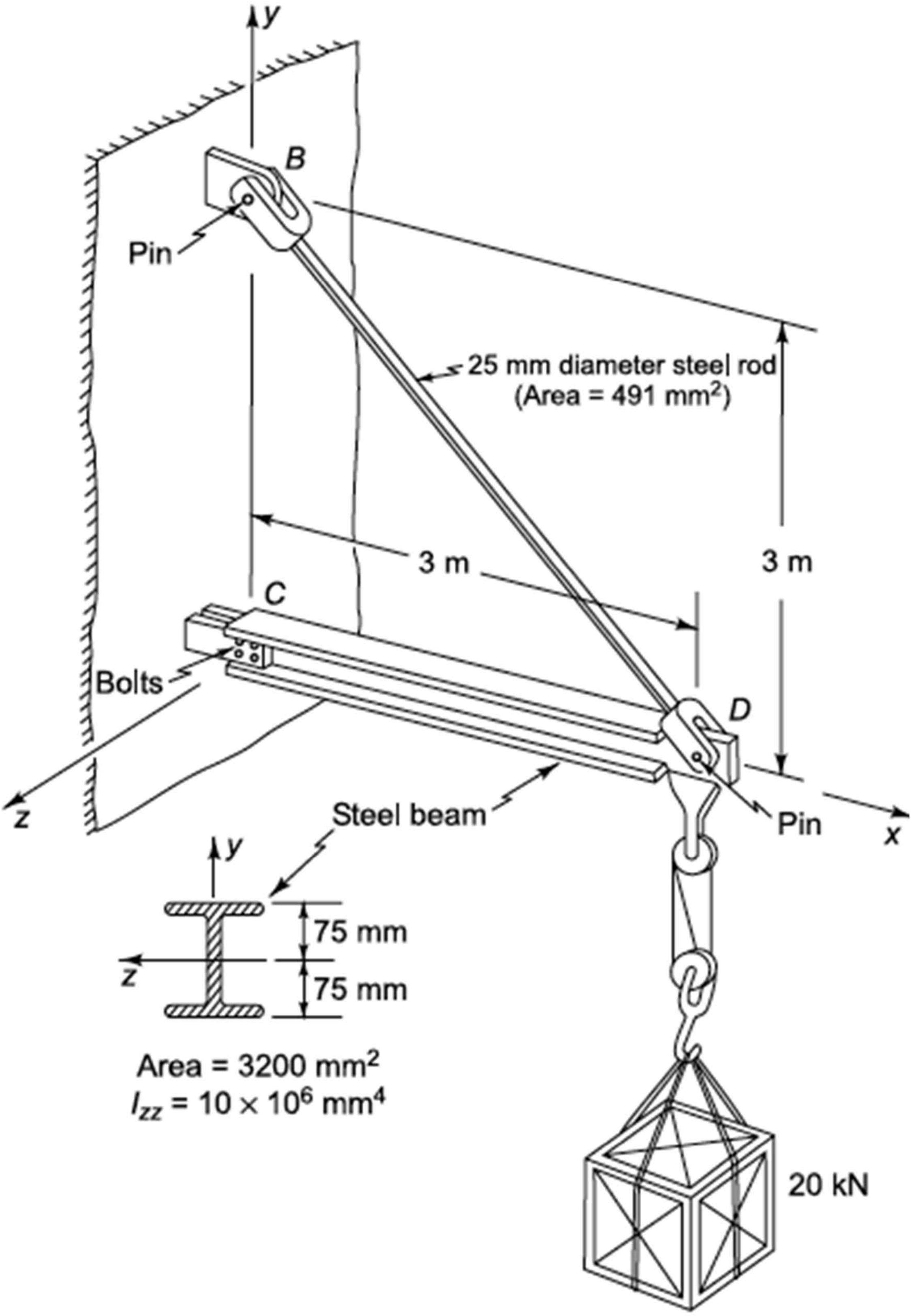

Fig. 8.14 *Example 8.8*

Sol)

▶ Idealization

i) The bolt-joint is completely effective in clamping the beam at C.

ii) The axial compressive force of the beam doesn't affect the bending.

cf. In case of ii), we next consider the case that the axial compressive force affect the bending.

