# Cryogenic Engineering

2017 Fall Semester

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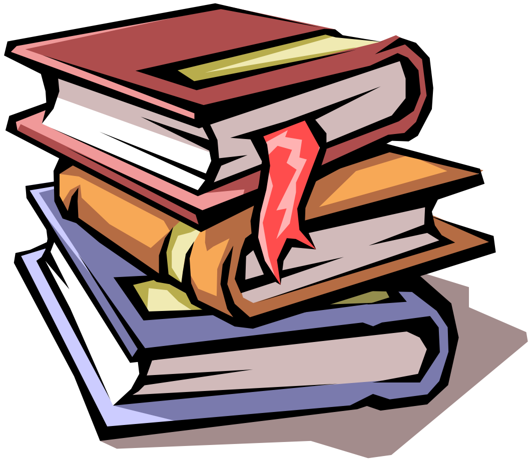
# Chapter 6.

# Measurement Systems for Low Temperatures



# 6.1 Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

① Mercury in a capillary tube

② Pt (platinum) wire resistance

③ Ideal gas pressure

④ Thermoelectric emf (voltage)

⑤ Equilibrium pressure of gas

⑥ Difference in thermal expansion

⑦ Speed of sound

⑧ Magnetic susceptibility

## ▪ Temperature measurement from temperature property

① Mercury in a capillary tube



< Mercury in a capillary tube >

| | |
|---|---|
| Mercury in glass | -39 °C to +357 °C |
| Pressurised mercury in glass | -39 °C to +500 °C |
| Pressurised mercury in quartz | -39 °C to +800 °C |
| Alcohol in glass | -120 °C to +60 °C |
| Pentane in glass | -200 °C to +30 °C |

< The ranges of the most common liquid-in-glass thermometers >

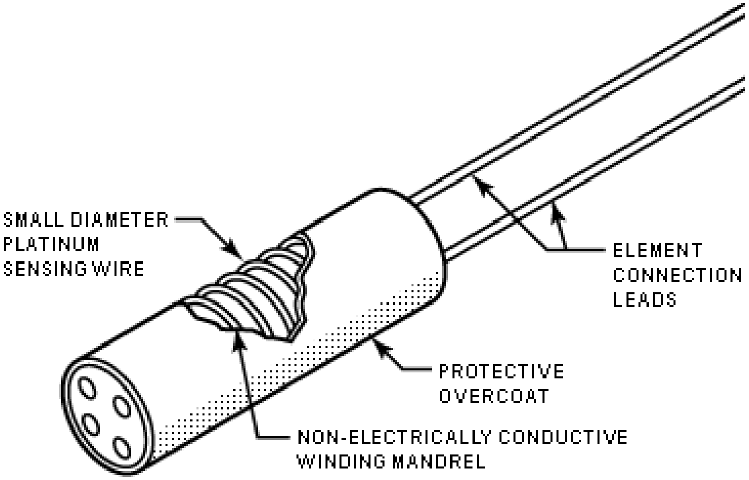
# 6.1 Theoretical plate calculations for columns

# 6.1 Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

② Pt (Platinum) wire resistance

< Platinum >




< wire-wound sensing element >

# 6.1 Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

③ Ideal gas pressure

**Properties**

Density = $\rho$ Pressure = p Temperature = T Volume = V Mass = M

**Observations**

*Boyle:* For a given mass, at constant temperature, the pressure times the volume is a constant. $pV = C_1$

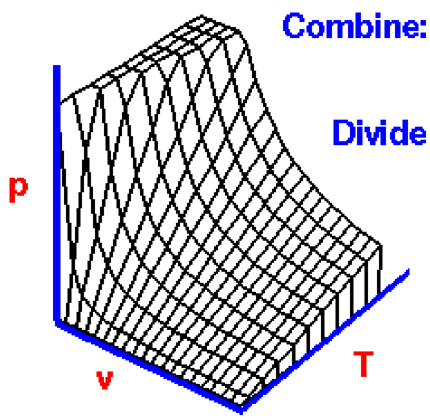
*Charles and Gay-Lussac:* For a given mass, at constant pressure, the volume is directly proportional to the temperature. $V = C_2 T$

**Combine:** $pV/T = n\bar{R}$ $\bar{R}$ = 8.31 J / mole / K (Universal)

$pV = n\bar{R}T$ n = number of moles

**Divide by mass:** $pv = \dfrac{n\bar{R}T}{M}$ Specific Volume = v

$v = \dfrac{\text{volume}}{\text{mass}} = \dfrac{1}{\rho}$



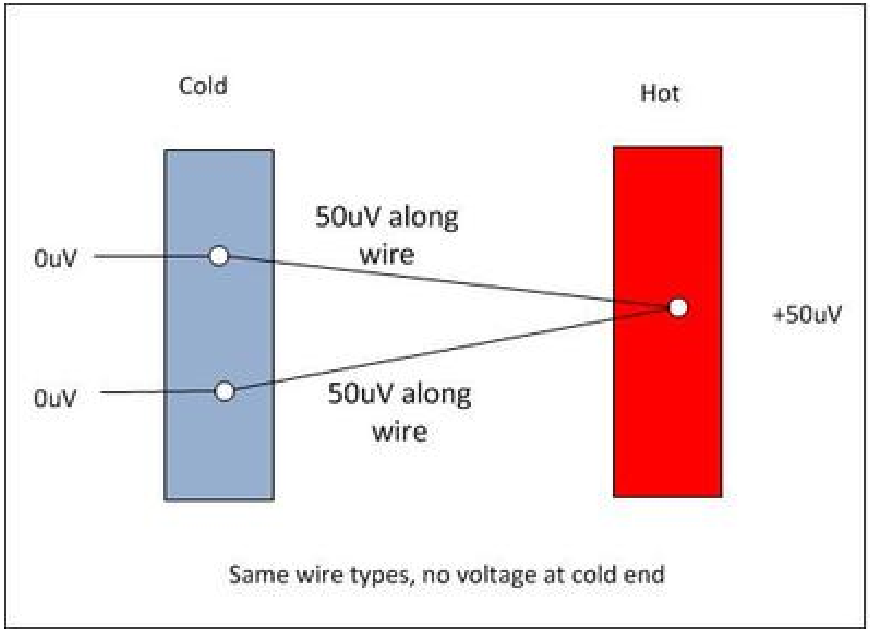

$pv = RT$ or $p = R\rho T$

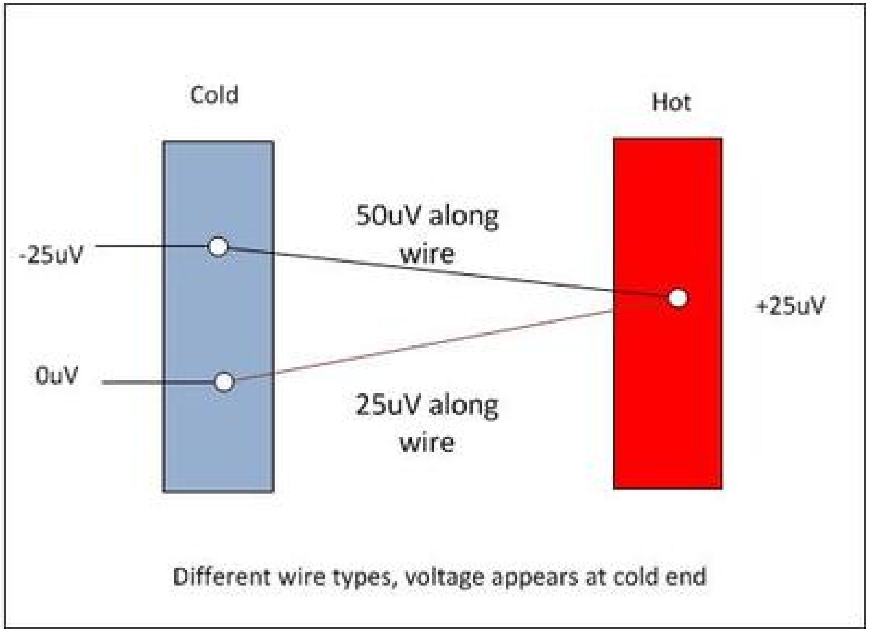
R = Constant value for each gas

= .286 kJ/ kg / K (for air)

