Optimal Design of Energy Systems (M2794.003400)

Chapter 7. Optimization

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Chapter 7. Optimization

7.1 Introduction

- Definition : Finding the conditions that **give max. or min. values of a function**
- Target : Selecting which criterion is to be optimized ex) size, weight, cost
- Components and system simulation are preliminary steps to optimization

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7.2 Levels of Optimization

- Two levels of optimization : comparison of alternate concepts & optimization within a concept
- A complete optimization procedure ;

Proposal

➢ Proposing all reasonable alternate concepts

Optimization

➢ Optimizing the design of each concepts

Choice

➢ Choosing the best design

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7.3 Mathematical Representation of Optimization Problems

- Objective function : Meaning the function to be optimized ('y' below equation)
- Independent variables : Constituting an objective function but independent with each variable ('$x_1 \cdots x_n$' below equation)

$$y = y(x_1, \cdots, x_n) \longrightarrow \text{optimize}$$

Objective function   Independent variables

7.3 Mathematical Representation of Optimization Problems

- In many physical situations there are constraints, some of which may be **equality constraints** as well as **inequality constraints**.

- Equality constraints (등호) ;

$$\phi_i = \phi_i(x_1, \cdots, x_n) = 0$$

- Inequality constraints (부등호) ;

$$\psi_i = \psi_i(x_1, \cdots, x_n) \leq L_j$$

7.3 Mathematical Representation of Optimization Problems

- **An additive constant** in the objective function does **not affect** the values of **independent variables at which the optimum occurs.**

$$if \qquad y = \underbrace{\mathrm{a}}_{\text{Additive constant}} + Y(x_1, \cdots, x_n)$$

$$then \qquad \min y = \mathrm{a} + \min Y$$

- **The maximum** of a function occurs at the **same state point** at which **the minimum of the negative** of the function occurs

$$\max[y(x_1, \cdots, x_n)] = -\min[-y(x_1, \cdots, x_n)]$$

7.4 A Water-Chilling System

- A water-chilling system, shown schematically in the figure, will be used to illustrate the mathematical statement.
- **Task : Minimize the first cost to satisfy the cooling system's requirements**

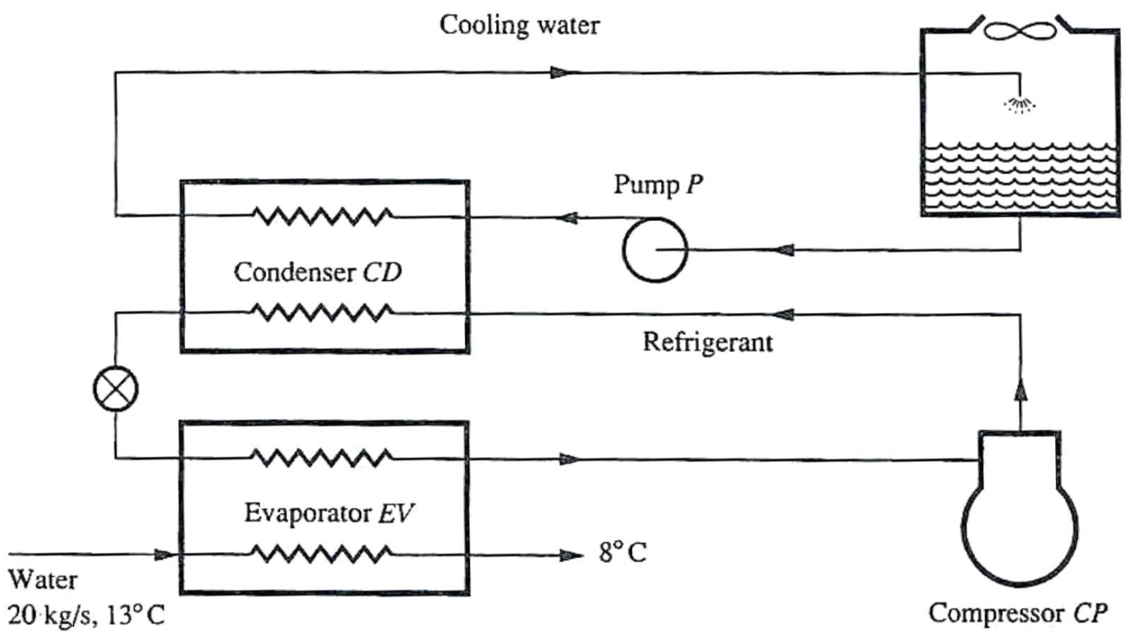

Fig. Water-chilling unit being optimized for minimum first cost

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7.4 A Water-Chilling System

(Given)