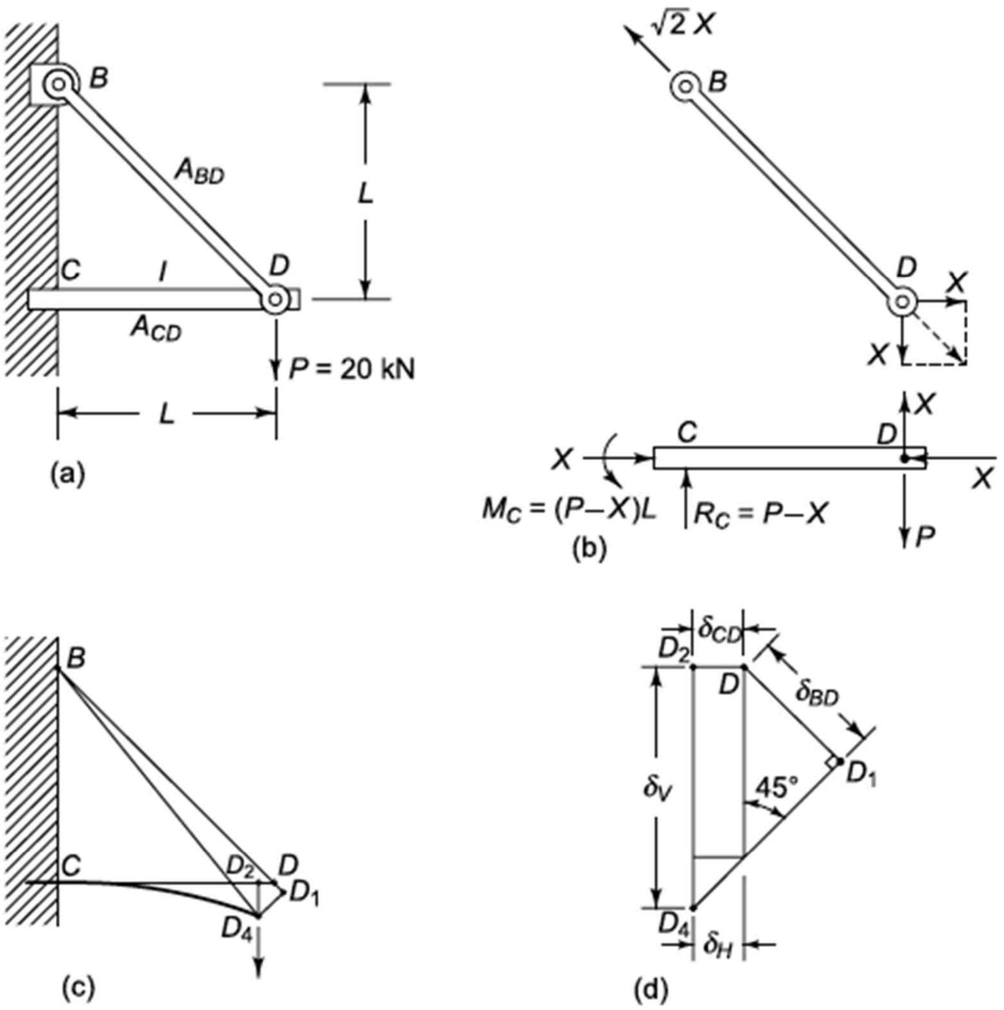

Fig. 8.15 *Example 8.8. Force analysis and geometric analysis for a model based on clamping assumption at C*

▶ Analysis

From Fig. 8.15 (d),

$$\delta_V = \delta_{CD} + \sqrt{2}\delta_{BD} \qquad \text{(a)}$$

From Fig. 8.15 (b),

$$\begin{cases} \delta_{BD} = \dfrac{\sqrt{2}X}{EA_{BD}}\sqrt{2}L = \dfrac{2XL}{EA_{BD}} \\ \delta_{CD} = \dfrac{XL}{EA_{CD}} \end{cases} \qquad \text{(b)(c)}$$

From Table 8.1 – Case 1,

$$\delta_V = \frac{(P-X)L^3}{3EI} \qquad \text{(d)}$$

Insert (b), (c), and (d) into the compatibility relation (a) :

$$X = \frac{P}{1+3I/(A_{CD}L^2)+6\sqrt{2}I/(A_{BD}L^2)} \qquad \text{(e)}$$

$$= \frac{20}{1+0.0014+0.0192} = 19.60\ kN \qquad \text{(f)}$$

$$\therefore \begin{cases} \delta_H = 0.090\ mm \\ \delta_V = 1.760\ mm \end{cases} \qquad \text{(g)}$$

▶ Effect of the compressive force on the bending.