- **Temperature measurement from temperature property**

④ Thermoelectric EMF(Electro Motive Force)

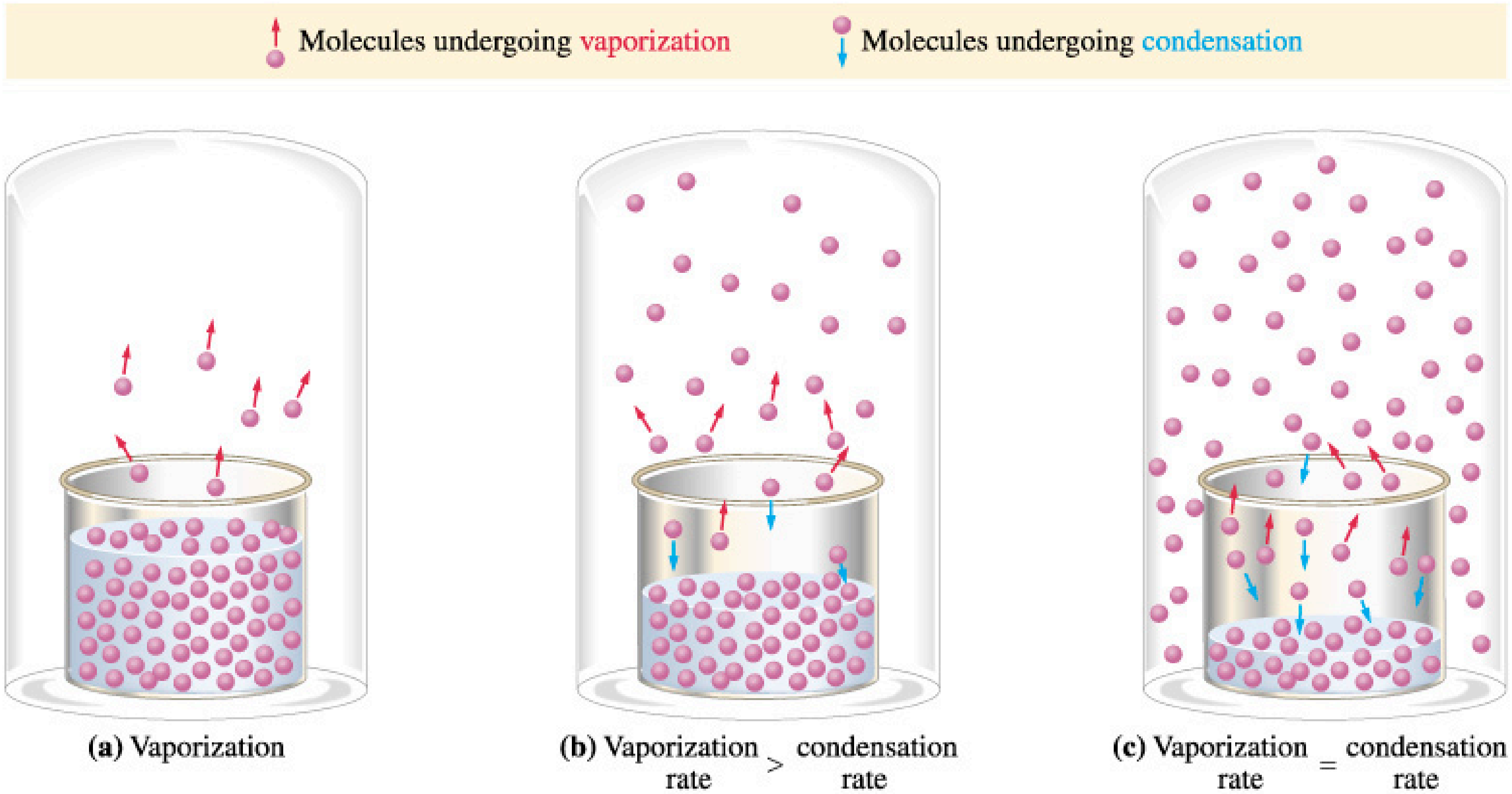


# 6.1 Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

⑤ Equilibrium pressure of gas



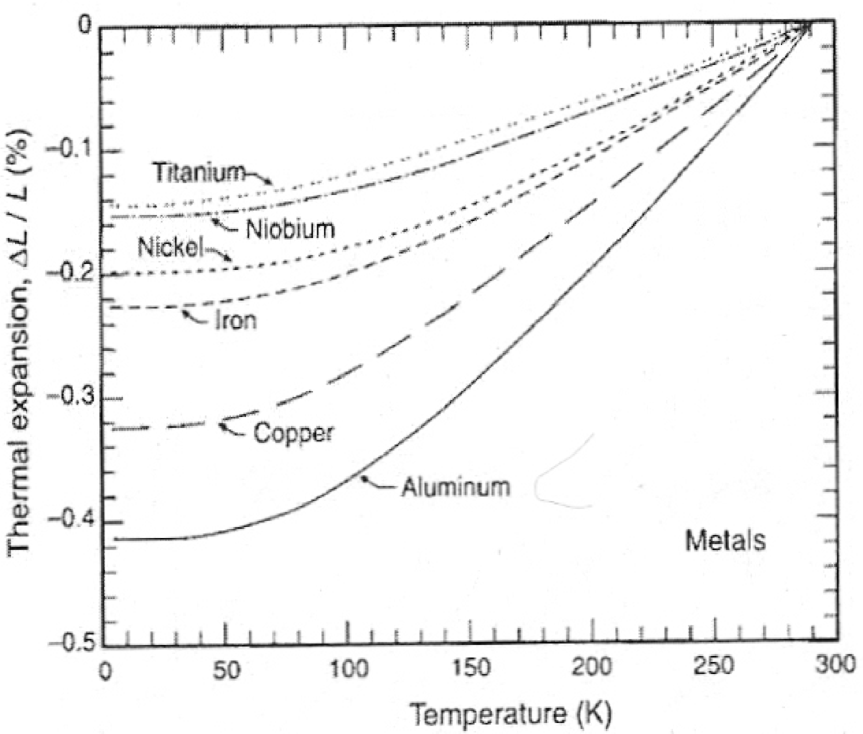
# 6.1 Theoretical plate calculations for columns

▪ **Temperature measurement from temperature property**

⑥ Difference in thermal expansion

| Material | Fractional expansion per degree C x10^-6 | Fractional expansion per degree F x10^-6 |
|---|---|---|
| Glass, ordinary | 9 | 5 |
| Glass, pyrex | 4 | 2.2 |
| Quartz, fused | 0.59 | 0.33 |
| Aluminum | 24 | 13 |
| Brass | 19 | 11 |
| Copper | 17 | 9.4 |
| Iron | 12 | 6.7 |
| Steel | 13 | 7.2 |
| Platinum | 9 | 5 |
| Tungsten | 4.3 | 2.4 |
| Gold | 14 | 7.8 |
| Silver | 18 | 10 |

