Optimal Design of Energy Systems (M2794.003400)

Chapter 15. Dynamic Behavior of Thermal Systems

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Chapter 15. Dynamic Behavior of Thermal Systems

15.1 In What Situations is Dynamic Analysis Important?

Steady-state	Dynamic
More frequently than dynamic simulations	Address transient problems
Can be justified in the design	Can be corrected in in the field
Ex. Part-load efficiency, Potential operating problems	Ex. System shutdown, Damage the plant, Imprecise control

Dynamic Analysis : with respect to time, on/off, under control, disturbance

Chapter 15. Dynamic Behavior of Thermal Systems

15.2 Scope and Approach of This Chapter

Intention

- Concentration on thermal components
- Emphasis of behavior in the time domain
- The translation of physical situations into symbolic or mathematical representation

Object

- More comfortable in making dynamic analysis
- Representation of the performance in the time domain
- Experience in block diagram

15.3 One Dynamic Element in a Steady-State Simulation

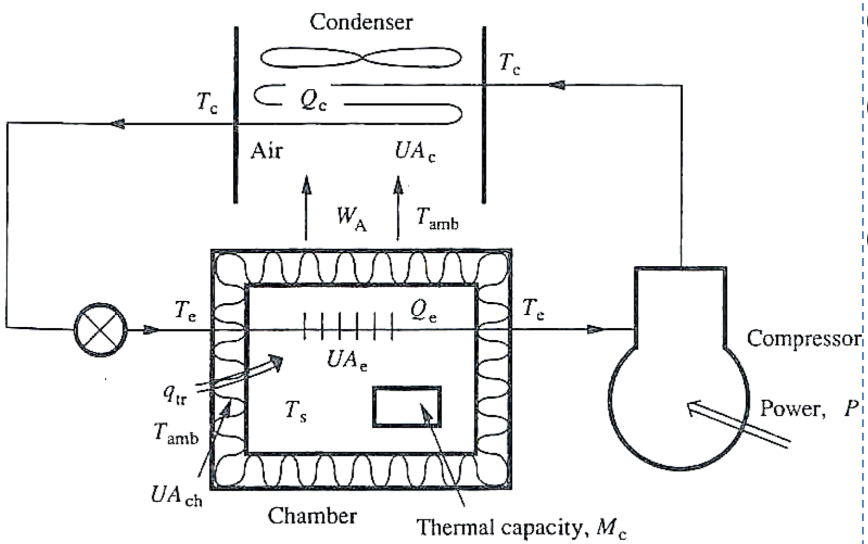


Fig. System with one dynamic element (refrigeration plant serving a cold room)

Compressor ref. capacity $q_e = f_1(T_e, T_c)$

Compressor power $P = f_2(T_e, T_c)$

Condenser $q_c = \dot{m}c_{p,a}(T_c - T_{amb})\underbrace{(1 - e^{\frac{UA}{\dot{m}c_{p,a}}})}_{\varepsilon}$

Evaporator $q_e = (T_s - T_e)(UA_e)$

Energy balance $q_c = P + q_e$

Heat transfer to chamber

$$q_e = q_{tr} = UA_{ch}(T_{amb} - T_s)$$

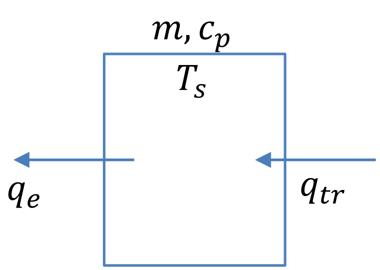
steady-state

15.3 One Dynamic Element in a Steady-State Simulation

Pull-down $$q_{tr} = UA_{ch}(T_{amb}, -T_s)$$

$$q_{tr} = q_e + mc_p \frac{dT_s}{dt}$$

Dynamic : during pull-down $q_{tr} \neq q_e$



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15.4 Laplace transform

- Powerful tool in predicting dynamic behavior
- One way to solve ODE

$$L\{F(t)\} = \int_0^\infty F(t)e^{-st}dt = f(s)$$

$$L\{F'(t)\} = \int_0^\infty F'(t)e^{-st}dt$$

$$= e^{-st}F(t)]_0^\infty - \int_0^\infty F(t)(-s)\,e^{-st}dt$$

$$= -F(0) + sf(s)$$

$$L\{F''(t)\} = \int_0^\infty F''(t)e^{-st}dt$$

$$= e^{-st}F'(t)]_0^\infty - \int_0^\infty F'(t)(-s)\,e^{-st}dt$$

$$= -F'(0) + s[-F(0) + sf(s)]$$

$$= -F'(0) - sF(0) + s^2 f(s)$$

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15.4 Laplace Transforms

Example 15.1 : What is the Laplace transform of the constant *c* ?

(Solution)

$$\mathcal{L}\{c\} = \int_0^\infty c\, e^{-st} dt = -\frac{c}{s} e^{-st}\Big]_0^\infty = \frac{c}{s}$$

Example 15.2 : What is the Laplace transform of *bt* ?

(Solution)

$$\mathcal{L}\{bt\} = \int_0^\infty bt\, e^{-st} dt = -b\frac{d}{ds}\int_0^\infty e^{-st}\, dt = -b\frac{d(1/s)}{ds} = b/s^2$$

15.5 Inversion of Laplace Transforms $L^{-1}\{f(s)\} = F(t)$

Example 15.4 : Invert $\dfrac{s+10}{(s-2)^2(s+1)}$

(Solution) $$\frac{s+10}{(s-2)^2(s+1)} = \frac{A}{(s+1)} + \frac{B}{(s-2)^2} + \frac{B'}{s-2}$$

$$constants : 10 = 4A + B - 2B'$$

$$s : \quad 1 = -4A + B - B'$$

$$s^2 : \quad 0 = A + B'$$

$$A = 1, \quad B = 4, \quad B' = -1$$

$$\therefore \; L^{-1}\left\{\frac{s+10}{(s-2)^2(s+1)}\right\} = e^{-t} + 4te^{2t} - e^{2t}$$

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15.5 Inversion of Laplace Transforms $L^{-1}\{f(s)\} = F(t)$

(Another Solution of Example 15.4)

- ✓ **For non-repeated roots**

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{s-b} + \cdots \qquad A = \frac{N(s)(s-a)}{D(s)}\Big|_{s\to a} \qquad B = \frac{N(s)(s-b)}{D(s)}\Big|_{s\to b}$$

- ✓ **For repeated roots**

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{(s-b)^2} + \frac{B'}{s-b} \qquad B = \frac{N(s)(s-b)^2}{D(s)}\Big|_{s\to b} \qquad B' = \frac{d}{ds}\left[\frac{N(s)(s-b)^2}{D(s)}\right]_{s\to b}$$

$$\to A = \frac{s+10}{(s-2)^2}\Big|_{s\to -1} = 1 \qquad B = \frac{s+10}{s+1}\Big|_{s\to 2} = 4 \qquad B' = \left(\frac{s+10}{s+1}\right)\Big|_{s\to 2} = -1$$

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15.6 Solution of ordinary differential equations

Example 15.6 : Solve $\boldsymbol{Y''(t) + k^2 Y(t) = 0}$

(boundary conditions : $Y(0) = A, Y'(0) = B$)

(Solution)

Transform the differential equation

$s^2 y(s) - sY(0) - Y'(0) + k^2 y(s) = 0$

Boundary conditions

$$y(s) = \frac{As}{s^2 + k^2} + \frac{B}{s^2 + k^2}$$

Invert y(s)

$Y(t) = Acos(kt) + (B/k)sin(kt)$

15.7 Blocks, Block diagrams, and transfer functions

- Variable in S domain (not in time domain)