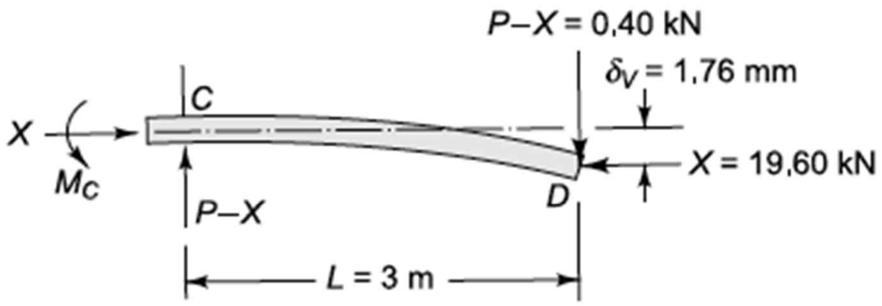

Fig. 8.16 *Example 8.8. Estimation of interaction between compression and bending*

$$\frac{M_C(\text{Due to Compressive Load})}{M_C(\text{Due to Transverse Load})} = X\cdot\frac{\delta_V}{(P-X)L}$$

$$= 19.60\frac{0.00176}{0.4\ (3)} = 0.02875$$

$$\fallingdotseq 2.9\ \%$$

∴ The bending from compressive load can be ignored.

▶ Comparison of peak stress between the pin joint and clamping.

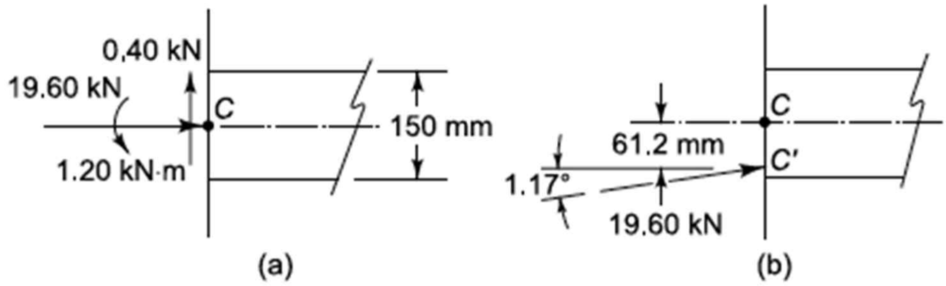

Fig. 8.17 *Example 8.8. Reactions at C shown as a statically equivalent, single force resultant*

i) σ_{CD} at the pin joint

$$\rightarrow \sigma_{CD} = \frac{20.00}{3.2\ (10)^{-3}} = 6.25\ M\ N/m$$

ii) σ_{CD} at the clamping

$$\rightarrow \sigma_{CD} = \frac{19.60}{3.2(10)^{-3}} + \frac{(1.20)(0.075)}{10\ (10)^{-6}} = 15.12\ M\ N/m^2$$

∴ The peak stress can be main factor of local deformation in the slender member.

▶ Truss

→ If the joints are pinned, the structure is called a truss

▶ Frame

→ If the joints are rigid, the structure is called a frame.

cf. The structure in Fig.8-15 is the one example of the mixed structure.

8.5 Load-Deflection Differential Equation

→ As an alternative to using the moment-curvature equation (8.4) to solve beam-deflection problems, we can make use of an equation which directly relates the external loading to the beam deflection.

▶ Load-deflection differential equation

From Chapter 3,

$$\frac{dV}{dx} + q = 0 \tag{3.11}$$

$$\frac{dM_b}{dx} + V = 0 \tag{3.12}$$

$$\therefore \quad \frac{d^2 M_b}{dx^2} = q \tag{8.5}$$

$$\rightarrow \frac{d^2}{dx^2}\left(EI\,\frac{d^2 v}{dx^2}\right) = EIv'''' = q \tag{8.6}$$

$$\& \quad \frac{d}{dx}\left(EI\,\frac{d^2 v}{dx^2}\right) = EIv''' = -V \tag{8.7}$$