< Thermal Expansion Coefficients at 20 ℃ >

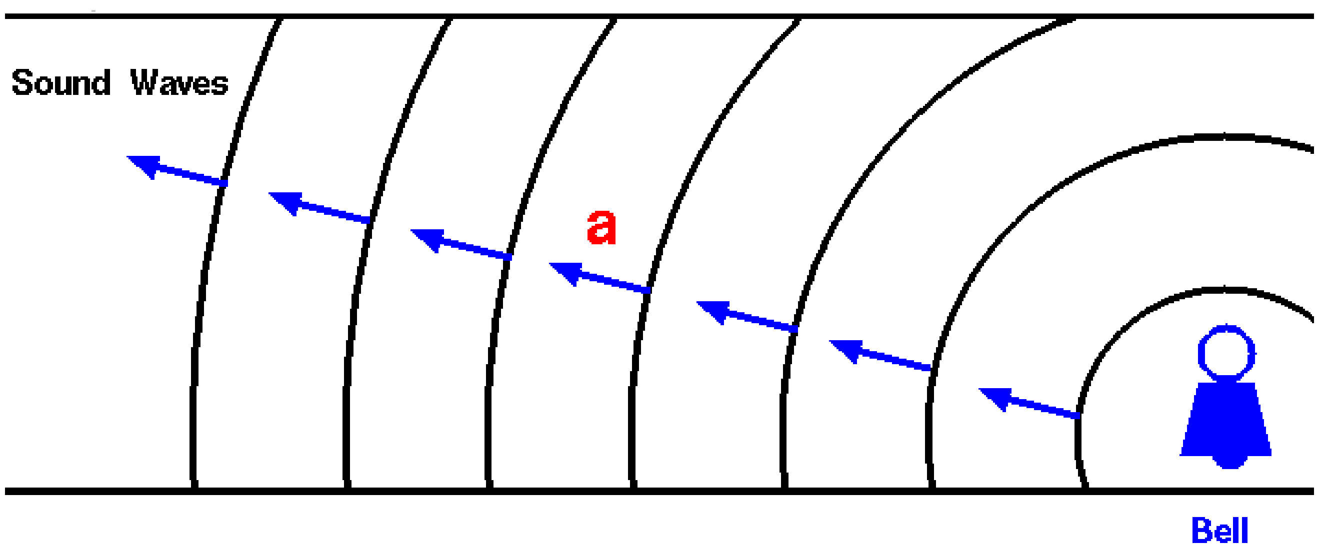



< Thermal linear expansion of common metals>

# 6.1 Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

⑦ Speed of sound




Speed of sound (a) depends on the type of medium and the temperature of the medium.

$$a = \mathrm{sqrt}(\gamma\ R\ T)$$

$\gamma$ = ratio of specific heats (1.4 for air at STP)

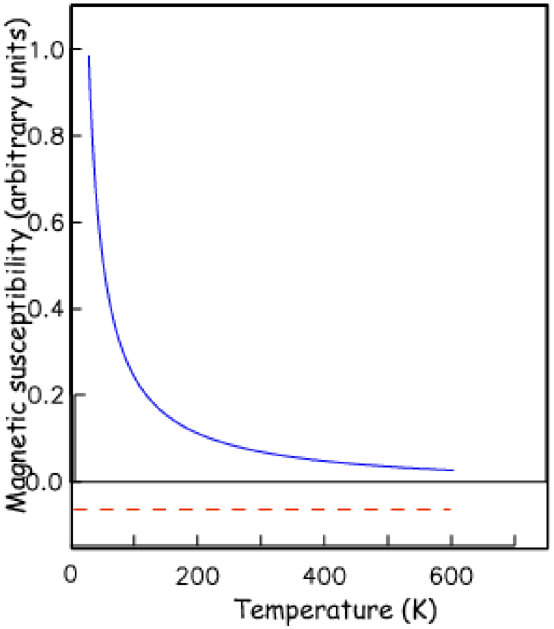
R = gas constant (286 $m^2/s^2/K$ for air)

T = absolute temperature (273.15 + °C)

## ▪ Temperature measurement from temperature property

### ⑧ Magnetic susceptibility

Magnetic susceptibility, quantitative measure of the extent to which a material may be magnetized in relation to a given applied magnetic field. The magnetic susceptibility of a material, commonly symbolized by $\chi m$, is equal to the ratio of the magnetization M within the material to the applied magnetic field strength H, or $\chi m = M/H$. This ratio, strictly speaking, is the volume susceptibility, because magnetization essentially involves a certain measure of magnetism (dipole moment) per unit volume.




< Schematic drawings of paramagnetic (solid line) and diamagnetic (dashed line) magnetic susceptibility as a function of temperature >

# 6.2 Temperature scales and fixed points

- **ITS-27 (International Temperature scale)**
  - The ITS-27 was based on six fixed points.
  - ITS-27 was defined only down to the NBP of oxygen or about - 190℃.

- **ITS-48 (1948)**
  - Adopted several changes in the international temperature scale
  - The same six fixed points were used.

- **ITS-48 (1960)**
  - Triple point of water became a standard fixed point instead of the freezing point of water

# 6.2 Temperature scales and fixed points

- **IPTS-68**

Table 6.1. Temperatures of primary fixed points on the IPTS-68

| Fixed Point | Temperature °C | K | °R |
|---|---|---|---|
| NMP[a] of gold (gold point) | 1064.43 | 1337.58 | 2407.64 |
| NMP of silver | 961.93 | 1235.08 | 2223.14 |
| NMP of zinc | 419.58 | 692.73 | 1246.91 |
| NBP[b] of water (steam point) | 100.00 | 373.15 | 671.67 |
| Standard—triple point of water | 0.01 | 273.16 | 491.69 |
| NBP of oxygen | −182.962 | 90.188 | 162.338 |
| TP[c] of oxygen | −218.789 | 54.361 | 97.850 |
| NBP of neon | −246.048 | 27.102 | 48.784 |
| NBP of hydrogen | −252.87 | 20.28 | 36.50 |
| B.P. of hydrogen at 25 torr | −256.108 | 17.042 | 30.676 |
| TP of hydrogen | −259.34 | 13.81 | 24.86 |

[a]NMP = normal melting point or freezing point

[b]NBP = normal boiling point

[c]TP = triple point

# 6.2 Temperature scales and fixed points

- **ITS-90**

**Table 7-1. Defining Fixed Points of the ITS-90**

| Description | K | °C |
|---|---|---|
| Vapor pressure (VP) point of helium | 3 to 5 | -270.15 to -268.15 |
| Equilibrium hydrogen at triple point (TP) | 13.8033 | 259.3467 |
| Equilibrium hydrogen at VP point | ≈17 | ≈-256.15 |
| Equilibrium hydrogen at VP point | ≈ 20.3 | ≈-252.85 |
| Neon at TP | 24.5561 | 248.5939 |
| Oxygen at TP | 54.3584 | 218.7916 |
| Argon at TP | 83.8058 | 189.3442 |
| Mercury at TP | 234.3156 | 38.8344 |
| Water at TP | 273.16 | 0.01 |
| Gallium at melting point (MP) | 302.9146 | 29.7646 |
| Indium at freezing point (FP) | 429.7485 | 156.5985 |
| Tin at FP | 505.078 | 231.928 |
| Zinc at FP | 692.677 | 419.527 |
| Aluminum at FP | 933.473 | 660.323 |
| Silver at FP | 1234.93 | 961.78 |
| Gold at FP | 1337.33 | 1064.18 |
| Copper at FP | 1357.77 | 1084.62 |