Transfer function : $$TF = \frac{L\{O(t)\}}{L\{I(t)\}} = \frac{O(s)}{I(s)}$$

ratio of the output to the input

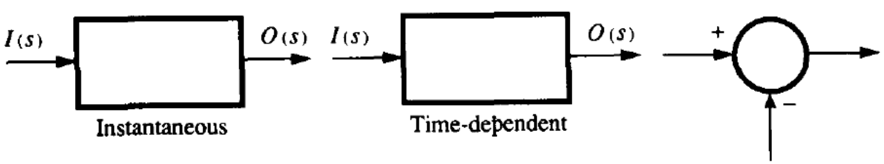


Fig. Symbols used in block diagrams

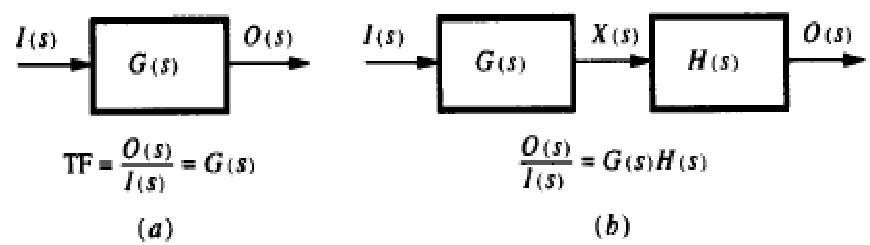


Fig. Transfer function and cascading of blocks

15.8 Feedback Control Loop

Unity feedback $TF = \frac{G(s)}{1+G(s)}$

Non-unity feedback $TF = \frac{G(s)}{1+G(s)H(s)}$

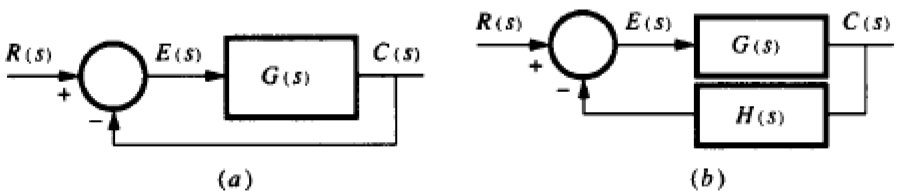


Fig. (a) Unity feedback loop

(b) Nonunity feedback loop

15.9 Time Constant Blocks

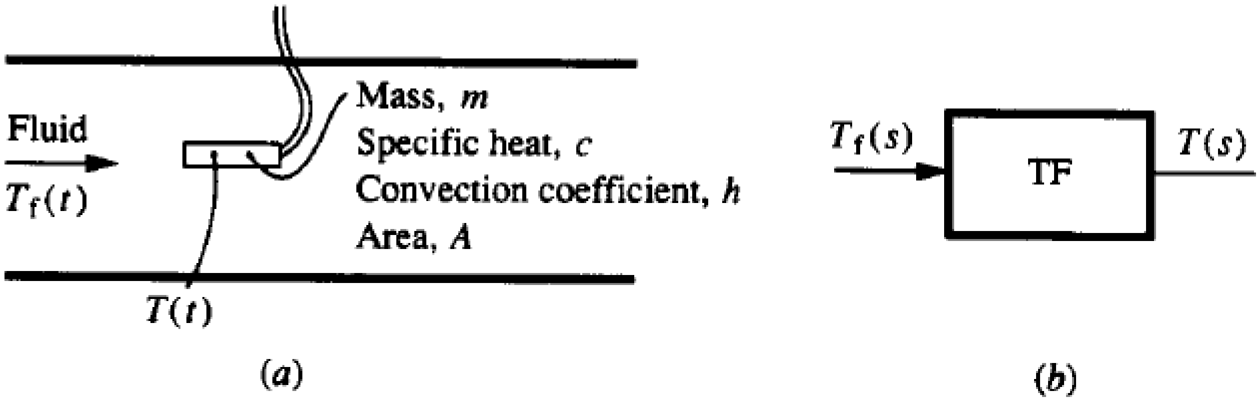


Fig. (a) Response of a temperature-sensing bulb to a change in fluid temperature
(b) Transfer function of this time-constant block

Standard technique for developing transfer function

1. Write differential equation $mc\frac{dT}{dt} = (T_f - T)hA$
2. Transform equation $\frac{mc}{hA}[sL(T) - T(0)] = L(T_f) - L(T)$
3. Solve for transfer function ($L\{O\}/L\{I\}$) $TF = \frac{T(s)}{T_f(s)} = \frac{1+T(0)\frac{B}{T_f(s)}}{1+Bs}$ $(B = \frac{mc}{hA})$

For special case $T(0) = 0 : TF = \frac{1}{1+Bs}$

15.9 Time Constant Blocks

$$mc\frac{d(T-T_0)}{dt} = [(T_f - T_0) - (T - T_0)]hA$$

$$TF = \frac{L\{T-T_0\}}{L\{T_f - T_0\}} = \frac{1}{Bs+1}$$

T_f : unit step increase $\quad T_f(s) = \frac{\Delta}{s}$

$\alpha\beta - \beta = 0, \alpha = 1, \beta = B$

$$L\{T-T_0\} = L\{T_f - T_0\}\frac{1}{(Bs+1)} = \frac{\Delta}{s(Bs+1)} = \Delta\left(\frac{\alpha}{s} - \frac{\beta}{Bs+1}\right) = \Delta\left(\frac{1}{s} - \frac{B}{Bs+1}\right)$$

$$T - T_0 = \Delta\left(1 - e^{-\frac{t}{B}}\right)$$

$\left(B = \frac{mc}{hA}\right)$: time constant

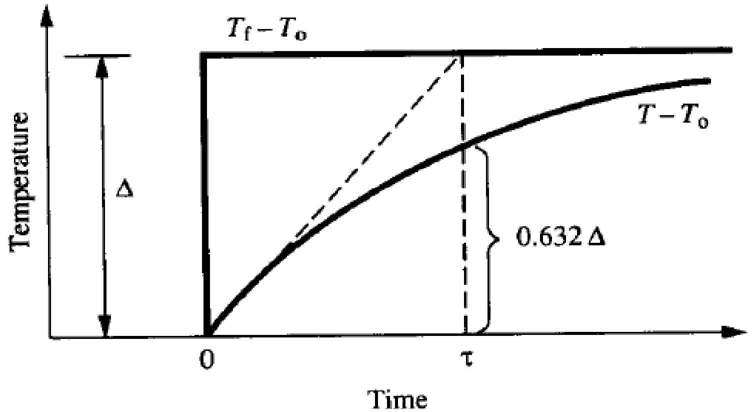
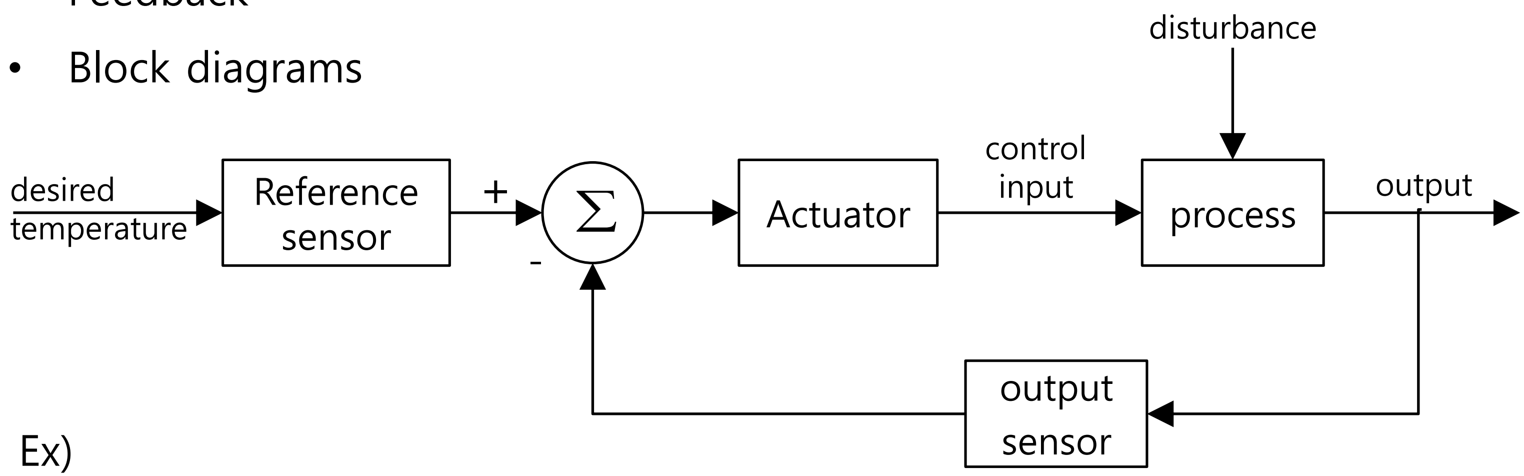