▶ Digest

$$EIv'' = M_b \tag{8.4}$$

$$EIv''' = -V \tag{8.7}$$

$$EIv'''' = q \tag{8.6}$$

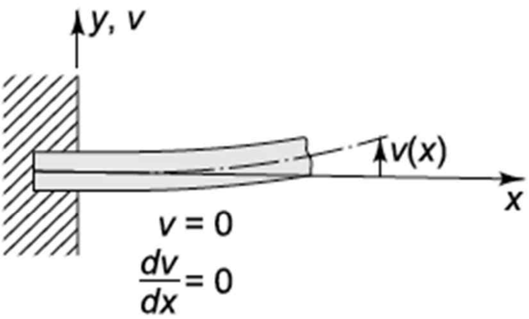

Fig. 8.19 *Built-in or clamped end*

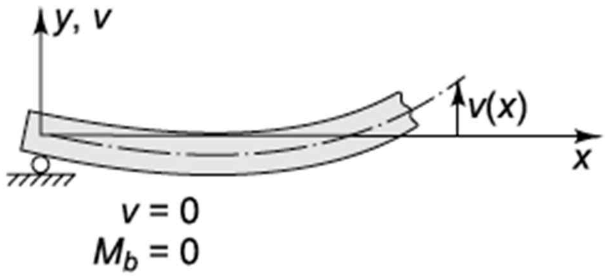

Fig. 8.20 *Simply supported end*

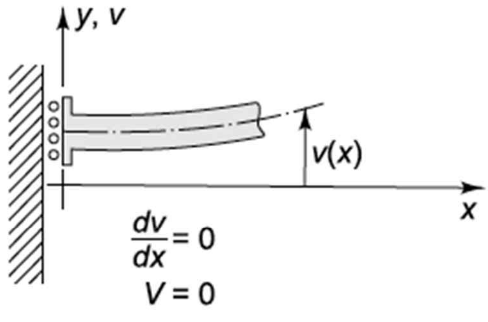

Fig. 8.21 *End restrained against rotation but free to displace*

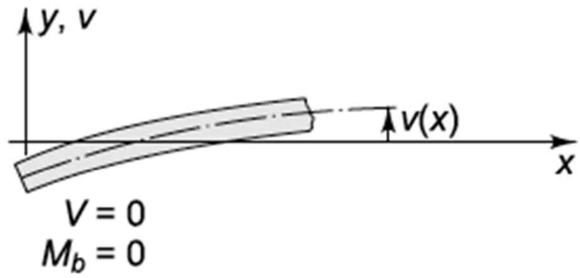

Fig. 8.22 *Free end*

cf. The Fig. 8.19, 8.20, 8.21 and 8.22 represent the support condition at the support point.

▶ Problem solving process

i) Set up the loading intensity equation q(x). It's efficient to use singularity function.

ii) Integrate the governing equation and find the four constants of integration.

iii) This procedure is available regardless of whether it is determinate or not.

cf. According to 'Timoshenko & Gere', the constant of integration is always zero if the governing equation q(x) contains all reaction forces. However, in 'Crandall' it can have non zero value, and actually it is true. See the example 3.9

▶ Example 8.9

Determine the deflection at the point B in offset arm. Ignore the weight
Sol)

