Optimal Design of Energy Systems (M2794.003400)

Chapter 3. ECONOMICS

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Chapter 3. ECONOMICS

3.1 Introduction

- Basis of engineering decision

Economics

- Minimum investment cost
- Minimum total lifetime cost

Non-economic factors

- Legal concerns
- Social concerns
- Environmental concerns
- Aesthetic concerns

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3.2. Interest

- **Interest** is the rental charge of the use of money
- **Simple interest** is calculated only on the principal amount, or on the portion of the principal amount that remains.
- **Compound interest** includes interest earned on the interest which was previously accumulated.

Example 3.1 : Simple interest, lump sum

Simple interest of 8% per year is charged on a 5-year loan of $500. How much does the borrower pay to the lend?

(Solution)

Annual interest : $(\$500)(0.08) = \$40$

Total interest $= (Annual\ interest)(year) = (\$40)(5) = \$200$

Repayment $= \$200 + \$500 = \$700$

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Example 3.2 : Compound interest, lump sum

What amount must be repaid on the $500 loan in Example 3.1, if the interest of 8% is compounded annually?

(Solution)

Repayment after n year $=(\$500)(1+i)^n$

$i = 0.08 \qquad n = 5$

→ Repayment$= \$500(1+0.08)^5 = \$734.66$

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Example 3.3 : Compounded more often than annually, lump sum

What amount must be repaid on a 5-year $500 loan at 8% annual interest compounded quarterly?

(Solution)

Repayment $= P(1+\frac{i}{m})^{m*n}$

$P = \$500 \quad i = 0.08 \quad m = 4 \quad n = 5$

$\Rightarrow$ $\$500(1+\frac{0.08}{4})^{20} = \$742.97$

$$S = P(1+\frac{i}{m})^{m \times n}$$

Where i = nominal annual interest rate

n = number of years

m = number of compounding periods per year

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3.5 Compound-Amount Factor (f/p) and Present-Worth Factor (p/f)

- Future worth and can be mutually converted
- **Compund amount factor** (CAF or f/p)

$$(\text{Future worth } S) = (\text{Present worth } P) \cdot (\text{f/p}) \Leftarrow \text{f/p} = \left(1 + \frac{i}{m}\right)^{mn}$$

- **Present-worth factor** (PWF or p/f)

$$(\text{Present worth P}) = (\text{future worth } S) \cdot (\text{p/f}) \Leftarrow \text{p/f} = \frac{1}{(1 + i/m)^{mn}}$$

Where i = nominal annual interest rate

n = number of years

m = number of compounding periods per year

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Example 3.4 : Compound-Amount Factor (f/p)

You invest \$5000 in a credit union which compounds 5% interest quarterly. What is the value of the investment after 5 years?

(Solution)

Future worth = (present worth, p/a)(f/p), f/p = $(1+\frac{i}{m})^{m*n}$

$p/a = \$5000 \quad i = 0.05 \quad m = 4 \quad n = 5$

$\Rightarrow$ $\$5000(1+\frac{0.05}{4})^{20} = \$6410$

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Example 3.5 : Present-Worth Factor (p/f)

You wish to invest a sum of money so that accumulated amount will be $10,000 12 years later. The money can be invested at 8%, compounded semiannually. What amount must be invested?

(Solution)

Present worth = (Future worth, f/a)(p/f), p/f $= \frac{1}{(1+\frac{i}{m})^{n*m}}$

$f/a = \$10{,}000 \quad i = 0.08 \quad m = 2 \quad n = 12$

$$(\$10{,}000) * \frac{1}{\left(1+\frac{0.08}{2}\right)^{24}} = \$3901.20$$

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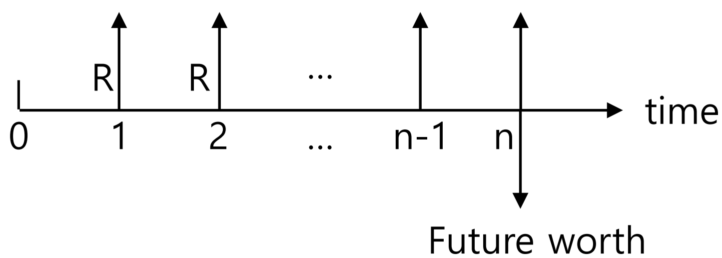
3.6 Future worth (f/a) of a uniform series of amounts