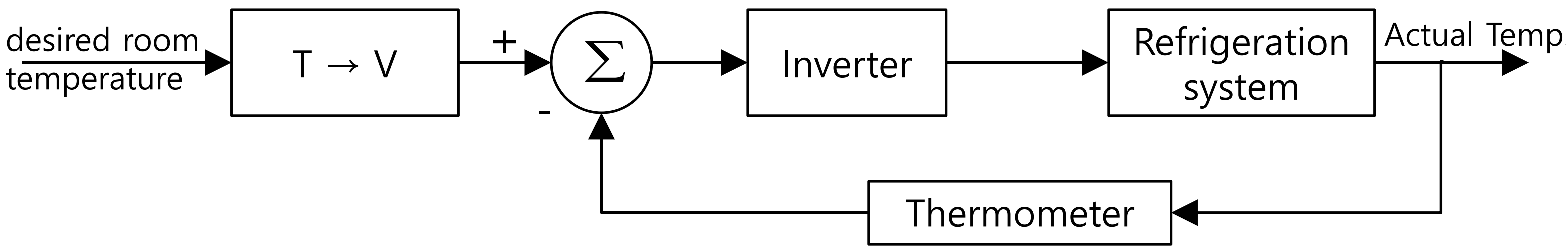
Fig. Step increase in fluid temperature T_f and response of the bulb temperature

15.9 Time Constant Blocks - additional

- Control
- Feedback
- Block diagrams



Ex)



15.10 Cascade Time-constant Blocks

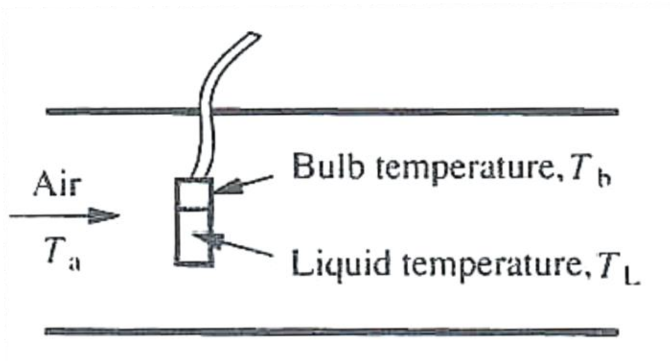


Fig. Response of liquid temperature T_L to a change in air temperature T_A

Heat balance equation :

$$(T_a - T_b)h_1A_1 = mc\frac{dT_b}{dt} + (T_b - T_L)h_2A_2$$

Neglect in order for the heat transfer from the air to the bulb to be represented by the time constant

$$(T_b - T_L)h_2A_2 = mc\frac{dT_L}{dt}$$

$$Let\ \tau_1 = \frac{mc}{h_1A_1}, \tau_2 = \frac{mc}{h_2A_2}$$

subscript 1 : air to bulb
subscript 2 : bulb to liquid

Suppose that T_a experiences a step increase of magnitude Δ from T_0

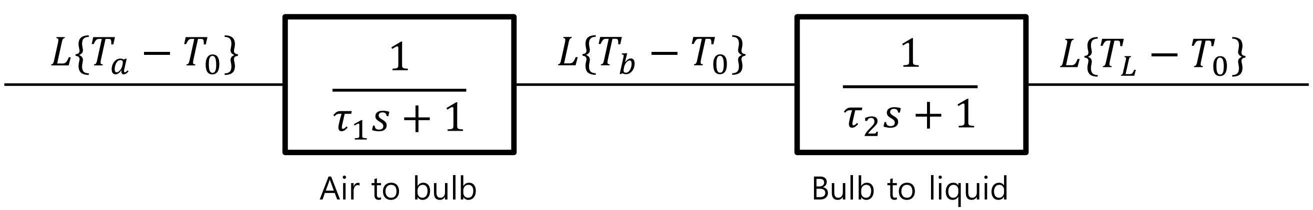


Fig. Cascaded time-constant blocks to represent the dynamic process

15.10 Cascade Time-constant Blocks

For unit step input

$$L\{T_L - T_0\} = \frac{\Delta}{s}\left(\frac{1}{\tau_1 s + 1}\right)\left(\frac{1}{\tau_2 s + 1}\right)$$

Inversion

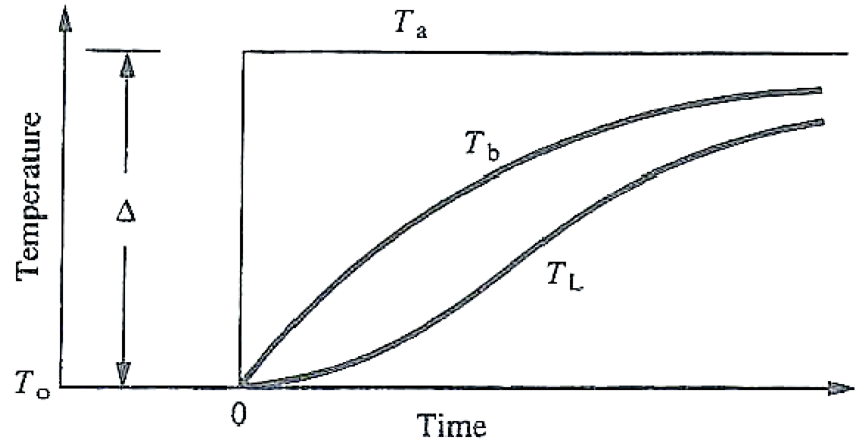
$$\frac{T_L - T_0}{\Delta} = 1 - \frac{\tau_1}{\tau_1 + \tau_2} e^{-\frac{t}{\tau_1}} - \frac{\tau_2}{\tau_2 + \tau_1} e^{-\frac{t}{\tau_2}}$$

① $t = 0,\ T_L - T_0 = 0$

② $\dfrac{d(T_L - T_0)}{dt} = 0\ at\ t = 0$

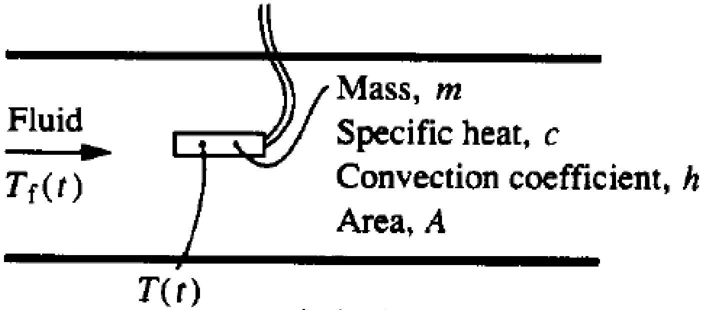
③ $if\quad \tau_2 \ll \tau_1 \quad T_L - T_0 = \Delta(1 - e^{-t/\tau})$

④ $if\quad \tau_2 = \tau_1 \quad \frac{T_L - T_0}{\Delta} = 1 - e^{-t/\tau} - \frac{te^{-t/\tau}}{\tau}$