$$q = \frac{WL}{3}\langle x - L/3\rangle_{-2} - .W\langle x - L/3\rangle_{-1}$$

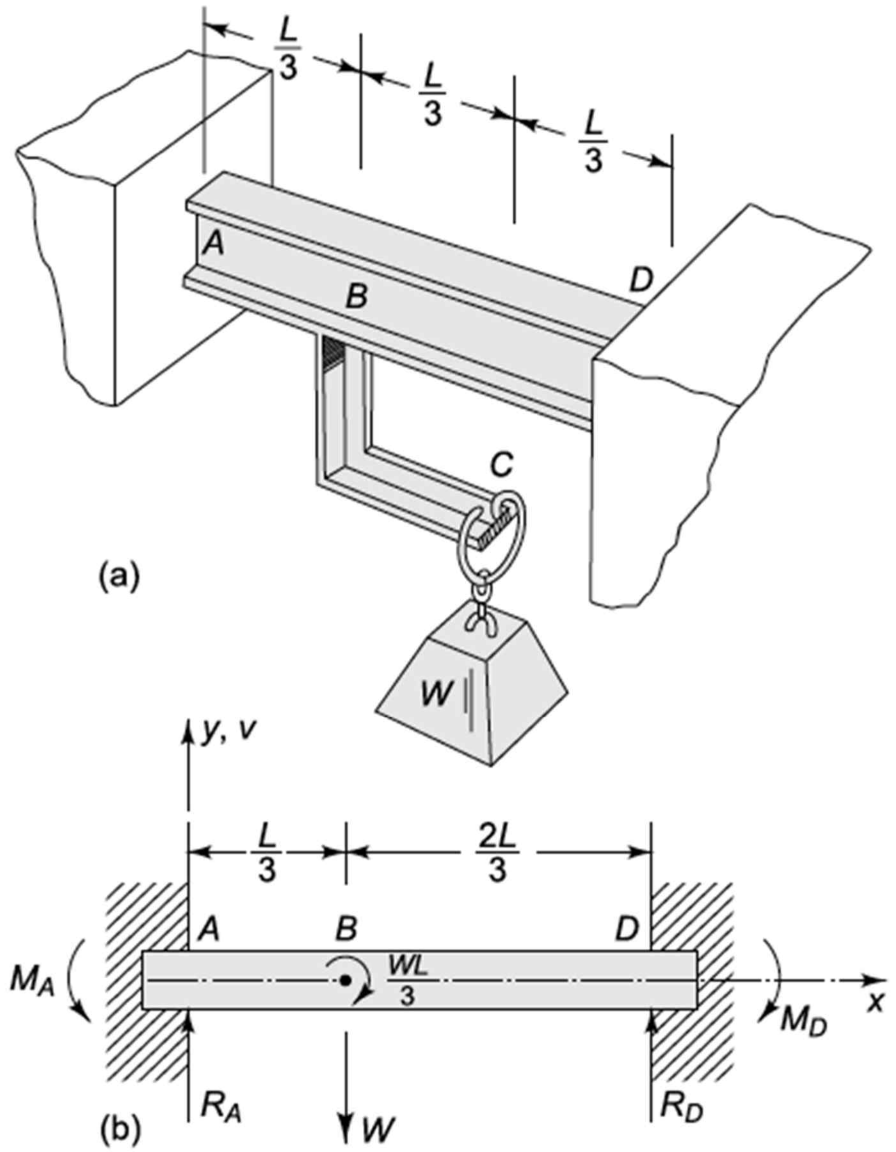

Fig. 8.23 *Example 8.9. Offset loading is equivalent to a force and a couple at B*

The load intensity function q for $0 < x < L$ is

$$q = \frac{W\,L}{3}\langle x - L/3\rangle_{-2} - W\langle x - L/3\rangle_{-1} \qquad \text{(a)}$$

B.C.)

$$\begin{cases} v(0) = v(L) = 0 \\ v'(0) = v'(L) = 0 \end{cases} \tag{b}$$

Insertion of (a) into the load-deflection differential equation (8.6) yields

$$EI\,\frac{d^4v}{dx^4} = W\left[\frac{L}{3} < X - L/3 >_{-2} - < x - L/3 >_{-1}\right] \tag{c}$$

Expressions for dv/dx and v are obtained by integrating (c).

$$\frac{dv}{dx} = \frac{W}{EI}\left[\frac{L}{3}\langle x - L/3\rangle^1 - \frac{\langle x-L/3\rangle^2}{2} + c_1\frac{x^2}{2} + c_2 x + c_3\right] \tag{d}$$

$$v = \frac{W}{EI}\left[\frac{L}{6}\langle x - L/3\rangle^2 - \frac{\langle x-L/3\rangle^3}{6} + c_1\frac{x^3}{6} + c_2\frac{x^2}{2} + c_3 x + c_4\right] \tag{e}$$

Substitution of (d) and (c) into the boundary conditions (b) gives four simultaneous equations for the constants of integration. Their solution is

$$c_1 = \frac{8}{27}\ ,\ c_2 = -\frac{4}{27}L\ ,\ c_3 = 0\ ,\ c_4 = 0 \tag{f}$$

Inserting these in (e) we find

$$v = \frac{W}{27\,EI}\left[\frac{9}{2}L\langle x - L/3\rangle^2 - \frac{9}{2}\langle x - L/3\rangle^3 + \frac{4}{3}x^3 - 2Lx^2\right] \tag{g}$$

We obtain the desired deflection by setting $x = L/3$

$$\therefore\quad \delta_B = -(v)_{x=L/3} = \frac{14\,WL^3}{2{,}187\,EI} \tag{h}$$