- System's requirements

  mass flow rate : **20 kg/s of water** inserted

  cooling temperature : **from 13℃ to 8 ℃**

  Rejecting the heat to the atmosphere through cooling tower

- Target : **total cost y** ⇒ objective function
- Variables : x_{CP}, x_{EV}, x_{CD}, x_P, x_{CT} ⇒ individual variables

(meaning **the size** of the compressor, evaporator, condenser, pump, cooling tower, respectively)

7.4 A Water-Chilling System

(Optimization)

- **The total cost y** ; $y = y(x_{CP},\ x_{EV},\ x_{CD},\ x_P,\ x_{CT}) \ \Rightarrow$ minimize
- Prior to optimization, the water-chilling assignment can be calculated as below;

$$20\ \mathrm{kg/s} \times (13-8)^{\circ}\mathrm{C} \times \left[\frac{4.19\ \mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right] = 419\ \mathrm{kW}$$

- **The cooling capacity** φ ; $\varphi = \varphi(x_{CP},\ x_{EV},\ x_{CD},\ x_P,\ x_{CT}) \geq 419\ kW$
- t_{ev} is above $0^{\circ}\mathrm{C}$ to prevent water from freezing on the tube surface.
- **The evaporating temperature** t_{ev} ; $t_{ev}(x_{CP},\ x_{EV},\ x_{CD},\ x_P,\ x_{CT}) \geq 0^{\circ}\mathrm{C}$

- There may be other inequality constraints, such as limiting the condenser cooling water flow, discharge temperature of the refrigerant leaving the compressor.

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7.5 Optimization Procedures

- The objective function is dependent upon more than one variable.
- Some thermal systems may have many variables which demand sophisticated **optimization techniques**.
- Several optimization methods will be listed in the next sections.

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7.6 Calculus Methods : Lagrange Multipliers (Ch.8 & 16)

- **Using derivatives** to indicate the optimum. (presented in Ch.8 and Ch.16)
- The method of Lagrange multipliers performs an optimization where **equality constraints** exist.

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7.7 Search Methods (Ch.9 & 17)

- These method involve **examining many combinations of the independent variables**
- Then, drawing conclusions from the magnitude of **the objective function at these combinations** (presented in Ch.9 & 17)
- Usually inefficient, but it can be proper when optimizing systems where the components are available only **in finite steps** of sizes.

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7.8 Dynamic Programming (Ch.10 & 18)

- Not meaning computer programming, but optimization technique
- The result of this method is **an optimum function**, relating several variables, rather than an optimum state point. (covered in Ch.10 & 18)
- E.g. the best route of a gas pipeline

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7.9 Geometric Programming (Ch.11)

- Optimizing a function that consists of **a sum of polynomials** wherein the variables appear to integer and non-integer exponents. (covered in Ch.11)

7.10 Linear Programming (Ch.12)

- Widely used and well-developed discipline applicable **when a given equation is linear.** (covered in Ch.12)

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7.11 Setting up the Mathematical Statement of optimization Problem

- Strategy

  1) Specify all direct constraints e.g. capacity, temperature, pressure
  2) Describe in equation form the component characteristics
  3) Write mass and energy balances

Example 7.1 : Equations for Optimizing Air-Cooling System

- Hot air is cooled by two-stage water cooling system. Develope (a) the objective function and (b) the constraint equations for an optimization to provide **minimum first cost.**