# 6.3 Metallic resistance thermometers

- **Callendar-van Dusen equation**

$$R_e / R_0 = 1 + At + Bt^2 + Ct^3 (t - 100)$$

Typical values for platinum thermometers are

$R_0 = 25\ \Omega$, $A = 3.946 \times 10^{-3}\ °C^{-1}$, $B = -1.108 \times 10^{-6}\ °C^{-2}$

$C = 3.33 \times 10^{-12}\ °C^{-4}$

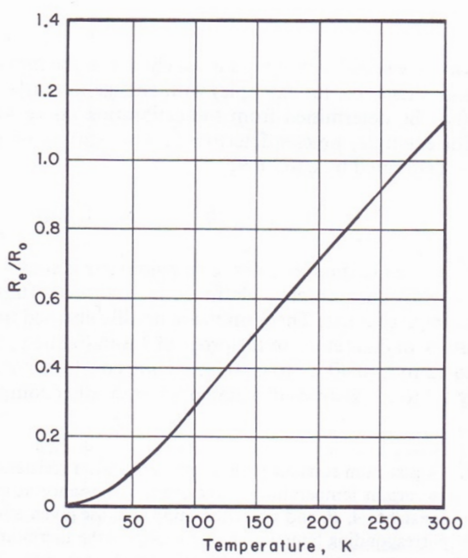


Fig. 6.1. Reduced electric resistance ratio for platinum. $R_0$ is the electric resistance at 0°C.

Choose materials for temperature range

- Platinum (Pt): below 630.74℃

- Indium (In): low temperature range

< Reduced electric resistance ratio for platinum. $R_0$ is the electric at 0 ℃ >

- **Platinum resistance thermometer**

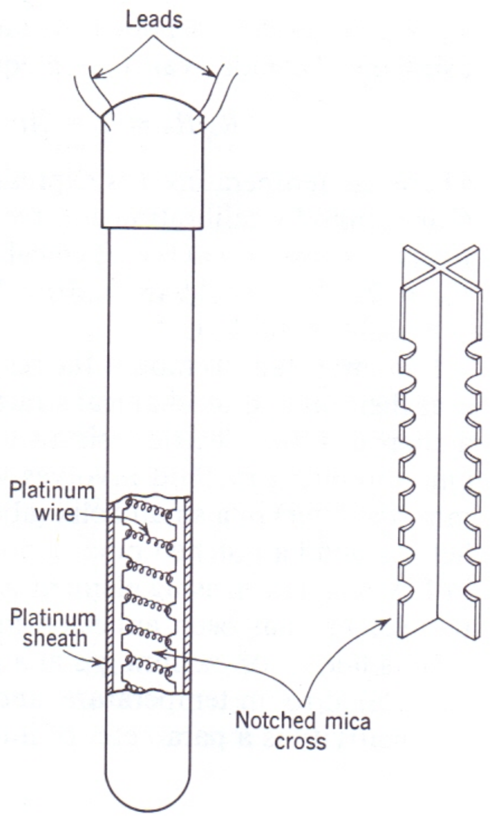
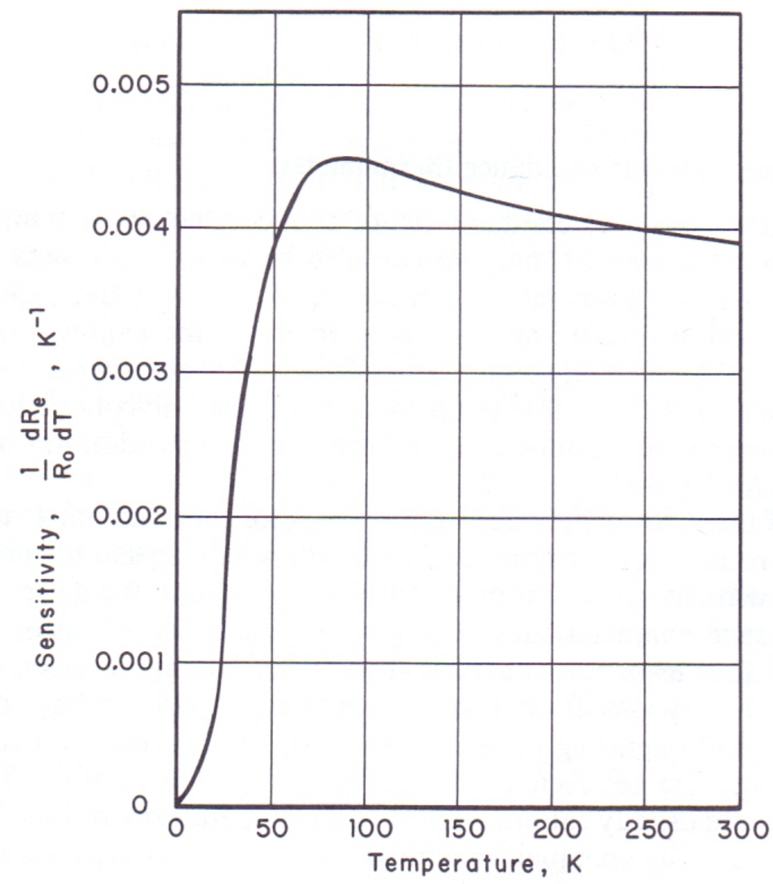



Fig. 6.2. Platinum resistance thermometer.

Fig. 6.3. Sensitivity of a platinum resistance thermometer.



# 6.4 Semiconductor resistance thermometers

- **Electrical conductivity of semiconductors is temperature dependent**

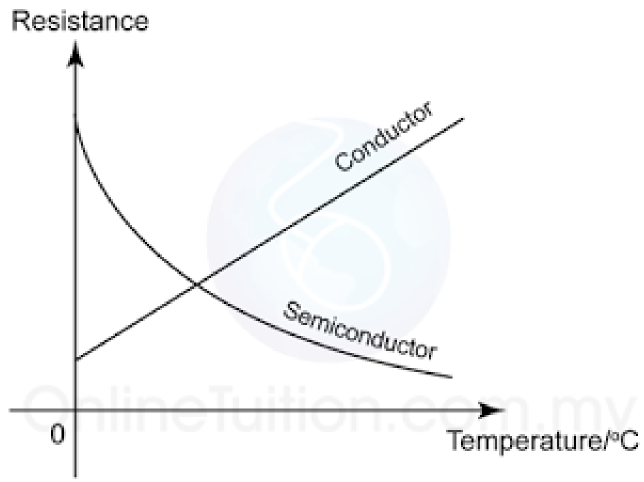
  - High temperature : exponential proportion of the absolute temperature

  - Low temperature : due to the presence of impurities

- **Material of semiconductor thermometer**

  - Germanium : widely used

  - Carbon : for low temperature thermometer




< The resistivity change of a conductor and semiconductor against the temperature >

# 6.5 Thermocouples

One junction of thermocouple pair is placed at the point to be measured, the other junction is placed in a reference temperature region.