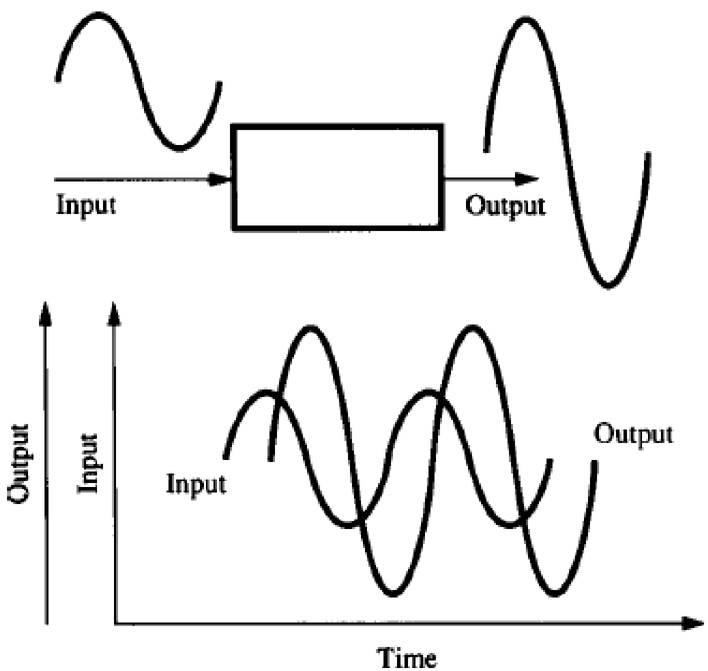


15.11 Stability Analysis

- Frequency response / Bode plot (diagram)
  └ Response to sinusoidal input



<Example in 15.9>



Sinusoidal input : $T_f(t) - T_0 = \Delta sin(2\pi\, ft)$

$L\{\Delta sin(2\pi\ ft)\} = \Delta \frac{a}{s^2+a^2},\ a = 2\pi f$

<From example in 15.9>

$$L(T - T_0) = \frac{1}{\tau s+1} \cdot \frac{\Delta a}{s^2+a^2} = \frac{A}{\tau s+1} + \frac{Bs+C}{s^2+a^2}$$

$$A = \frac{\Delta a}{1/\tau^2+a^2},\ B = \frac{-\Delta a}{\tau a^2+1/\tau},\ C = \frac{\Delta a}{\tau^2 a^2+1}$$

$$L(T - T_0) = \frac{\frac{\Delta a \tau^2}{a^2\tau^2+1}}{\tau s+1} + \frac{\frac{-\Delta a}{\tau a^2+1/\tau}s + \frac{\Delta a}{\tau^2 a^2+1}}{s^2+a^2}$$

Fig. Frequency response in which an sinusoidal input provides an output lagging the input and also exhibiting a different amplitude than the input.

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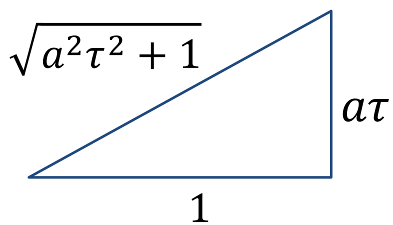
15.11 Stability Analysis

$$T - T_0 = \frac{\Delta a}{a^2\tau^2 + 1} L^{-1}\{\frac{\tau^2}{\tau s + 1} + \frac{-\tau s + 1}{s^2 + a^2}\}$$

$$= \frac{\Delta a}{a^2\tau^2+1}\{\tau e^{-t/\tau} - \tau cos(at) + \frac{1}{a} sin(at)\}$$

$as \quad t \rightarrow \infty$



$$T - T_0 = \frac{\Delta}{a^2\tau^2 + 1} \{sin(at) - a\tau cos(at)\}$$

$$= \frac{\Delta a}{a^2\tau^2+1}\{\sqrt{a^2\tau^2 + 1} \cdot sin(at + \emptyset)\} = \frac{\Delta}{a^2\tau^2+1} sin(at - \emptyset)$$

$$\emptyset = tan^{-1}(a\tau)$$

Amplification ratio : $\frac{1}{\sqrt{a^2\tau^2+1}} \sin(at - \emptyset)$

Phase lag : $\emptyset = tan^{-1}(2\pi f\tau)$

15.11 Stability Analysis

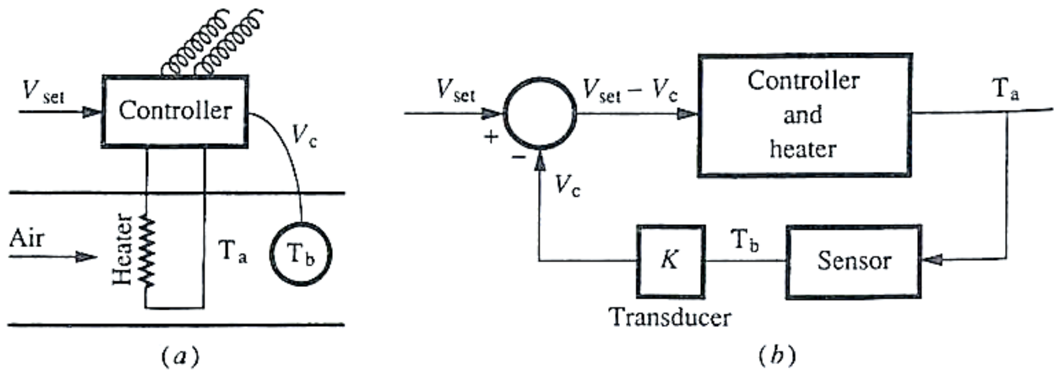


Fig. (a) Air heater
(b) control block diagram

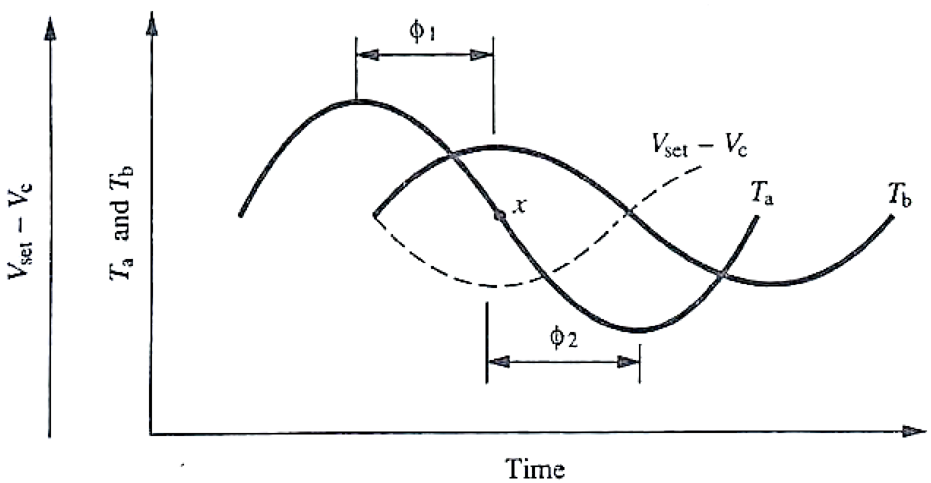


Fig. Perpetuation of sinusoidal disturbances throughout the control loop of the air heater

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15.11 Stability Analysis

- Stability criterion

  At the frequency f where sum of phase lags = 180° Fig.(a)

  product of amplitude ratio > 1 Fig.(b)

  → the loop is unstable

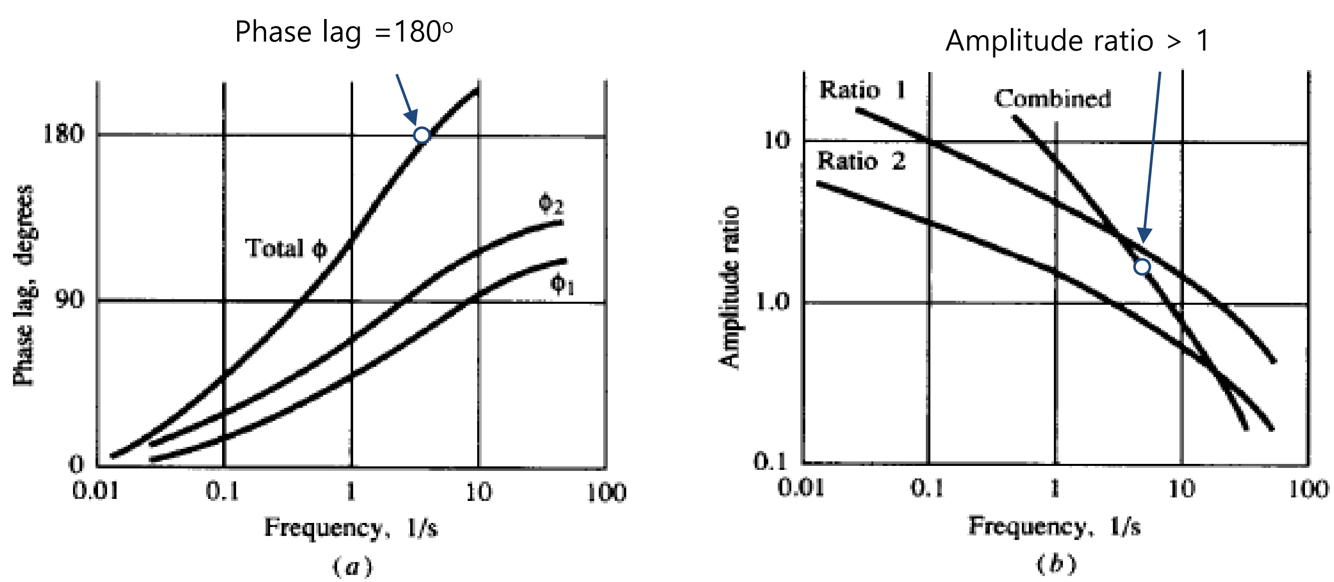

Fig. (a) Sum of the phase lags in the Bode diagram
(b) Product of the amplification ration