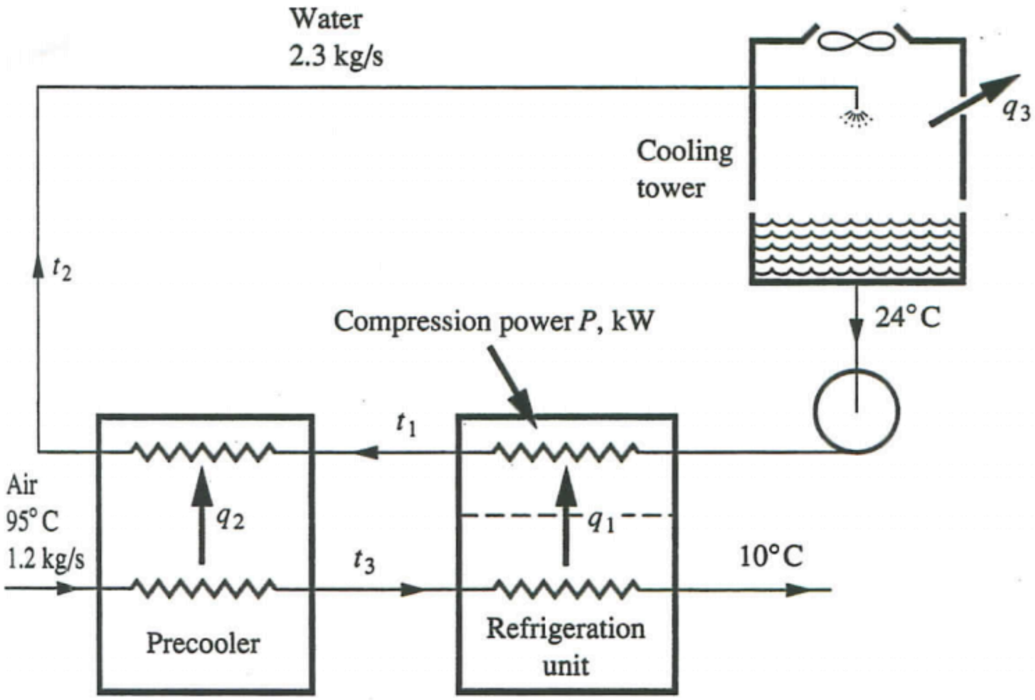

Fig. Air-cooling system in Example 7.1

Example 7.1 : Setting up equations for optimizing cooling system

(Variables)

- Cost of refrigeration unit, pre-cooler, cooling tower : x_1, x_2, x_3 [$] (respectively)
- Rate of heat transfer of refrigeration unit, pre-cooler, cooling tower : q_1, q_2, q_3 [kW]
- Compression power required by refrigeration unit : P [kW]
- Local temperature : t_1, t_2, t_3 (covered in Fig.)

(Given)

- Air : $\dot{m}_a = 1.2\,\mathrm{kg/s}$ and $C_{p,a} = 1.0\,\mathrm{kJ/(kg \cdot K)}$ $95°\mathrm{C} \rightarrow 10°\mathrm{C}$ (cooled down)
- Water : $\dot{m}_w = 2.3\,\mathrm{kg/s}$ and $C_w = 4.19\,\mathrm{kJ/(kg \cdot K)}$
- Cooling tower : $t_c = 24°\mathrm{C}$ (leaving from)

Example 7.1 : Setting up equations for optimizing cooling system

(Given)

- Relation between variables :

Refrigeration unit : $x_1 = 48q_1$

Precooler : $x_2 = \frac{50q_2}{t_3 - t_1}$ (applicable when $t_3 > t_1$)

Cooling tower : $x_3 = 25q_3$

Compression power : $P = 0.25q_1$

- q_1 **and** P must be **absorbed** by the condenser cooling water passing through **the refrigeration unit**

- Pipelines, which consist of the system, are adiabatic.