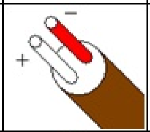
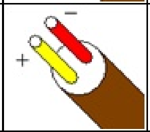
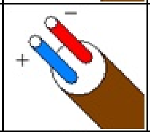
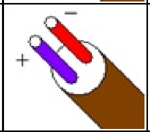
$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

$$t = b_1 e + b_2 e^2 + b_3 e^3 + b_4 e^4$$

t : the difference in temperature

e : the thermocouple output E.M.F value

* Disadvantage : Output E.M.F is quite small

| Type | Material | | Color Code | Range (°C) | |
|---|---|---|---|---|---|
| Thermocouple Grade | Positive Wire | Negative Wire | | Minimum | Maximum |
| J | Iron | Constantan |  | 0 | 750 |
| K | Chromel | Alumel |  | -200 | 1250 |
| T | Copper | Constantan |  | -200 | 350 |
| E | Chromel | Constantan |  | -200 | 900 |

# 6.6 Constant-volume gas thermometer

The constant-volume thermometer may be calibrated by measuring the gas pressure $T_s$ at a standard temperature $p_s$.

From ideal gas law : $T = p(T_s/p_s)$

For accurate work : $$T = \frac{p(T_s/p_s)}{1+(1-p/p_s)(V_0/V)(T_s/T_0)} = K_1 p(T_s/p_s)$$

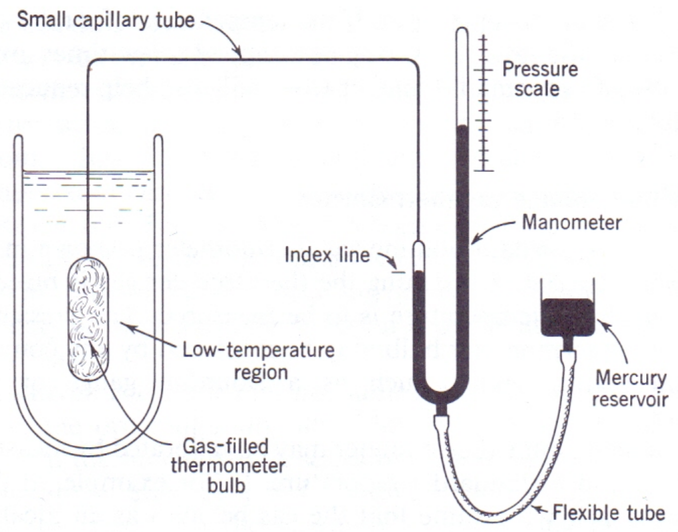



Fig. 6.4. Constant-volume gas thermometer. Other types of pressure gauges could be used instead of the manometer.

# 6.7 Vapor-pressure thermometer

The vapor pressure(saturation pressure) is a definite function of the temperature of the liquid, so this property can be used in determining the temperature.

$$\ln(p/p_0) = C_1 - C_2/T - C_3\ln(T/T_0) - C_4T + C_5T^2$$

- Advantage : Great sensitivity in the applicable temperature range
- Disadvantage :Limited range

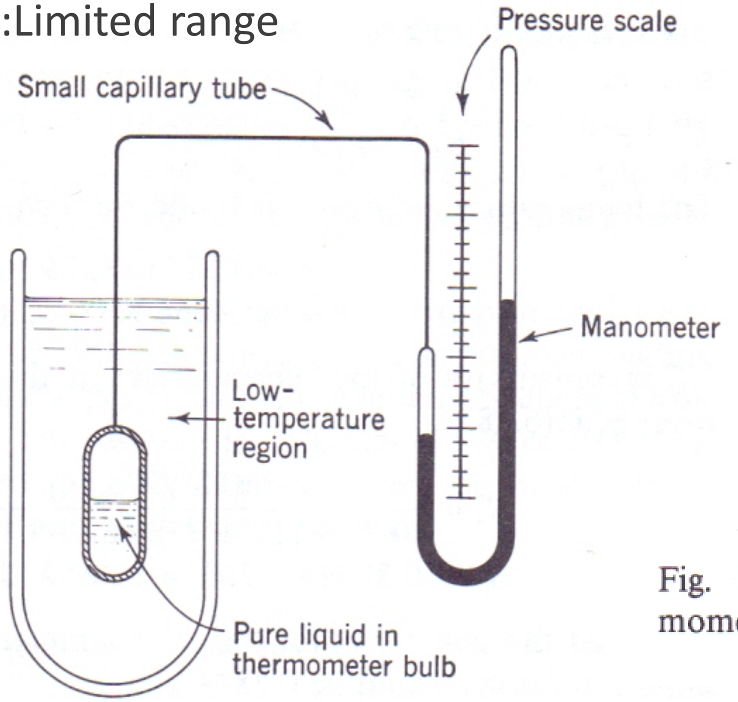



Fig. 6.6. Vapor-pressure thermometer.

# 6.8 Magnetic thermometer

For temperature measurement below 1.0K.

From Curie law, define a magnetic temperature $T^*$

$$T^* = \frac{C}{\chi}\ ; C = The\ Curie\ constant, \chi = the\ magnetic\ susceptibility$$

Following are some correlations of magnetic temperature with the absolute temperature.