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15.11 Stability Analysis - additional

$* \ \frac{Y(s)}{U(s)} = G(s)$

$u(t) = U_0 \sin \omega t$: input

$$Y(s) = G(s)\frac{U_0\omega}{s^2+\omega^2}$$

$$= \frac{\alpha_1}{s-a_1} + \cdots + \frac{\alpha_n}{s-a_n} + \frac{\alpha_0}{s+j\omega} + \frac{\alpha_0^*}{s-j\omega}$$

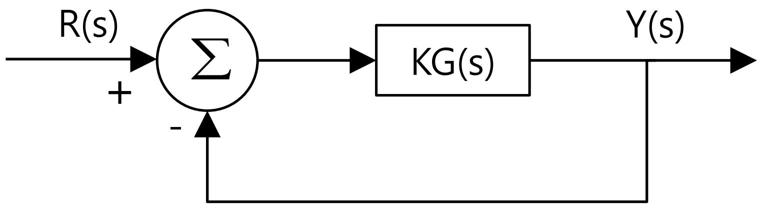
$$y(t) = \alpha_1 e^{a_1 t} + \cdots + \alpha_n e^{a_n t} + 2|\alpha_0|\sin(\omega t + \emptyset), \qquad \emptyset = tan^{-1}\frac{Im(\alpha_0)}{Re(\alpha_0)}$$

$$as \quad t \rightarrow \infty, \quad y(t) = U_0 A sin(\omega t + \emptyset)$$

magnitude $A = |G(j\omega)| = |G(s)| = \sqrt{\{Re[G(j\omega)]\}^2 + \{IM[G(j\omega)]\}^2}$

phase $\emptyset = \angle G(j\omega) = tan^{-1}\frac{Im[G(j\omega)]}{Re[G(j\omega)]}$

15.12 Stability Analysis Using Loop Transfer Function



$$\frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)}$$

Roots are the poles of the transfer function $1+KG(s)$

→ root locus : graph of all possible roots

$root < 0$	e^{-at}
〃 $= 0$	1
〃 > 0	e^{at}
〃 $-a+ib$	$e^{(a+ib)t}$

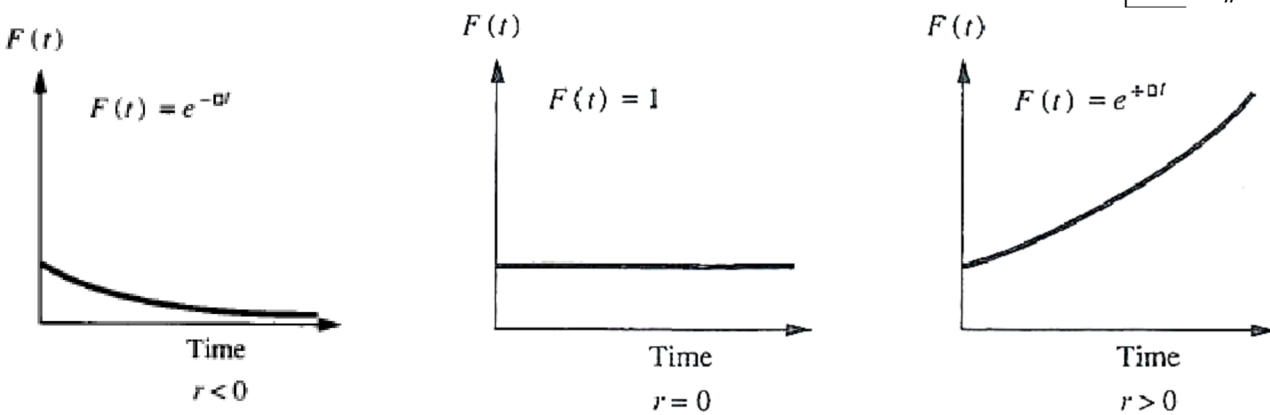


Fig. Inverses F(t) are dependent upon the sign of the roots in (const)/(s-r) terms.

15.12 Stability Analysis Using Loop Transfer Function

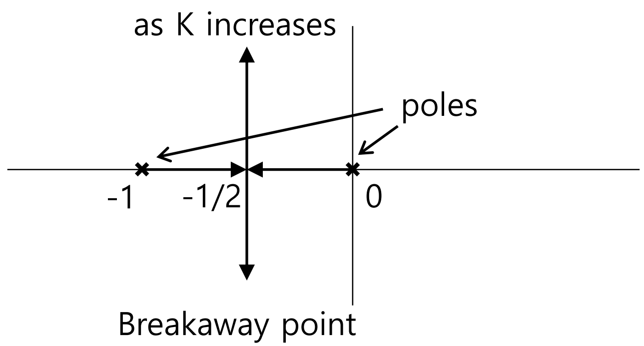
$$KG(s) = \frac{Kb(s)}{a(s)} = \frac{K(s^m + b_1 s^{m-1} + \cdots + b_m)}{s^n + a_1 s^{n-1} + \cdots + a_n} \qquad n > m$$

$$b(s) = (s - z_1)(s - z_2)\cdots(s - z_m)$$

$$b(s) = (s - p_1)(s - p_2)\cdots(s - p_n)$$

Root locus form : $1 + KG(s) = 1 + K\frac{b(s)}{a(s)} = 0 \quad \rightarrow \quad a(s) + Kb(s) = 0$

Ex) $KG(s) = \frac{K}{s(s+1)} \qquad Denominator = 1 + \frac{K}{s(s+1)} \rightarrow s^2 + s + K = 0$