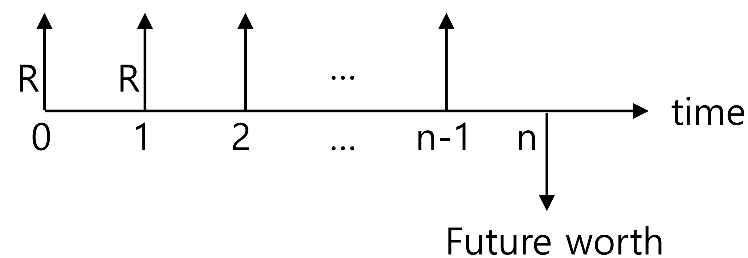
- Uniform amount is paid at each time period
- There are two types for a uniform series of amounts

$$S = R(1+i)^{n-1} + R(i+i)^{n-2} + \cdots + R(1+i) + R$$



First payment at the end of the first period

$$S = R(1+i)^{n} + R(i+i)^{n-1} + \cdots + R(1+i)$$



First payment at the start of the first period

- R : uniform amount at each time period
- S : Future worth

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3.6 Future worth (f/a) of a uniform series of amounts

- If first payment is at the end of the first payment
- **Series compound amount factor** (SCAF or f/a)

$$(\text{Future worth } S) = (\text{Regular amount } R) \cdot (\text{f/a}) \quad \Leftarrow \quad \text{f/a} = \frac{(1+i)^n - 1}{i}$$

- **Sinking fund factor** (SFF or a/f)

$$(\text{Regular amount } R) = (\text{future worth } S) \cdot (\text{a/f}) \quad \Leftarrow \quad \text{a/f} = \frac{i}{(1+i)^n - 1}$$

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3.6 Future worth (f/a) of a uniform series of amounts

- If first payment is at the start of the first payment
- **Series compound amount factor** (SCAF or f/a)

$$(\text{Future worth } S) = (\text{Regular amount } R) \cdot (\text{f/a})_{shift} \Leftarrow (\text{f/a})_{shift} = \frac{(1+i)^n - 1}{i/(1+i)}$$

- **Sinking fund factor** (SFF or a/f)

$$(\text{Regular amount } R) = (\text{future worth } S) \cdot (\text{a/f})_{shift} \Leftarrow (\text{a/f})_{shift} = \frac{i/(1+i)}{(1+i)^n - 1}$$

Example 3.6

The management to set aside equal amounts of investment each year starting 1 year from now so that \$16,000 will be available in 10 years for the replacement of the machine. The compound interest is 8% annually. How much must be provided each year?

(Solution)

$$16000 = R[(1+0.08)^9 + (1+0.08)^8 + \cdots + (1+0.08) + 1]$$

interest : $i = 0.08$ sum : $S = 16000$ year : $n = 10$

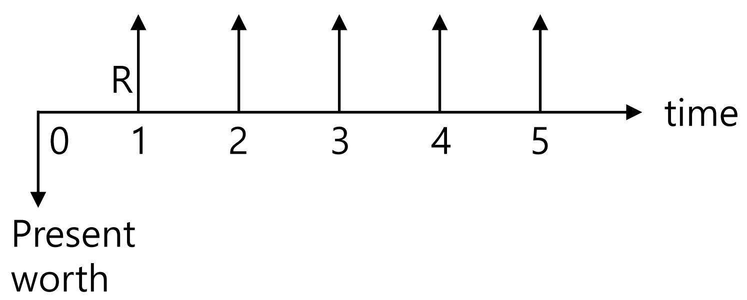
$$R = (\text{future worth})(\text{SFF}) = S \cdot a/f = S\frac{i}{(1+i)^n - 1} = 16000\frac{0.08}{(1+0.08)^{10} - 1}$$