- Cerium magnesium nitrate, for $T \geq 0.004\mathrm{K}$

$$T^* = T + (0.236 + 0.004137/T)(10^{-3})$$

- Chromic methylammonium alum, for $T \geq 0.070\mathrm{K}$

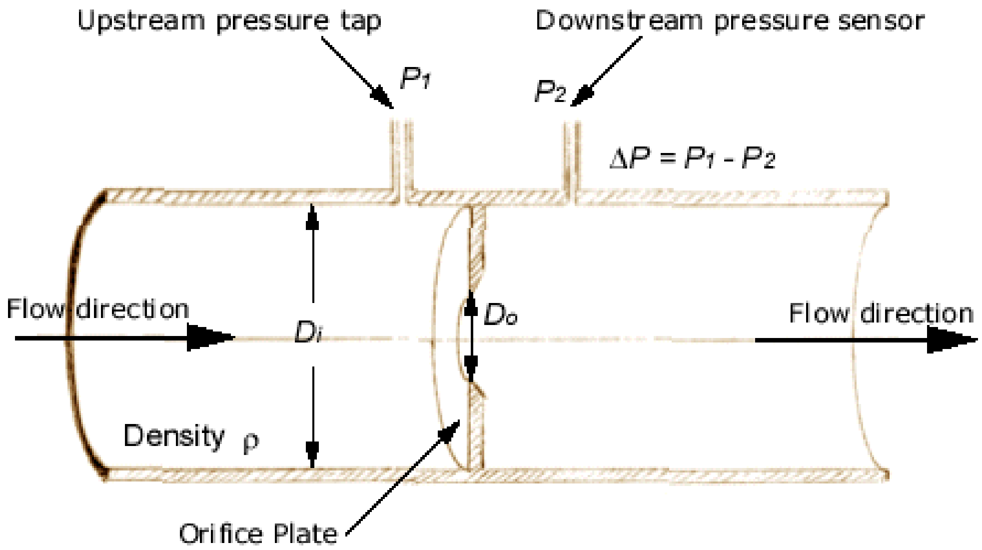
$$T^* = T + 0.00250 + 0.002422/T$$

- Chromium potassium alum, for $T \geq 0.10\mathrm{K}$

$$T^* = T + 0.000862 + 0.002057/T$$

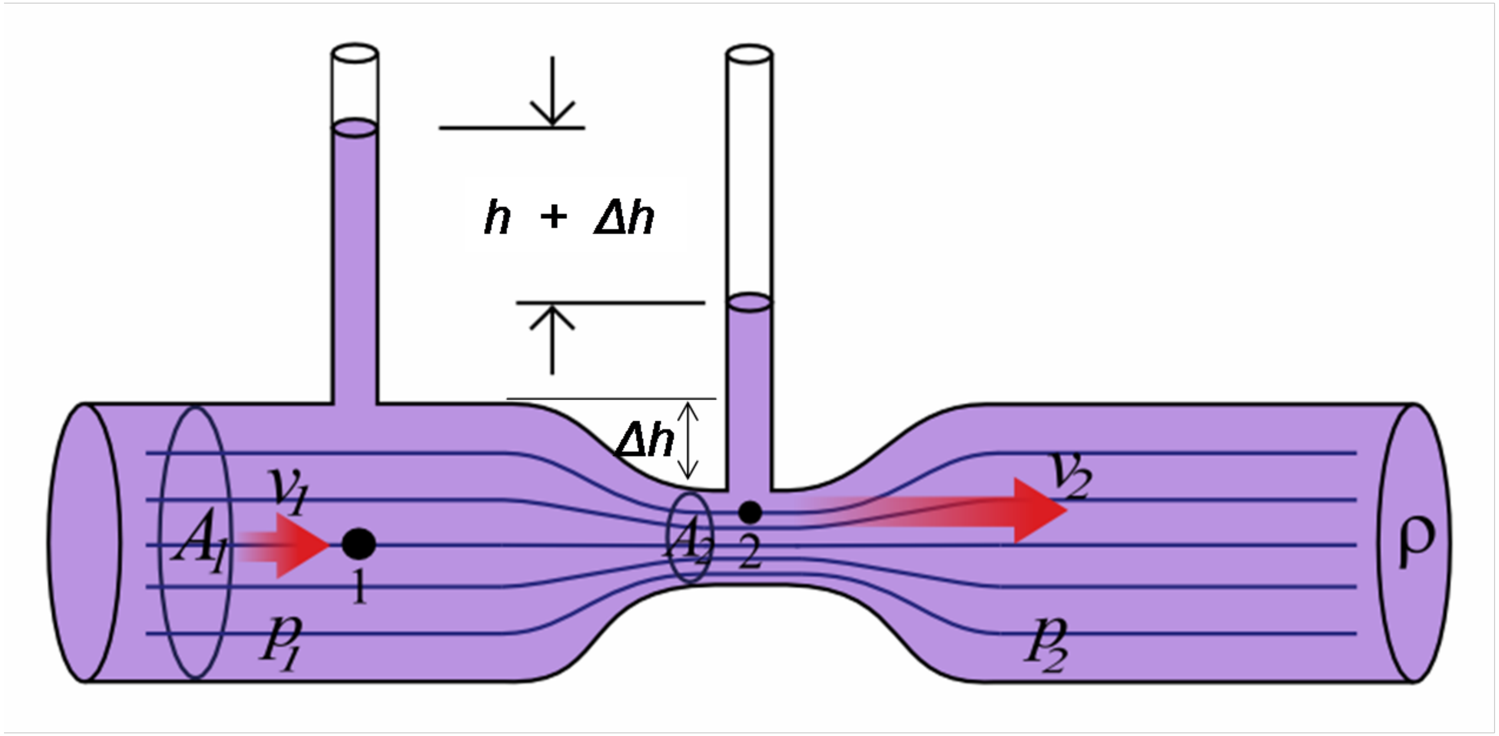
# 6.9 Orifice meters

A fluid passing though an orifice constriction will experience a drop in pressure across the orifice. This change can be used to measure the flowrate of the fluid. To calculate the flowrate of a fluid passing through an orifice plate, enter the parameters below.




$$\Delta p = \frac{1}{2}\,\rho\, Q^2\, \frac{1}{{A_0}^2}\left[1 - \left(\frac{A_o}{A_i}\right)^2\right]$$

# 6.10 Venturi meter




$$P_1 - P_2 = \frac{\rho}{2}\left({v_2}^2 - {v_1}^2\right)$$

# 6.11 Turbine flowmeters

- **Advantages of turbine flow meter**

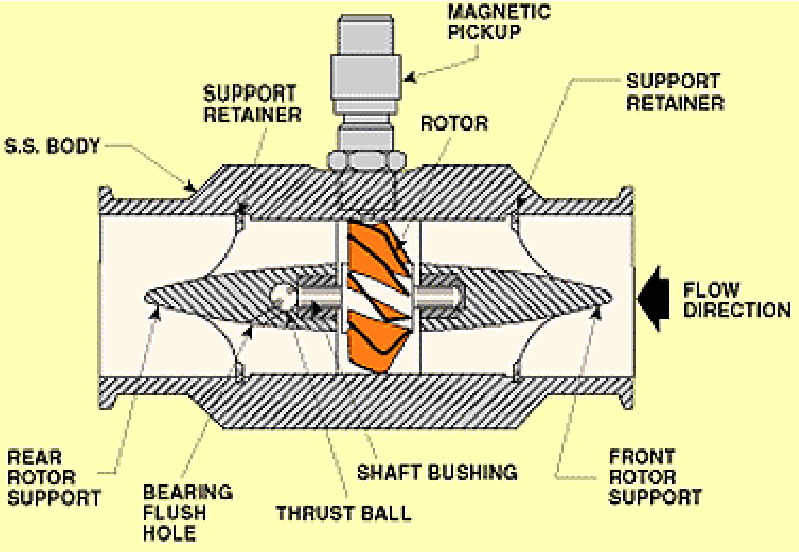
  - Simple, durable structure

  - Easy to install and maintain

  - Turbine meters are able to operate under a wide range of temperatures and pressures

  - Low pressure drop across the flow meter

  - Most effective in applications with steady, high-speed flows



# 6.11 Turbine flowmeters

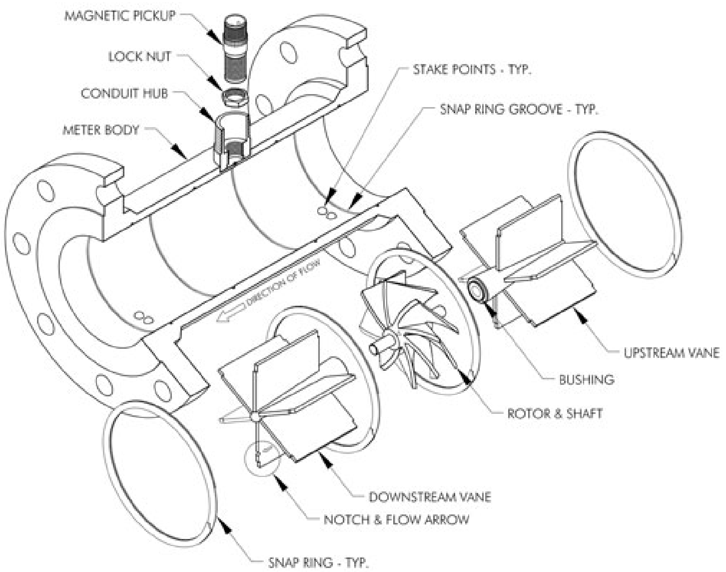
- **Disadvantages of turbine flow meter**