Example 7.1 : Setting up equations for optimizing cooling system

(Solution)

(a) Total cost (objective function) : $y = x_1 + x_2 + x_3$

- However it can be also written in terms of **Rate of heat transfer** (the q 's) or even **The temperature** (the t 's), not only by the individual costs (the x 's)

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Example 7.1 : Setting up equations for optimizing cooling system

(Solution)

(b) Refrigeration unit :

1) $q_1 + P = (2.3\ \mathrm{kg/s})[4.19\ \mathrm{kJ/(kg \cdot K)}](t_1 - 24)$ (water side)

2) $q_1 = \dot{m}_a C_p \Delta T = (1.2\ \mathrm{kg/s})[1.0\ \mathrm{kJ/(kg \cdot K)}](t_3 - 10)$ (air side)

Pre-cooler :

3) $(1.2)(1.0)(95 - t_3) = (2.3)(4.19)(t_2 - t_1)$ (from energy balance)

4) $q_2 = \dot{m}_a C_p \Delta T = (1.2 kg/s)[1.0\ \mathrm{kJ/(kg \cdot K)}](95 - t_3)$

Cooling tower :

5) $q_3 = \dot{m}_w C_w \Delta T = (2.3\ \mathrm{kg/s})[4.19\ \mathrm{kJ/(kg \cdot K)}](t_2 - 24)$

Example 7.1 : Setting up equations for optimizing cooling system

(Solution)

(b) The given relations :

6) Refrigeration unit : $x_1 = 48q_1$

7) Precooler : $x_2 = \frac{50q_2}{t_3 - t_1}$ (applicable when $t_3 > t_1$)

8) Cooling tower : $x_3 = 25q_3$

9) Compression power : $P = 0.25q_1$

Unknowns : $q_1 \cdots q_3,\ x_1 \cdots x_3,\ t_1 \cdots t_3,\ P$

⇒ There are 9 equations in the set and 10 unknowns

Example 7.1 : Setting up equations for optimizing cooling system

(Solution)

(b) The number of equations : 9 ⇒ 2 (one less than the # of equations)

The number of unknowns : 10 ⇒ 3 (the individual costs, $x_1 \cdots x_3$)

- Eliminating the variables, two equations finally remain as follows;

$$\emptyset_1(x_1, x_2, x_3) = 0.01466x_1x_2 - 14x_2 + 1.042x_1 - 5100 = 0$$

$$\emptyset_2(x_1, x_2, x_3) = 7.69x_3 - x_1 - 19{,}615 = 0$$

Example 7.1 : Setting up equations for optimizing cooling system

(Answer)

Minimize

(a) $$y = x_1 + x_2 + x_3$$

Subject to

(b) $$\emptyset_1(x_1, x_2, x_3) = 0.01466x_1x_2 - 14x_2 + 1.042x_1 - 5100 = 0$$

$$\emptyset_2(x_1, x_2, x_3) = 7.69x_3 - x_1 - 19{,}615 = 0$$

7.12 Discussion of Example 7.1

- The optimum value of x :

$$(x_1, x_2, x_3) = (\$1450, \$496, \$2738)$$

With the methods in the subsequent chapters (not covered in here)

- Limitation of temperature
  - If $t_3 < t_1$, x_2 becomes negative, physically impossible ($x_2 = \frac{50q_2}{t_3 - t_1}$)
  - From heat transfer consideration, the precooler can cool the air no lower than 24℃

7.12 Discussion of Example 7.1

- The equation permits to $x_2 \rightarrow 0$, when all cooling is performed by the refrigeration unit (x_1).

$$0.01466x_1x_2 - 14x_2 + 1.042x_1 - 5100 = 0$$

7.12 Discussion of Example 7.1

- The constraint equation below imposes a minimum value of the cooling tower x_3.
- The refrigeration unit $x_1 \uparrow \; \rightarrow \;$ the cooling tower $x_3 \uparrow$

  because of the compression power associated with the refrigeration unit.

$$7.69x_3 - x_1 - 19{,}615 = 0$$

$$P = 0.25q_1$$

$$x_1 = 48q_1$$

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7.13 Summary

- This chapter is to introduce procedures for **setting up the mathematical statement** of the optimization problem.
- In the next five chapters, specific optimization techniques are suggested
- The optimization is available when the characteristics of the physical system have been converted into **the equations for the objective function and constraints.**