➡ $R = \$1104.5$

3.7 Present worth (p/a) of a uniform series of amounts

- The value of a series of uniform amounts R can be translated into the present worth

$$R\left[\frac{1}{1+i}+\frac{1}{(1+i)^2}+\cdots+\frac{1}{(1+i)^n}\right]$$



First payment at the end
of the first period

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3.7 Present worth (p/a) of a uniform series of amounts

- If first payment is <u>at the end of the first payment</u>
- **Series present worth factor** (SPWF or p/a)

$$\mathrm{p/a} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

- **Capital recovery factor** (CRF or a/p)

$$\mathrm{p/a} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

Example 3.7 : Present worth (p/a) of a uniform series of amounts

You borrow \$1000 from a loan company that charges 15% nominal <u>annual interest compounded monthly</u>. How many month will it take to repay the loan if you pay off \$38 per month?

(Solution)

$\$1000 = (\$38)(\text{p/a})$, p/a $= \dfrac{\left(1+\frac{i}{m}\right)^n - 1}{\frac{i}{m}(1+\frac{i}{m})^n}$

$i = 0.15 \quad m = 12$

$1000 = 38 * \dfrac{(1.0125)^n - 1}{0.0125(1.0125)^n}$

➡ $n = 32.1\ month$

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3.8 Gradient present worth factor (GPWF)

- Non uniform amounts in the series (ex: maintenance cost is being increased)
- No cost during the first year
- cost G at the end of the 2nd year, and 2G at the end of the 3rd year…

$$(Present\ worth\ P) = \frac{G}{(1+i)^2} + \frac{2G}{(1+i)^3} + \cdots + \frac{(n-1)G}{(1+i)^n}$$

$$= G\left\{\frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right\}\right.$$

$$\therefore GPWF = \left\{\frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right\}\right.$$

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3.10 Bonds

- Bond is an instrument of indebtedness of the bond issuer to the holders.
- Face value and its interest is paid by the issuers to holder.
- Interest is usually semiannual.
- It is possible to sell and buy the bond.

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3.10 Bonds

$$P_b(1+\frac{i_c}{2})^{2n} = FV + FV\,\frac{i_b}{2}\,\frac{(1+i_c/2)^{2n}-1}{i_c/2}$$

Future worth of investment

Future worth of uniform series of the **semiannual interest** payment on the bond

- FV : face value
- P_b : price to be paid for bond now
- i_c : current interest rate
- i_b : interest rate on bond
- n : years to maturity

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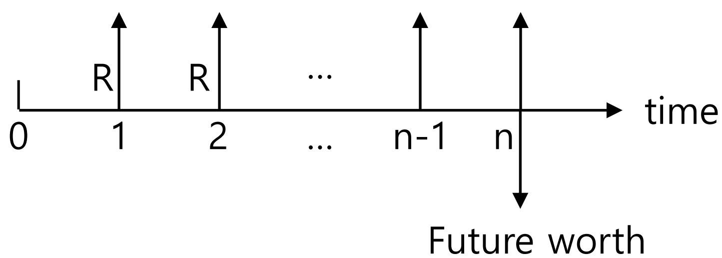
3.11 Shift in time of a series

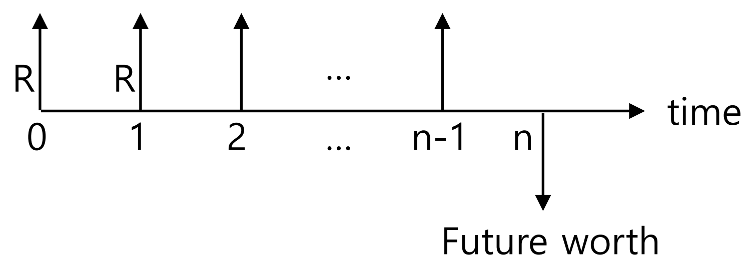
- Unlike the previous examples, first payment is at the start of the first period

$$S = R(1+i)^{n-1} + R(i+i)^{n-2} + \cdots + R(1+i) + R$$



First payment at the end
of the first period

$$S = R(1+i)^{n} + R(i+i)^{n-1} + \cdots + R(1+i)$$



First payment at the start
of the first period

- R : uniform amount at each time period
- S : Future worth

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3.11 Shift in time of a series