  - Require constant backpressure in order to avoid cavitation

  - Accuracy adversely affected by bubbles in liquids

  - Sensitive to changes in fluid viscosity

**Turbine Meter Kits**



# 6.12 Fluid-quality measurement

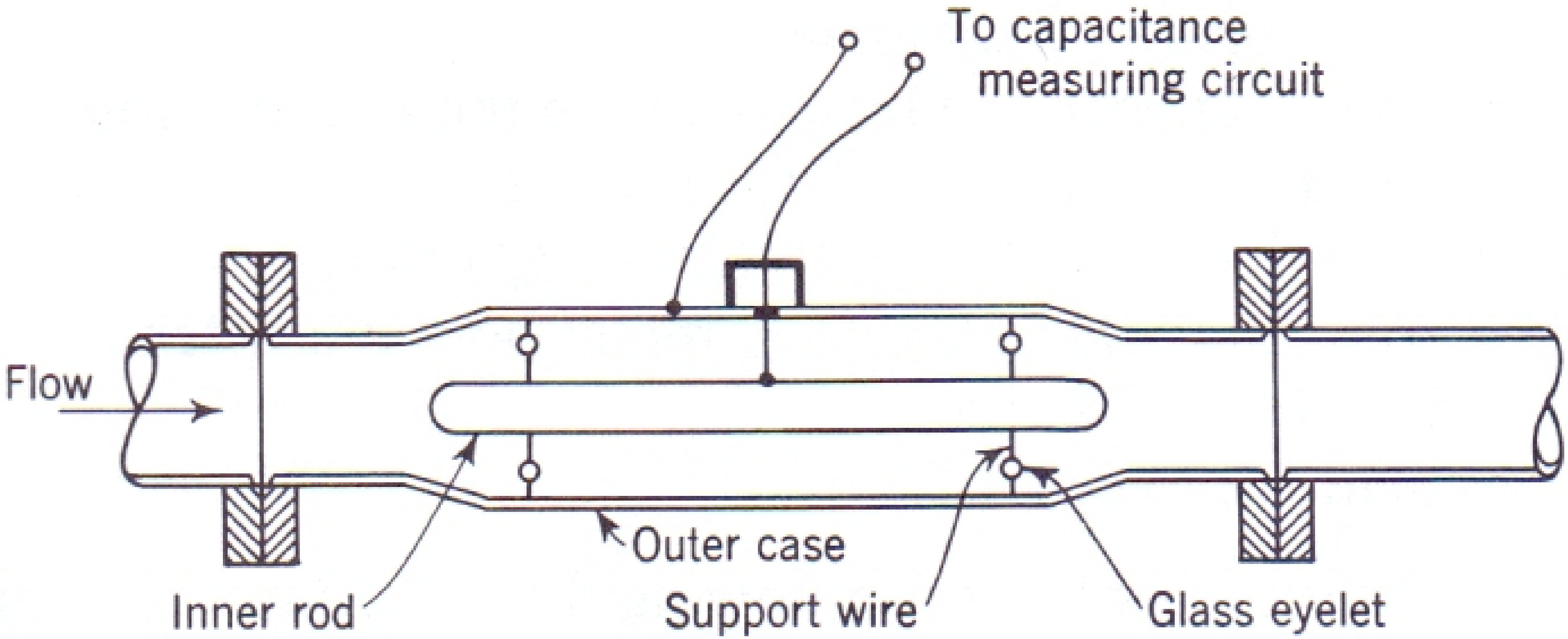



Fig. 6.12. Capacitance quality meter (Killian and Simpson 1960).

# 6.13 Hydrostatic gauges

- **Hydrostatic pressure is related to the liquid level $L_f$ by**

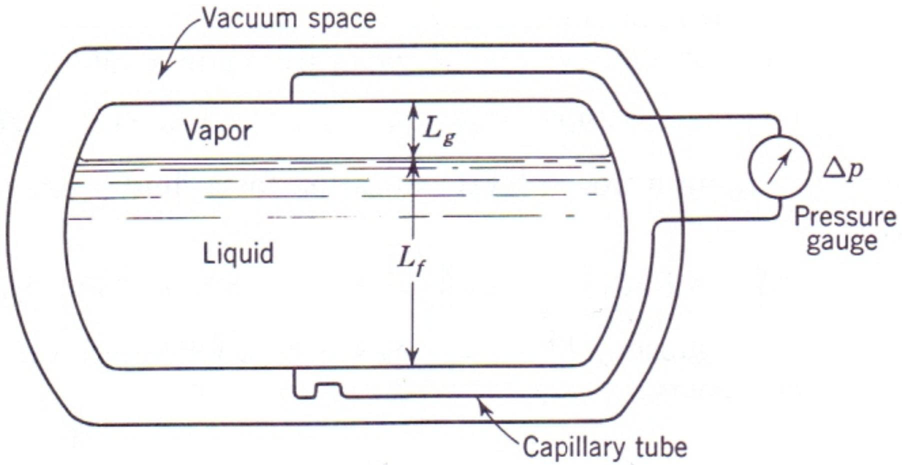



Fig. 6.13. Hydrostatic liquid-level gauge.

$$\Delta P = \frac{\rho_f L_f g}{g_c} + \frac{\rho_g L_g g}{g_c}$$

$L_f$ = height of the liquid column
$L_g$= L-Lf = height of the vapor column
L = inside diameter of the vessel
$\rho_f$ = liquid density
$\rho_g$ = liquid density(saturated)
g = local acceleration due to gravity
$g_c$ = conversion factor in Newton's Second Law of Motion

# 6.14 Electric-resistance gauges

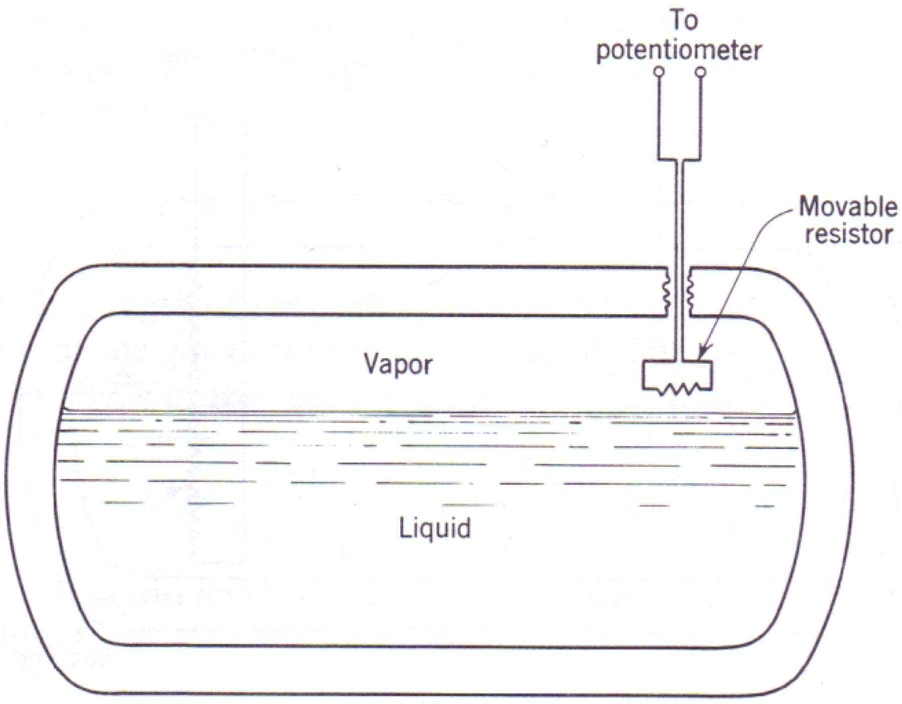



Fig. 6.14. Movable electric resistance liquid-level gauge.