Root locus is a graph of the roots of the quadratic equation

$$r_1, r_2 = -\frac{1}{2} \pm \frac{\sqrt{1-4K}}{2}$$

$$0 \leq K \leq \frac{1}{4} \qquad -1 < \ < 0$$

$$K > \frac{1}{4} \qquad complex$$

15.14 Restructuring the Block Diagram

- converting complex block diagram → feedback loop
- simplifying

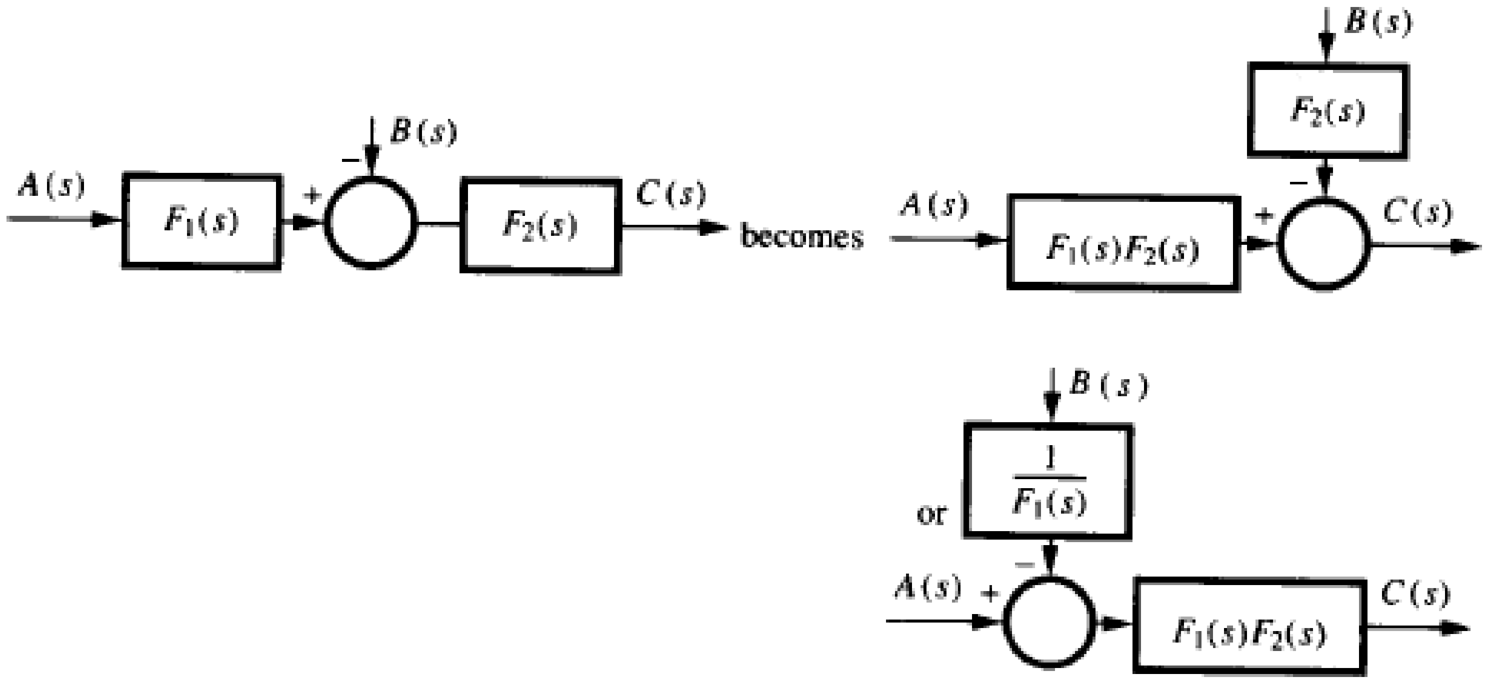


Fig. Moving a summing point around a block

15.14 Restructuring the Block Diagram

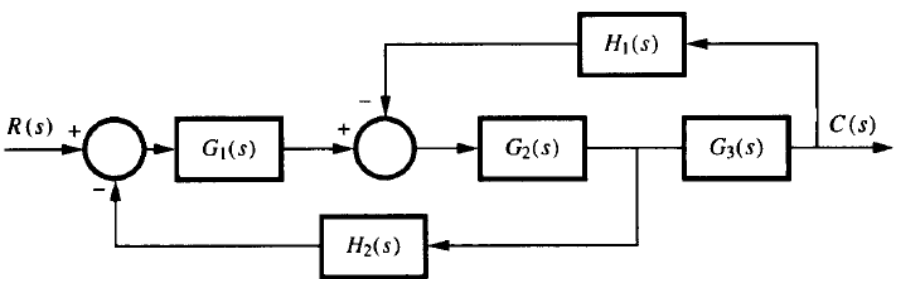


Fig. Control block diagram in Example 15. 10

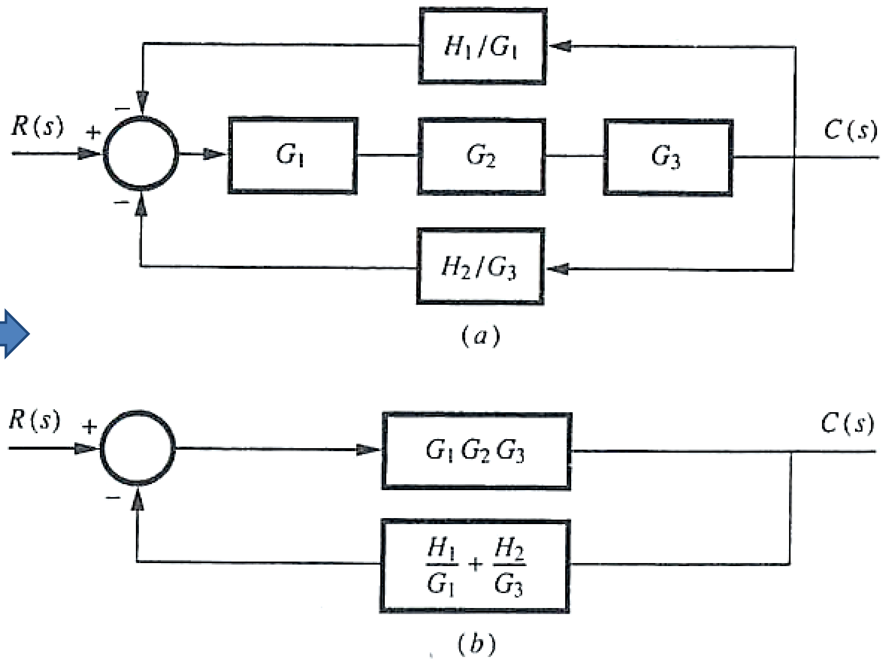


Fig. Modification of loop in Example 15. 10.

15.15 Translating the Physical situation into a Block Diagram

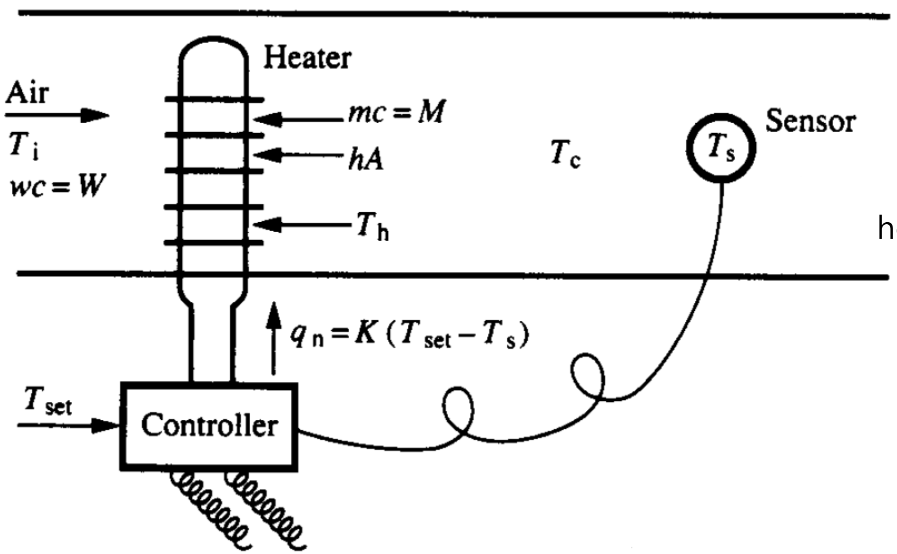


Fig. Air heating system and its control

$$q_h = K(T_{set} - T_s)$$

$$q_a = wc(T_h - T_i)\left(1 - e^{-\frac{hA}{wc}}\right) \underset{wc = W}{\underset{\uparrow}{=}} W(T_h - T_i)\varepsilon$$

heater→air

$$q_h - q_a = mc\frac{dT_h}{dt}$$

$$q_h(s) - q_a(s) = M[sT_h(s) - T_h(0)]$$

15.15 Translating the Physical situation into a Block Diagram

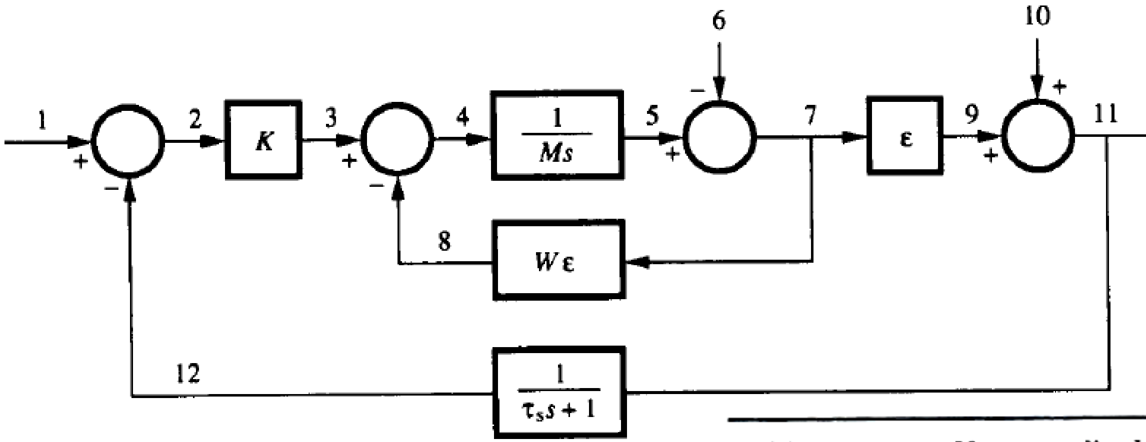


Fig. Air heating system and its control

Position	Nonnormalized	Normalized
1	T_{set}	$T_{set} - T_{set,0}$
2	$T_{set} - T_s$	$(T_{set} - T_s) - (T_{set,0} - T_{s,0})$
3	q_h	$q_h - q_{h,0}$
4	$q_h - q_a$	$(q_h - q_a) - (q_{h,0} - q_{a,0})$
5	T_h	$T_h - T_{h,0}$
6	T_i	$T_i - T_{i,0}$
7	$T_h - T_i$	$(T_h - T_i) - (T_{h,0} - T_{i,0})$
8	q_a	$q_a - q_{a,0}$
9	$T_c - T_i$	$(T_c - T_i) - (T_{c,0} - T_{i,0})$
10	T_i	$T_i - T_{i,0}$
11	T_c	$T_c - T_{c,0}$
12	T_s	$T_s - T_{s,0}$

Table. Designations of variables in block diagram of Fig.