- If first payment is at the start of the first payment
- **Series compound amount factor** (SCAF or f/a)

$$(\text{Future worth } S) = (\text{Regular amount } R)\cdot(\text{f/a})_{shift} \quad \Leftarrow \quad (\text{f/a})_{shift} = \frac{(1+i)^n - 1}{i/(1+i)}$$

- **Sinking fund factor** (SFF or a/f)

$$(\text{Regular amount } R) = (\text{future worth } S)\cdot(\text{a/f})_{shift} \quad \Leftarrow \quad (\text{a/f})_{shift} = \frac{i/(1+i)}{(1+i)^n - 1}$$

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Example 3.9 : Bonds

A \$1000 bond that has 10 years to maturity pays interest semiannually at a nominal annual rate of 8%. An investor wishes to earn 9% on investment. What price could investor pay for the bond to achieve this 9% interest rate?

(Solution)

$$P_b(1+\frac{i_c}{2})^{2n} = FV + FV\frac{i_b}{2}\frac{(1+i_c/2)^{2n}-1}{i_c/2}$$

$$P_b = ? \quad i_c = 0.09 \quad n = 10 \quad FV = \$1000 \quad i_b = 0.08$$

$$\Rightarrow \quad P_b = \$934.96$$

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3.14 Evaluating Potential Investments

- Four elements of consideration in **investment analysis**

① first cost

② Income

③ Operating expense

④ Salvage value

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Example 3.12 : Evaluating Potential Investments

You have a choice of buying building A of building B to operate the building for 5 years and then sell it. Building A's expected value is to be 20% higher in 5 years, while building B is expected to drop in value of 10% in 5 years. Other data are shown in Table below. What will be the rate of return on each building?

Economic data	Building A	Building B
First cost	$800,000	$600,000
Annual income from rent	160,000	155,000
Annual operating and maintenance cost	73,000	50,300
Anticipated selling price	960,000	540,000

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(Solution)

First cost = (Annual income - Annual operating and maintenance cost)(p/a)

+ (Anticipated selling price)(p/f)

Recall $p/a = \frac{(1+i)^n - 1}{i(1+i)^n}$ $p/f = \frac{1}{(1+i)^n}$

Building A : $800{,}000 = (160{,}000 - 73{,}000)\left(\frac{(1+i)^5 - 1}{i(1+i)^5}\right) + (960{,}000)\left(\frac{1}{(1+i)^5}\right)$

Building B : $600{,}000 = (155{,}000 - 50{,}300)\left(\frac{(1+i)^5 - 1}{i(1+i)^5}\right) + (540{,}000)\left(\frac{1}{(1+i)^5}\right)$

$\Rightarrow$ $i = \begin{cases} 13.9\% & building\ A \\ 16.0\% & building\ B \end{cases}$

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3.18 Continuous compounding

- High frequency of compounding is quite realistic in business operation.
- Businesses control their money more on a flow basis than on a batch basis.

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3.18 Continuous compounding

if m approaches infinity,

$$f/p = (1 + \frac{i}{m})^{mn} \qquad (f/p)_{const} = (1 + \frac{i}{m})^{mn}\Big|_{m \to \infty}$$

by taking the logarithm and using tailor expansion,

$$\ln((f/p)_{const}) = mn\,[\ln(1 + \frac{i}{m})]\Big|_{m \to \infty} = mn\,[0 + \frac{i}{m} + a_2 \frac{i^2}{m^2}]\Big|_{m \to \infty}$$

cancling m and letting m approaches infinity,

$$\ln((f/p)_{const}) = in \quad \Rightarrow \quad (f/p)_{const} = e^{in}$$

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Example 3.13 : Continuous compounding

Compare the values of (f/p, 8%, 10) and [$(f/p)_{cont}$, 8%, 10]

(Solution)

(f/p, 8%, 10) = $(1 + 0.08)^{10}$ = 2.1589

[$(f/p)_{cont}$, 8%, 10] = $e^{0.8}$ = 2.2255