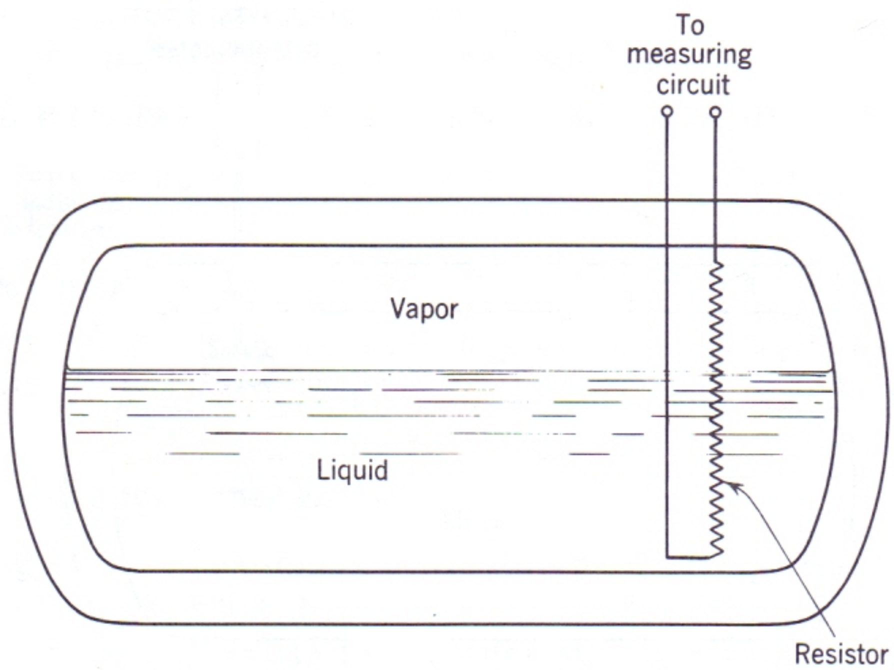
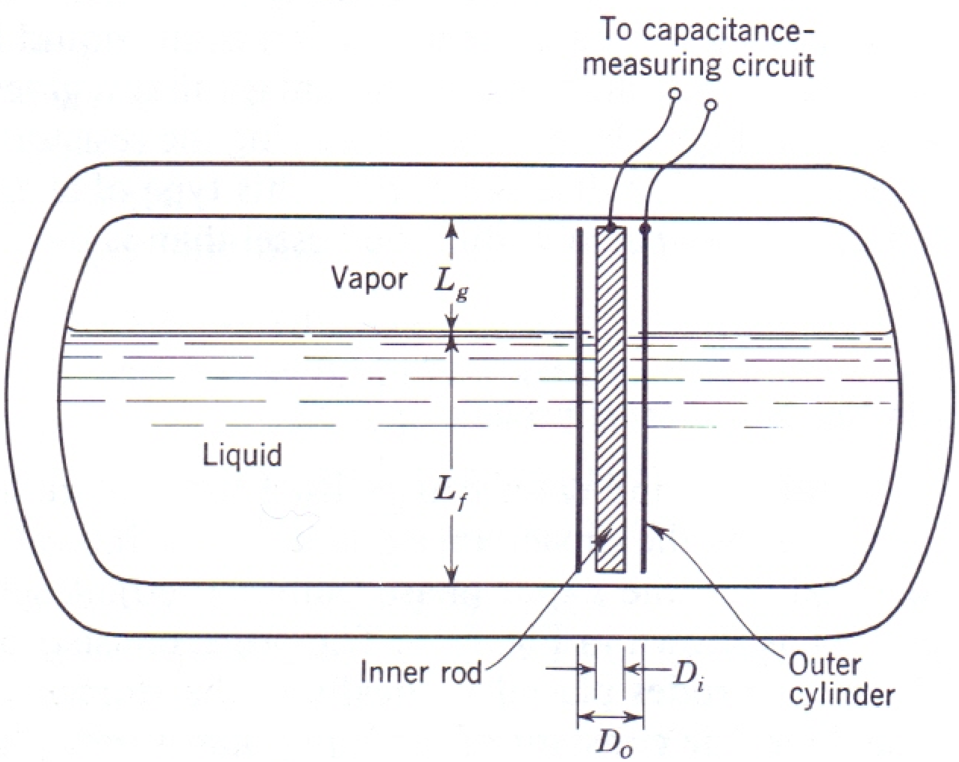



Fig. 6.15. Fixed electric resistance liquid-level gauge.

# 6.15 Capacitance liquid-level probes

Fig. 6.16. Capacitance liquid-level gauge.




$$L_f = \frac{C\ ln\left(\frac{D_0}{D_i}\right)}{2\pi\left(\varepsilon_f - \varepsilon_g\right)\varepsilon_0} - \frac{\varepsilon_g L}{\varepsilon_f - \varepsilon_g}$$

$\varepsilon_0 = 8.8542 * 10^{-12}\frac{\mathrm{F}}{\mathrm{M}} = \text{permittibity of free space}$

$\varepsilon_g = The\ dielectric\ constants\ for\ the\ vapor$

$\varepsilon_f = The\ dielectric\ constants\ for\ the\ fluid$

$C = The\ total\ capacitance\ for\ the\ gauge$

# 6.16 Thermodynamic liquid-level gauge

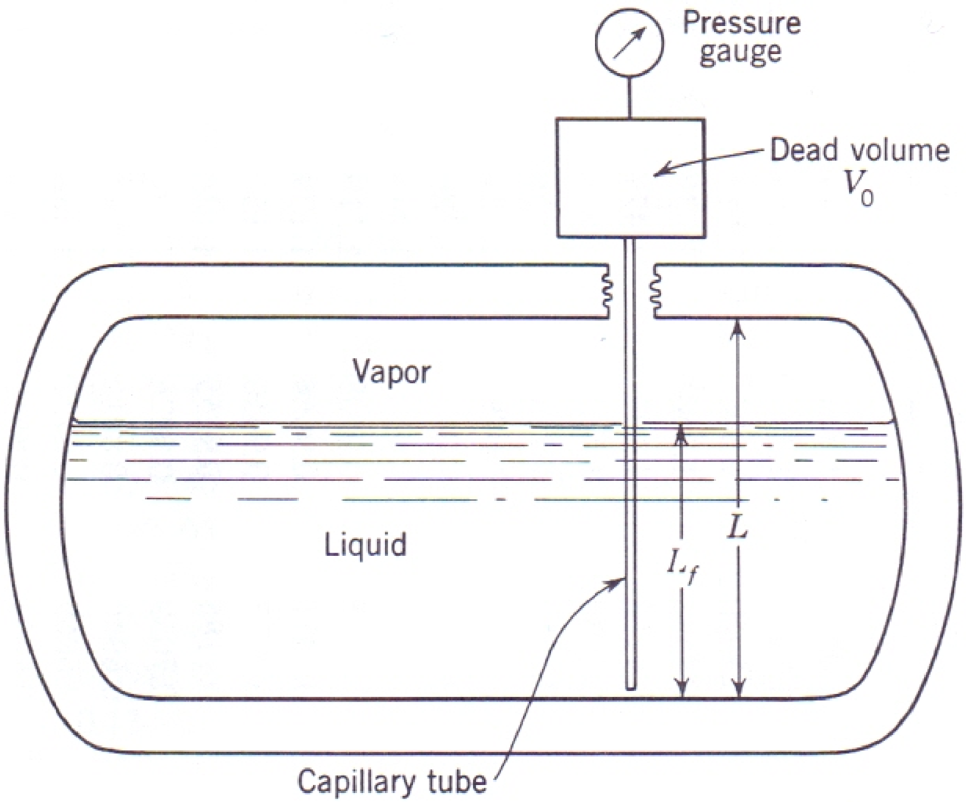



Fig. 6.17. Thermodynamic liquid-level gauge.

$$\frac{L_f}{L} = \frac{[(m - \rho_0 V_0)/A_c L] - \rho_g}{\rho_f - \rho_g}$$