15.15 Translating the Physical situation into a Block Diagram

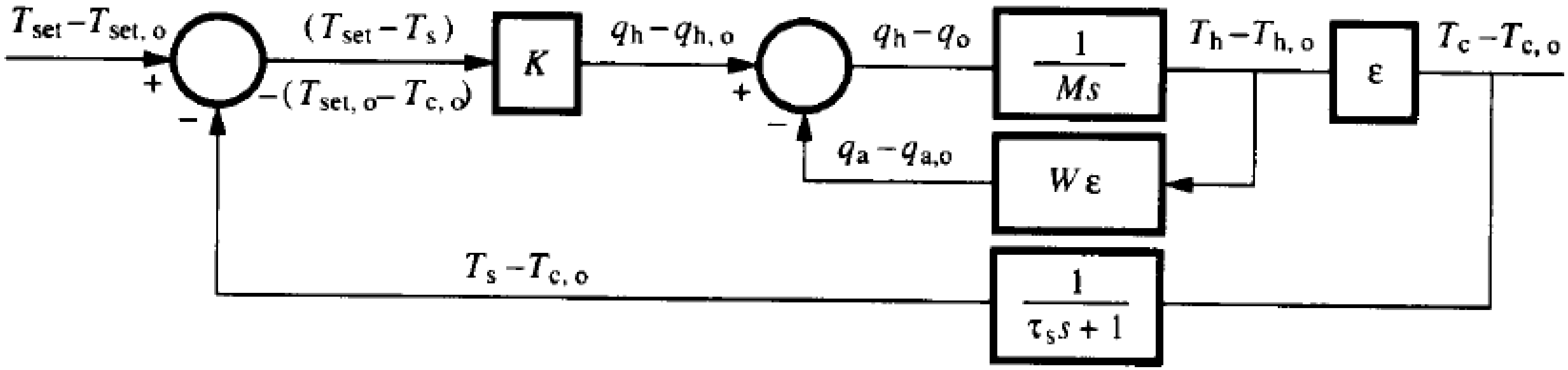


Fig. Diagram after elimination of two summing points

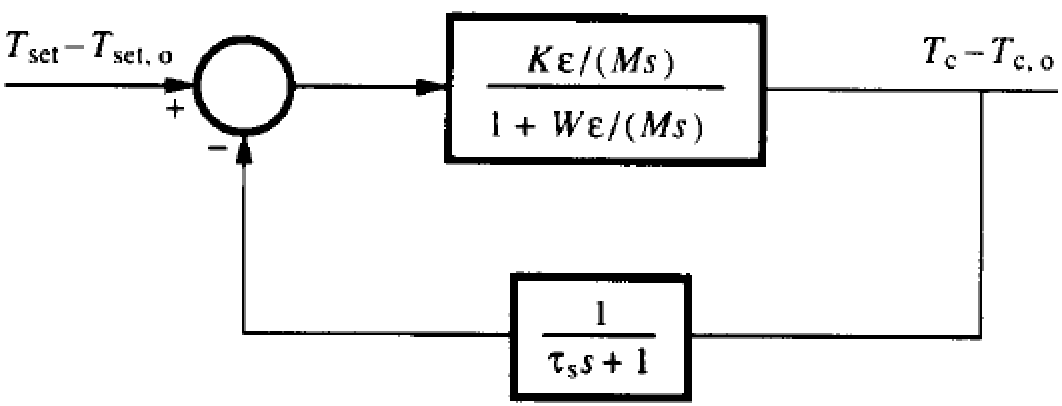


Fig. Simplified nonunity feedback loop for air heater controller

15.16 Proportional Control

$$q_h = K_p \underbrace{(T_{set} - T_s)}_{\text{error}}$$

K↑ unstable

K↓ offset

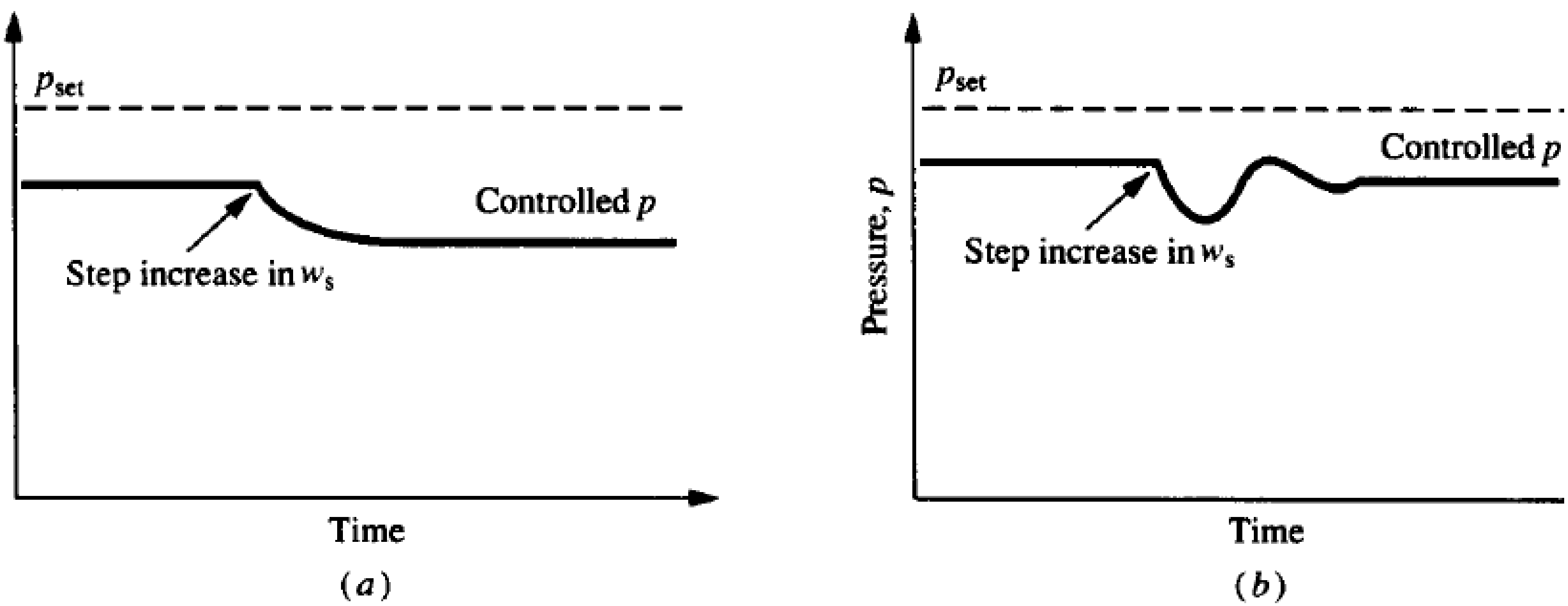


Fig. Pressure controller (a) with low gain (b) with high gain

15.17 Proportional – Integral (PI) Control

- to eliminate the offset

$$K_I \int (error)dt$$

$$TF = \frac{K_I}{s} \leftarrow \frac{K_I \Delta / s^2}{\Delta / s}$$

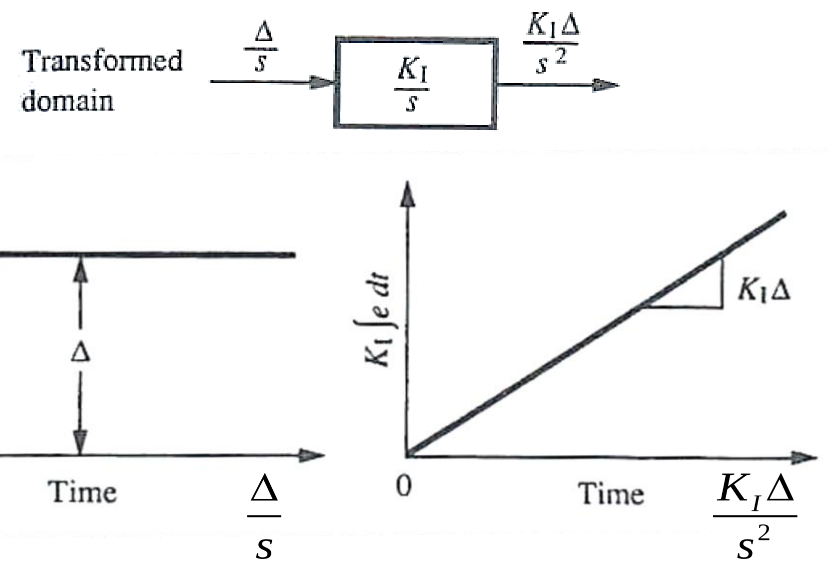


Fig. Transfer function of the I-mode

- PI control

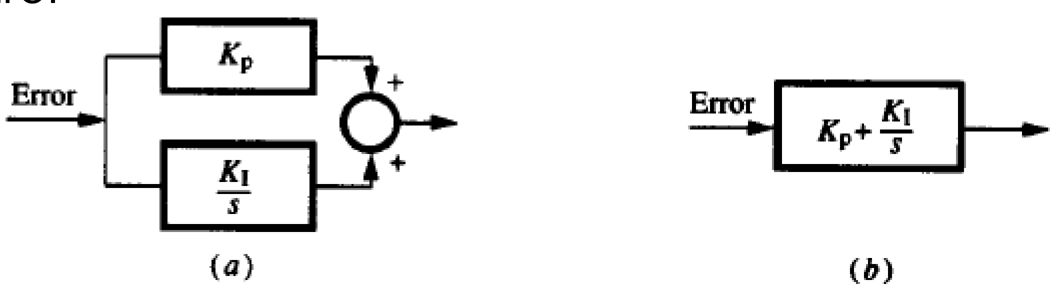


Fig. Block diagram symbols of the PI control

15.18 Proportional-Integral-Derivative(PID) Control

$$K_p + \frac{K_I}{s} + K_D s$$

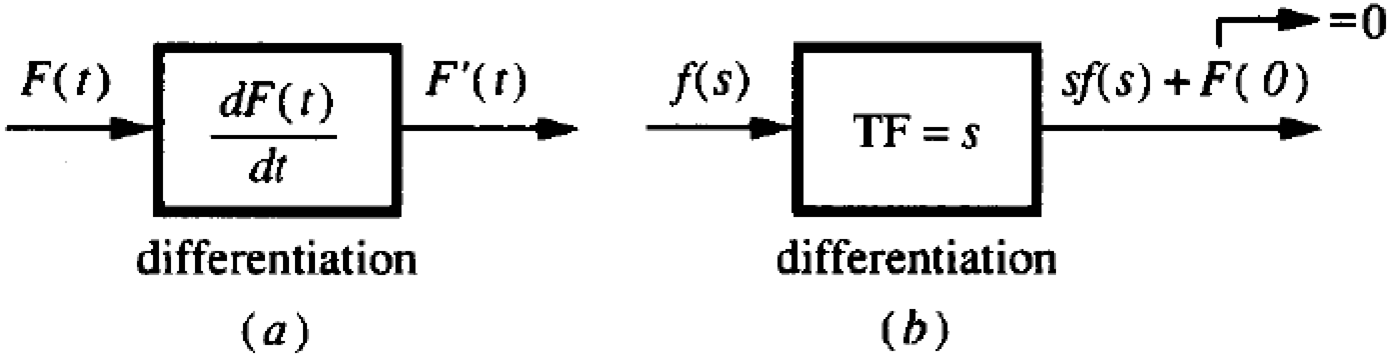
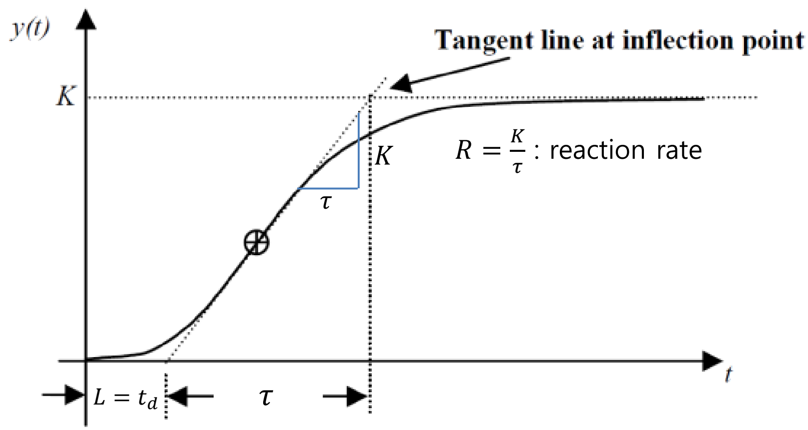


Fig. The differentiation process in (a) the time domain,
(b) the transformed domain

15.18 Proportional-Integral-Derivative(PID) Control

※ Ziegler-Nichols Tuning of PID controller(1942)



Time delay of t_d seconds

TF may be approximated by $TF = \dfrac{Ke^{-tdS}}{\tau s+1}$

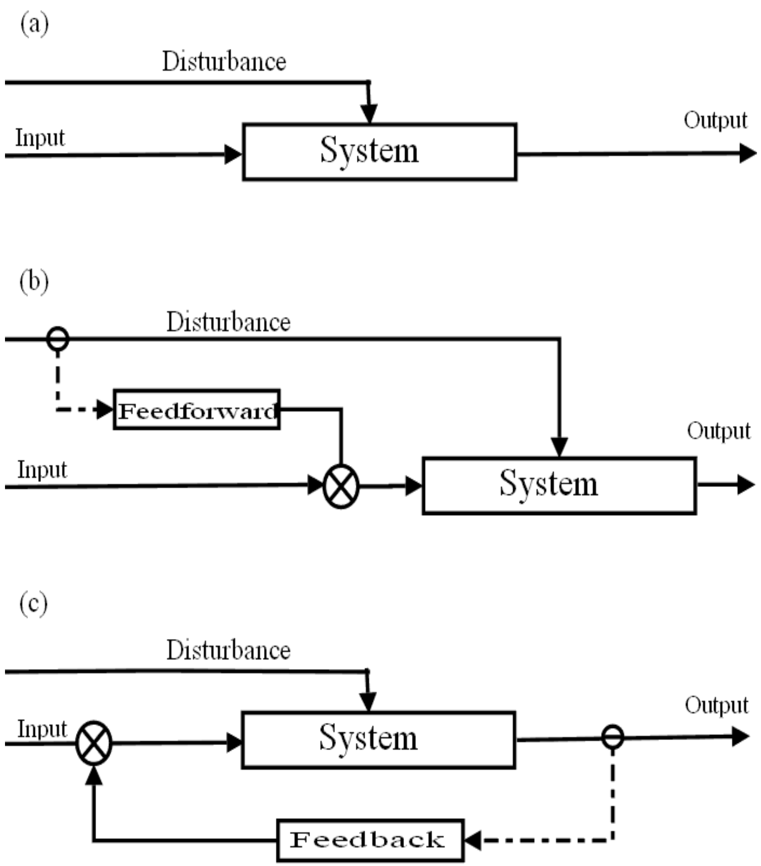
$e^{-t/\tau}$

$$D(s) = K\left(1 + \frac{1}{T_1 s} + T_D s\right)$$

$$= K + \frac{K}{T_I s} + KT_D s = K_p + \frac{K_I}{s} + K_D s$$

15.19 Feedforward control

└ open loop control



(a) No control

(b) Feed forward control

(c) Feedback control